EPGY P051 Practice Final Examination SOLUTIONS

You should try to take this exam under test conditions: no book, no notes, and give yourself 3 hours.

Multiple Choice questions. :	21 pts.
Problem 1	8 pts
Problem 2	8 pts
Problem 3	8 pts
Problem 4	8 pts
Problem 5	8 pts
Problem 6	8 pts
Problem 7	8 pts
Problem 8	8 pts
Short response questions	15 pts
Total:	100 pts

Constants

$$g = 9.81\frac{m}{s^2} \quad \text{(near Earth's surface)}$$
$$G = 6.67\times 10^{-11}\frac{Nm^2}{kg^2}$$

Quadratic Equation

$$Ax^2+Bx+C=0$$
$$x=-\frac{B}{2A}\pm\frac{1}{2A}\sqrt{B^2-4AC}$$

Kinematical relations

$$\vec{v} = \frac{d\vec{x}}{dt},$$
$$\vec{a} = \frac{d\vec{v}}{dt}=\frac{d^2\vec{x}}{dt^2}.$$

Expressions for *constant* acceleration

$$x = x_0+v_{0x}t+\frac{1}{2}a_xt^2$$
$$v_x = v_{0x}+a_xt$$
$$v_x^2 = v_{0x}^2+2a_x\Delta x$$

(equivalent equations for the y and z directions)

$$a_c = \frac{v^2}{r}=\omega^2 r \quad \text{centripetal accel.}$$

Newton's Laws

1. Every body continues in its state of rest or uniform velocity unless acted on by a nonzero net force.
2. $\vec{F}_{net} = \Sigma\vec{F} = I\vec{a} = \frac{\Delta\vec{p}}{\Delta t}$. The resulting acceleration is in the direction of the net force.
3. Whenever one object exerts a force on a second object, the second exerts an equal (in magnitude) but opposite (in direction) force on the first.

Problem solving with Newton's Laws

1. Draw a picture.
2. Draw a detailed Free Body Diagram for the body in question.
3. Choose a coordinate system and origin.
4. Resolve forces into the x, y, and z directions.
5. Apply Newton's second law for each direction.
   ask first whether the object is accelerating and, if so, in which direction(s)
6. Solve equations.
7. Check answer to see if it makes sense.
8. Round to appropriate number of sigfigs, include units, and box answer.

Formulas; Work, Energy, etc.

$$F_{fr} = \mu_k F_N \quad \text{kinetic friction}$$
$$F_{fr} \leq \mu_s F_N \quad \text{static friction}$$
$$\vec{F} = -k\Delta\vec{x} \quad \text{force due to a spring}$$

Multiple choice

The speed v of an automobile moving on a straight road is given in meters per second as a function of time t in seconds by the following equation:

$$v = 4 + 2t^3$$

1. What is the acceleration of the automobile at $t = 2s$?

a) $12m/s^2$

b) $16m/s^2$

c) $20m/s^2$

d) $24m/s^2$

e) $28m/s^2$

2. How far has the automobile traveled in the interval between t= 0 and t= 2 s?

a) $16m$

b) $20m$

c) $24m$

d) $32m$

e) $72m$

3. Consider the International Space Station (ISS) that orbits at an altitude of roughly $250km$ above the surface of the Earth. If the force of gravity on the ISS due to Earth has magnitude F_0 when it is on the Earth's surface, which value is closest in magnitude to the force of gravity, due to Earth, when it is in orbit?

a) 0

b) 0.1 F_0

c) 0.5 F_0

d) **0.9** F_0

e) F_0

4. The sum of all the external forces on a system of particles is zero. Which of the following must be true of the system?

a) The total mechanical energy is constant.

b) The total potential energy is constant.

c) The total kinetic energy is constant.

d) The total linear momentum is constant.

e) It is in static equilibrium.

5. A disk X rotates freely with angular velocity, ω, on frictionless bearings about its center. A second identical disk Y, initially not rotating, is placed on top of X so that both disks rotate together without slipping. The two disks are collinear (symmetry axes are the same). When the disks are rotating together, which of the following is half what it was before?

a) Moment of inertia of X

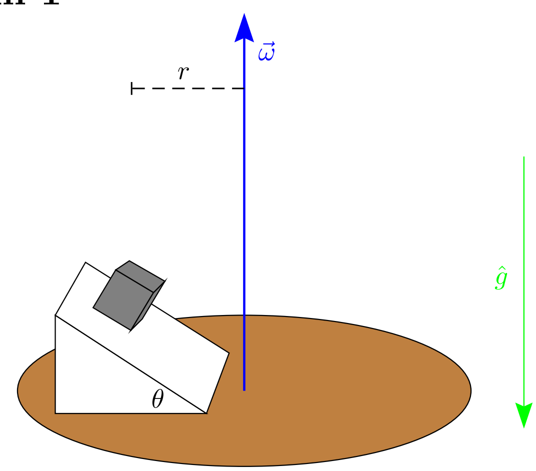
b) Moment of inertia of Y

c) Angular velocity of X

d) Angular velocity of Y

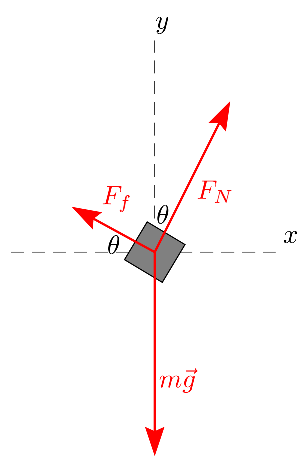
e) Angular momentum of both disks

Problem 1

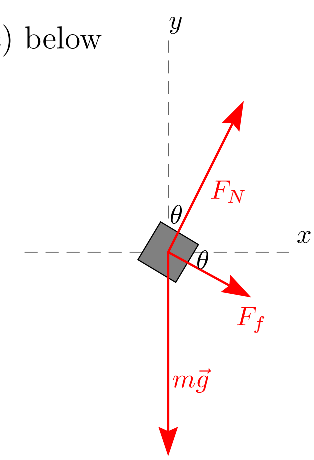


An inclined plane of angle θ is fixed to a rotating disk. A block is placed on the inclined plane a distance r from the axis of rotation of the disk. The turntable is rotating at an angular velocity of ω. There is friction between the block and the ramp and the coefficient of static friction is μ.

a) Draw a free body diagram for the block.



FBD for part c) below



We choose the axes x, y as such because the block *is accelerating* in the $+x$ direction. This choice just simplifies the algebra a little bit.

b) What is the minimum value of angular velocity ω_{min} that will keep the block from sliding down the plane. Give your answer in terms of r, g, μ, and θ.

Resolving the forces in the diagram above and using $F_f = \mu F_N$ we have,

$$\sum F_y = 0 = +F_N \cos\theta + \mu F_N \sin\theta - mg \quad \longrightarrow \quad F_N = \frac{mg}{\cos\theta + \mu \sin\theta}$$

$$\sum F_x = ma = +mr\omega_{min}^2 = +F_N \sin\theta - \mu F_N \cos\theta = mg\frac{\sin\theta - \mu\cos\theta}{\cos\theta + \mu\sin\theta}$$

$$\omega_{min} = \sqrt{\frac{g}{r}\left(\frac{\sin\theta - \mu\cos\theta}{\cos\theta + \mu\sin\theta}\right)}$$

c) What is the maximum value of angular velocity ω_{max} such that the block does not slide *up* the plane?

For this case we see that if the rotation rate is increased greatly, the friction will try to prevent the block from sliding up the plane. All we need to do is switch the sign on the frictional force.

$$\begin{aligned}
\sum F_y &= 0 = +F_N \cos\theta - \mu F_N \sin\theta - mg \qquad \longrightarrow \quad F_N = \frac{mg}{\cos\theta - \mu \sin\theta} \\
\sum F_x &= ma = +mr\omega_{max}^2 = +F_N \sin\theta + \mu F_N \cos\theta = mg\frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta} \\
\omega_{max} &= \sqrt{\frac{g}{r}\left(\frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}\right)}
\end{aligned}$$

Problem 2

An hour glass filled with sand sits on a weight scale. Initially all of the sand (of total mass m) in the glass (of mass M) is held in the upper reservoir. At time $t = 0$ the sand is released and falls at a constant rate of $\lambda = \frac{dm}{dt}$ to the bottom of the lower reservoir. Find the reading of the scale as a function of time for the time below.

a) From the time $t = 0$ when the sand is first released until time $t = t_1$ at which it just starts to arrive at the bottom of the reservoir.

Note that the value of the glass (M) always remains the same, so let's concentrate on the grains only.

For this time, the weight decreases as the amount of sand in free fall increases. Thus the mass of sand contributing to the weight starts out at m and decreases from $t = 0 \to t_1$ by λt. Or,

$$\begin{aligned} m_a(t) &= m - \lambda t \\ W_a(t) &= Mg + mg - \lambda g t \end{aligned}$$

b) From time $t = t_1$ to time $t = t_2$ at which the last grain just falls from the upper reservoir.

During this time, the amount of sand in free fall is constant and equal to the amount of sand that is in free fall at the time t_1 (which spans the length from top to bottom).

$$\begin{aligned} m_b &= m - \lambda t_1 \\ W_b &= Mg + mg - \lambda g t_1 \end{aligned}$$

c) From time $t = t_2$ until time $t = t_3$ when the last grain arrives at the bottom.

During this period the mass that contributes to the weight increases from the previous constant value back to the original, full, weight. The amount of sand in free fall is $\lambda(t - t_2)$. Thus we have,

$$\begin{aligned} m_c(t) &= m - \lambda t_1 + \lambda(t - t_2) \\ W_c(t) &= Mg + mg + \lambda g(t - t_1 - t_2) \end{aligned}$$

d) After time t_3. Use this to obtain a relation between t_1, t_2, t_3.

At time t_3 the weight returns to the initial value, however we can insert this time into the last expression to get a relation between the times.

$$m_d = m \quad = \quad m + \lambda(t_3 - t_2 - t_1) \quad \longrightarrow \quad t_3 = t_1 + t_2$$

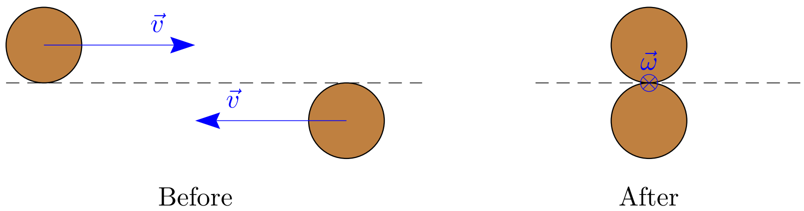
The weight is then,

$$W_d = (M + m)g \tag{1}$$

e) Sketch the reading of the scale as a function of time, noting that $m < M$.

The graph simply starts at W_a then decreases linearly from $t = 0 \to t_1$, is flat between $t_1 \to t_2$, then increases linearly back to the original value at t_3.

Problem 3



Two cylindrical disks, each of mass M and radius R, slide towards each other on a frictionless table. Initially they are traveling in opposite directions at speed v. They are offset so that when they have a glancing collision, they stick to each other at their edges.

a) What is the total angular momentum of this system before the collision about the center of mass of the system?

The center of mass lies on the dotted line, halfway between the two disks. Each one contributes an amount MRv to the angular momentum. Thus,

$$L_i = 2MRv$$

b) What is the momentum of inertia of the two disks, stuck together, after the collision has occurred? (about their center of mass).

Using the parallel axis theorem we have,

$$I = 2 \times [\frac{1}{2}MR^2 + MR^2] = 3MR^2$$

c) What is the angular velocity ω of the two pucks after the collision?

Using conservation of angular momentum we have,

$$\begin{aligned} L_i = 2MRv &= L_f = I\omega \\ 2MRv &= 3MR^2\omega \quad \longrightarrow \quad \omega = \frac{2}{3}\frac{v}{R} \end{aligned}$$

d) How much energy was lost in the collision?

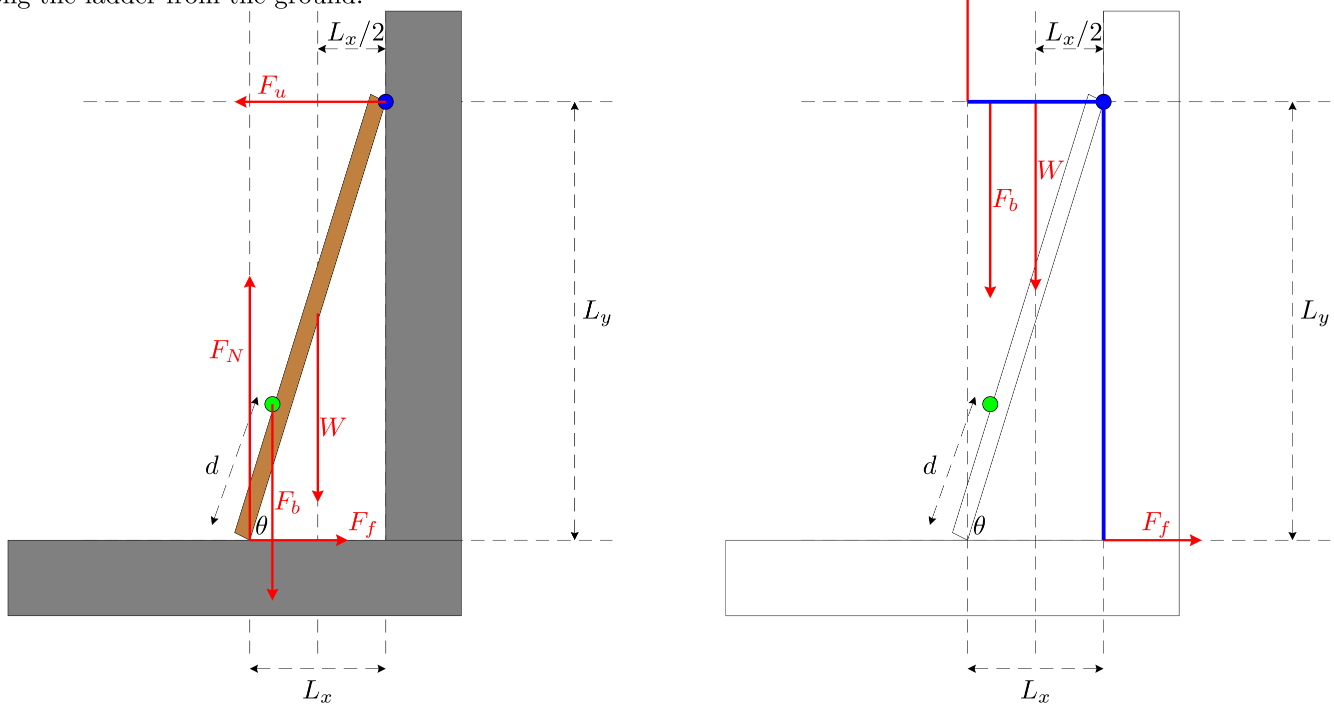
The initial energy was (note final linear velocity is zero from conservation of linear momentum),

$$\begin{aligned} E_i &= 2 \times \frac{1}{2}Mv^2 = Mv^2 \qquad E_f = \frac{1}{2}I\omega^2 \\ \Delta E &= Mv^2 - \frac{3}{2}MR^2\omega^2 = Mv^2[1 - \frac{3}{2}\frac{4}{9}] = \frac{1}{3}Mv^2 \end{aligned}$$

Problem 4

A ladder, of mass $10kg$ and length $5m$, leans against a wall at an initial angle of $\theta = 75^o$ from the horizontal. There is no friction between the ladder and the wall but there is friction between the ladder and the floor, with coefficient of static friction $\mu = 0.1$. A boy of mass $50kg$ begins to climb the ladder.

How far up the ladder does he get before the ladder starts to slide down. Give your distance as measured along the ladder from the ground.



The left hand figure above is the FBD for the boy on the ladder. In order to keep the diagram as uncluttered as possible we've expressed the forces and lengths as simply as possible. The only unknown in this diagram is d.

In the right hand diagram we've slid the forces along parallel lines until they intersect the moment arm at right angles. We are allowed to do this and it is equivalent to resolving the forces parallel and perpendicular to the ladder. We've chosen the upper contact point as our point to take torques about since *we do not know, nor do we care about, the force* F_u. This is why we deleted it in the second figure. This allows for a simple determination of the torques since all of the lengths are easily found. Explicitly we have $L_x = L\cos 75^o$, $L_y = L\sin 75^o$.

Before moving on to the torque equation let's quickly do the linear second law to determine the normal force. We only need to consider the vertical direction. We have directly,

$$0 = F_N - F_b - W \qquad \longrightarrow \qquad F_N = W + F_b = (60kg)g$$

Considering clockwise torques as negative we have, proceeding down the ladder from the torque point,

$$\begin{aligned} 0 &= +W\frac{L_x}{2} + F_b(L-d)\cos 75^o - F_N L_x + F_f L_y \\ d &= \frac{+W\frac{L_x}{2} + F_b L\cos 75^o - F_N L_x + F_f L_y}{F_b \cos 75^o} \end{aligned}$$

In the second line we have solved this expression for d. Inserting the values for L_x and L_y and the normal force we get,

$$\begin{aligned} d &= \left[\frac{+W\frac{\cos 75^o}{2} + F_b\cos 75^o - (W+F_b)\cos 75^o + \mu(W+F_b)\sin 75^o}{F_b\cos 75^o}\right] L \\ d &= \left[\frac{1}{2}\frac{W}{F_b} + 1 - (\frac{W}{F_b} + 1) + \mu(\frac{W}{F_b} + 1)\tan 75^o\right] L \end{aligned}$$

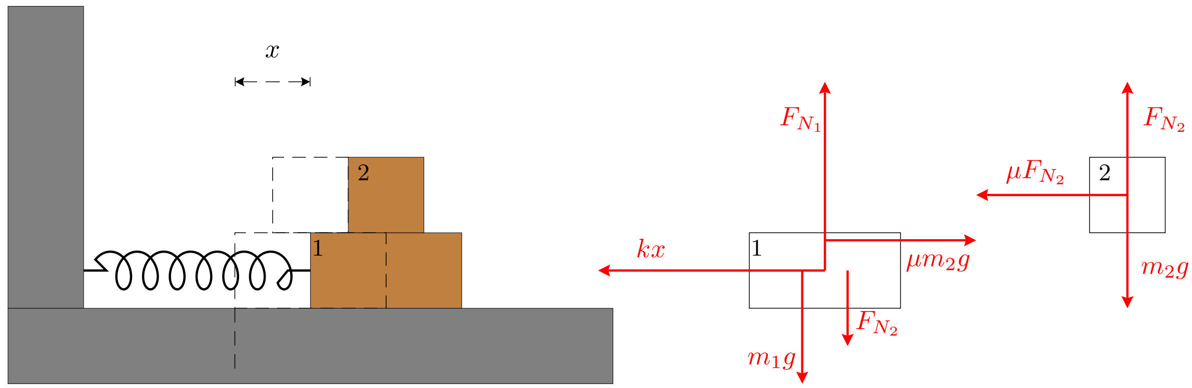
Noting that $\frac{W}{F_b} = \frac{1}{5} = 0.2$ we get,

$$d = [0.1 + 1 - (1.2) + (0.1)(1.2)\tan 75^o]\, L = 0.348L = 1.74m$$

Problem 5

A large block of mass m_1 undergoes simple harmonic motion as it slides across a frictionless floor while connected to a spring (on the left) with spring constant k. A block of mass m_2 rests on top of the m_1 block. The coefficient of static friction between the two blocks is μ. The upper block initially does not slip with respect to the lower block.

a) Draw a diagram for the situation and make free body diagrams for each block when the spring is stretched a distance x from its equilibrium point to the right.



b) What is the angular frequency of oscillation, ω of the system?

This is pretty straightforward since we can just treat it as a block of mass $m_1 + m_2$ that is oscillating on a spring. Thus,

$$\omega = \sqrt{\frac{k}{m_1 + m_2}} \quad \text{or} \quad k = \omega^2(m_1 + m_2)$$

c) What is the maximum amplitude, A, that the system can have if m_2 does not slip relative to m_1. (Answer in terms of ω).

From the FBD we have the following Newton Law equations, vertically,

$$\begin{aligned} \sum F_{2_y} &= 0 = F_{N_2} - m_2 g \quad \longrightarrow \quad F_{N_2} = m_2 g \\ \sum F_{1_y} &= 0 = F_{N_1} - m_1 g - F_{N_2} \quad \longrightarrow \quad F_{N_1} = (m_1 + m_2) g \end{aligned}$$

The x direction yields (with $x = A$),

$$\begin{aligned} m_1 a &= -kA + \mu m_2 g \\ m_2 a &= -\mu m_2 g \quad \longrightarrow \quad a = -\mu g \end{aligned}$$

Inserting this acceleration into the first expression we get,

$$\begin{aligned} -\mu m_1 g &= -kA + \mu m_2 g \\ A &= \frac{\mu g}{k}(m_1 + m_2) = \frac{\mu g}{\omega^2} \end{aligned}$$

Problem 6

Consider a forced harmonic oscillator consisting of a block of mass m resting on a frictionless surface attached to a spring with spring constant k (on the left). An outside entity supplies a periodic driving force to the block (this could be a simple motor with an arm attached to the block) of the form $F(t) = F_0 \cos \omega t$ (F_0 a positive constant). The equilibrium position of the spring is at $x = 0$ and the block moves along the x axis only.

a) Show (explicitly) how you obtain the following differential equation from your free body diagram,

$$\ddot{x} + \omega_0^2 x = \frac{F_0}{m} \cos \omega t,$$

where $\omega_0^2 \equiv \frac{k}{m}$ and $\ddot{x} \equiv \frac{d^2x}{dt^2}$.

From the free body diagram you should come up with Newton's second law for the x direction as,

$$\begin{aligned} ma = m\frac{d^2x}{dt^2} &= -kx + F_0 \cos \omega t \quad \text{or} \\ \frac{d^2x}{dt^2} + \frac{k}{m}x &= \frac{F_0}{m} \cos \omega t \\ \ddot{x} + \omega_0^2 x &= \frac{F_0}{m} \cos \omega t \end{aligned}$$

b) From this differential equation come up with a general solution, $x(t) =$

You should have determined that a cosine function would yield the right hand side, so try,

$$x(t) = A \cos \omega t$$

(any phase within the cosine would eventually have to agree with the driving term so ignore it).

Inserting this general form back into the differential equation, determine any unknown constants you may have introduced.

$$\begin{aligned} \frac{d^2}{dt^2} A \cos \omega t + \omega_0^2 A \cos \omega t &= \frac{F_0}{m} \cos \omega t \\ -\omega^2 A \cos \omega t + \omega_0^2 A \cos \omega t &= \frac{F_0}{m} \cos \omega t \\ (-\omega^2 + \omega_0^2) A \cos \omega t &= \frac{F_0}{m} \cos \omega t \quad \longrightarrow \quad A = \frac{F_0}{m(\omega_0^2 - \omega^2)}. \end{aligned}$$

c) When $\omega < \omega_0$ how are the driving force $F(t)$ and the position $x(t)$ related in phase? What about when $\omega > \omega_0$?

When $\omega < \omega_0$ both $x(t)$ and $F(t)$ are of the same sign, thus they're in phase. When $\omega > \omega_0$, $x(t)$ and $F(t)$ are of the different signs, thus they are π out of phase.

d) Given your solution to this driven harmonic oscillator, find the mechanical energy of the spring-block part of the system, $E = KE + PE_{elastic}$.

First find the velocity,

$$v(t) = \dot{x}(t) = -A\omega \sin \omega t$$

$$\begin{aligned} E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 &= \frac{1}{2}mA^2\omega^2 \sin^2 \omega t + \frac{1}{2}A^2 k \cos^2 \omega t \\ &= \frac{1}{2}\frac{F_0^2}{m^2(\omega_0^2 - \omega^2)^2}\left[m\omega^2 + (k - m\omega^2)\cos^2 \omega t\right] \\ &= \frac{1}{2}\frac{F_0^2}{m(\omega_0^2 - \omega^2)^2}[\omega^2 + (\omega_0^2 - \omega^2)\cos^2 \omega t] \end{aligned}$$

e) Discuss briefly what happens when the driving frequency matches the natural frequency of this system, that is when $\omega = \omega_0$. What happens to the energy found in the previous part?

When the driving frequency equals the natural frequency, resonance occurs. In this case we see that the amplitude and the energy become infinite. A more realistic analysis needs to include friction, which makes the divergence go away and yields a finite (but large) amplitude.

Optional further discussion

Notice also that when $\omega < \omega_0$ the second term is positive and the energy of oscillation is larger than when the driver is not there. When $\omega > \omega_0$ the energy is less and the driving force tends to dampen the oscillation. You can test this answer out yourself with a simple pendulum.

Take a ball of string unravel until it is about the height of your hand when standing. Tape the string to the ball so it does not unravel further, you now have a pendulum. Try the following:

Give the pendulum a small amplitude. Find something to measure the displacement of your hand as you hold the top of the string -this insures that you always swing your hand (the force driver) through the same distance. Start by moving your had slowly from side to side (this is $\omega < \omega_0$) so that the pendulum goes through more than one period before your goes from one side and back. Notice what happens to the amplitude of the pendulum. It should dampen or stay the same.

Now move your hand very quickly from side to side with the same initial amplitude, you should notice the motion damps out rather quickly. Try it with the pendulum having a large initial amplitude, you'll see the amplitude still decays.

Lastly, try to swing the pendulum with a frequency about equal to the natural frequency. You should find the amplitude get larger, this is resonance. It, obviously, does not become infinite since there is resistance in this system (not taken care of in the above). You can do this experiment with a mass on a spring if you have one, just jiggle the free end of the spring.

To include friction you add a force that is in the opposite direction to the velocity ($\dot{x}$) to the second law. You get then,

$$\ddot{x} = -\omega_0^2 x - \frac{\gamma}{m}\dot{x} + \frac{F_0}{m}\cos\omega t$$

Problem 7

Consider a projectile launched from the surface of the Earth (fired from a canon) of mass m. However some evil madman has turned on a forcefield that envelopes the Earth that has a potential energy function for the projectile of the form,

$$U_{evil} = \frac{am}{r^2}$$

This potential is in addition to the normal gravitational potential due to the Earth ($r = 0$ is the center of the Earth). Use the following values [$R_{Earth} = 6.37 \times 10^6 m$, $M_{Earth} = 5.98 \times 10^{24} kg$, and $a = 1.00 \times 10^{21} \frac{Jm^3}{kg}$]

a) Find the escape velocity of the projectile with this additional potential in place and compare it with the escape velocity without the the evil forcefield.

The escape velocity is defined as the velocity that *just* gets the projectile to infinity with no kinetic energy. Note that the energy at infinity is zero since both PEs vanish at infinity. Thus,

$$\begin{aligned}
E_i &= \frac{1}{2}mv^2 - \frac{GMm}{R_E} + \frac{am}{R_E^2} = E_f = 0 \\
v^2 &= \frac{2GM}{R_E} - \frac{2a}{R_E^2} \\
v_{escape} &= \sqrt{\frac{2GM}{R_E} - \frac{2a}{R_E^2}} = \sqrt{1.25 \times 10^8 - 4.93 \times 10^7}\frac{m}{s} = 2750\frac{m}{s}
\end{aligned}$$

With out the evil forcefield escape velocity is $11,200\frac{m}{s}$.

b) Find the distance r_{equil} where the net force on the projectile vanishes. Locate where this position is in relation to the Earth.

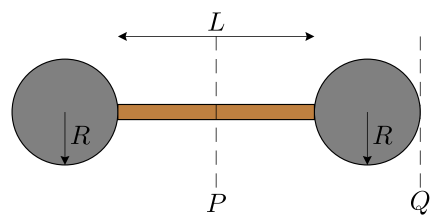
Just where the force vanishes.

$$\begin{aligned}
F = -\nabla U &= -\frac{d}{dr}[-\frac{GMm}{r} + \frac{am}{r^2}] = -\frac{GMm}{r^2} + \frac{2am}{r^3} \\
F = 0 &\longrightarrow -\frac{GMm}{r_{equil}^2} + \frac{2am}{r_{equil}^3} = 0 \\
r_{equil} &= \frac{2a}{GM} = 5.0 \times 10^6 \frac{m}{s}
\end{aligned}$$

Since this is within the Earth, the projectile does not encounter this point.

Problem 8

Consider the following object,



The spheres have mass M and radius R and the rod connecting them is of length L and has mass m. Find the moment of inertia about the two points indicated on the diagram.

a) Moment of inertia about the central axis P.

First note the Is for axes of rotation through their CMs,

Sphere $I = \frac{2}{5}MR^2$

Rod $I = \frac{1}{12}mL^2$

You will need to use the parallel axis theorem for these two cases. For this first one we add the CM I for the rod to the two spheres including the Mx^2 terms. Note that the distance from the axis to each sphere is $x = \frac{L}{2} + R$. Thus we have

$$\begin{aligned} I_P &= \frac{1}{12}mL^2 + 2\frac{2}{5}MR^2 + 2Mx^2 \\ &= \frac{1}{12}mL^2 + \frac{4}{5}MR^2 + 2M(\frac{L}{2} + R)^2 \\ &= \frac{1}{12}mL^2 + \frac{4}{5}MR^2 + \frac{1}{2}ML^2 + 2MLR + 2MR^2 \\ &= \frac{1}{12}(m + 6M)L^2 + \frac{14}{5}MR^2 + 2MLR \end{aligned}$$

b) Moment of inertia about the edge Q.

Now all three terms require the parallel axis theorem, and with different distances to each CM.

$$\begin{aligned} I_{Rsphere} &= \frac{2}{5}MR^2 + MR^2 \\ I_{rod} &= \frac{1}{12}mL^2 + m(2R + \frac{L}{2})^2 \\ I_{Lsphere} &= \frac{2}{5}MR^2 + M(3R + L)^2 \end{aligned}$$

Adding all this up you get,

$$\begin{aligned} I_Q &= \frac{9}{5}MR^2 + \frac{1}{12}mL^2 + m(2R + \frac{L}{2})^2 + M(3R + L)^2 \\ &= \frac{54}{5}MR^2 + 6MLR + ML^2 + \frac{1}{3}mL^2 + 2mRL + 4mR^2 \end{aligned}$$

Short Response Questions

Question 9

As the Earth orbits the Sun its axis of rotation (which currently points towards the north star *Polaris*) is tilted by 11.25^o from the normal to the plane of Earth's orbit. The Earth is not a perfect sphere but is an ellipsoid (oblate spheroid), it is bulged around the equator by a small amount.

Use these two facts to *qualitatively* describe why the axis of rotation of the Earth precesses (with a period of 25,800 years). Make a free body diagram of the Earth and the forces due to the Sun acting on it at different points. (The summer solstice is a nice point to analyze this from -that is when the north pole points toward the Sun and the south pole away from it).

Since the Earth is oblate and is at an angle, the upper equator (on the far side of the Sun) and the lower equator (on the near side) would exert a net torque on the spinning Earth. Thus, this torque makes the Earth precess.

Question 10

What would happen if the Sun instantaneously turned into a black hole. The Earth will not become a black hole but for sake of this problem let's say it can. A black hole is an object whose mass has been compressed down beyond the *Schwarzschild radius* defined by

$$R_S = \frac{2GM}{c^2}$$

For the Sun, this value is $R_S = 3000m$. That means ***if*** you could compress the Sun into a ball $3km$ in radius, it would form a black hole.

Describe any potential change in the orbits of the planets (say Mercury and Earth) if this were to happen. Where would a change be noticeable? (Neglect other effects like it would be dark, etc. Just concentrate on the orbits).

Basically nothing would change to Earth or Mercury's orbits if the Sun were turned into a black hole. Clearly if you were at a distance closer than the Sun's previous (normal) radius you would see deviations. The extreme effects of a black hole are not felt until you get to about 10 Schwarzschild radii or so. Thus, if the Sun were to become a black hole (magically), we would **not** get "sucked in".