Universal Control

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Abstract.

0.1 Syntax

$$
\begin{aligned}
t, \ell, A, B ::= {} & x \\
& \mid \lambda(x : A) \to t \\
& \mid t_1\ t_2 \\
& \mid \forall(x : A) \to B \\
& \mid t_1 \equiv_A t_2 \\
& \mid \text{refl}\ t \\
& \mid \text{Level} \\
& \mid 0 \\
& \mid \omega \uparrow \ell_1 +_{\{t\}} \ell_2 \\
& \mid \ell_1 \sqcup \ell_2 \\
& \mid \ell_1 <_\ell \ell_2 \\
& \mid\ <_{\ell_1} \\
& \mid\ <_{\ell_2}\ \ t_1 \\
& \mid\ <_{\ell_3}\ \ t_1\ t_2 \\
& \mid \text{Level}[\ell] \\
& \mid \ell,_\ell t \\
& \mid \text{proj}_\ell\ t \\
& \mid \text{proj}_{<_\ell}\ t \\
& \mid \text{Set}[\ell] \\
& \mid \text{Set}\varepsilon_{0+i} \text{ for all } i \in \mathbb{N}
\end{aligned}
$$

Less then or equal proof t in $\omega \uparrow \ell_1 +_{\{t\}} \ell_2$ omitted when it follows from the context.

0.2 Laws

Idempotence: $\ell \sqcup \ell \equiv \ell$
Associativity: $(\ell_1 \sqcup \ell_2) \sqcup \ell_3 \equiv \ell_1 \sqcup (\ell_2 \sqcup \ell_3)$
Commutativity: $\ell_1 \sqcup \ell_2 \equiv \ell_2 \sqcup \ell_1$
Distributivity: $\omega \uparrow \ell +_{\{t\}} (\ell_1 \sqcup \ell_2) \equiv \omega \uparrow \ell +_{\{t_1\}} \ell_1 \sqcup \omega \uparrow \ell +_{\{t_2\}} \ell_2$
Neutrality: $\ell \sqcup 0 \equiv \ell$
Subsumption: $\ell \sqcup \omega \uparrow \ell_1 + .. + \omega \uparrow \ell_n + \ell \equiv \omega \uparrow \ell_1 + .. + \omega \uparrow \ell_n + \ell$

0.3 Typing

$$\frac{(x : T) \in \Gamma}{\Gamma \vdash x : T} \text{ T-Var}$$

$$\frac{\Gamma \vdash A : Set[o_1] \quad \Gamma, x : t_1 \vdash e : \forall (x : t_1) \to t_2}{\Gamma \vdash \lambda (x : A) \to t : B} \text{ T-Abs}$$

$$\frac{\Gamma \vdash e_1 : \forall (x : t_1) \to t_2 \quad \Gamma \vdash e_2 : t_1}{\Gamma \vdash e_1 e_2 : t_2[x/e_2]} \text{ T-App}$$

$$\frac{\Gamma \vdash t_1 : Set[o_1] \quad \Gamma, x : t_1 \vdash t_2 : Set[o_2]}{\Gamma \vdash \forall (x : t_1) \to t_1 : Set[o_1 \sqcup suc\ o_2]} \text{ T-All}$$

$$\frac{\Gamma \vdash e : t_1 \quad t_1 \equiv t_2}{\Gamma \vdash e : t_2} \text{ T-Conv}$$