

A Second Look at Overloading

```

class Eq a where
  eq :: a → a → Bool

instance Eq Nat where
  eq Zero    Zero    = True
  eq (Suc x) (Suc y) = eq x y
  eq _       _       = False

instance Eq a ⇒ Eq [a] where
  eq []      []      = True
  eq (x : xs) (y : ys) = eq x y &&
                           eq xs ys
  eq _       _       = False

isEq :: Bool
isEq = eq [Zero] [Zero]

```

Haskell

```

trait Eq
  fn eq(&self, rhs: &Self) → Bool

impl Eq for Nat
  fn eq(&self, rhs: &Self) → Bool
  match (self, rhs)
    (Zero, Zero)      ⇒ True,
    (Suc(x), Suc(y)) ⇒ x.eq(y),
    (_, _)            ⇒ False

impl<A: Eq> Eq for [A]
  fn eq(&self, rhs: &Self) → Bool
  match (self, rhs)
    ([], []) ⇒ True,
    ([x, xs@..], [y, ys@..])
      ⇒ x.eq(y) && xs.eq(ys),
    (_, _) ⇒ False

fn is_eq() → Bool
  [Zero].eq(&[Zero])

```

Rust

```

inst eq :: Nat → Nat → Bool
  eq Zero    Zero    = True
  eq (Suc x) (Suc y) = eq x y
  eq _       _       = False

inst eq :: (eq :: a → a → Bool) ⇒ [a] → [a] → Bool
  eq []      []      = True
  eq (x:xs) (y:ys)   = eq x y && eq xs ys
  eq _       _       = False

fun isEq :: Bool
  isEq = [Zero] = [Zero]

```

Pseudocode

$$\begin{aligned}
 e &::= x \\
 &| \lambda x. e \\
 &| e e \\
 &| \mathbf{let} \ x = e \ \mathbf{in} \ e
 \end{aligned}$$

$$\begin{aligned}
 \tau &::= \alpha \\
 &| \tau \rightarrow \tau \\
 \sigma &::= \tau \\
 &| \forall \alpha. \sigma
 \end{aligned}$$

```

let id    = λx. x
let cons = λx. λlst. x : lst in .. :: ∀α. α → α
let cons = λx. λlst. x : lst in .. :: ∀α. α → [α] → [α]

```

```

let evil = λi. id. id i in .. :: Int → (∀α. α → α) → Int

```

$e := x$

| o (if overloaded)

| k ($k \in \{\text{unit}, 42, [e_1, \dots, e_n], \dots\}$)

| $\lambda x. e$

| $e e$

| **let** $x = e$ **in** e

$p := e$

| **inst** $o : \sigma_T = \sigma$ **in** p

```
inst eq : Nat → Nat → Bool = λx. λy. x ≐ y in
inst eq : ∀a. (eq : a → a → Bool) ⇒ [a] → [a] → Bool =
  | λ[]. λ[]. True
  | λ[x : xs]. λ[y : ys]. eq x y && eq xs ys in
eq [0] [0]
```

$$\begin{aligned}
\tau &::= \alpha \\
&| \tau \rightarrow \tau \\
&| D \ \tau_1 \ \dots \ \tau_n \quad (D \in \{\text{Unit}, \text{Nat}, \text{List } \tau, ..\}, \text{arity}(D) = n) \\
\pi_\alpha &::= o_1 : \alpha \rightarrow \tau_1, \ \dots \ , o_n : \alpha \rightarrow \tau_n \quad (n \in \mathbb{N}, o_i \neq o_j) \\
\sigma &::= \tau \\
&| \forall \alpha. \pi_\alpha \Rightarrow \sigma_T \\
\sigma_T &::= T \ \alpha_1 \ \dots \ \alpha_n \rightarrow \tau \quad (T \in D \cup \{\rightarrow\}, \text{tv}(\tau) \subseteq \{\alpha_1, \dots, \alpha_n\}) \\
&| \forall \alpha. \pi_\alpha \Rightarrow \sigma_T \quad (\text{tv}(\pi_\alpha) \subseteq \text{tv}(\sigma_T))
\end{aligned}$$

$$\begin{array}{c}
(\text{LET}) \quad \dfrac{\Gamma \vdash e_1 : \sigma \quad \Gamma, x : \sigma \vdash e_2 : \tau}{\Gamma \vdash \mathbf{let} \ x = e_1 \ \mathbf{in} \ e_2 : \tau}
\end{array}$$

$$\begin{array}{c}
(\text{INST}) \quad \dfrac{\Gamma \vdash e : \sigma_T \quad \Gamma, o : \sigma_T \vdash p : \sigma \quad \forall (o : \sigma_{T'}) \in \Gamma : T \neq T'}{\Gamma \vdash \mathbf{inst} \ o : \sigma_T = e \ \mathbf{in} \ p : \sigma}
\end{array}$$

$$\begin{array}{c}
(\forall \text{I}) \quad \dfrac{\Gamma, \pi_\alpha \vdash e : \sigma \quad \text{fresh } \alpha}{\Gamma \vdash e : \forall \alpha. \pi_\alpha \Rightarrow \sigma}
\end{array}$$

$$\begin{array}{c}
(\forall \text{E}) \quad \dfrac{\Gamma \vdash e : \forall \alpha. \pi_\alpha \Rightarrow \sigma \quad \Gamma \vdash [\tau / \alpha] \pi_\alpha}{\Gamma \vdash e : [\tau / \alpha] \sigma}
\end{array}$$

$$\Gamma = \{\text{eq} : \text{Nat} \rightarrow \text{Nat} \rightarrow \text{Bool}, \\ \text{eq} : \forall \alpha. (\text{eq} : \alpha \rightarrow \alpha \rightarrow \text{Bool}) \Rightarrow [\alpha] \rightarrow [\alpha] \rightarrow \text{Bool}\}$$

$$\frac{\text{eq} : \forall \alpha. (\text{eq} : \alpha \rightarrow \alpha \rightarrow \text{Bool}) \\ \Rightarrow [\alpha] \rightarrow [\alpha] \rightarrow \text{Bool} \in \Gamma}{\Gamma \vdash \text{eq} : \forall \alpha. (\text{eq} : \alpha \rightarrow \alpha \rightarrow \text{Bool}) \\ \Rightarrow [\alpha] \rightarrow [\alpha] \rightarrow \text{Bool}} \quad \frac{\text{Nat} \rightarrow \text{Nat} \rightarrow \text{Bool} \in \Gamma}{\Gamma \vdash \text{Nat} \rightarrow \text{Nat} \rightarrow \text{Bool}} \quad \dots$$

$$\Gamma \vdash \text{eq} : [\text{Nat}] \rightarrow [\text{Nat}] \rightarrow \text{Bool} \quad \dots$$

$$\Gamma \vdash \text{eq} [0] : [\text{Nat}] \rightarrow \text{Bool} \quad \dots$$

$$\Gamma \vdash \text{eq} [0] [0]$$

System 0 — Constraint Solving

$$\begin{aligned}
& \llbracket \text{inst } eq : \mathbb{N} \rightarrow \mathbb{N} \rightarrow \mathbb{B} = e_1 \text{ in} \\
& \quad \text{inst } eq : \forall \alpha. (eq : \alpha \rightarrow \alpha \rightarrow \mathbb{B}) \Rightarrow [\alpha] \rightarrow [\alpha] \rightarrow \mathbb{B} = e_2 \text{ in} \\
& \quad eq [0] [0] \rrbracket_{\emptyset} \\
= & \llbracket \text{inst } eq : \forall \alpha. (eq : \alpha \rightarrow \alpha \rightarrow \mathbb{B}) \Rightarrow [\alpha] \rightarrow [\alpha] \rightarrow \mathbb{B} = .. \text{ in} \\
& \quad eq [0] [0] \rrbracket_{\{eq := \lambda x. \text{ if } x \text{ is } \mathbb{N} \text{ then } e_1 \ x\}} \\
= & \llbracket eq [0] [0] \rrbracket_{\{eq := \lambda x. \text{ if } x \text{ is List then } e_2 \ x \text{ else } \lambda x. \text{ if } x \text{ is } \mathbb{N} \text{ then } e_1 \ x\}} \\
= & (\lambda x. \text{ if } x \text{ is } [\alpha] \text{ then } e_2 \ x \text{ else } \lambda x. \text{ if } x \text{ is } \mathbb{N} \text{ then } e_1 \ x) [0] [0] \\
= & e_2 [0] [0]
\end{aligned}$$

System 0 — Semantics

```

inst eq : Nat → Nat → Bool = λ.. in
inst eq : ∀a. (eq : a → a → Bool)
    ⇒ [a] → [a] → Bool = λ.. in
eq [0] [0]

```

System 0

```

let eq0 :: Nat → Nat → Bool
    = λ.. in
let eq1 :: ∀a. (a → a → Bool) → [a] → [a] → Bool
    = λeq0. λ.. in
eq1 eq0 [0] [0]

```

Hindley Milner

System 0 — Translation to Hindley Milner

```

let max :: ∀β. (gte : β → β → Bool) ⇒
    ∀α. (α ≤ {key: β}) ⇒ α → α → α
    = λx. λy. if gte x.key y.key then x else y in
max {field: "a", key: 1} {field: "b", key: 2}

```

Records + Subtyping

```

inst field : ∀α. ∀β. R0 α β → α = λR0 x y. x in
inst key   : ∀α. ∀β. R0 α β → β = λR0 x y. y in
let max :: ∀β. (gte : β → β → Bool) ⇒
    ∀α. (key : α → β) ⇒ α → α → α
    = λx. λy. if gte (key x) (key y) then x else y in
max (R0 "a" 1) (R0 "b" 2)

```

System 0

System 0 — Relationship with Record Typing

Repository

github.com/Mari-W/pop1

Literatur

- [A Second Look at Overloading](#) 1995
Martin Odersky, Philip Wadler, Martin Wehr
- [A Theory of Type Polymorphism in Programming](#) 1978
Hindley Milner