From Debruijn to co-Debruijn using Category Theory

Everybody's Got To Be Somewhere[1]

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Abstract. We explore the connection between De Bruijn indices used for variable representation in an intrinsically scoped syntax and the adoption of co-De Bruijn representation. To comprehend the duality between De Bruijn and co-De Bruijn, we will look into the categorical concepts underpinning scopes, binders, and intrinsically scoped syntaxes in general. Lastly, we will address challenges encountered in expressing these concepts and investigate the practical application of co-De Bruijn to specific syntaxes within theorem proving contexts.

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References

[1]   Conor McBride. "Everybody's Got To Be Somewhere". In: *Electronic Proceedings in Theoretical Computer Science* 275 (July 2018), pp. 53–69. ISSN: 2075-2180. DOI: `10.4204/eptcs.275.6`. URL: `http://dx.doi.org/10.4204/EPTCS.275.6`.