HMW. 4

P 1

$$f(x) = \begin{cases} cx^2 & |x| \le 1 \\ 0 & \text{н. ур.} \end{cases}$$

а) $c - ?$

$$\int_{-\infty}^{+\infty} f_X(x)\,dx = 1$$

$$\int_{-\infty}^{+\infty} f_X(x)\,dx = \int_{-\infty}^{-1} 0 \cdot dx + \int_{-1}^{1} cx^2\,dx + \int_{1}^{+\infty} 0\,dx = \int_{-1}^{1} cx^2\,dx = \frac{cx^3}{3}\Big|_{-1}^{1} =$$

$$= c \cdot 2 \cdot \frac{1}{3} \qquad c = \frac{3}{2}$$

б) $E(X) = \int_{-\infty}^{+\infty} x\, f(x)\,dx = \int_{-1}^{1} x \cdot \frac{3}{2} x^2 = \frac{3}{2} \frac{x^4}{4}\Big|_{-1}^{1} = 0$

$Var(X) = \int_{-\infty}^{+\infty} (x-\mu)^2 f(x)\,dx = \int_{-1}^{1} x^2 \frac{3}{2} x^2\,dx = \frac{3}{2} \frac{x^5}{5}\Big|_{-1}^{1} = \frac{3}{2} \cdot \frac{2}{5} = \frac{3}{5}$

cdf

$$F(x) = \begin{cases} 0 & x < -1 \\ \frac{x^2}{2} & -1 \le x < 1 \\ 1 & 1 \end{cases}$$

$$P(x \ge 12) = 1 - P(x < 12) = 1 - F(12) = 1 - 1 = \underline{\underline{0}}$$

P2

$$f_X(x) = \frac{1}{2} e^{-|x|}$$

$$Y = X^2$$

$$F_Y(y) = P(Y \le y) = P(X^2 \le y) = P(-\sqrt{y} \le X \le \sqrt{y}) = \boxed{(y > 0)}$$

$$= \int_{-\sqrt{y}}^{\sqrt{y}} \frac{1}{2} e^{-|t|} dt = 2 \cdot \int_0^{\sqrt{y}} \frac{1}{2} e^{-|t|} dt = \int_0^{\sqrt{y}} e^{-t} dt =$$

$$= -e^{-t} \Big|_0^{\sqrt{y}} = -e^{-\sqrt{y}} - (-e^0) = -e^{-\sqrt{y}} + 1 = \underline{\underline{1 - e^{-\sqrt{y}}}}$$

$$F_Y(y) = \begin{cases} 1 - e^{-\sqrt{y}} & y \ge 0 \\ 0 & \text{h.ч.} \end{cases}$$

P.3

$X \sim N(3, 9)$

$\sigma = 3$

1. $P(X > 0)$

$P(X \leq 0) = P\left(\frac{X-3}{3} \leq \frac{0-3}{3}\right) = P(Z \leq -1) = 0{,}1586\ldots$

[illegible] [illegible] st. norm. cdf(-1) [illegible]

st. norm. cdf(0, loc = 3, scale = 3)

$1 - P(X \leq 0) = P(X > 0) \approx 0{,}8413$

2. $P(-3 < X < 8) = F_X(8) - F_X(-3) = P\left(\frac{8-3}{3}\right) -$

$- P\left(\frac{-3-3}{3}\right) = P\left(\frac{5}{3}\right) - P(-2) = 0{,}952 - 0{,}0227 = 0{,}9294$

3. $P(X > 5 \mid X < 3) = \frac{P(X > 5 \cup X < 3)}{P(X \leq 3)} = \frac{0}{P(X \leq 3)} = \underline{\underline{0}}$

P4

$$EX = \int_0^\infty P(X \geq x)\,dx$$

$$P(X \geq x) = \int_x^\infty f_X(t)\,dt$$

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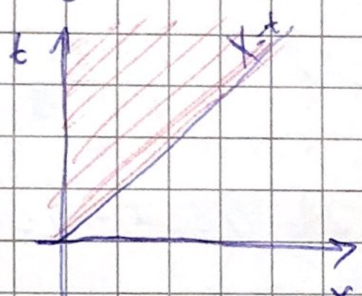
$$EX = \int_0^\infty \int_x^\infty f_X(t)\,dt\,dx$$

$$R = \{(x, t) \mid 0 \leq x < +\infty,\ x \leq t < +\infty\}$$



$$\int_0^\infty \int_0^t f_X(t)\,dx\,dt = \int_0^\infty f_X(t) \int_0^t dx\,dt = \int_0^\infty f_X(t)\,t\,dt =$$

$$= \underline{\underline{EX}}$$