Elaboration on HM(X): Type Inference with Constraint Types

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Abstract. We investigate HM(X) [[CITE]], a family of type systems designed to accommodate polymorphism, full type inference and constraint types. HM(X) extends the Hindley-Milner type system (HM) [[CITE]], which already limits System F, to ensure decidability and unambiguity of full type inference. The constraint system X utilized in HM(X) remains abstract, allowing instantiating X with arbitrary constraint systems that meet specific criteria. This abstraction empowers HM(X) to serve as a model for analyzing various constraint-related type features commonly encountered in practice. Notable examples include subtyping, substructural types and type classes. HM(X) encompasses a sound and complete type inference algorithm that remains independent of the actual constraint system X. As a result, the work for proving theoretical properties and designing inference algorithms for novel constraint-based type systems within an HM context is notably simplified.

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1 Introduction

1.1 Polymorphism and Full Type Inference in HM

The HM type system represents a well known and understood typing discipline that refines System F [[CITE]] by establishing constraints that allow type inference to be decidable. HM serves as the foundation for numerous real-world functional programming languages, including languages like Haskell and Rust.

In System F, we can introduce variables for both expressions and types. The type $\forall\alpha.\ T$, where α binds a new type in T, indicates that an expression of this type is polymorphic over some arbitrary type α. Unfortunately it is undecidable for arbitrary programs to determine when to introduce and eliminate $\forall$ types and in consequence type inference in System F is undecidable [[CITE]]. In consequence, System F is equipped with explicit type abstraction ($\Lambda\alpha.\ e$) and type application ($e\ T$) on the syntax level.

The HM type system imposes several restrictions that make type inference decidable in a polymorphic context. Moreover, HM ensures that the most general type (the *principal type*) of a any given program is inferred. Consequently, there's no need for extra syntax to introduce or eliminate type variables. Instead, HM employs let bindings **let** $x = e_2$ **in** e_1 where e_2 is the only expression that is allowed to have a polymorphic type. Other constructs, such as applications or variables bound by a lambda abstractions, cannot inherit polymorphic types. This constraint is commonly referred to as 'let polymorphism'.

In the future, we will label polymorphic types as σ, where a $\forall$-type exists within σ, and we will refer to them as 'poly types'. On the contrary, all other types, including base types, functions, and type variables, will be designated as 'mono types' and denoted as τ.

Poly types are further constrained to exclusively permit $\forall$ binders at the top level. Therefore, a type like $\forall\alpha.\alpha \rightarrow \forall\alpha.\alpha$ would not be a valid poly type. Consequently, we establish that poly types consistently adhere to the structure $\forall\bar{\alpha}.\ \tau$, where $\bar{\alpha} = \alpha_1, .., \alpha_n$. By upholding these two regulations, that is let polymorphism and the exclusion of higher-order polymorphism, type inference remains decidable and yields a principal type.

Example 1 (Concatination of Lists).

```
concat : ∀α. [α] → [α] → [α]
concat = λ[].      λys. ys
         λ[x:xs]. λys. x : (concat xs ys)
```

Examples assume the extension of HM with various language features such as lists and pattern matching. Further, Haskell like notation is used instead of let bindings. For convenience some inferred types are given.

In this example, HM would be capable of deducing the type $\forall\alpha.[\alpha] \rightarrow [\alpha] \rightarrow [\alpha]$ for the function 'concat'. This type is in fact the most general type for 'concat'.

1.2 Introducing Constraints on Types

Although parametric polymorphism is already a powerful abstraction, there are instances where we desire to constrain type variables solely to instantiations to types that satisfy specific constraints. We refer to such types as *constraint types*. Constraints can exhibit various different forms depending on the type features present in the actual language. As an illustration, consider HM extended with polymorphic records ($\mathrm{HM}_{\mathcal{R}}$), as outlined in [[CITE]]. In this scenario, it becomes valuable to have the capability to specify that a type variable α should solely be instantiated as a record type containing specific fields.

Example 2 (Alternative to Selector Syntax on Records for Field 'key').

```
key : ∀β.∀α.(α ≤ {key : β}) ⇒ α → β
key = λ{key : β, ..}. key
```

This example simulates the extraction of a particular field from a record, conventionally represented as $e.l$ where l denotes a label. We simulate the extraction of the field 'key'. Instead of the notation $e.key$, the alternative syntax $key\ e$ could then be employed. [1]

We introduce two type variables, namely α and β. While β exhibits parametric polymorphism, α functions as a constraint type. The constraint imposed on α, denoted as $\alpha \leq \{\text{key} : \beta\}$, signifies that α is exclusively permitted to take on a type that corresponds to a record featuring a 'key' field of type β. For introducing constraints on type variables α, we will adopt the notation $\forall\bar{\alpha}.C \Rightarrow \tau$. Multiple constraints will be combined using conjunction.

Naturally, we can envision entirely distinct constraints as well. Consider a language with overloading and constraints in the format $ident : \alpha \to \tau$, where an overloaded identifier 'ident' with type $\alpha \to \tau$ is expected to be present. For instance, consider the overloaded 'eq' function designed for list equality: $eq : \forall\alpha.(eq : \alpha \to \alpha \to bool) \Rightarrow [\alpha] \to [\alpha] \to bool$. This formulation allows us to convey that list equality is feasible only when the elements of the list can be compared.

A in-depth investigation into both polymorphic records and overloading will be undertaken subsequently REF 4. Instead of focusing exclusively on these individual systems, our exploration will center on $\mathrm{HM}(X)$, a HM-based system that remains detached from the actual constraint domain X. Subsequently, we will proceed to instantiate X using the two showcased instances of constraints, namely overloading and polymorphic records.

[1] In fact, desugaring label syntax as seen in the example forms a component of a transformation from $\mathrm{HM}_{\mathcal{R}}$ to a calculus incorporating data types and overloading [[CITE System O Chapter 5]].

Fig. 1. Syntax

Fig. 2. Typing $(C, \Gamma \vdash e : \sigma)$

2 HM(X)

2.1 Introduction

2.2 Syntax

2.3 Typing

2.4 Type Inference

3 Instantiating HM(X)

3.1 HM($\mathscr{R}$): Extension with Polymorphic Records

Extensions

Example

3.2 HM($\mathscr{O}$): Extension with Overloading

Extensions

Example

4 Metatheory

4.1 Soundness

4.2 Type Inference

5 Related Work & Conclusion

5.1 Related Work

5.2 Conclusion

References

[1] Martin Odersky, Martin Sulzmann, and Martin Wehr. “Type Inference with Constrained Types”. In: *TAPOS* 5 (Jan. 1999), pp. 35–55. DOI: 10.1002/(SICI)1096-9942(199901/03)5:1<35::AID-TAPO4>3.0.CO;2-4.

Fig. 3. Syntax

Fig. 4. Constraints

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[3] Christian Skalka and François Pottier. “Syntactic Type Soundness for HM(X)”. In: *Electronic Notes in Theoretical Computer Science* 75 (2003). TIP’02, International Workshop in Types in Programming, pp. 61–74. ISSN: 1571-0661. DOI: `https://doi.org/10.1016/S1571-0661(04)80779-5`. URL: `https://www.sciencedirect.com/science/article/pii/S1571066104807795`.

Fig. 5. Syntax

Fig. 6. Constraints