

1 Part 1: probability review

Exercise 1.1

Let X be a random variable taking values between 0 and π with pdf given by $f(x) = c \sin(x)$, $x \in [0, \pi]$. What is the value of c ?

Solution: Since the integral of the pdf is always 1 (by definition),

$$1 = \int_0^\pi f(x) \, dx = \int_0^\pi c \sin x \, dx = -c \Big|_0^\pi \cos x = -c(\cos \pi - \cos 0) = -c(-2) = 2c.$$

And so, $2c = 1$ and $c = \frac{1}{2}$. \boxed{B} .

Exercise 1.2

What is $\mathbb{E}[X]$?

Solution: By the definition of expectation,

$$\mathbb{E}[X] = \int_0^\pi x f(x) \, dx = \int_0^\pi cx \sin x \, dx = c \Big|_0^\pi (\sin x - x \cos x) = c(-\pi \cos \pi + \sin \pi - 0 \cos 0 + \sin 0) = \pi c.$$

We know from the previous problem that $c = \frac{1}{2}$, so $\mathbb{E}[X] = \frac{\pi}{2}$. \boxed{A} .