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Sizing of a fleet of cooperative robots for the transport of homogeneous loads

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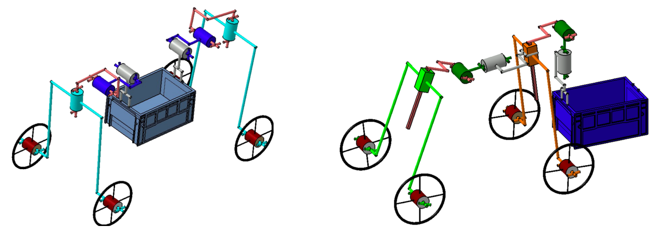
***Abstract*— We consider the problem of determining the number of robots necessary to transport a set of homogeneous loads in a given time interval from a zone A to a zone B, at minimum cost. The cost is function of the number of robots and of the distance travelled by robots. The operations are divided into several phases: loading, loaded travel, unloading, empty travel and battery charging. The case of non-cooperative robots is considered for which we derive a closed-form expression for the optimal number of robots. We then consider the case of cooperative robots where loads can be carried either by a single robot (mono-robot) or by several robots that cooperate (poly-robot). The fleet sizing problem can be formulated as a mathematical programming. We distinguish several scenarios, depending on the respective carrying capacity of mono-robots and poly-robots. Finally, the infinite horizon problem is also addressed, which models a fleet of vehicles and leads to simpler results.**

I. INTRODUCTION

A warehouse of a manufacturing company today is characterized by dynamic production processes governed by the demands of a rapidly changing global economy, such as the increasing number of product variants, customization of products and responsiveness to changing market conditions [1]. In order to be competitive, companies are forced to seek innovative robotic solutions to operate their warehouses. Thus, certain giants of the online trade organize competitions to develop autonomous robots for the pick-and-place tasks [2]. A particular development over the last decade has taken place for AGVs (Automated Guided Vehicles) and AMRs (Autonomous Mobile Robots) [3], especially in logistics warehouses and industrial production [4]. In our work, we use the term AGV in both cases, considering the robots individually and also in cooperation.

The benefits of cooperation are clearly demonstrated by the animal world. Collaboration can be of different types, for example, the execution of a task by several subordinates under the direction of a leader, or the execution of the same task with the same level of responsibility [5]. The cooperative approach can be applied not only in nature, but in industry [6]. Cooperative robots capable of working in parallel on the same task open wide perspectives [7]. For example, a large load can be transported by several small robots connected to the load (co-manipulation mode) or one robot can transport the load while connecting to another robot to increase stability (connection mode) [8] as illustrated in Figure 1.



(a) co-manipulation mode (b) connection mode

Fig. 1: Different modes of cooperation [8]

We use the concepts and terminology developed in [9]:

- Mono-robot (m-bot) = An elementary robot, which is designed to work on its own or with others;
- Poly-robot (p-bot) = A set of p m-bots, which cooperate on the same task.

Different questions arise when operating a fleet of robots, such as the design of the warehouse architecture [10], trajectory planning with obstacle and collision avoidance [11], [12], service policy [13] and battery charging [14].

In this article, we are interested in the sizing of a fleet of cooperative robots for the transport of standardized loads that are all identical and referred as "homogeneous loads".

AGVs are generally expensive and determining the right type and number of vehicles is crucial. Egbelu [15] highlights several factors influencing the required number of vehicles, such as system layout, location of load transfer points, trips frequency, vehicle-dispatching strategy, system reliability and speed of travel.

Different works are interested in the sizing of robot fleets. They can be divided schematically into two categories: the stochastic models [16], [17], [18] and the deterministic models [19], [15]. As in our work we are developing a deterministic mathematical model, we focus on the last two works in more detail. The work [15] proposes four analytical approaches to estimate the number of robots, giving examples for each of them. The author proposes to consider add dispatching rules and then simulate his models with

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varying incoming material flow. These models are optimistic according to the author. Rjeb et al [19] refine the results of Egbelu [15] in the case of homogeneous loads by providing an analytical formula for the optimal number of robots. In the case of heterogeneous loads, the authors formulate the problem as a bin packing problem.

Different objective functions are considered in the literature. In particular, several works are interested in minimizing the cost or in maximizing the profit. Beaujon and Turnquist [20] maximizes the total profit (difference between revenues and total transportation costs, including penalty costs for unmet demand). Etezadi and Beasley [21] minimizes the cost of a fleet of purchased or leased vehicles. Sinriech and Tanchoco [22] minimizes the cost by applying penalties if performance is not achieved in terms of quality of service.

Contributions

To our knowledge, this article is the first to focus on the sizing of a fleet of cooperative robots. To simplify, we first consider that the loads are homogeneous. The section II considers the sizing of a fleet of non-cooperative robots and extends the results of [19] by adding the concept of transport capacity. The section III considers the problem of sizing a fleet of cooperative robots. Finally, the section IV offers a conclusion and research perspectives, in particular concerning the problem with heterogeneous loads.

II. Sizing of a fleet of non-cooperative robots

In this first part, we determine the number of robots necessary to transport a set of loads over a time interval $[0,T]$ where T is the horizon.

A. Assumptions and notations

The following notations are used:

- d: round trip distance from A to B
- τ: cycle time
- v_l: travelling speed of a loaded robot
- v_e: travelling speed of a empty robot
- t_l: loading time
- t_u: unloading time
- t_b: time when a robot is immobilized
- c: robot capacity
- D: total distance traveled by all the robots
- α: fixed cost per unit of time of a robot
- β: cost per meter traveled by a robot
- γ: fixed cost per unit of time, independent of the number of robots
- N: number of robots for loads transportation
- n: number of loads transported
- r: number of round trips for a robot
- T: planning horizon

In this paper, we consider a fleet of N identical mobile robots which must transport a set of n identical loads unloaded in B and return empty to A, as shown in Figure 2.

Robot capacity is denoted c. The capacity of a robot is related to the size and mass of the loads and the number of loads that a robot can carry simultaneously.

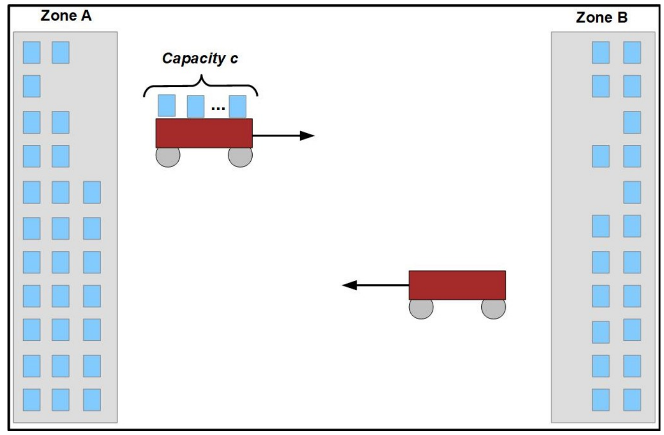


Fig. 2: Transport of identical loads by a fleet of homogeneous robots

The cycle time τ represents the sum of the loaded travel time ($d/2v_l$ where v_l is the travelling speed of a loaded robot), the empty travel time ($d/2v_e$ where v_e is the displacement speed of an empty robot), the loading time (t_l)and the unloading time (t_u):

$$\tau = t_l + \frac{d}{2v_l} + t_u + \frac{d}{2v_e} \tag{1}$$

Over the time interval $[0,T]$, the robot is immobilized during t_b (battery recharge, maintenance, failure, etc.). The remaining available time is then $(T - t_b)$.

The cost per unit of time of a fleet of $N > 0$ robots traveling a total distance D over the $[0,T]$ is

$$f(N) = \alpha N + \frac{\beta D}{T} + \gamma \tag{2}$$

where α represents the fixed cost of a robot per unit of time (cost related to maintenance, purchase or rental), β the cost per meter traveled by one robot and γ the cost per unit of time independent of the number of robots (e.g. hardware and software infrastructure).

Note that the total distance traveled D is directly related to the number of loads n to be transported. It takes $r = \frac{n}{c}$ round trips to transport the n loads. Then $D = dr$ and the cost function is

$$f(N,r) = \alpha N + \frac{\beta d}{T} r + \gamma. \tag{3}$$

The objective is to determine the number of required robots, N^*, to transport the set of n loads over time interval $[0,T]$ at minimum cost, considering A as the starting point and return point of the robots. This simple problem is equivalent to determining the minimum number of round trips and robots allowing all loads to be transported over the time interval. To have feasible solutions, we assume that $T - t_b \geq \tau$.

The following assumptions are also made:

- The robot storage place is located at point A. There is no waiting to load in A or to unload in B (the loads are

available immediately to be loaded and the robots do not hinder each other).
- The problem of traffic jams for robots is not taken into account. These different elements could nevertheless be taken into account by introducing an efficiency coefficient, as proposed in [15].

B. Optimal number of robots (finite horizon)

One robot can make at most $\left\lfloor \frac{T-t_b}{\tau} \right\rfloor$ round trips over the time interval $[0,T]$ where $\lfloor x \rfloor$ denotes the greatest integer less than or equal to x. Thus N robots with capacity c can carry at most $Nc \left\lfloor \frac{T-t_b}{\tau} \right\rfloor$ loads over the time interval.

To transport the n loads, it is therefore necessary that $Nc \left\lfloor \frac{T-t_b}{\tau} \right\rfloor \geq n$ and therefore that $N \geqslant \frac{n}{c\lfloor (T-t_b)/\tau \rfloor}$.

The number of robots being an integer, the minimum number of robots to transport n loads during the time interval $[0,T]$ is then

$$N^* = \left\lceil \frac{n}{c\lfloor (T-t_b)/\tau \rfloor} \right\rceil \tag{4}$$

The minimum number of round trips for robots to transport n loads is

$$r^* = \left\lceil \frac{n}{c} \right\rceil, \tag{5}$$

where $\lceil x \rceil$ is the least integer greater than or equal to x.

The minimum cost is then

$$f^* = f(N^*, r^*) = \alpha N^* + \frac{\beta d}{T} r^* + \gamma \tag{6}$$

$$= \alpha \left\lceil \frac{n}{c\lfloor (T-t_b)/\tau \rfloor} \right\rceil + \frac{\beta d}{T}\left\lceil \frac{n}{c} \right\rceil + \gamma \tag{7}$$

Consider the following example: $n = 5, c = 2, \tau = 0.4, t_b = 0, T = 1 \alpha = 10, \beta d = 2, \gamma = 1$.Then $N^* = 2,\ f^* = 27$ and a possible scheduling is represented as a Gantt diagram in Figure 3.

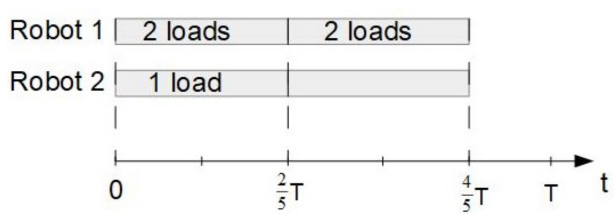


Fig. 3: Gantt diagram for optimal transport of 5 loads

C. Optimal number of robots (infinite horizon)

We are also interested in the limit case where the time horizon T tends to infinity. This allows, on the one hand, to avoid side effects (if horizon T is not a multiple of cycle time τ) and, on the other hand, to model a fleet of vehicles operating permanently.

We denote by $\mu = c/\tau$ the maximum flow rate of loads per robot (maximum number of loads that a robot can carry per unit of time), by $\lambda = n/T$ the demand flow of loads to transport and by $\delta = t_b/T$ the immobilization rate.

If we make T tend to infinity, keeping λ and δ constant, we get:

$$N^* = \left\lceil \frac{\lambda}{\mu(1-\delta)} \right\rceil \tag{8}$$

$$f^* = \alpha \left\lceil \frac{\lambda}{\mu(1-\delta)} \right\rceil + \beta d \frac{\lambda}{c} + \gamma \tag{9}$$

A detailed proof of this result is provided in the Appendix.

III. Sizing of a fleet of cooperative robots

In this section, we assume that robots can cooperate to transport loads. A m-bot is an elementary robot which can work on its own or with others while a p-bot is a set of p m-bots that cooperate together.

A. Assumptions and notations

The following notations are used:

- p: number of m-bots constituting one p-bot
- c_m: m-bot capacity
- c_p: p-bot capacity
- $c'_m = c_p/p$: virtual m-bot capacity when it evolves as part of a p-bot
- τ_m: m-bot cycle time
- τ_p: p-bot cycle time (including possible cooperation time)
- α: fixed cost per unit of time of a m-bot
- β: cost per meter traveled by a m-bot
- γ: fixed cost per unit of time, independent of the number of m-bots
- N_m: number of m-bots working alone
- N_p: number of p-bots
- n_m: number of loads transported by m-bots working alone
- n_p: number of loads transported by p-bots

The other assumptions remain unchanged from Section II:

- T: planning horizon
- r_m: number of round trips for m-bots
- r_p: number of round trips for p-bots
- n: number of loads to be transported from A to B

$$n = n_m + n_p$$

- N: number of m-bots in the fleet (including those working in a p-bot)

$$N = N_m + pN_p$$

We will make the following additional assumptions:

- $\tau_m \leq \tau_p$ as a p-bot may waste time in cooperation.
- There is no additional cost associated to a p-bot. The costs of a p-bot are simply those induced by the m-bots constituting it.
- There is no possible reconfiguration. A p-bot always remains a p-bot and a m-bot that is alone always remains alone.

B. Optimal fleet (finite horizon)

The fleet sizing problem can then be modeled by the following mathematical program which aims at minimizing the cost function for cooperative robots $f_c(r_m, r_p, N_m, N_p)$:

$$f_c = \alpha(N_m + pN_p) + \frac{\beta d}{T}(r_m + pr_p) + \gamma \tag{10}$$

subject to:

$$n_m \leq N_m c_m \left\lfloor \frac{T - t_b}{\tau_m} \right\rfloor \tag{11}$$

$$n_p \leq N_p c_p \left\lfloor \frac{T - t_b}{\tau_p} \right\rfloor \tag{12}$$

$$n = n_m + n_p \tag{13}$$

$$\frac{n_m}{c_m} \leqslant r_m \tag{14}$$

$$\frac{n_p}{c_p} \leqslant r_p \tag{15}$$

$$r_m, r_p, n_m, n_p, N_m, N_p \in \mathbb{N} \tag{16}$$

Constraints interpretation :

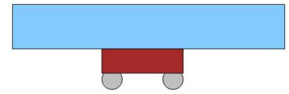
- Constraint (11): the number of loads carried by m-bots must be less than or equal to the maximum number of loads that m-bots can transport on interval $[0, T]$;
- Constraint (12): the number of loads carried by p-bots must be less than or equal to the maximum number of loads that p-bots can transport on interval $[0, T]$;
- Constraint (13): the sum of the number of loads carried by m-bots and p-bots must be equal to the total number of loads to be transported;
- Constraint (14): the number of round trips for the transport of all the loads designated by m-bots must be rounded up to allow the transport of all loads;
- Constraint (15): the number of round trips for the transport of all loads designated by p-bots must be rounded up to allow the transport of all loads.

When $\beta = 0$, the problem comes down to determining the minimum number of robots allowing all loads to be transported. When $\alpha = 0$, the problem comes down to achieving the optimal number of round trips to transport all the loads (a round trip from a p-bot counts as p round trips): $r_m^* = \left\lceil \frac{n_m^*}{c} \right\rceil$ and $r_p^* = \left\lceil \frac{n_p^*}{c} \right\rceil$, where n_m^* and n_p^* are the optimal number of loads transported by m-bots and p-bots, respectively, at which the cost is minimal.

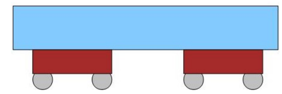
We will distinguish three cases linked to the respective capacities of m-bot and p-bot.

a) $c_m = 0$: In this first case, we assume that a m-bot can't carry a load on its own ($c_m = 0$). This scenario may appear for a load of great mass, great volume or even great length. For example, as shown in Figure 4, the robot cannot transport a load much larger than itself for stability reasons.

The problem then consists in determining the number of p-bots needed to carry all the loads and the results of Section II can be re-used. So we have:



(a) 1 m-bot (not stable)

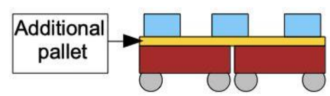


(b) p-bot with $p = 2$

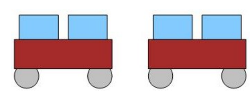
Fig. 4: Illustration for case $c_m = 0$

$$N_p^* = \left\lceil \frac{n}{c_p \lfloor (T - t_b)/\tau_p \rfloor} \right\rceil, \qquad N_m^* = 0.$$

b) $c_m \geq c_m'$: In this second scenario, a single m-bot has a greater capacity than a m-bot in p-bot configuration. Let us give a first example where this scenario occurs. If the capacity constraint is related to the transported mass and an additional pallet is needed in p-bot mode, then we lose mass capacity in p-bot mode. Another example would be the case where the load is transported by manipulator arms installed on the mobile platform in p-bot mode. Figure 5 shows the case when the robot loses its mass capacity due to the pallet, and thus the m-bots transport more than the p-bots.



(a) p-bot with $p = 2$



(b) 2 m-bots

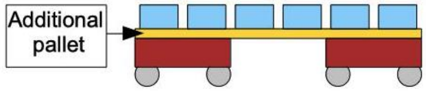
Fig. 5: Illustration for case $c_m \geq c_m'$

We can easily show that it is optimal to use exclusively m-bots. Assume that we use a p-bot for a round-trip in time interval $[t, t + \tau_p]$ to transport c_p loads. As $\tau_m \leq \tau_p$, we could use p individual m-bot to transport these loads in the same interval, as $c_m \geq c_m'$ (or equivalently $pc_m \geq c_p$). Hence, it is optimal to use exclusively m-bots.

We can then use again the results from the previous section. So we have

$$N_p^* = 0, \qquad N_m^* = \left\lceil \frac{n}{c_m \lfloor (T - t_b)/\tau_m \rfloor} \right\rceil.$$

c) $0 < c_m < c_m'$: In this 3rd case, a single m-bot has a lower capacity than a m-bot in p-bot configuration. This scenario may arise for the transport of long objects (for example tubes) or even objects of large volumes but of low density. Figure 6 shows an example where a p-bot can have a capacity greater than a m-bot, when we can not stack loads on top of each other.



(a) p-bot with $p = 2$

(b) 2 m-bots

Fig. 6: Illustration for case $0 < c_m < c'_m$

In this case, unlike previously, the optimal solution can consist of a mix of p-bots and m-bots. This appears in particular if the cost linked to the distance traveled is significant. Consider the following example: $n = 4, p = 2, c_m = 1, c_p = 3, \tau_m = \tau_p = T/2, t_b = 0, \alpha = 1, \beta d/T = 10, \gamma = 0$. Table I presents the optimal solution according to the type of authorized robots.

TABLE I: Optimal solutions according to the types of authorized robots

	N_m	N_p	n_m	n_p	total cost
m-bots only	2	0	4	0	42
p-bot only	0	1	0	4	42
Mix of m-bots and of p-bots	1	1	1	3	33

Figure 7 shows the Gantt diagram of the optimal solution when both configurations are allowed.

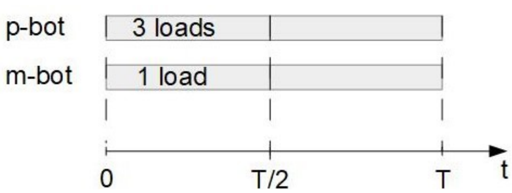


Fig. 7: Gantt diagram of the optimal solution of 4 loads

C. *Optimal fleet (infinite horizon)*

We use the following additional notations:

- $\mu_m = \dfrac{c_m}{\tau_m}$: m-bot flow rate
- $\mu_p = \dfrac{c_p}{\tau_p}$: p-bot flow rate
- λ: demand flow rate of loads to be transported
- $\lambda_m = \dfrac{n_m}{T}$: flow rate of loads carried by m-bots alone
- $\lambda_p = \dfrac{n_p}{T}$: flow rate of loads carried by p-bots
- $\delta = t_b/T$ immobilization rate

The mathematical program can then be written as a MILP (Mixed-Integer Linear Programming) :

$$\min \quad \alpha(N_m + pN_p) + \beta d \left(\frac{\lambda_m}{c_m} + p\frac{\lambda_p}{c_p} \right) + \gamma \tag{17}$$

$$s.t. \quad \lambda_m \leq N_m \mu_m (1-\delta) \tag{18}$$

$$\lambda_p \leq N_p \mu_p (1-\delta) \tag{19}$$

$$\lambda = \lambda_m + \lambda_p \tag{20}$$

$$\lambda_m, \lambda_p \in \mathbb{R}, \quad N_m, N_p \in \mathbb{N} \tag{21}$$

In two cases, we can re-use the results of Section II-C.

a) $c_m = 0$ *:*

$$N_m^* = 0, \qquad N_p^* = \left\lceil \frac{\lambda}{\mu_p(1-\delta)} \right\rceil .$$

b) $c_m \geq c'_m$*:*

$$N_m^* = \left\lceil \frac{\lambda}{\mu_m(1-\delta)} \right\rceil, \qquad N_p^* = 0.$$

IV. CONCLUSION

This paper describes a deterministic mathematical framework for the sizing of a fleet of identical robots, named m-bots, which have the possibility to cooperate. A set of p m-bots that cooperate on a given task constitute a p-bot.

When cooperation is not allowed, we obtain a closed-form expression for the optimal number of m-bots. When cooperation is allowed and reconfiguration forbidden, we formulate the fleet sizing problem by a mathematical program as an ILP (Integer Linear Programming).

Our mathematical model allows us to determine the most profitable number of robots that should cooperate. If the capacity of p m-bots is smaller than the capacity of a single p-bot, then using exclusively p-bots or a mix of m-bots can lead to a significant cost decrease. Otherwise, it is optimal to use exclusively m-bots.

The next step of our work will be to consider the case where a p-bot can be reconfigured into p separate m-bots leading to a higher use rate of all robots in the fleet. We will also consider the case of heterogeneous loads in mass and dimensions, where a m-bot can be used for a small load and a p-bot for a large load.

APPENDIX

Using the fact that $x-1 < \lfloor x \rfloor \leq x$, we can limit the optimal number of robots obtained in (5):

$$\left\lceil \frac{n}{c\frac{T-t_b}{\tau}} \right\rceil \leq N^* < \left\lceil \frac{n}{c\left(\frac{T-t_b}{\tau}-1\right)} \right\rceil \tag{22}$$

$$\Leftrightarrow \left\lceil \frac{\frac{n}{T}}{\frac{c}{\tau}\left(1-\frac{t_b}{T}\right)} \right\rceil \leq N^* < \left\lceil \frac{\frac{n}{T}}{\frac{c}{\tau}\left(1-\frac{t_b}{T}\right)-\frac{c}{T}} \right\rceil \tag{23}$$

Using the notations λ, μ, δ, this frame is re-written

$$\left\lceil \frac{\lambda}{\mu(1-\delta)} \right\rceil \leq N^* < \left\lceil \frac{\lambda}{\mu\left(1-\delta-\frac{c}{T}\right)} \right\rceil \tag{24}$$

If we tend T to infinity, keeping λ and δ constant, we get

$$N^* \xrightarrow[T\to+\infty]{\frac{n}{T}\to\lambda,\, \frac{t_b}{T}\to\delta} \left\lceil \frac{\lambda}{\mu(1-\delta)} \right\rceil \tag{25}$$

Similarly, we have the framing

$$\frac{n}{cT} \leq \frac{1}{T}\left\lceil \frac{n}{c} \right\rceil < \frac{1}{T}\left(\frac{n}{c}+1\right) \tag{26}$$

and

$$\frac{1}{T}\left\lceil \frac{n}{c} \right\rceil \xrightarrow[T\to+\infty]{\frac{n}{T}\to\lambda} \frac{\lambda}{c} \tag{27}$$

Then

$$f^* \xrightarrow[T\to+\infty]{\frac{n}{T}\to\lambda,\, \frac{t_b}{T}\to\delta} \alpha\left\lceil \frac{\lambda}{\mu(1-\delta)} \right\rceil + \beta d\frac{\lambda}{c} + \gamma \tag{28}$$

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