Monty Hall Problem

Probability Distribution Tables

P(MyChoice)

MyChoice	A	B	C
	1/3	1/3	1/3

P(ContainsPrice)

ContainsPrize	A	B	C
	1/3	1/3	1/3

P(OpenedByOfficial | ContainsPrice, MyChoice)

MyChoice	A			B			C		
ContainsPrice	A	B	C	A	B	C	A	B	C
OpenedByOfficial: A	0	0	0	0	0.5	1	0	1	0.5
OpenedByOfficial: B	0.5	0	1	0	0	0	1	0	0.5
OpenedByOfficial: C	0.5	1	0	1	0.5	0	0	0	0

The table above is to be read in the following manner: Given MyChoice = {A, B, C}, and the Price being behind one in {A, B, C}, then the probabilty of each door being opened by the official is either {0, 0.5, 1}.

Now computing P(ContainsPrize | OpenedByOfficial, MyChoice) = P(OpenedByOfficial | ContainsPrice, MyChoice) * P(ContainsPrice) / P(OpenedByOfficial | MyChoice). Where P(OpenedByOfficial | MyChoice) = sum over P(OpenedByOfficial | ContainsPrize, MyChoice)*P(ContainsPrice)

Eg: P(A |B, A) = P(B | A, A)*P(A) / P(B | A)
P(B | A) = P(B | A, A) * P(A) + P(B | C, A) * P(C) = 1/2 * 1/3 + 1 * 1/3 = 1/2
=> P(A | B, A) = (1/2 * 1/3) / (1/2) = 1/3

P(OpenedByOfficial | MyChoice)

MyChoice	A	B	C

OpenedByOfficial: A	0	0.5	0.5
OpenedByOfficial: B	0.5	0	0.5
OpenedByOfficial: C	0.5	0.5	0

P(ContainsPrize | OpenedByOfficial, MyChoice)

MyChoice	**A**		**B**		**C**	
OpenedByOfficial	B	C	A	C	A	B
ContainsPrice: A	1/3	1/3	0	2/3	0	2/3
ContainsPrice: B	0	2/3	1/3	1/3	2/3	0
ContainsPrice: C	2/3	0	2/3	0	1/3	1/3