

# HM( $\mathcal{X}$ ): Type Inference with Constraint Types

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# Outline

- 1 HM: Polymorphic Lambda Calculus with Full Type Inference
- 2 Constraint Systems: Formulate Constraints on Types
- 3  $\text{HM}(X)$ : HM Parametrized over Constraint Systems  $X$
- 4  $\text{HM}(\mathcal{R})$ : Instantiating  $\text{HM}(X)$  with Polymorphic Records
- 5 Conclusion: Properties of  $\text{HM}(X)$

# Hindley Milner Basics

- Let Polymorphism
- Full Type Inference
- Principle Type Property

## Pseudocode Example

```
let id =  $\lambda x. x$  in           ::  $\alpha \rightarrow \alpha$   
id 42                        :: Int  
id "Hello World"            :: Str
```

# Hindley Milner Syntax

$$\begin{array}{l} e ::= x \\ \quad | \lambda x. e \\ \quad | e \ e \\ \quad | \mathbf{let} \ x = e \ \mathbf{in} \ e \end{array}$$
$$\tau ::= \alpha \mid \tau \rightarrow \tau$$
$$\sigma ::= \tau \mid \forall \alpha. \sigma$$

# Hindley Milner Typing

$$\frac{\Gamma \vdash e : \sigma \quad \Gamma, x : \sigma \vdash e' : \tau'}{\Gamma \vdash \mathbf{let} \ x = e \ \mathbf{in} \ e' : \tau'} \text{ Let}$$

$$\frac{\Gamma \vdash e : \sigma \quad \text{fresh } \alpha}{\Gamma \vdash e : \forall \alpha. \sigma} \forall \text{ Intro}$$

$$\frac{\Gamma \vdash e : \sigma' \quad \sigma' \sqsubseteq \sigma}{\Gamma \vdash e : \sigma} \forall \text{ Elim}$$

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# HM( $X$ ) Syntax

$$\begin{array}{l} C ::= x \\ \quad | \lambda x. e \\ \quad | e \ e \\ \quad | \mathbf{let} \ x = e \ \mathbf{in} \ e \end{array}$$
$$\begin{array}{l} \tau ::= \alpha \mid \tau \rightarrow \tau \\ \sigma ::= \tau \mid \forall \alpha. C \Rightarrow \sigma \end{array}$$

# HM( $X$ ) Typing

$$\frac{C, \Gamma \vdash e : \sigma \quad C, (\Gamma, x : \sigma) \vdash e' : \tau'}{C, \Gamma \vdash \mathbf{let} \ x = e \ \mathbf{in} \ e' : \tau'} \text{ Let}$$

$$\frac{C \wedge D, \Gamma \vdash e : \tau \quad \text{fresh } \vec{\alpha}}{C \wedge \exists \vec{\alpha}. D, \Gamma \vdash e : \forall \vec{\alpha}. D \Rightarrow \tau} \forall \text{ Intro}$$

$$\frac{C, \Gamma \vdash e : \forall \vec{\alpha}. D \Rightarrow \tau' \quad C \vdash^e [\vec{\tau}/\vec{\alpha}] D}{C, \Gamma \vdash e : [\vec{\tau}/\vec{\alpha}] \tau'} \forall \text{ Elim}$$



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