HW 7

P1

$P(x,y)$		y: 0	1	2
X	0	0,1	0,04	0,02
	1	0,08	0,2	0,06
	2	0,06	0,14	0,3

1) $P(X=1 \cap y=1) = 0,2$

2) $P(X \leq 1 \cap Y \leq 1) = 0,1 + 0,04 + 0,08 + 0,2 = 0,42$

3) $P(X \neq 0 \cap Y \neq 0) = 0,06 + 0,2 + 0,14 + 0,3 = 0,7$

this means that ther is at least one hose being used on the self-service isl. and at least one hose being used on full-service isl.

4) marginal pmf.

X $\quad$ Y

$P_x(0) = 0,1 + 0,04 + 0,02 = 0,16$ $\quad$ $P_y(0) = 0,1 + 0,08 + 0,06 = 0,24$

$P_x(1) = 0,08 + 0,2 + 0,06 = 0,34$ $\quad$ $P_y(1) = 0,04 + 0,2 + 0,14 = 0,38$

$P_x(2) = 0,06 + 0,14 + 0,3 = 0,5$ $\quad$ $P_y(2) = 0,02 + 0,06 + 0,3 = 0,38$

$P(X \leq 1) = P_x(0) + P(1) = 0,34 + 0,16 = 0,5$

5.

$P(x, y) = P_x(x) \cdot P_y(y)$

$P(0,0) = P_x(0) \cdot P_y(0)$

$0,1 \neq 0,24 \cdot 0,16$

[illegible]

P2 $P(Brown) = 0,51$

$P(Blue) = 0,32$

$P(G) = 0,17$

1. $C_{10}^{5} \cdot (0,51)^5 \cdot C_5^3 (0,32)^3 \cdot C_2^2 (0,17)^2$

2. $C_{10}^{1} \cdot 0,32 \; C_9^1 \cdot 0,17 \cdot C_8^8 \; (0,51)^8$

3. $C_{10}^{7} \cdot (0,51)^7 \cdot (0,49)^3 + C_{10}^{8} \cdot (0,51)^8 \cdot (0,49)^2 +$
$+ C_{10}^{9} (0,51)^9 \cdot (0,49)^1 + C_{10}^{10} (0,51)^{10} \cdot (0,49)^0$

P 3

$$f(x,y) = x e^{-x(1+y)} \quad x \geq 0 \;\; y \geq 0$$

1) $P(X>3) = \int_3^{\infty}\int_0^{\infty} x e^{-x(1+y)}\, dy\, dx = \int_3^{\infty} \left[\left(\frac{x e^{-x(1+y)}}{-x} \right) \Big|_0^{\infty} \right] dx =$

$$= \int_3^{\infty} e^{-x} dx = -e^{-x} \Big|_3^{\infty} = \underline{\underline{e^{-3}}}$$

2)

$f_x(x,y) = \int_{-\infty}^{\infty} f(x,y)\,dy = \int_0^{\infty} x e^{-x(1+y)}\, dy = \frac{x e^{-x(1+y)}}{-x} \Big|_{y=0}^{\infty} =$

$$= -e^{-x(1+y)} \Big|_{y=0}^{+\infty} = \lim_{y \to \infty} \left(-e^{-x(1+y)} \right) - \left(-e^{-x} \right) =$$

$$= 0 - (-e^{-x}) = \underline{\underline{e^{-x}}}$$

$f_y(y) = \int_{-\infty}^{\infty} f(x,y)\,dx = \int_0^{+\infty} x e^{-x(1+y)}\, dx = \frac{x e^{-x(1+y)}}{-(1+y)} \Big|_{x=0}^{\infty} +$ (,0)

$$+ \frac{1}{1+y} \int_0^{+\infty} e^{-x(1+y)}\, dx = - \frac{e^{-x(1+y)}}{(1+y)^2} \Big|_0^{\infty} = \underline{\underline{\frac{1}{(1+y)^2}}}$$

3) $f(x,y) = f_x(x) \; f_y(y)$

$$x e^{-x(1+y)} \neq (e^{-x}) \frac{1}{(1+y)^2}$$

[illegible]

5) $P(X>3$ [illegible] $Y>3) = 1 - P(X \le 3, Y \le 3) =$

$$= 1 - \int_0^3 \int_0^3 x e^{-x(1+y)} \, dy \, dx = 1 - \int_0^3 \left(\frac{x e^{-x(1+y)}}{-x} \Big|_0^3 \right) dx =$$

$$= 1 + \int_0^3 e^{-x(1+y)} \Big|_{y=0}^{y=3} dx = 1 + \int_0^3 \left(e^{-4x} - e^{-x} \right) dx =$$

$$= 1 + \int_0^3 e^{-4x} dx - \int_0^3 e^{-x} dx = 1 + \frac{e^{-4x}}{-4} \Big|_0^3 +$$

$$+ e^{-x} \Big|_0^3 = \cancel{1} + \frac{e^{-12}}{4} + \frac{1}{4} + e^{-3} \cancel{- 1} =$$

$$= \frac{1}{4} + e^{-3} - \frac{e^{-12}}{4} \;//$$

P4

$P(X,Y)$		Y: 0	5	10	15
X	0	0,02	0,06	0,02	0,1
	5	0,04	0,15	0,2	0,1
	10	0,01	0,15	0,14	0,01

1) $E(X+Y) = E(X) + E(Y) = 5.55 + 8.55 = 14.1$

$E(X) = 0 \cdot 0{,}2 + 5 \cdot 0{,}49 + 10 \cdot 0{,}31 = 5{,}55$

$E(Y) = 0 \cdot 0{,}07 + 5 \cdot 0{,}36 + 10 \cdot 0{,}36 + 15 \cdot 0{,}21 = 8{,}55$

2) M - max-h event-n

$M = 0$

$\max(X,Y) = 0 \quad P(M=0) = P(X=0, Y=0) = 0{,}02$

$M = 5$

$P(M=5) = P(5,0) + P(0,5) + P(5,5) = 0{,}04 + 0{,}06 + 0{,}15 = 0{,}25$

$M = 10$

$P(M=10) = P(10,0) + P(10,5) + P(10,10) + P(0,10) + P(5,10) =$

$= 0{,}01 + 0{,}15 + 0{,}14 + 0{,}02 + 0{,}02 + 0{,}2 = 0{,}52$

$M = 15$

$P(M=15) = P(0,15) + P(5,15) + P(10,15) = 0{,}1 + 0{,}1 + 0{,}01 = 0{,}21$

M	0	5	10	15
P	0,02	0,25	0,52	0,21

$E(M) = 0{,}02 \cdot 0 + 5 \cdot 0{,}25 + 10 \cdot 0{,}52 + 15 \cdot 0{,}21 =$

$= 1{,}25 + 5{,}2 + 3{,}15 = 9{,}6$

P.5

$\mu_1 = 200 \qquad \mu_2 = 250 \qquad \mu_3 = 100$

$\sigma_1 = 10 \qquad \sigma_2 = 12 \qquad \sigma_3 = 8$

$Volume = 27X_1 + 125X_2 + 512X_3$

$E(V) = 27E(X_1) + 125E(X_2) + 512E(X_3) =$

$= 27 \cdot 200 + 125 \cdot 250 + 512 \cdot 100 =$

$= 5400 + 31250 + 51200 = 87850$

$V(V) = 27^2 V(X_1) + 125^2 V(X_2) + 512\ V(X_3) =$

$= 27^2 \cdot 100 + 250 \cdot 144 + 512 \cdot 64 = 72900 + 36000 +$

$+ 32768 = 141.668$

[illegible]
[illegible]
[illegible]

P5-1 $\quad f(x,y) = 2e^{-(x+y)} \quad 0 < x < y < \infty$

1) $f_x(x) = \int_{-\infty}^{+\infty} f(x,y)\,dy = \int_{x}^{+\infty} 2e^{-(x+y)}\,dy = -2e^{-(x+y)}\Big|_x^{\infty} =$

$= 0 - (-2e^{-2x}) = \underline{\underline{2e^{-2x}}}$

2) $f_{y|x}(y|x) = \dfrac{f_{x,y}(x,y)}{f_x(x)} = \dfrac{2e^{-(x+y)}}{2e^{-2x}} = e^{-(x+y)+2x} = \underline{\underline{e^{x-y}}}$

3) $P(Y>2 \mid X=1) = \dfrac{f_{x,y}(x=1, y>2)}{f_x(x=1)} = \dfrac{2e^{-3}}{2e^{-2}} = \underline{\underline{e^{-1}}}$

$f_{x,y}(x=1, y>2) = \int_2^{\infty} e^{-(1+y)}\,dy = -2e^{-(1+y)}\Big|_2^{\infty} = 2e^{-3}$

$f_x(x=1) = 2e^{-2}$

4) $E(Y \mid X=x) = \int_{-\infty}^{+\infty} y \cdot f_{y|x}(y|x=x)\,dy = \int_0^{+\infty} y\,e^{x-y}\,dy =$

$= -e^{x-y}\,y\Big|_0^{+\infty} + \int_0^{\infty} e^{x-y}\,dy = \int_0^{\infty} e^{x-y}\,dy = -e^{x-y}\Big|_0^{\infty} = \underline{\underline{e^{x}}}$