A Second Look at Overloading

Marius Weidner — Lehrstuhl für Programmiersprachen — Seminar '22

```haskell
instance Eq a ⇒ Eq [a] where
  eq []       []       = True
  eq (x : xs) (y : ys) = eq x y &&
                         eq xs ys
  eq _        _        = False



isEq :: Bool
isEq = eq [Zero] [Zero]
```

<p class="subtitle">Haskell</p> </div>
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```rust
trait Eq
  fn eq(&self, rhs: &Self) → Bool;

impl Eq for Nat {
  fn eq(&self, rhs: &Self) → Bool
    match (self, rhs) {
      (Zero, Zero)       ⇒ True,
      (Suc(x), Suc(y)) ⇒ x.eq(y),
      (_, _)             ⇒ False
```

```
inst eq :: Nat → Nat → Bool
  eq Zero     Zero     = True
  eq (Suc x) (Suc y) = x == y
  eq _        _        = False

inst eq :: (eq :: a → a → Bool) ⇒ [a] → [a] → Bool
  eq []      []        = True
  eq (x:xs) (y:ys)    = (x == y) && (xs == ys)
  eq _       _         = False

isEq :: Bool
isEq = [Zero] == [Zero]
```

<p class="subtitle">Pseudocode</p>

$$
\begin{aligned}
&\mid \lambda x.\, e \\
&\mid \textbf{let } x = e \textbf{ in } e
\end{aligned}
$$

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Types

$$
\begin{aligned}
\tau ::=\ & \alpha \\
&\mid b \in B \\
&\mid \tau \to \tau \\
\sigma ::=\ & \tau \\
&\mid \forall \alpha.\, \sigma
\end{aligned}
$$

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$$\text{(TAUT)} \quad \frac{x : \sigma \in \Gamma}{\Gamma \vdash x : \sigma} \qquad \frac{c \text{ is constant of } b \in B}{\Gamma \vdash c : b} \quad \text{(BASE)}$$

$$(\to \mathrm{I}) \quad \frac{\Gamma,\ x : \tau \vdash e : \tau'}{\Gamma \vdash \lambda x.\ e : \tau \to \tau'} \qquad \frac{\Gamma \vdash e : \tau \to \tau' \qquad \Gamma \vdash e' : \tau}{\Gamma \vdash e\ e' : \tau'} \quad (\to \mathrm{E})$$

$$(\forall \mathrm{I}) \quad \frac{\Gamma \vdash e : \sigma \qquad \text{fresh } \alpha}{\Gamma \vdash e : \forall \alpha.\ \sigma} \qquad \frac{\Gamma \vdash e : \sigma \qquad \sigma \sqsubseteq \sigma'}{\Gamma \vdash e : \sigma'} \quad (\forall \mathrm{E})$$

$$\text{(LET)} \quad \frac{\Gamma \vdash e' : \sigma \qquad \Gamma,\ x : \sigma \vdash e : \tau}{\Gamma \vdash \textbf{let } x = e' \textbf{ in } e : \tau}$$

<p class="subtitle">Hindley Milner —— Typing</p>

Let $\Gamma = \{\mathbf{id} : \forall \alpha.\ \alpha \to \alpha,\ \mathbf{n} : \mathbb{N}\}$.

$$\dfrac{\dfrac{\mathbf{id} : \forall \alpha.\ \alpha \to \alpha \in \Gamma \qquad \forall \alpha.\ \alpha \to \alpha \sqsubseteq \mathbb{N} \to \mathbb{N}}{\Gamma \vdash \mathbf{id} : \mathbb{N} \to \mathbb{N}} \qquad \dfrac{\mathbf{n} : \mathbb{N} \in \Gamma}{\Gamma \vdash \mathbf{n} : \mathbb{N}}}{\Gamma \vdash \mathbf{id}\ \mathbf{n}}$$

<p class="subtitle">Hindley Milner —— Instantiation</p>

$$
\dfrac{
  \dfrac{
    \dfrac{
      \dfrac{x : \alpha \in \{x : \alpha\}}{x : \alpha \vdash x : \alpha}
    }{\vdash \lambda x.\ x : \alpha \to \alpha}
  }{\vdash \lambda x.\ x : \forall \alpha.\ \alpha \to \alpha \quad \text{fresh } \alpha}
  \qquad
  \dfrac{\mathrm{id} : \forall \alpha.\ \alpha \to \alpha \in \{\mathrm{id} : \forall \alpha.\ \alpha \to \alpha\}}{\mathrm{id} : \forall \alpha.\ \alpha \to \alpha \vdash \mathrm{id} : \forall \alpha.\ \alpha \to \alpha}
}{\vdash \mathbf{let}\ \mathrm{id} = \lambda x.\ x\ \mathbf{in}\ \mathrm{id} : \forall \alpha.\ \alpha \to \alpha}
$$

<p class="subtitle">Hindley Milner -- Generalization</p>

$$| \ \mathbf{inst}\ \sigma = \sigma\ \mathbf{in}\ p$$

$$e ::= \ x$$

$$| \ e\ e$$

$$| \ \lambda x.\ e$$

$$| \ \mathbf{let}\ x = e\ \mathbf{in}\ e$$

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Types

$$\tau ::= \ \alpha$$

$$| \ \tau \to \tau$$

$$\sigma ::= \ \tau$$

$$| \ \forall\alpha.\ \pi \Rightarrow \sigma$$

Example

(T-Var) $\dfrac{x : \sigma \in \Gamma}{\Gamma \vdash x : \sigma}$

$\dfrac{\Gamma \vdash e' : \sigma \qquad \Gamma,\ x : \sigma \vdash e : \tau}{\Gamma \vdash \textbf{let}\ x = e'\ \textbf{in}\ e : \tau}$ (T-Let)

$\to$ E $\dfrac{\Gamma,\ x : \tau \vdash e : \tau'}{\Gamma \vdash \lambda x.\ e : \tau \to \tau'}$

$\dfrac{\Gamma \vdash e : \tau \to \tau' \qquad \Gamma \vdash e' : \tau}{\Gamma \vdash e\ e' : \tau'}$ $\to$ E

$\forall$I $\dfrac{\Gamma \vdash e : \sigma \qquad \text{fresh } \alpha}{\Gamma \vdash e : \forall \alpha.\ \sigma}$

$\dfrac{\Gamma \vdash e : \sigma \qquad \sigma \sqsubseteq \sigma'}{\Gamma \vdash e : \sigma'}$ $\forall$E

SET $\dfrac{\Gamma \vdash x_1 : \sigma_1 \quad \dots \quad \Gamma \vdash x_n : \sigma_n}{\Gamma \vdash x_1 : \sigma_1 \quad \dots \quad x_n : \sigma_n}$

$\dfrac{\Gamma \vdash e : \sigma \qquad \Gamma,\ o : \sigma \vdash p : \sigma' \qquad \forall (o : \sigma'') \in \Gamma \Rightarrow \sigma'' \neq}{\Gamma \vdash e : \forall \alpha.\ \sigma}$ INST

<p class="subtitle">System O –– Typing</p>

Semantics