# Simulacro 1er Parcial

No. Métodos 1
Date. 23/03/2024

*a)* Compruebe si ésta es una buena definición de producto interno.
*b)* A partir de esa definición de producto interno construya la definición de norma asociada. Esta definición de norma se conoce como norma de Frobenious.
*c)* A partir de la definición de norma de Frobenious, encuentra la expresión para la definición de distancia entre dos matrices $2 \times 2$
*d)* Considere las Matrices de Pauli

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \sigma_0 \equiv \mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

y compruebe si esas matrices son ortogonales bajo la definición de producto interno de Frobenious
*e)* Cuál es la distancia entre las Matrices de Pauli
*f)* Muestre que las matrices de Pauli $\{\sigma_0, \sigma_1, \sigma_2, \sigma_3\}$ forman una base para ese espacio vectorial.
*g)* Compruebe que esa base es ortogonal bajo la definición de producto interno $\langle a | b \rangle \rightleftharpoons \mathrm{Tr}(\mathbb{A}^\dagger \mathbb{B})$.
*h)* Explore si se pueden construir subespacios vectoriales de matrices reales e imaginarias puras.

**a.** Si $A, B, C \in M_{m\times n}(\mathbb{C})$ y $\alpha \in \mathbb{C}$

$\langle \alpha(A+B), C\rangle = tr(\alpha A^{*T} + \alpha B^{*T})(C)$
$= tr(\alpha(A^* + B^*)^T)(C) = \alpha\, tr(A^* + B^*)^T (C)$
$= \alpha\, tr(A^{*T}C) + \alpha\, tr(B^{*T}C)$
$= \alpha\langle A, C\rangle + \alpha\langle B, C\rangle$

Ahora si $\langle A, \alpha(B+C)\rangle = tr((A^*)^T(\alpha B + \alpha C))$
$= tr(\alpha(A^*)^T(B+C)) = \alpha\, tr(A^{*T}(B+C))$
$= \alpha\, tr(A^{*T}B) + \alpha\, tr(A^{*T}C)$
$= \alpha\langle A, B\rangle + \alpha\langle A, C\rangle$

$\overline{\langle B, A\rangle} = tr((B^{*T}A)^*) = tr(A^* B^T) = \langle A, B\rangle$
Por lo cual es una función hermítica.

**b.** $\langle A, A\rangle = tr(A^{*T}A) = \sum_{k=1}^{n} (A^{*T}A)_{kk}$

$= \sum_{k=1}^{n}\left(\sum_{j=1}^{m} (A^*)_{kj}(A_{jk})\right)$ ; $A_{jk} = A^{*k}_{\ j} A^{j}_{\ i}$

$\langle A, A\rangle = \sum_{k=1}^{n}\sum_{j=1}^{m} |A_{jk}|^2$

**C.** $A = \begin{bmatrix} 1+2i & 3+i \\ 5+6i & 7+8i \end{bmatrix}$ $B = \begin{bmatrix} 2-3i & -1+9i \\ 6-7i & 8-9i \end{bmatrix}$

$d(|A\rangle, |B\rangle) = \|A - B\|$ Norma de Frobenious

$A - B = \begin{bmatrix} -1+5i & -1+9i \\ -1+13i & -1+17i \end{bmatrix}$

$\|A-B\|^2 = |-1+5i|^2 + |-1+13i|^2 + |-1+17i|^2$
$\|A-B\|^2 = 26 + 82 + 170 + 290 = 568$
$\|A-B\| = \sqrt{568} = 23{,}85$

**d.** $\sigma_0\begin{pmatrix} 1 & 0 \\ 0 & 1\end{pmatrix}$ $\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0\end{pmatrix}$ $\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0\end{pmatrix}$ $\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1\end{pmatrix}$

$\langle\sigma_0|\sigma_1\rangle = Tr\left(\begin{pmatrix} 1 & 0 \\ 0 & 1\end{pmatrix}^*\begin{pmatrix} 0 & 1 \\ 1 & 0\end{pmatrix}\right) = Tr\begin{pmatrix} 0 & 1 \\ 1 & 0\end{pmatrix} = 0$

$\langle\sigma_0|\sigma_2\rangle = Tr\left(\begin{pmatrix} 1 & 0 \\ 0 & 1\end{pmatrix}^*\begin{pmatrix} 0 & -i \\ i & 0\end{pmatrix}\right) = Tr\begin{pmatrix} 0 & -i \\ i & 0\end{pmatrix} = 0$

$\langle\sigma_0|\sigma_3\rangle = Tr\left(\begin{pmatrix} 1 & 0 \\ 0 & 1\end{pmatrix}^*\begin{pmatrix} 1 & 0 \\ 0 & -1\end{pmatrix}\right) = Tr\begin{pmatrix} 1 & 0 \\ 0 & -1\end{pmatrix} = 0$

$\langle\sigma_1|\sigma_2\rangle = Tr\left(\begin{pmatrix} 0 & 1 \\ 1 & 0\end{pmatrix}^*\begin{pmatrix} 0 & -i \\ i & 0\end{pmatrix}\right) = Tr\begin{pmatrix} i & 0 \\ 0 & -i\end{pmatrix} = 0$

$\langle\sigma_1|\sigma_3\rangle = Tr\left(\begin{pmatrix} 0 & 1 \\ 1 & 0\end{pmatrix}^*\begin{pmatrix} 1 & 0 \\ 0 & -1\end{pmatrix}\right) = Tr\begin{pmatrix} 0 & -1 \\ 1 & 0\end{pmatrix} = 0$

$\langle\sigma_2|\sigma_3\rangle = Tr\left(\begin{pmatrix} i & 0 \\ 0 & -i\end{pmatrix}^*\begin{pmatrix} 1 & 0 \\ 0 & -1\end{pmatrix}\right) = Tr\begin{pmatrix} 0 & -i \\ -i & 0\end{pmatrix} = 0$

Por lo cual son ortogonales bajo la definición de producto interno de Frobenious

**e.** $d(|\sigma_0\rangle, |\sigma_1\rangle)^2 = |1-0|^2 + |0-1|^2 + |0-1|^2$
$+ |1-0|^2 = 4$
$d(|\sigma_0\rangle, |\sigma_1\rangle) = \sqrt{4} = 2$

$d(|\sigma_0\rangle, |\sigma_2\rangle)^2 = |1-0|^2 + |i|^2 + |-i|^2 + |1|^2$
$d(|\sigma_0\rangle, |\sigma_2\rangle)^2 = 4$
$d(|\sigma_0\rangle, |\sigma_2\rangle) = \sqrt{4} = 2$

$d(|\sigma_0\rangle, |\sigma_3\rangle)^2 = |1-(-1)|^2 = \sqrt{4} = 2$

$d(|\sigma_1\rangle, |\sigma_2\rangle)^2 = |1+i|^2 + |1-i|^2 = \sqrt{4} = 2$

$d(|\sigma_1\rangle, |\sigma_3\rangle)^2 = |-1|^2 + |1|^2 + |1|^2 + |-1|^2 = \sqrt{4} = 2$

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$$d(|\sigma_2\rangle, |\sigma_3\rangle)^2 = |-1|^2 + |-i|^2 + |i|^2 + |1|^2 = \sqrt{4} = 2$$

La distancia entre las matrices de Pauli es 2.

**f.** Para ser una base deben ser linealmente independientes

$$\alpha_1\sigma_0 + \alpha_2\sigma_1 + \alpha_3\sigma_2 + \alpha_4\sigma_3 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} \alpha_1 & 0 \\ 0 & \alpha_1 \end{pmatrix} + \begin{pmatrix} 0 & \alpha_2 \\ \alpha_2 & 0 \end{pmatrix} + \begin{pmatrix} 0 & -\alpha_3 i \\ \alpha_3 i & 0 \end{pmatrix} + \begin{pmatrix} \alpha_4 & 0 \\ 0 & -\alpha_4 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} \alpha_1 + \alpha_4 & \alpha_2 - \alpha_3 i \\ \alpha_2 + \alpha_3 i & \alpha_1 - \alpha_4 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

* $\alpha_1 - \alpha_4 = 0$, $\alpha_1 = \alpha_4 = 0$
* $\alpha_2 - \alpha_3 i = 0$, $\alpha_2 = \alpha_3 i = 0$

La única solución es que $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 0$

Ahora demostramos que cualquier matriz del espacio vectorial de matrices complejas 2x2 hermíticas puede escribirse como una combinación lineal de las matrices de Pauli.

Si $A \in M_{2x2}(\mathbb{C})$ hermítica y $\{\sigma_0, \sigma_1, \sigma_2, \sigma_3\}$ son una base del E.V $M_{2x2}(\mathbb{C})$, entonces

$$A = \begin{pmatrix} z_1 & z_2 \\ z_3 & z_4 \end{pmatrix} = z_1\sigma_0 + z_2\sigma_1 + z_3\sigma_2 + z_4\sigma_3$$

$$= z_1\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + z_2\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + z_3\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} + z_4\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} z_1 & 0 \\ 0 & z_1 \end{pmatrix} + \begin{pmatrix} 0 & z_2 \\ z_2 & 0 \end{pmatrix} + \begin{pmatrix} 0 & -z_3 i \\ z_3 i & 0 \end{pmatrix} + \begin{pmatrix} z_4 & 0 \\ 0 & -z_4 \end{pmatrix}$$

$$= \begin{pmatrix} z_1 + z_4 & z_2 - z_3 i \\ z_2 + z_3 i & z_1 - z_4 \end{pmatrix}$$

$z_1$ y $z_4$ son reales, $z_3$ y $z_2$ son complejos y $z_2^* = z_3$

**g.** $\langle\sigma_0|\sigma_1\rangle = Tr(\sigma_0 \cdot \sigma_1) = Tr\left(\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\right) = Tr\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = 0$

por lo cual $\sigma_0$ y $\sigma_1$ son ortogonales.

$\langle\sigma_0|\sigma_2\rangle = Tr(\sigma_0 \cdot \sigma_2) = Tr\left(\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\right) = Tr\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = 0$

Por lo cual $\sigma_0$ y $\sigma_2$ son ortogonales

$\langle\sigma_0|\sigma_3\rangle = Tr(\sigma_0 \cdot \sigma_3) = Tr\left(\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\right) = Tr\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = 1 + (-1) = 0$

Por lo cual $\sigma_0$ y $\sigma_3$ son ortogonales

$\langle\sigma_1|\sigma_2\rangle = Tr(\sigma_1 \cdot \sigma_2) = Tr\left(\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\right) = Tr\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = 0$

Por lo cual $\sigma_1$ y $\sigma_2$ son ortogonales

$\langle\sigma_1|\sigma_3\rangle = Tr(\sigma_1 \cdot \sigma_3) = Tr\left(\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\right) = Tr\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = 0$

Por lo cual $\sigma_1$ y $\sigma_3$ son ortogonales

$\langle\sigma_2|\sigma_3\rangle = Tr(\sigma_2 \cdot \sigma_3) = Tr\left(\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\right) = Tr\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = 0$

Por lo cual $\sigma_2$ y $\sigma_3$ son ortogonales

**h.** Si se pueden construir estos subespacios pues

* El conjunto generado por la combinación lineal de $\sigma_0, \sigma_1, \sigma_3$ con coeficientes reales dará matrices netamente reales
* El conjunto generado por la combinación de $\sigma_0$ y $\sigma_2$ con coeficientes imaginarios dará matrices imaginarias

a) Considere el polinomio $p^{\mathcal{P}}(x) = x^2+x+3$ y expréselo en término de la base de polinomios de Legendre $\{|e_i^{\mathcal{P}}\rangle\} \leftrightarrow \{|P_i(x)\rangle\}$ (2ptos)

b) Seleccione ahora dos polinomios $p^{\mathcal{P}}(x) = x^2+x+3$ y $p^{\mathcal{G}}(y) = y+1$. Construya el tensor, $p^{\mathcal{P}\otimes\mathcal{G}}(x,y) = p^{\mathcal{P}}(x) \otimes p^{\mathcal{G}}(y)$, mediante el producto exterior de esos polinomios. (2ptos)

c) Elija las bases de monomios $\{1, x, x^2\}$ y $\{1, y, y^2\}$ e identifique las componentes $c^{ij}$ del tensor $p^{\mathcal{P}\otimes\mathcal{G}}(x,y)$ al expandir ese tensor respecto a estas bases en el espacio tensorial $\mathcal{T}_2(xy) = \mathcal{P}_2(x) \otimes \mathcal{G}_2(y)$. (2ptos)

d) Ahora suponga las bases de polinomios de Legendre, $\{|e_i^{\mathcal{P}}\rangle\} \leftrightarrow \{|P_i(x)\rangle\}$ y $\left\{\left|e_j^{\mathcal{G}}\right\rangle\right\} \leftrightarrow \{|P_j(y)\rangle\}$, para $\mathcal{P}_2(x)$ y $\mathcal{G}_2(y)$. Calcule las componentes $\tilde{c}^{ij}$ del tensor $p^{\mathcal{P}\otimes\mathcal{G}}(x,y)$ respecto a estas bases en el espacio tensorial $\mathcal{T}_2(xy) = \mathcal{P}_2(x) \otimes \mathcal{G}_2(y)$. (4ptos)

**a.** $p^P(x) = x^2 + x + 3 = a^1|e_1^P\rangle + a^2|e_2^P\rangle + a^3|e_3^P\rangle$

$$|p^P\rangle = b^1|P_1\rangle + b^2|P_2\rangle + b^3|P_3\rangle + \dots$$

$$\langle P_1|p^P\rangle = (a^1\langle P_1|e_1^P\rangle + a^2\langle P_1|e_2^P\rangle + a^3\langle P_1|e_3^P\rangle) = b^1\langle P_1|P_1\rangle$$

* $b^1 = \dfrac{a^1\langle P_1|e_1^P\rangle + a^2\langle P_1|e_2^P\rangle + a^3\langle P_1|e_3^P\rangle}{\langle P_1|P_1\rangle}$
* $b^2 = \dfrac{a^1\langle P_2|e_1^P\rangle + a^2\langle P_2|e_2^P\rangle + a^3\langle P_2|e_3^P\rangle}{\langle P_2|P_2\rangle}$
* $b^3 = \dfrac{a^1\langle P_3|e_1^P\rangle + a^2\langle P_3|e_2^P\rangle + a^3\langle P_3|e_3^P\rangle}{\langle P_3|P_3\rangle}$

$\vdots$

* $b^k = \dfrac{a^i\langle P_k|e_i^P\rangle}{\langle P_k|P_k\rangle}$

**b.** $p^G(y) = y + 1$

$$p^{P\otimes G}(x,y) = p^P(x) \otimes p^G(y)$$

$$p^P(x) = a^i|e_i^P\rangle, \quad p^G(y) = g^i|e_i^G\rangle$$

$$p^{P\otimes G}(x,y) = a^i|e_i^P\rangle \otimes g^i|e_i^G\rangle = a^i g^i|e_i^P, e_i^G\rangle = c^{ij}|e_i^P, e_i^G\rangle$$

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**c.** Teniendo en cuenta la base de los monomios respectivos de $M_i^\rho$ $N_i^6$, entonces

$$P^{\rho\otimes 6}(x,y) = c^{ij}\,|e_i^\rho, e_j^6\rangle = d^{ij}\,|M_i^\rho, M_i^6\rangle$$

después lo proyectamos y nos queda

$$c^{ij} = d^{ij}\,\frac{\langle e_i^\rho, e_i^6 \,|\, M_i, M_j\rangle}{\langle e_i^\rho, e_i^6 \,|\, e_c^\rho, e_j^6\rangle}$$

**d.** Tomando los polinomios de legendre

$$P^{\rho\otimes 6}(x,y) = c^{ij}\,|e_i^\rho, e_j^6\rangle = \tilde{c}^{ij}\,|P_K^\rho, P_M^6\rangle$$

y proyectando queda

$$\tilde{c}^{ij} = c^{ij}\,\frac{\langle P_K, P_M \,|\, e_i^\rho, e_j^6\rangle}{\langle P_K^\rho, P_M^6 \,|\, P_K^\rho\, P_M^6\rangle}$$

*a*) A partir de las condiciones de ortogonalidad para la tétrada $\left\{\tilde{\mathbf{v}}, \tilde{\mathbf{k}}, \tilde{\mathbf{l}}, \tilde{\mathbf{s}}\right\}$ en coordenadas esféricas, $(t, r, \theta, \phi) \equiv (\tilde{x}^0, \tilde{x}^1, \tilde{x}^2, \tilde{x}^3)$ encontrar sus componentes contravariantes (2ptos).

*b*) Suponga las siguientes componentes cartesiana para un cuadrivector $a^\alpha = (5, 3, 2, 1)$ y encuentre las componentes $(\tilde{a}^0, \tilde{a}^1, \tilde{a}^2, \tilde{a}^3)$, en coordenadas esféricas. (2ptos)

*c*) Compruebe que, en coordenadas cartesianas, se cumplen las siguientes proyecciones

$$F_{\mu\alpha}v^\mu v^\alpha = F_{\mu\alpha}k^\mu k^\alpha = F_{\mu\alpha}l^\mu l^\alpha = F_{\mu\alpha}s^\mu s^\alpha = 0;$$

$$F_{\mu\alpha}v^\mu k^\alpha = E^x; \quad F_{\mu\alpha}v^\mu l^\alpha = E^y; \quad F_{\mu\alpha}v^\mu k^\alpha = E^z\,.$$

Además complete las proyecciones faltantes. (2ptos)

*d*) Encuentre la expresión del tensor **mixto** de Maxwell, $\tilde{F}^\mu_\alpha$, en coordenadas esféricas (4ptos).

*e*) Compruebe que, en coordenadas esféricas, se cumplen proyecciones del $\tilde{F}^{\mu\alpha}$ equivalentes a las cartesianas, pero en este caso con las componentes contravariantes. (4ptos)

**a.** Considerando una base de vectores en coordenadas esféricas $\{\langle e^t|, \langle e^r|, \langle e^\theta|, \langle e^\phi|\}$ construimos una tetrada $\{\tilde v = \tilde v_\alpha \langle \tilde e^\alpha|,\ \tilde k = \tilde k_\alpha \langle \tilde e^\alpha|,\ \tilde l = \tilde l_\alpha \langle \tilde e^\alpha|,\ \tilde s = \tilde s_\alpha \langle \tilde e^\alpha|\}$ con componentes:

$\tilde v_\alpha = (-1,0,0,0)$ $\tilde k_\alpha = (0,1,0,0)$ $\tilde l_\alpha = (0,0,r,0)$ $\tilde s_\alpha = (0,0,0,r\,\mathrm{sen}\,\theta)$

Dado que cumplen las condiciones de ortogonalidad

$$-\tilde v_\alpha v^\alpha = \tilde k_\alpha \tilde k^\alpha = \tilde l_\alpha \tilde l^\alpha = \tilde s_\alpha \tilde s^\alpha = 1$$

$$\tilde v_\alpha \tilde k^\alpha = \tilde v_\alpha \tilde l^\alpha = \tilde v_\alpha \tilde s^\alpha = \tilde k_\alpha \tilde l^\alpha = \tilde k_\alpha \tilde s^\alpha = \tilde l_\alpha \tilde s^\alpha = 0$$

$$* \ -\tilde v_\alpha \tilde v^\alpha = 1 \to -(-1,0,0,0)\begin{pmatrix}v^0\\ v^1\\ v^2\\ v^3\end{pmatrix} = 1 \to 1v^0 + \cancel{0v^1} + \cancel{0v^2} + \cancel{0v^3} = 1$$

$1v^0 = 1$

$v^0 = 1$

$$\tilde k_\alpha \tilde v^\alpha = (0,1,0,0)\begin{pmatrix}1\\ v^1\\ v^2\\ v^3\end{pmatrix} = \cancel{0\cdot 1} + 1v^1 + \cancel{0v^2} + \cancel{0v^3} = 0$$

$1v^1 = 0$

$v^1 = 0$

$$\tilde l_\alpha \tilde v^\alpha = (0,0,r,0)\begin{pmatrix}1\\ 0\\ v^2\\ v^3\end{pmatrix} = \cancel{0\cdot 1} + \cancel{0\cdot 0} + rv^2 + \cancel{0v^3} = 0$$

$rv^2 = 0$

$v^2 = 0$

$$\tilde s_\alpha \tilde v^\alpha = (0,0,0,r\,\mathrm{sen}\,\theta)\begin{pmatrix}1\\ 0\\ 0\\ v^3\end{pmatrix} = \cancel{0\cdot 1} + \cancel{0\cdot 0} + \cancel{0\cdot 0} + r\,\mathrm{sen}\,\theta\, v^3 = 0$$

$r\,\mathrm{sen}\,\theta\, v^3 = 0$

$v^3 = 0$

entonces $\tilde v^\alpha = \begin{pmatrix}1\\ 0\\ 0\\ 0\end{pmatrix}$

$$* \ \tilde k_\alpha \tilde k^\alpha = (0,1,0,0)\begin{pmatrix}k^0\\ k^1\\ k^2\\ k^3\end{pmatrix} = \cancel{0k^0} + 1k^1 + \cancel{0k^2} + \cancel{0k^3} = 1$$

$1k^1 = 1$

$k^1 = 1$

$$\tilde v_\alpha \tilde k^\alpha = (-1,0,0,0)\begin{pmatrix}k^0\\ 1\\ k^2\\ k^3\end{pmatrix} = -1k^0 + \cancel{0\cdot 1} + \cancel{0k^2} + \cancel{0k^3} = 0$$

$-1k^0 = 0$

$k^0 = 0$

$$\tilde l_\alpha \tilde k^\alpha = (0,0,r,0)\begin{pmatrix}0\\ 1\\ k^2\\ k^3\end{pmatrix} = \cancel{0\cdot 0} + \cancel{0\cdot 1} + rk^2 + \cancel{0k^3} = 0$$

$rk^2 = 0$

$k^2 = 0$

$$\tilde s_\alpha \tilde k^\alpha = (0,0,0,r\,\mathrm{sen}\,\theta)\begin{pmatrix}0\\ 1\\ 0\\ k^3\end{pmatrix} = \cancel{0\cdot 0} + \cancel{0\cdot 1} + \cancel{0\cdot 0} + r\,\mathrm{sen}\,\theta\, k^3 = 0$$

$r\,\mathrm{sen}\,\theta\, k^3 = 0$

$k^3 = 0$

entonces $\tilde k^\alpha = \begin{pmatrix}0\\ 1\\ 0\\ 0\end{pmatrix}$

$$* \ \tilde l_\alpha \tilde l^\alpha = (0,0,r,0)\begin{pmatrix}l^0\\ l^1\\ l^2\\ l^3\end{pmatrix} = \cancel{0l^0} + \cancel{0l^1} + rl^2 + \cancel{0l^3} = 1$$

$rl^2 = 1$

$l^2 = 1/r$

$$\tilde v_\alpha \tilde l^\alpha = (-1,0,0,0)\begin{pmatrix}l^0\\ l^1\\ 1/r\\ l^3\end{pmatrix} = -1l^0 + \cancel{0l^1} + \cancel{0(1/r)} + \cancel{0l^3} = 0$$

$-1l^0 = 0$

$l^0 = 0$

$$\tilde k_\alpha \tilde l^\alpha = (0,1,0,0)\begin{pmatrix}0\\ l^1\\ 1/r\\ l^3\end{pmatrix} = \cancel{0\cdot 0} + 1l^1 + \cancel{0(1/r)} + \cancel{0l^3} = 0$$

$1l^1 = 0$

$l^1 = 0$

$$\tilde s_\alpha \tilde l^\alpha = (0,0,0,r\,\mathrm{sen}\,\theta)\begin{pmatrix}0\\ 0\\ 1/r\\ l^3\end{pmatrix} = \cancel{0\cdot 0} + \cancel{0\cdot 0} + \cancel{0(1/r)} + r\,\mathrm{sen}\,\theta\, l^3 = 0$$

$r\,\mathrm{sen}\,\theta\, l^3 = 0$

$l^3 = 0$

entonces $\tilde l^\alpha = \begin{pmatrix}0\\ 0\\ 1/r\\ 0\end{pmatrix}$

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* $\tilde{S}_\alpha \tilde{S}^\alpha = (0,0,0,r\,\mathrm{sen}\,\theta)\begin{pmatrix}S^0\\S^1\\S^2\\S^3\end{pmatrix} = 0S^0+0S^1+0S^2+r\,\mathrm{sen}\,\theta\, S^3 = 1$

$r\,\mathrm{sen}\,\theta\, S^3 = 1$

$S^3 = 1/r\,\mathrm{sen}\,\theta$

$\tilde{V}_\alpha \tilde{S}^\alpha = (-1,0,0,0)\begin{pmatrix}S^0\\S^1\\S^2\\1/r\,\mathrm{sen}\,\theta\end{pmatrix} = -1S^0+\cancel{0S^1}+\cancel{0S^2}+\cancel{0(1/r\,\mathrm{sen}\,\theta)} = 0$

$-1S^0 = 0$

$S^0 = 0$

$\tilde{K}_\alpha \tilde{S}^\alpha = (0,1,0,0)\begin{pmatrix}0\\S^1\\S^2\\1/r\,\mathrm{sen}\,\theta\end{pmatrix} = \cancel{0\,0}+1S^1+\cancel{0S^2}+\cancel{0(1/r\,\mathrm{sen}\,\theta)} = 0$

$1S^1 = 0$

$S^1 = 0$

$\tilde{L}_\alpha \tilde{S}^\alpha = (0,0,r,0)\begin{pmatrix}0\\0\\S^2\\1/r\,\mathrm{sen}\,\theta\end{pmatrix} = \cancel{0\,0}+\cancel{0\,0}+rS^2+\cancel{0(1/r\,\mathrm{sen}\,\theta)} = 0$

$rS^2 = 0$

$S^2 = 0$

entonces $\tilde{S}^\alpha = \begin{pmatrix}0\\0\\0\\1/r\,\mathrm{sen}\,\theta\end{pmatrix}$

b. $a^\alpha = a_v V^\alpha + a_k K^\alpha + a_L L^\alpha + a_S S^\alpha = \tilde{a}_v \tilde{V}^\alpha + \tilde{a}_k \tilde{K}^\alpha + \tilde{a}_L \tilde{L}^\alpha + \tilde{a}_S \tilde{S}^\alpha$

$a^\alpha = (5,3,2,1)^t$ en coordenadas cartesianas es

$a^\alpha = 5\begin{pmatrix}1\\0\\0\\0\end{pmatrix}+3\begin{pmatrix}0\\1\\0\\0\end{pmatrix}+2\begin{pmatrix}0\\0\\1\\0\end{pmatrix}+1\begin{pmatrix}0\\0\\0\\1\end{pmatrix}$; en coordenadas esféricas $a^\alpha = \tilde{a}_v\begin{pmatrix}1\\0\\0\\0\end{pmatrix}+\tilde{a}_k\begin{pmatrix}0\\1\\0\\0\end{pmatrix}+\tilde{a}_L\begin{pmatrix}0\\0\\1/r\\0\end{pmatrix}+\tilde{a}_S\begin{pmatrix}0\\0\\0\\1/r\,\mathrm{sen}\,\theta\end{pmatrix}$

$\begin{pmatrix}5\\3\\2\\1\end{pmatrix} = \tilde{a}_v\begin{pmatrix}1\\0\\0\\0\end{pmatrix}+\tilde{a}_k\begin{pmatrix}0\\1\\0\\0\end{pmatrix}+\tilde{a}_L\begin{pmatrix}0\\0\\1/r\\0\end{pmatrix}+\tilde{a}_S\begin{pmatrix}0\\0\\0\\1/r\,\mathrm{sen}\,\theta\end{pmatrix}$

$\begin{pmatrix}5\\3\\2\\1\end{pmatrix} = \begin{pmatrix}\tilde{a}_v\\\tilde{a}_k\\\tilde{a}_L/r\\\tilde{a}_S/r\,\mathrm{sen}\,\theta\end{pmatrix} \Rightarrow \tilde{a}_v = 5,\ \tilde{a}_k = 3,\ \dfrac{\tilde{a}_L}{r} = 2,\ \dfrac{\tilde{a}_S}{r\,\mathrm{sen}\,\theta} = 1$

$\tilde{a}_L = 2r$

$\tilde{a}_S = r\,\mathrm{sen}\,\theta$

$\begin{pmatrix}5\\3\\2\\1\end{pmatrix} = 5\begin{pmatrix}1\\0\\0\\0\end{pmatrix}+3\begin{pmatrix}0\\1\\0\\0\end{pmatrix}+2r\begin{pmatrix}0\\0\\1/r\\0\end{pmatrix}+r\,\mathrm{sen}\,\theta\begin{pmatrix}0\\0\\0\\1/r\,\mathrm{sen}\,\theta\end{pmatrix}$

c. $V^\alpha = \begin{pmatrix}1\\0\\0\\0\end{pmatrix}$ $K^\alpha = \begin{pmatrix}0\\1\\0\\0\end{pmatrix}$ $L^\alpha = \begin{pmatrix}0\\0\\1\\0\end{pmatrix}$ $S^\alpha = \begin{pmatrix}0\\0\\0\\1\end{pmatrix}$

$F_{\nu\alpha} = \begin{pmatrix}0 & E^x & E^y & E^z\\ -E^x & 0 & -B^z & B^y\\ -E^y & B^z & 0 & -B^x\\ -E^z & B^y & B^x & 0\end{pmatrix} = \begin{pmatrix}F_{00} & F_{01} & F_{02} & F_{03}\\ F_{10} & F_{11} & F_{12} & F_{13}\\ F_{20} & F_{21} & F_{22} & F_{23}\\ F_{30} & F_{31} & F_{32} & F_{33}\end{pmatrix}$

Dado a que $V^\alpha = (V^0,V^1,V^2,V^3) = (1,0,0,0)$

$V^1 = V^2 = V^3 = 0$

$F_{\nu\alpha}V^\nu V^\alpha = F_{00}V^0V^0 = F_{00}(1)(1) = 0(1)(1) = 0$

Dado a que $K^\alpha = (K^0,K^1,K^2,K^3) = (0,1,0,0)$

$K^0 = K^2 = K^3 = 0$

$F_{\nu\alpha}K^\nu K^\alpha = F_{11}K^1K^1 = F_{11}(1)(1) = 0(1)(1) = 0$

Dado a que $L^\alpha = (L^0,L^1,L^2,L^3) = (0,0,1,0)$

$L^0 = L^1 = L^3 = 0$

$F_{\nu\alpha}L^\nu L^\alpha = F_{22}L^2L^2 = F_{22}(1)(1) = 0(1)(1) = 0$

Dado a que $S^\alpha = (S^0,S^1,S^2,S^3) = (0,0,0,1)$

$F_{\nu\alpha}S^\nu S^\alpha = F_{33}S^3S^3 = F_{33}(1)(1) = 0(1)(1) = 0$

Entonces $F_{\nu\alpha}V^\nu V^\alpha = F_{\nu\alpha}K^\nu K^\alpha = F_{\nu\alpha}L^\nu L^\alpha = F_{\nu\alpha}S^\nu S^\alpha = 0$

Ahora para V

$F_{\nu\alpha}V^\alpha K^\alpha = F_{01}V^0K^1 = E^x(1)(1) = E^x$

$F_{\nu\alpha}V^\alpha L^\alpha = F_{02}V^0L^2 = E^y(1)(1) = E^y$

$F_{\nu\alpha}V^\alpha S^\alpha = F_{03}V^0S^3 = E^z(1)(1) = E^z$

Para K

$F_{\nu\alpha}K^\nu V^\alpha = F_{10}K^1V^0 = -E^x(1)(1) = -E^x$

$F_{\nu\alpha}K^\nu L^\alpha = F_{12}K^1L^2 = -B^z(1)(1) = -B^z$

$F_{\nu\alpha}K^\nu S^\alpha = F_{13}K^1S^3 = B^y(1)(1) = B^y$

Para L

$F_{\nu\alpha}L^\nu V^\alpha = F_{20}L^2V^0 = -E^y(1)(1) = -E^y$

$F_{\nu\alpha}L^\nu K^\alpha = F_{21}L^2K^1 = B^z(1)(1) = B^z$

$F_{\nu\alpha}L^\nu S^\alpha = F_{23}L^2S^3 = -B^x(1)(1) = -B^x$

Para S

$F_{\nu\alpha}S^\nu V^\alpha = F_{30}S^3V^0 = -E^z(1)(1) = -E^z$

$F_{\nu\alpha}S^\nu K^\alpha = F_{31}S^3K^1 = B^y(1)(1) = B^y$

$F_{\nu\alpha}S^\nu L^\alpha = F_{32}S^3L^2 = B^x(1)(1) = B^x$

No. Métodos 1

Date. 23/03/2024

# Simulacro 1er Parcial

**d.** El tensor mixto $\tilde{F}^{N}_{\alpha}$ es covariante en $\alpha$ y contravariante en $N$

$$F_{N\alpha} = \begin{pmatrix} 0 & E^x & E^y & E^z \\ -E^x & 0 & -B^z & B^y \\ -E^y & B^z & 0 & -B^x \\ -E^z & B^y & B^x & 0 \end{pmatrix}$$

El tensor de Maxwell en su forma covariante en coordenadas esféricas

$$\tilde{F}_{N\alpha} = \begin{pmatrix} 0 & -Er & -E\theta & -E\phi \\ Er & 0 & -B\phi & B\theta \\ E\theta & B\phi & 0 & -Br \\ E\phi & -B\theta & Br & 0 \end{pmatrix}$$ donde $E = (Er, E\theta, E\phi)$, $B = (Br, B\theta, B\phi)$

Son los campos eléctricos y magnéticos respectivamente, medidos en coordenadas esféricas para un observador O

Para encontrar $\tilde{F}^{N}_{\alpha}$ debemos operar $\tilde{F}_{N\alpha}\, \tilde{n}^{N\alpha}$

Sabemos que $\tilde{n}_{N\alpha} = -\tilde{V}_N \tilde{V}_\alpha + \tilde{K}_N \tilde{K}_\alpha + \tilde{L}_N \tilde{L}_\alpha = \tilde{S}_N \tilde{S}_\alpha$

donde

$\tilde{V}_N = (-1,0,0,0)$ $\tilde{K}_N = (0,1,0,0)$ $\tilde{L}_N = (0,0,r,0)$ $\tilde{S}_N = (0,0,0,r\,\text{sen}\theta)$

$\tilde{n}_{N\alpha} = -(-1,0,0,0)(-1,0,0,0) + (0,1,0,0)(0,1,0,0) + (0,0,r,0)(0,0,r,0)$
$+ (0,0,0,r\,\text{sen}\theta)(0,0,0,r\,\text{sen}\theta)$

$$\tilde{n}_{N\alpha} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & r^2\text{sen}^2\theta \end{pmatrix}$$

$$\tilde{n}_{N\alpha} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2\text{sen}^2\theta \end{pmatrix}$$

además $\tilde{n}_{N\alpha}\, \tilde{n}^{N\alpha} = \delta^{N}_{\alpha} \rightarrow \tilde{n}^{N\alpha} = (\tilde{n}_{N\alpha})^{-1} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2\text{sen}^2\theta \end{pmatrix}^{-1}$

$$\tilde{n}^{N\alpha} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/r^2 & 0 \\ 0 & 0 & 0 & 1/r^2\text{sen}^2\theta \end{pmatrix}$$ Ahora multiplicamos con $\tilde{F}_{N\alpha}$

$$\tilde{F}_{N\alpha} \cdot \tilde{n}^{\alpha\beta} = \begin{pmatrix} 0 & -Er & -E\theta & -E\phi \\ Er & 0 & -B\phi & B\theta \\ E\theta & B\phi & 0 & -Br \\ E\phi & -B\theta & Br & 0 \end{pmatrix} \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/r^2 & 0 \\ 0 & 0 & 0 & 1/r^2\text{sen}^2\theta \end{pmatrix}$$

$$\tilde{F}_{N\alpha} \cdot \tilde{n}^{\alpha\beta} = \begin{pmatrix} 0 & -Er & -E\theta/r^2 & -E\phi/r^2\text{sen}^2\theta \\ -Er & 0 & -B\phi/r^2 & B\theta/r^2\text{sen}^2\theta \\ -E\theta & B\phi & 0 & -Br/r^2\text{sen}^2\theta \\ -E\phi & -B\theta & Br/r^2 & 0 \end{pmatrix} = \tilde{F}^{\beta}_{N}$$

**e.** Para encontrar $\tilde{F}^{N\alpha}$, partimos de $\tilde{F}^{\beta}_{N}$

$$\tilde{F}^{\beta}_{\mu} \cdot \tilde{n}^{\mu\alpha} = \begin{pmatrix} 0 & -Er & -E\theta/r^2 & -E\phi/r^2\text{sen}^2\theta \\ -Er & 0 & -B\phi/r^2 & B\theta/r^2\text{sen}^2\theta \\ -E\theta & B\phi & 0 & -Br/r^2\text{sen}^2\theta \\ -E\phi & -B\theta & Br/r^2 & 0 \end{pmatrix} \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/r^2 & 0 \\ 0 & 0 & 0 & 1/r^2\text{sen}^2\theta \end{pmatrix}$$

$$\tilde{F}^{\beta}_{\mu} \cdot \tilde{n}^{\mu\alpha} = \begin{pmatrix} 0 & -Er & -E\theta/r^4 & -E\phi/r^4\text{sen}^4\theta \\ Er & 0 & -B\phi/r^4 & B\theta/r^4\text{sen}^4\theta \\ E\theta & B\phi & 0 & -Br/r^4\text{sen}^4\theta \\ E\phi & -B\phi & Br/r^4 & 0 \end{pmatrix} = \tilde{F}^{B\alpha} = \tilde{F}^{N\alpha}$$

$$= \begin{pmatrix} F^{00} & F^{01} & F^{02} & F^{03} \\ F^{10} & F^{11} & F^{12} & F^{13} \\ F^{20} & F^{21} & F^{22} & F^{23} \\ F^{30} & F^{31} & F^{32} & F^{33} \end{pmatrix}$$

$V^\alpha = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$ $K^\alpha = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$ $L^\alpha = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$ $S^\alpha = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$

$\tilde{F}^{N\alpha} V^N V^\alpha = F^{00} V^0 V^0 = F_{00}(1)(1) = 0(1)(1) = 0$

Dado a que $K^\alpha = (K^0, K^1, K^2, K^3) = (0,1,0,0)$
$K^0 = K^2 = K^3 = 0$

$\tilde{F}^{N\alpha} K^N K^\alpha = F^{11} K^1 K^1 = F_{11}(1)(1) = 0(1)(1) = 0$

Dado a que $L^\alpha = (L^0, L^1, L^2, L^3) = (0,0,1,0)$
$L^0 = L^1 = L^3 = 0$

$\tilde{F}^{N\alpha} L^N L^\alpha = F^{22} L^2 L^2 = F_{22}(1)(1) = 0(1)(1) = 0$

Dado a que $S^\alpha = (S^0, S^1, S^2, S^3) = (0,0,0,1)$

$\tilde{F}^{N\alpha} S^N S^\alpha = F^{33} S^3 S^3 = F_{33}(1)(1) = 0(1)(1) = 0$

Entonces $\tilde{F}^{N\alpha} V^N V^\alpha = \tilde{F}^{N\alpha} K^N K^\alpha = \tilde{F}^{N\alpha} L^N L^\alpha = \tilde{F}^{N\alpha} S^N S^\alpha = 0$

Ahora para V

$\tilde{F}^{N\alpha} V^\alpha K^\alpha = \tilde{F}^{01} V^0 K^1 = -E_r(1)(1) = -Er$

$\tilde{F}^{N\alpha} V^\alpha L^\alpha = \tilde{F}^{02} V^0 L^2 = -E\theta/r^4 (1)(1) = -E\theta/r^4$

$\tilde{F}^{N\alpha} V^\alpha S^\alpha = \tilde{F}^{03} V^0 S^3 = -E\phi/r^4\text{sen}^4\theta\,(1)(1) = -E\phi/r^4\text{sen}^4\theta$

Para K

$\tilde{F}^{N\alpha} K^N V^\alpha = \tilde{F}^{10} K^1 V^0 = Er(1)(1) = Er$

$\tilde{F}^{N\alpha} K^N L^\alpha = \tilde{F}^{12} K^1 L^2 = -B\phi/r^4 (1)(1) = -B\phi/r^4$

# Simulacro 1er Parcial

$\tilde{F}^{\nu\alpha} k^{\nu} S^{\alpha} = \tilde{F}^{13} k^{1} S^{3} = -B\theta / r^4 \text{sen}^4\theta \,(1)(1) = -B\theta / r^4 \text{sen}^4\theta$

Para $L$

$\tilde{F}^{\nu\alpha} L^{\nu} V^{\alpha} = \tilde{F}^{20} L^{2} V^{0} = E\theta \,(1)(1) = E\theta$

$\tilde{F}^{\nu\alpha} L^{\nu} k^{\alpha} = \tilde{F}^{21} L^{2} k^{1} = B\phi \,(1)(1) = B\phi$

$\tilde{F}^{\nu\alpha} L^{\nu} S^{\alpha} = \tilde{F}^{23} L^{2} S^{3} = -Br / r^4 \text{sen}^4\theta \,(1)(1) = -Br / r^4 \text{sen}^4\theta$

Para $S$

$\tilde{F}^{\nu\alpha} S^{\nu} V^{\alpha} = \tilde{F}^{30} S^{3} V^{0} = E\phi \,(1)(1) = E\phi$

$\tilde{F}^{\nu\alpha} S^{\nu} k^{\alpha} = \tilde{F}^{31} S^{3} k^{1} = -B\phi \,(1)(1) = -B\phi$

$\tilde{F}^{\nu\alpha} S^{\nu} L^{\alpha} = \tilde{F}^{32} S^{3} L^{2} = Br / r^4 \,(1)(1) = Br / r^4$