18.650 Homework 1

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1   Random Variables

Exercise 1.1

Exercise 1.2

Exercise 1.3

Exercise 1.4

Let X have probability density function

$$f_X(x) = \begin{cases} 1/4 & 0 < x < 1 \\ 3/8 & 3 < x < 5 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the cumulative distribution function of X.

(b) Let $Y = \frac{1}{X}$. Find the probability density function $f_Y(y)$ for Y.

Solution:

(a) There are five cases we have to consider: $x \leq 0$, $0 < x < 1$, $1 \leq x \leq 3$, $3 < x < 5$, and $x \geq 5$. The first and last cases are straight-forward, when $x \leq 0$, $F_X(x) = 0$ and when $x \geq 5$, $F_X(x) = 1$. As for the case when $0 < x < 1$,

$$F_X(x) = \int_0^x f_X(x)\mathrm{d}x = \int_0^x \frac{1}{4}\mathrm{d}x = \frac{1}{4}x.$$

When $1 \leq x \leq 3$, the cdf is $\frac{1}{4}$, and when $3 < x < 5$, the CDF is

$$F_X(x) = \int_0^x f_X(x)\mathrm{d}x = \frac{1}{4} + \int_3^x f(x)\mathrm{d}x = \frac{1}{4} + \frac{3}{8} \cdot (x - 3) = \frac{3x - 7}{8}.$$

Putting this all together,

$$F_X(x) = \begin{cases} 0 & x \leq 0 \\ x/4 & 0 < x < 1 \\ 1/4 & 1 \leq x \leq 3 \\ (3x - 7)/8 & 3 < x \leq 5 \\ 1 & x > 5 \end{cases} .$$

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