

(17)

$$(88901, 10987) = d$$

$$d = 88901u + 10987v, \quad u, v \in \mathbb{Z}$$

$$x_{88901} = (1, 0) \quad (u, v) \in \mathbb{Z}^2$$

$$x_{10987} = (0, 1)$$

$$88901 = 10987 \cdot 8 + 1005$$

$$x_{1005} = x_{88901} - 8x_{10987}$$

$$x_{1005} = (1, 0) - 8(0, 1) = \cancel{(1, -8)} (1, -8)$$

$$10987 = 1005 \cdot 10 + 937$$

$$x_{937} = x_{10987} - 10x_{1005}$$

$$x_{937} = (0, 1) - 10(1, -8) = (-10, 81)$$

$$1005 = 937 \cdot 1 + 68$$

$$x_{1005} = x_{937} \cdot 1 + x_{68}$$

$$x_{68} = x_{1005} - x_{937} = (1, -8) - (-10, 81) = (11, -89)$$

$$937 = 68 \cdot 13 + 53$$

$$x_{53} = x_{937} - 13x_{68}$$

$$\begin{aligned} x_{53} &= (-10, 81) - 13(11, -89) = (-10 - 143, 81 + 1157) \\ &= (-153, 1238) \end{aligned}$$

$$68 = 53 \cdot 1 + 15$$

$$\begin{aligned} x_{15} &= x_{68} - x_{53} = (11, -89) - (-153, 1238) \\ &= (11 + 153, -89 - 1238) \\ &= (164, -1327) \end{aligned}$$



$$53 = 15 \cdot 3 + 8$$

$$x_8 = x_{53} - 3x_{15}$$

$$x_8 = (-153, 1238) - 3(164, -1324)$$

$$= (-153 - 492, 1238 + 3981)$$

$$= (-645, 5219)$$

$$\begin{array}{r} 53 \\ -45 \\ \hline 8 \end{array}$$

$$15 = 8 \cdot 1 + 7$$

$$x_{15} = x_8 + x_7$$

$$x_7 = x_{15} - x_8 = (164, -1324) - (-645, 5219)$$

$$= (164 + 645, -1324 - 5219)$$

$$= (809, -6546)$$

$$8 = 7 \cdot 1 + 1$$

$$x_1 = x_8 - x_7 = (-645, 5219) - (809, -6546)$$

$$= (-645 - 809, 5219 + 6546)$$

$$= (-1454, 11765)$$

$$7:7 = 7$$

$$7 = 1 \cdot 7 + 0$$

→ CMMDC

$$x_2 = x_1 + x_0$$

$$x_0 = x_1 - 7x_1 = (809, -6546) - 7(-1454, 11765)$$

$$d = u \cdot a + v \cdot b$$

$$(1 = -1454 \cdot 88901 + 11765 \cdot 10987)$$

$$1 = \underbrace{-1454 \cdot 88901} + \underbrace{11765 \cdot 10987}$$

Coefficientii Berout sunt:  $u; v$

(17)

$$a = 36$$

$$m = 67$$

$$x_{36} = (0, 1)$$

$$x_{67} = (1, 0)$$

$$67 = 36 \cdot 1 + 31$$

$$x_{31} = x_{67} - x_{36} = (1, 0) - (0, 1) = (1, -1)$$

$$36 = 31 \cdot 1 + 5$$

$$x_{36} = x_{31} \cdot 1 + x_5$$

$$x_5 = x_{36} - x_{31} = (0, 1) - (1, -1) = (-1, 2)$$

$$31 = 5 \cdot 6 + 1$$

$$x_{31} = x_5 \cdot 6 + x_1$$

$$x_1 = x_{31} - 6x_5 = (1, -1) - 6(-1, 2) = (1+6, -1-12) = (7, -13)$$

$$5 = \textcircled{1} \cdot 5 + 0$$

→ c.m.m.d.c

$$1 = 67u + 36v = 67 \cdot 7 + 36 \cdot (-13) = 469 + (-468) = 1$$

$$u = 7$$

$$v = -13$$

$$1 = \underbrace{67 \cdot 7}_{\equiv 0} + 36 \cdot (-13) \pmod{67}$$

$$1 \equiv 36 \cdot (-13) \pmod{67} \Rightarrow 36^{-1} = -13$$

$$36^{-1} = 67 - 13 = 54 \text{ in } \mathbb{Z}_{67}.$$