

Sizing of a fleet of cooperative and reconfigurable robots for the transport of heterogeneous loads*

Mari Chaikovskaia¹, Jean-Philippe Gayon² and Mairtin Marjolle³

Abstract—We consider a fleet of elementary robots that can be connected in different ways to transport loads of different types. For instance, a single robot can transport a small load and the association of two robots can either transport a large load or two small loads. We seek to determine the number of robots necessary to transport a set of loads in a given time interval, at minimum cost. The cost is function of the number of robots and of the distance travelled by robots. The fleet sizing problem can be formulated by an integer linear problem. In the special case of two types of loads and two configurations, closed-form expressions for the minimum number of robots can be derived. Finally, we show how reconfigurability can allow to diminish the number of required robots.

I. INTRODUCTION

A particular development over the last decade has taken place for AGVs (Automated Guided Vehicles) and AMRs (Autonomous Mobile Robots) [1], especially in logistics warehouses and industrial production [2]. In this work, we consider elementary robots that can be connected in different ways in order to transport loads of different types. The objective of this work is to determine the number of elementary robots allowing to transport a set of heterogeneous loads over a given time horizon. In the rest of this section, we introduce the notions of cooperation between robots and of reconfiguration, before presenting a state of the art on the sizing of a fleet of robots and summarizing our contributions.

A. Cooperation of robots to transport loads

There are different definitions for cooperation in Multi Robot Systems (MRS). Cooperation can be defined as the joint performance of a task [3]. For some authors, there is cooperation if the robots share a common goal, even if there is no interaction between the robots. For other authors, there is cooperation if the task cannot be performed sequentially by a single robot and requires coordination of actions and communication between robots [4]. Communication is explicit when the robots communicate directly with each other, and implicit when they communicate through an object which is, for example, transported.

Concerning the cooperation for the transport of objects, one can distinguish three categories of cooperation [4] :

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¹Mari Chaikovskaia is member of LIMOS, INP Clermont Auvergne, France, mari.chaikovskaia@doctorant.uca.fr

²Jean-Philippe Gayon is member of LIMOS, INP Clermont Auvergne, France, j-philippe.gayon@uca.fr

³Mairtin Marjolle is student of ISIMA, INP Clermont Auvergne, France, mairtin.marjolle@etu.uca.fr

- Pushing-only strategy: robots are not physically attached to the object, and transport is achieved by pushing the object. This method can be used when robots cannot pull an object. Kube and Zhang [5] demonstrated the ability to move an object without direct communication between the robots. A more complex model with obstacles has been studied by [6].
- Grasping strategy: robots are physically attached to the object, and transport can be achieved by pushing or pulling (or both) the object. This strategy assumes that the robots are equipped with a gripping tool. Algorithms have been developed to find the position of the robots to ensure maximum stability of the robots carrying the load [7], [8].
- Caging strategy: robots surround the object and block it during transport, unlike the pushing-only strategy. It is essential to position the robots correctly according to the shape and size of the object, so that the object does not escape from the cage [9].

B. Reconfigurable robotic systems

Reconfigurable robotic systems consist of identical modules that can attach and detach from each other to change their overall topology [10]. They can dynamically adapt their shape to meet the needs of the task at hand, for example for manipulation, locomotion and the creation of static structures [10]. In our work, a module corresponds to an elementary robot. For example, a small load can be transported by a single elementary robot while a voluminous or heavy load requires the collaboration of several elementary robots.

Several works have considered reconfigurable robotic systems. The work of Castano et al [11] was the first to describe a module for reconfigurable robots. The CORNO modules are autonomous, homogeneous and miniature. Using the modules, the robot can change shape like a building block, make a snake to get into tight spaces, a hexapod to carry loads, or the modules can work separately to perform parallel tasks. They use an information retrieval system for communication that also serves as a tracking system. A similar concept of modules is used for PolyBot, a system in which each module must fit in a cube of 5 cm side [12]. Reconfigurable and cooperative robots to transport loads were described in [13]. In this work, a module is a mobile manipulator. The number of modules may grow with the size of the product or the complexity of the task. Recently, companies have been created to develop cooperative and reconfigurable robots to transport loads [14], [15]. Figure 1 shows an example of transport of loads by reconfigurable robots of MecaBotiX.

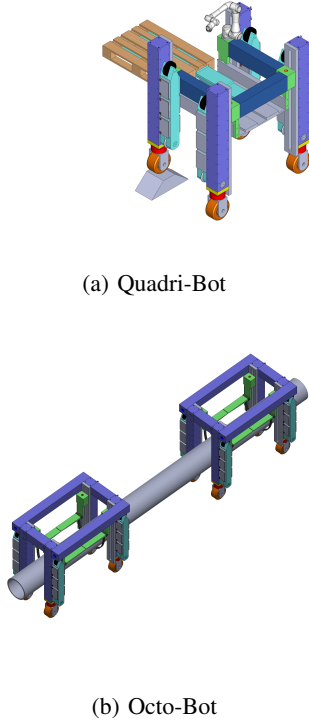


Fig. 1: Examples of MecaBotiX robots [14]

The quadri-bot in Figure 1a can transport a pallet while the octo-bot in Figure 1b can carry a long object. These robots communicate through the load being carried.

There also exists a literature on configurable vehicle capacity where the vehicle capacity depends on the chosen configuration (see e.g. [16], [17], [18]). Unlike the previous example where elementary robots are associated to form a poly-bot, the reconfiguration concerns the vehicle itself and not the combination of vehicles. For instance, Tellez et al [19] consider several types of users who do not occupy the same space in the vehicle (e.g. passengers using seats or wheelchairs). They consider a variant of the dial-a-ride problem where en-route modifications of the vehicle's inner configuration are allowed. For the instance under consideration, the authors show that reconfigurable vehicles are advantageous for companies when their cost are no more than 20 % of the cost of standard non-reconfigurable vehicles.

C. Sizing of a fleet of robots

Robots are generally expensive and determining the right type and number of vehicles is crucial. Egbelu [20] highlights several factors influencing the required number of vehicles, such as system layout, location of load transfer points, trips frequency, vehicle-dispatching strategy, system reliability and speed of travel.

The literature on the sizing of a fleet of robots can be divided schematically into two categories: the stochastic models (see e.g. [21], [22], [23]) and the deterministic models (see e.g. [20], [24], [25]). A survey of recent works

on the fleet sizing problem can be found in [26], which also reviews other issues on planning and control of autonomous mobile robots for intralogistics.

Different objective functions are considered in the literature. Most of the time, the objective is to minimize the number of robots required to achieve a set of transportation jobs in a time interval. Several works consider more elaborate objective functions. Beaujon and Turnquist [27] maximize the total profit (difference between revenues and total transportation costs, including penalty costs for unmet demand). Etezadi and Beasley [28] minimize the cost of a fleet of purchased or leased vehicles. Sinriech and Tanchoco [29] minimize the cost by applying penalties if performance is not achieved in terms of quality of service.

There is a single work that considers the fleet sizing problem for robots that can cooperate physically to transport loads [30]. In this study, robots can cooperate to transport homogeneous loads. A load can be carried either by a single robot (mono-robot) or by several robots that cooperate (poly-robot). However, reconfiguration is forbidden. The fleet sizing problem is formulated as a mathematical programming. If the capacity of the poly-robot is greater than the capacity of the robots that compose it, then using a poly-robot, or a mix of mono-robots and poly-robots, can lead to a significant cost decrease.

Contributions

To the best of our knowledge, our paper is the first to consider the problem of sizing a fleet of cooperative and reconfigurable robots for transport of loads. Section II introduces the assumptions and notations of our model. Section III propose Integer Linear Programs (ILPs) formulations for the problems with or without reconfiguration. Section IV derive closed-form expressions for the minimum number of robots in the special case of two types of loads. We also show that reconfiguration allows to reduce the number of robots required by a factor of two or less.

II. ASSUMPTIONS AND NOTATIONS

We consider a fleet of N mobile elementary robots able to cooperate to transport loads of different types. An elementary robot is abbreviated as bot. A p -bot is a set of p elementary robots cooperating on the same transportation task. A 1-bot is an elementary robot working alone. A maximum of P elementary robots can cooperate.

There are n_k loads of type k to be transported ($k = 1, \dots, K$). All the loads to be moved are located in a loading area A of the warehouse and must be transported to unloading areas. The p -bots transport the loads between the loading zone and the unloading zones. A p -bot can only carry one type of load at a time and can simultaneously carry c_{pk} loads of type k .

The time horizon is divided into T periods ($t = 1, \dots, T$). At the beginning of each period, the robots are located in loading area A and can be reconfigured. For example, a 3-bot and a 2-bot, i.e. 5 elementary robots, can turn into a 4-bot

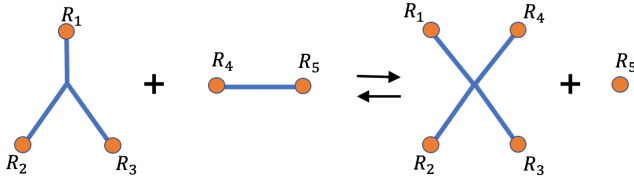


Fig. 2: Reconfiguration from a 3-bot and a 2-bot into a 4-bot and a 1-bot

and a 1-bot. This example is illustrated in Figure 2 where R_i denotes the i -th elementary robot.

In each period, after the reconfiguration of robots, p -bots have time to complete the following four steps of a delivery, namely: loading in area A, loaded trip, unloading and empty return trip to area A.

We consider three types of cost: a fixed cost per elementary robot linked to the acquisition or rental of the robot, a variable cost linked to the distance traveled and a fixed cost independent of the size of the fleet (hardware and software infrastructure). The fixed cost of an elementary robot is denoted α . The cost of a round trip for an elementary robot is denoted β and it follows that the cost of a round trip for a p -bot is $p\beta$. Finally, the fixed cost of a fleet is denoted γ . Thus, the cost of a fleet of N elementary robots performing M round trips is $\alpha N + \beta M + \gamma$. In the following, we won't consider γ because it is a constant in the optimization problem.

The objective is to determine the number of robots needed to transport all the loads over the time horizon at minimum cost. We consider two variants. In the first variant, reconfiguration is prohibited and the configurations are fixed over the entire horizon. The optimal number of elementary robots is denoted by N_W in this case. In the second variant, reconfiguration is allowed and the configurations can be changed at the start of each period. The optimal number of elementary robots is denoted N_R in this case. If we do not consider the cost related to the travelled distance, i.e. if we take $\beta = 0$, then N_R and N_W represent the minimum number of robots with or without reconfiguration respectively.

We now remind the main notations introduced in this section:

- $k = 1, \dots, K$: load types
- n_k : number of loads of type k
- $p = 1, \dots, P$: possible configurations
- c_{pk} : capacity of a p -bot carrying loads of type k
- $t = 1, \dots, T$: periods
- α : cost of an elementary robot
- β : cost of a round trip for an elementary robot
- N_W : minimum number of elementary robots when reconfiguration is prohibited
- N_R : minimum number of elementary robots when reconfiguration is possible

III. MATHEMATICAL FORMULATIONS

In this part, we propose two ILPs in order to formulate the two variants of the problem.

A. ILP without authorised reconfiguration

We first assume that reconfiguration is prohibited. We use the following decision variables:

- N_p : number of p -bots
- N_{pk}^t : number of robots in configuration p carrying loads of type k in period t

The number of elementary robots is then $N = \sum_{p=1}^P N_p$.

The optimisation problem can then be formulated by the following ILP.

$$\min \quad \alpha \sum_{p=1}^P p \cdot N_p + \beta \sum_{t=1}^T \sum_{k=1}^K \sum_{p=1}^P p \cdot N_{pk}^t \quad (1)$$

subject to

$$\sum_{t=1}^T \sum_{p=1}^P c_{pk} \cdot N_{pk}^t \geq n_k \quad \forall k \quad (2)$$

$$\sum_{k=1}^K N_{pk}^t \leq N_p \quad \forall t, \forall p \quad (3)$$

$$N_p \in \mathbb{N}, N_{pk}^t \in \mathbb{N} \quad \forall k, \forall p, \forall t \quad (4)$$

Constraint (2) means that the total capacity of the fleet must be able to transport all loads of each type. Constraint (3) means that the number of p -bots, N_p , must be greater than or equal to the number of p -bots used over each period.

B. ILP with authorised reconfiguration

When the reconfiguration is allowed, we use similar decision variables:

- N : number of elementary robots
- N_{pk}^t : number of p -bots carrying loads of type k in period t

The problem is formulated by the following ILP:

$$\min \quad \alpha N + \beta \sum_{t=1}^T \sum_{k=1}^K \sum_{p=1}^P p \cdot N_{pk}^t \quad (5)$$

subject to :

$$\sum_{t=1}^T \sum_{p=1}^P c_{pk} \cdot N_{pk}^t \geq n_k \quad \forall k \quad (6)$$

$$\sum_{k=1}^K \sum_{p=1}^P p N_{pk}^t \leq N \quad \forall t \quad (7)$$

$$N \in \mathbb{N}, N_{pk}^t \in \mathbb{N} \quad \forall k, \forall p, \forall t \quad (8)$$

Constraint (6) is similar to that of the problem without reconfiguration. Constraint (7) means that the number of elementary robots used, N , must be greater than or equal to the number of elementary robots used over each period.

IV. SPECIAL CASE OF TWO TYPES OF LOADS

In practice, there are often two types of loads, for example boxes and pallets. In this part, we obtain additional results for the special case of two load's types and two possible configurations (1-bot or p -bot with $p \geq 2$). We assume that there are $n_1 \geq 1$ loads of type 1 and $n_2 \geq 1$ loads of type 2. Capacities are summarized in Table I.

Configuration	Type of load	
	$k = 1$	$k = 2$
1-bot	c_{11}	0
p -bot	c_{p1}	c_{p2}

TABLE I: Capacities with two types of loads

A 1-bot can carry up to c_{11} loads of type 1 but no loads of type 2. A p -bot can carry up to c_{p1} loads of type 1 and up to c_{p2} loads of type 2. In order to always have feasible solutions, we assume that $c_{11} \geq 1$ and $c_{p2} \geq 1$. When $c_{p1} = 0$, it corresponds to a dedicated transport: loads of type 1 (respectively type 2) can only be transported by 1-bots (respectively p -bots).

Finally, we assume in this section that $\beta = 0$. Thus, N_R and N_W represent the minimum number of elementary robots needed to carry all the loads over the time horizon with or without reconfiguration respectively.

A. Closed-form solution

The problem simplifies considerably because there is only one way to transport loads of type 2 and we obtain analytical formulas for N_R and N_W . Moreover, when $c_{p1} \geq p \cdot c_{11}$ reconfigurability does not allow to economize robots. Theorem 1 summarizes these results.

Theorem 1: If $c_{p1} \geq p \cdot c_{11}$ then $N_W = N_R$. Unless,

$$N_W = p \cdot \left\lceil \frac{n_2}{T \cdot c_{p2}} \right\rceil + \left\lceil \frac{(n_1 - T_2 \cdot c_{p1})^+}{T \cdot c_{11}} \right\rceil \quad (9)$$

$$N_R = p \cdot \left\lceil \frac{n_2}{T \cdot c_{p2}} \right\rceil + \left\lceil \frac{(n_1 - T_2 \cdot p \cdot c_{11})^+}{T \cdot c_{11}} \right\rceil \quad (10)$$

$$T_2 = T \cdot \left\lceil \frac{n_2}{T \cdot c_{p2}} \right\rceil - \left\lceil \frac{n_2}{c_{p2}} \right\rceil. \quad (11)$$

In this theorem $x^+ = \max(0, x)$ designates the part positive of x .

Proof: We distinguish two cases. Whatever the case, we assume that the p -bots are fully filled before a new p -bot is loaded so that we have no more than one p -bot with free periods at the outcome of the assignment loads of type 2.

Case 1: $c_{p1} \geq p \cdot c_{11}$

We assume that a p -bot has more capacity than p 1-bots to carry loads of type 1. It is then always more interesting to use a p -bot than p 1-bots. Thus, there is no point for reconfiguration and $N_W = N_R$.

Case 2: $c_{p1} < p \cdot c_{11}$ In this case, note that it is always more interesting to use 1-bots to transport loads of type 1. Let's start by determining N_W . The general idea is as follows:

- 1) We first assign loads of type 2 to p -bots.
- 2) If the last p -bot is not used over all the periods, we assign as many loads of type 1 as possible to the last p -bot assigned for loads of type 2.
- 3) The remaining loads of type 1 are assigned to 1-bots.

For step (1), the number of p -bots needed is

$$\left\lceil \frac{n_2}{T \cdot c_{p2}} \right\rceil. \quad (12)$$

For step (2), let T_2 be the number of periods not used by the last p -bot:

$$T_2 = \left\lceil \frac{n_2}{T \cdot c_{p2}} \right\rceil T - \left\lceil \frac{n_2}{c_{p2}} \right\rceil. \quad (13)$$

We can therefore assign up to $T_2 \cdot c_{p1}$ loads of type 1 to the last p -bot.

For step (3), then $(n_1 - T_2 \cdot c_{p1})^+$ loads of type 1 remain to be transported. We conclude that

$$N_W = p \cdot \left\lceil \frac{n_2}{T \cdot c_{p2}} \right\rceil + \left\lceil \frac{(n_1 - T_2 \cdot c_{p1})^+}{T \cdot c_{11}} \right\rceil. \quad (14)$$

We now determine N_R . There are three steps as for the calculation of N_W . Step (2) is modified as follows: if the last p -bot is not used over all the periods, it is reconfigured into p 1-bots to which we assign as many loads of type 1 as possible (at most $T_2 \cdot p \cdot c_{11}$). For step (3), then $(n_1 - T_2 \cdot p \cdot c_{11})^+$ loads of type 1 remain to be transported. We conclude that

$$N_R = p \cdot \left\lceil \frac{n_2}{T \cdot c_{p2}} \right\rceil + \left\lceil \frac{(n_1 - T_2 \cdot p \cdot c_{11})^+}{T \cdot c_{11}} \right\rceil. \quad (15)$$

B. Number of robots economized due to reconfigurability

We now study the number of robots that can be saved thanks to the reconfiguration. We show that this gain is at most p elementary robots within the framework of the assumptions specified at the beginning of section IV.

Theorem 2: $N_W \leq N_R + p$

Proof: When $c_{p1} \geq p \cdot c_{11}$, the result is trivial because $N_W = N_R$.

In the following, we assume that $c_{p1} < p \cdot c_{11}$. According to Theorem 1, we have

$$N_W - N_R = \left\lceil \frac{(n_1 - T_2 \cdot c_{p1})^+}{T \cdot c_{11}} \right\rceil - \left\lceil \frac{(n_1 - T_2 \cdot p \cdot c_{11})^+}{T \cdot c_{11}} \right\rceil \quad (16)$$

$$\leq \frac{(n_1 - T_2 \cdot c_{p1})^+}{T \cdot c_{11}} + 1 - \frac{(n_1 - T_2 \cdot p \cdot c_{11})^+}{T \cdot c_{11}} \quad (17)$$

$$= \frac{(n_1 - T_2 \cdot c_{p1})^+ - (n_1 - T_2 \cdot p \cdot c_{11})^+}{T \cdot c_{11}} + 1. \quad (18)$$

Three cases are then possible. Assume first that $n_1 \leq T_2 \cdot c_{p1}$. Then $(n_1 - T_2 \cdot c_{p1})^+ = (n_1 - T_2 \cdot p \cdot c_{11})^+ = 0$. With (18), it comes $N_W - N_R \leq 1 \leq p$.

Assume now that $T_2 \cdot c_{p1} < n_1 \leq T_2 \cdot p \cdot c_{11}$. Then we have $(n_1 - T_2 \cdot c_{p1})^+ = n_1 - T_2 \cdot c_{p1}$, $(n_1 - T_2 \cdot p \cdot c_{11})^+ = 0$ and

$$N_W - N_R \leq \frac{n_1 - T_2 \cdot c_{p1}}{T \cdot c_{11}} + 1 \quad (19)$$

$$< \frac{n_1}{T \cdot c_{11}} + 1 \quad (20)$$

$$< \frac{n_1}{T_2 \cdot c_{11}} + 1 \quad (21)$$

$$\leq p + 1. \quad (22)$$

Strict inequality (20) comes from the assumption that $T_2 \cdot c_{p1} < n_1$. Strict inequality (21) comes from $T_2 < T$. Inequality (22) comes from the assumption that $n_1 \leq T_2 \cdot p \cdot c_{11}$. We have $N_W - N_R < p + 1$ and, As $N_W - N_R$ and p are integers, we conclude that $N_W - N_R \leq p$.

Finally, assume that $n_1 \geq T_2 \cdot p \cdot c_{11}$. We can therefore remove the positive parts in (18):

$$N_W - N_R \leq \frac{n_1 - T_2 \cdot c_{p1}}{T \cdot c_{11}} + 1 - \frac{n_1 - T_2 \cdot p \cdot c_{11}}{T \cdot c_{11}} \quad (23)$$

$$= \frac{T_2(p \cdot c_{11} - c_{p1})}{T \cdot c_{11}} + 1 \quad (24)$$

$$< \frac{p \cdot c_{11} - c_{p1}}{c_{11}} + 1 \quad (25)$$

$$\leq p + 1. \quad (26)$$

As $T_2 < T$: Again $N_W - N_R < p + 1$ implies that $N_W - N_R \leq p$.

In any case, we have $N_W \leq N_R + p$. ■

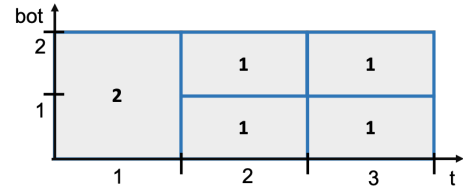
As a corollary to the 2 theorem, we show that we can at best halve the size of the fleet using reconfiguration.

Corollaire 1:

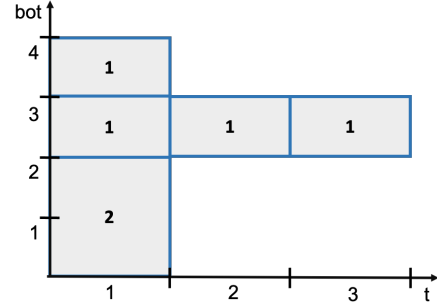
$$1 \leq \frac{N_W}{N_R} \leq 2$$

Proof: By assumption, there is at least one load of type 2 to transport. We therefore need at least one p -bot over a period, which implies that $N_R \geq p$ and $\frac{p}{N_R} \leq 1$. By theorem 2, we have $N_W \leq N_R + p$ which implies that $\frac{N_W}{N_R} \leq 1 + \frac{p}{N_R} \leq 2$. ■

We now provide an example where the upper bounds of theorem 2 and corollary 1 are reached. Let's take $c_{11} = c_{p2} = 1$, $c_{p1} = 0$, $T > p$, $n_1 = (T - 1)p$ and $n_2 = 1$. Then $N_R = p$ and $N_W = 2p$. Figure 3 represents the Gantt chart for this example when $p = 2$ and $T = 3$. The type of loads the robot is carrying is indicated in each rectangle. When reconfiguration is allowed, a 2-bot can carry one load of type 2, then it reconfigures into two 1-bots to carry 4 loads of type 1. If the reconfiguration is not allowed, the 2-bot is not be able to split into two independent robots, so it carries a single load of type 2. In the strategy with authorized reconfiguration, the minimum number of bots is therefore $N_R = 2$. In the strategy where reconfiguration is not allowed, the minimum number of bots is $N_W = 4$.



(a) With reconfiguration



(b) Without reconfiguration

Fig. 3: Gantt chart for a simple example where the number of robots is halved when reconfiguration is allowed.

The type of loads the robot is carrying is indicated in each rectangle. When reconfiguration is allowed, a 2-bot can carry one load of type 2, then it reconfigures into two 1-bots to carry 4 loads of type 1. If the reconfiguration is not allowed, the 2-bot is not be able to split into two independent robots, so it carries a single load of type 2. In the strategy with authorized reconfiguration, the minimum number of bots is therefore $N_R = 2$. In the strategy where reconfiguration is not allowed, the minimum number of bots is $N_W = 4$.

We have seen that the gain can be important in the previous example with a fleet divided by two. However, the gain in relative value is limited for large robot fleets. Indeed, according to Theorem 2, we have

$$\frac{N_W}{N_R} \leq 1 + \frac{p}{N_R}. \quad (27)$$

For instance, if $N_R = 100$ and $p = 4$, then $N_W/N_R \leq 1.04$ and $N_W \leq 104$. Hence reconfigurability allows to divide the size of the fleet by at most a factor 1.04.

V. CONCLUSION

This paper describes a deterministic mathematical framework for the sizing of a fleet of cooperative and reconfigurable robots. Two ILPs have been written to compare the cost of a fleet of robots with the possibility of reconfiguration and without this possibility. Then we have investigated a special case with two types of loads and two allowed configurations (1-bot and p -bot with $p > 1$). For this special case, closed-form expressions are derived for the minimum number of robots with or without reconfiguration. We show that the minimum number of elementary robots can be divided by a factor up to two by allowing reconfiguration. However, the number of robots saved by reconfigurability is

at most p robots. Hence, under our assumptions, the gain in relative value is limited for large robot fleets. We also show that reconfiguration is useless when a p -bot can transport more loads than $p-1$ bots.

In future research, it would be interesting to consider more complex warehouse topologies and take into account reconfiguration time.

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