LOW-PRIORITY QUEUE FLUCTUATIONS IN TANDEM OF QUEUING SYSTEMS UNDER CYCLIC CONTROL WITH PROLONGATIONS

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Abstract

A tandem of queuing systems is considered. Each system has a high-priority input flow and a low-priority input flow which are conflicting. In the first system, the customers are serviced in the class of cyclic algorithms. The serviced high-priority customers are transferred from the first system to the second one with random delays and become the high-priority input flow of the second system. In the second system, customers are serviced in the class of cyclic algorithms with prolongations. Low-priority customers are serviced when their number exceeds a threshold. A mathematical model is constructed in form of a multidimensional denumerable discrete-time Markov chain. The recurrent relations for partial probability generating functions for the low-priority queue in the second system are found.

Keywords: tandem of controlling queuing systems, cyclic algorithm with prolongations, conflicting flows, multidimensional denumerable discrete-time Markov chain

1. Introduction

Conflicting traffic flows control at a crossroad is one of classical problems in queuing theory. In the literature several algorithms were investigated: fixed duration cyclic algorithm, cyclic algorithm with a loop, cyclic algorithm with changing regimes, etc [1, 2, 3, 4, 5, 6]. However, several (two in our case) consecutive crossroads are of great interest, because in a real-life situation a vehicle having passed one highway intersection finds itself at another one. In other words, an output flow from the first intersection forms an input flow of the second intersection. Hence, the second input flow no longer has an *a priori* known simple probabilistic structure (for example, that of an non-ordinary Poison flow), and knowledge about the service algorithm should be taken into account to deduce formation conditions of the first output flow.

Tandems of intersections were considered by a few authors. In [7] a computer-aided simulation of adjacent intersection was carried out. In [8] a mathematical mode of two intersection in tandem governed by cyclic algorithms was investigates and stability conditions were found. In this paper we assume that the first intersection is governed by a cyclic algorithm while the econd intersection is governed by a cyclic algorithm with prolongations. In particular, we pay attention to the low-priority queue in the second intersection.

2. The problem settings

Consider a queuing system with a scheme shown in (see Fig. 1). There are four input flows of customers Π_1, Π_2, Π_3, and Π_4 entering the single server queueing system. Customers in the input flow Π_j, $j \in \{1, 2, 3, 4\}$ join a queue O_j with an unlimited capacity. For $j \in \{1, 2, 3\}$ the discipline of the queue O_j is FIFO (First In First Out). Discipline of the queue O_4 will be described later. The input flows Π_1 and Π_3 are generated by an external environment, which has only one state. Each of these flows is a nonordinary Poisson flow. Denote by λ_1 and λ_3 the intensities of bulk arrivals for the flows Π_1 and Π_3 respectively. The probability generating function of number of customers in a bulk in the flow Π_j is

$$f_j(z) = \sum_{\nu=1}^{\infty} p_\nu^{(j)} z^\nu, \quad j \in \{1, 3\}, \tag{1}$$

We assume that $f_j(z)$ converges for any $z \in \mathbb{C}$ such that $|z| < (1 + \varepsilon)$, $\varepsilon > 0$. Here $p_\nu^{(j)}$ is the probability of a bulk size in flow Π_j being exactly $\nu = 0, 1, \ldots$. Having been serviced the customers from O_1 come back to the system as the Π_4 customers. The Π_4 customers in turn after service enter the system as the Π_2 ones. The flows Π_2 and Π_3 are conflicting in the sense that their customers can't be serviced simultaneously. This implies that the problem can't be reduced to a problem with fewer input flows by merging the flows together.

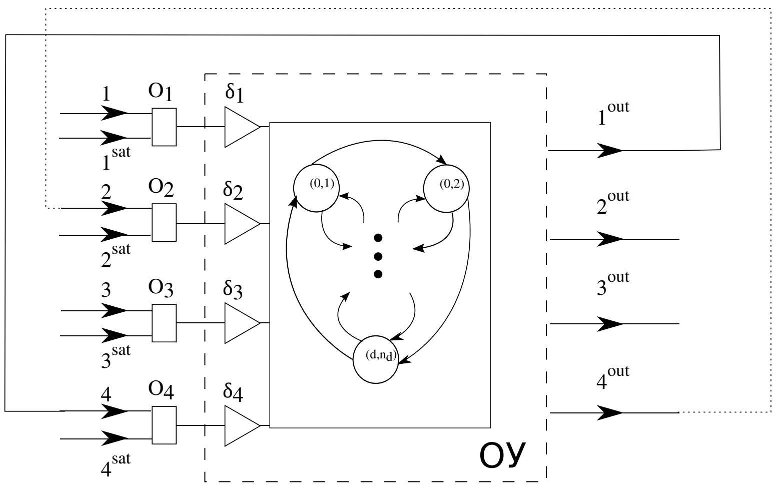

Figure 1: Scheme of the queuing system as a cybernetic control system

In order to describe the server behavior we fix positive integers d, n_0, n_1, …, n_d and we introduce a finite set $\Gamma = \{\Gamma^{(k,r)}\colon k = 0, 1, \ldots, d; r = 1, 2, \ldots n_k\}$ of states server can reside in. At the state $\Gamma^{(k,r)}$ sever stays during constant time $T^{(k,r)}$. Define disjoint subsets Γ^{I}, Γ^{II}, Γ^{III}, and Γ^{IV} of Γ as follows. In the state $\gamma \in \Gamma^{\mathrm{I}}$ only customers from the queues O_1, O_2 and O_4 are serviced. In the state $\gamma \in \Gamma^{\mathrm{II}}$ only customers from the queues O_2 and O_4 are serviced. In the state $\gamma \in \Gamma^{\mathrm{III}}$ only customers from queues O_1, O_3, and O_4 are serviced. In the state $\gamma \in \Gamma^{\mathrm{IV}}$ only customers from queues O_3 and O_4 are serviced. We assume that $\Gamma = \Gamma^{\mathrm{I}} \cup \Gamma^{\mathrm{II}} \cup \Gamma^{\mathrm{III}} \cup \Gamma^{\mathrm{IV}}$. Set also ${}^1\Gamma = \Gamma^{\mathrm{I}} \cup \Gamma^{\mathrm{III}}$, ${}^2\Gamma = \Gamma^{\mathrm{I}} \cup \Gamma^{\mathrm{II}}$, ${}^3\Gamma = \Gamma^{\mathrm{III}} \cup \Gamma^{\mathrm{IV}}$.

The server changes its state according to the following rules. We call a set $C_k = \{\Gamma^{(k,r)}\colon r = 1, 2, \ldots n_k\}$ the k-th cycle, $k = 1, 2, \ldots, d$. For $k = 0$ the state $\Gamma^{(0,r)}$ with $r = 1, 2, \ldots, n_0$ is called a prolongation state. Put $r \oplus_k 1 = r + 1$ for $r < n_k$, and $r \oplus_k 1 = 1$ for $r = n_k$ ($k = 0, 1, \ldots, d$). In the cycle C_k we select a subset C_k^{O} of input states, a subset C_k^{I} of output states, and a subset $C_k^{\mathrm{N}} = C_k \setminus (C_k^{\mathrm{O}} \cup C_k^{\mathrm{I}})$ of neutral states. After the state $\Gamma^{(k,r)} \in C_k \setminus C_k^{\mathrm{O}}$ the server switches to the state $\Gamma^{(k,r\oplus_k 1)}$ within the same cycle C_k. After the state $\Gamma^{(k,r)}$ in C_k^{O} the server switches to the state $\Gamma^{(k,r\oplus_k 1)}$ if number of customers in the queue O_3 at switching instant is greater than a predetermined threshold L. Otherwise, is the number of customers in the queue O_3 is less than or equals L then the new state is the prolongation one $\Gamma^{(0,r_1)}$ where $r_1 = h_1(\Gamma^{(k,r)})$ and $h_1(\cdot)$ is a given mapping of $\bigcup_{k=1}^{d} C_k^{\mathrm{O}}$ into $\{1, 2, \ldots, n_0\}$. After the state $\Gamma^{(0,r)}$ if the number of customers in O_3 is not above L the state of the same type $\Gamma^{(0,r_2)}$ is chosen where $r_2 = h_2(r)$ and $h_2(\cdot)$ is a given mapping of the set $\{1, 2, \ldots, n_0\}$ into itself; in the other case the new state is $\Gamma^{(k,r_3)} \in C_k^{\mathrm{I}}$ where $\Gamma^{(k,r_3)} = h_3(r)$ and $h_3(\cdot)$ is a given mapping of $\{1, 2, \ldots, n_0\}$ to $\bigcup_{k=1}^{d} C_k^{\mathrm{I}}$. We assume that each prolongation state $\Gamma^{(0,r)}$ belongs to the set ${}^2\Gamma$ and that relations $C_k^{\mathrm{O}} \subset {}^2\Gamma$ and $C_k^{\mathrm{I}} \subset {}^3\Gamma$ hold. We also assume that all the cycles have exactly one input and output state. Finally, we assume that all the prolongation states make a cycle, that is $h_2(r) = r \oplus_0 1$. Putting all together, we introduce a function which formalizes the server state changes:

$$h(\Gamma^{(k,r)}, y) = \begin{cases} \Gamma^{(k,r\oplus_k 1)} & \text{if } \Gamma^{(k,r)} \in C_k \setminus C_k^{\mathrm{O}} \text{ or } (\Gamma^{(k,r)} \in C_k^{\mathrm{O}}) \wedge (y > L); \\ \Gamma^{(0,h_1(\Gamma^{(k,r)}))} & \text{if } \Gamma^{(k,r)} \in C_k^{\mathrm{O}} \text{ and } y \leqslant L; \\ \Gamma^{(0,r\oplus_0 1)} & \text{if } k = 0 \text{ and } y \leqslant L; \\ h_3(r) & \text{if } k = 0 \text{ and } y > L. \end{cases} \tag{2}$$

In general, service durations of different customers can be dependent and may have different laws of probability distributions. So, saturation flows will be used to define the service process. A saturation flow Π_j^{sat}, $j \in \{1, 2, 3, 4\}$, is defined as a virtual output flow under the maximum usage of the server and unlimited number of customer in the queue O_j. The saturation flow Π_j^{sat}, $j \in \{1, 2, 3\}$ contains a non-random number $\ell(k, r, j) \geqslant 0$ of customers in the server state $\Gamma^{(k,r)}$. In particular, $\ell(k, r, j) \geqslant 1$ for $\Gamma^{(k,r)} \in {}^j\Gamma$ and $\ell(k, r, j) = 0$ for $\Gamma^{(k,r)} \notin {}^j\Gamma$. Let $\mathbb{Z}_+$ be the set of non-negative integer numbers. If the queue O_4 contains $x \in \mathbb{Z}_+$ customers the saturation flow Π_4^{sat} also contains the x customers. Finally, in the state $\Gamma^{(k,r)}$ every customer from queue O_4

with probability $p_{k,r}$ and independently of others ends servicing and joins Π_2 to go to O_2. With the complementary probability $1 - p_{k,r}$ the customer stays in O_4 until the next time slot. In the next time slot it repeats its attempt to join Π_2 with a proper probability.

A real-life example of just described queuing system is a tandem of two consecutive crossroads (Fig. 2). The input flows are flows of vehicles. The flows Π_1 and

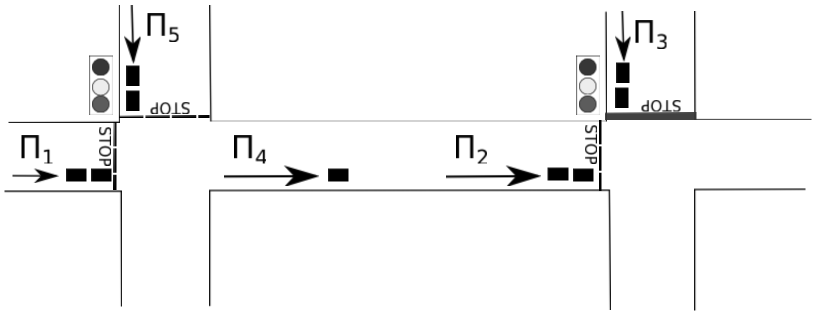

Figure 2: A tandem of crossroads, the physical interpretation of the queueing system under study

Π_5 at the first crossroad are conflicting; Π_2 and Π_3 at the second crossroad are also conflicting. Every vehicle from the flow Π_1 after passing first road intersection joint the flow Π_4 and enters the queue O_4. After some random time interval the vehicle arrives to the next road intersection. Such a pair of crossroads is an instance of a more general queuing model described above.

3. Mathematical model

The queueing system under investigation can be regarded as a cybernetic control system what helps to rigorously construct a formal stochastic model [8]. The scheme of the control system is shown in Fig. 1. There are following blocks present in the scheme: 1) the external environment with one state; 2) input poles of the first type — the input flows Π_1, Π_2, Π_3, and Π_4; 3) input poles of the second type — the saturation flows Π_1^{sat}, Π_2^{sat}, Π_3^{sat}, and Π_4^{sat}; 4) an external memory — the queues O_1, O_2, O_3, and O_4; 5) an information processing device for the external memory — the queue discipline units δ_1, δ_2, δ_3, and δ_4; 6) an internal memory — the server (OY); 7) an information processing device for internal memory — the graph of server state transitions; 8) output poles — the output flows Π_1^{out}, Π_2^{out}, Π_3^{out}, and Π_4^{out}. The coordinate of a block is its number on the scheme.

Let us introduce the following variables and elements along with their value ranges. To fix a discrete time scale consider the epochs $\tau_0 = 0$, τ_1, τ_2, … when the server changes its state. Let $\Gamma_i \in \Gamma$ be the server state during the interval $(\tau_{i-1}; \tau_i]$, $\varkappa_{j,i} \in \mathbb{Z}_+$ be the number of customers in the queue O_j at the instant τ_i, $\eta_{j,i} \in \mathbb{Z}_+$ be the number of customers arrived into the queue O_j from the flow Π_j during the interval $(\tau_i; \tau_{i+1}]$, $\xi_{j,i} \in \mathbb{Z}_+$ be the number of customers in the saturation flow Π_j^{sat} during the interval $(\tau_i; \tau_{i+1}]$, $\overline{\xi}_{j,i} \in \mathbb{Z}_+$ be the actual number of serviced customers from the queue O_j during the interval $(\tau_i; \tau_{i+1}]$, $j \in \{1, 2, 3, 4\}$.

The server changes its state according to the following rule:

$$\Gamma_{i+1} = h(\Gamma_i, \varkappa_{3,i}) \tag{3}$$

where the mapping $h(\cdot, \cdot)$ is defined by Formula (2). To determine the duration T_{i+1} of the next time slot it useful to introduce a mapping $h_T(\cdot, \cdot)$ by

$$T_{i+1} = h_T(\Gamma_i, \varkappa_{3,i}) = T^{(k,r)} \quad \text{where } \Gamma^{(k,r)} = \Gamma_{i+1} = h(\Gamma_i, \varkappa_{3,i}).$$

A functional relation

$$\overline{\xi}_{j,i} = \min\{\varkappa_{j,i} + \eta_{j,i}, \xi_{j,i}\}, \quad j \in \{1, 2, 3\}, \tag{4}$$

between $\overline{\xi}_{j,i}$ and $\varkappa_{j,i}$, $\eta_{j,i}$, $\xi_{j,i}$ describes the service strategy. Further, since

$$\varkappa_{j,i+1} = \varkappa_{j,i} + \eta_{j,i} - \overline{\xi}_{j,i}, \quad j \in \{1, 2, 3\},$$

and due to (4) it follows that

$$\varkappa_{j,i+1} = \max\{0, \varkappa_{j,i} + \eta_{j,i} - \xi_{j,i}\}, \quad j \in \{1, 2, 3\}. \tag{5}$$

We also have from the problem settings the following relations for the flow Π_4:

$$\eta_{4,i} = \min\{\xi_{1,i}, \varkappa_{1,i} + \eta_{1,i}\}, \quad \varkappa_{4,i+1} = \varkappa_{4,i} + \eta_{4,i} - \eta_{2,i}, \quad \xi_{4,i} = \varkappa_{4,i}. \tag{6}$$

Put $\varkappa_i = (\varkappa_{1,i}, \varkappa_{2,i}, \varkappa_{3,i}, \varkappa_{4,i})$. The non-local description of the input and saturation flows consists in specifying particular features of the conditional probability distribution of selected discrete components $\eta_i = (\eta_{1,i}, \eta_{2,i}, \eta_{3,i}, \eta_{4,i})$ and $\xi_i = (\xi_{1,i}, \xi_{2,i}, \xi_{3,i}, \xi_{4,i})$ of marked point processes $\{(\tau_i, \nu_i, \eta_i); i \geqslant 0\}$ and $\{(\tau_i, \nu_i, \xi_i); i \geqslant 0\}$ with marks $\nu_i = (\Gamma_i; \varkappa_i)$. Let $\varphi_1(\cdot, \cdot)$ and $\varphi_3(\cdot, \cdot)$ be defined by series expansions

$$\sum_{\nu=0}^{\infty} z^{\nu} \varphi_j(\nu, t) = \exp\{\lambda_j t (f_j(z) - 1)\}$$

with functions where $f_j(z)$ defined by (1), $j \in \{1, 3\}$. The function $\varphi_j(\nu, t)$ equals the probability of $\nu = 0, 1, \ldots$ arrivals in the flow Π_j during time $t \geqslant 0$. If $\nu < 0$ the value of $\varphi_j(\nu, t)$ is set to zero. Define function $\psi(\cdot, \cdot, \cdot)$ by

$$\psi(k; y, u) = C_y^k u^k (1 - u)^{y-k}.$$

Then $\psi(k; y, p_{k,r})$ is the probability of k arrival from flow Π_2 given the queue O_4 contains y customers and the server state is $\Gamma^{(k,r)}$. For values $k \notin \{0, 1, \ldots, y\}$ the value of $\psi(k; y, u)$ is set to zero.

Let $a = (a_1, a_2, a_3, a_4) \in \mathbb{Z}_+^4$ and $x = (x_1, x_2, x_3, x_4) \in \mathbb{Z}_+^4$. If the mark value is $\nu_i = (\Gamma^{(k,r)}; x)$ then the probability $\varphi(a, k, r, x)$ of simultaneous equalities $\eta_{1,i} = a_1$, $\eta_{2,i} = a_2$, $\eta_{3,i} = a_3$, $\eta_{4,i} = a_4$ according the the problem statement is

$$\varphi_1(a_1, h_T(\Gamma^{(k,r)}, x_3)) \cdot \psi(a_2, x_4, p_{\tilde{k},\tilde{r}}) \cdot \varphi_3(a_3, h_T(\Gamma^{(k,r)}, x_3)) \cdot \delta_{a_4,\min\{\ell(\tilde{k},\tilde{r},1),x_1+a_1\}}$$

where $\Gamma^{(\tilde{k},\tilde{r})} = h(\Gamma^{(k,r)}, x_3)$ and $\delta_{i,j}$ is the Kroneker's delta:

$$\delta_{i,j} = \begin{cases} 1 & \text{if } i = j, \\ 0 & \text{if } i \neq j. \end{cases}$$

Let $b = (b_1, b_2, b_3, b_4) \in \mathbb{Z}_+^4$. The probability $\zeta(b, k, r, x)$ of simultaneous equalities $\xi_{1,i} = b_1$, $\xi_{2,i} = b_2$, $\xi_{3,i} = b_3$, $\xi_{4,i} = b_4$ given the fixed label value $\nu_i = (\Gamma^{(k,r)}; x)$ is

$$\delta_{b_1,\ell(\tilde{k},\tilde{r},1)} \cdot \delta_{b_2,\ell(\tilde{k},\tilde{r},2)} \cdot \delta_{b_3,\ell(\tilde{k},\tilde{r},3)} \cdot \delta_{b_4,x_4}.$$

The assumptions on statistical properties of some blocks and function relations between blocks are not contradicting and sufficient to construct a formal probability model, as the following theorem first proven in [9] demonstrates.

Theorem 1. *Choose* $\gamma_0 = \Gamma^{(k_0,r_0)} \in \Gamma$ *and* $x^0 = (x_{1,0}, x_{2,0}, x_{3,0}, x_{4,0}) \in \mathbb{Z}_+^4$. *There exists a probability space* $(\Omega, \mathcal{F}, \mathbf{P}(\cdot))$, *random variables* $\eta_{j,i} = \eta_{j,i}(\omega)$, $\xi_{j,i} = \xi_{j,i}(\omega)$, $\varkappa_{j,i} = \varkappa_{j,i}(\omega)$, *and random elements* $\Gamma_i = \Gamma_i(\omega)$, $i \geqslant 0$, $j \in \{1, 2, 3, 4\}$ *defined on this space, such that: 1) equalities* $\Gamma_0(\omega) = \gamma_0$ *and* $\varkappa_0(\omega) = x^0$ *hold; 2) relations* (3), (5), (6) *hold; 3) for any a, b,* $x^t = (x_{1,t}, x_{2,t}, x_{3,t}, x_{4,t}) \in \mathbb{Z}_+^4$, $\Gamma^{(k_t,r_t)} \in \Gamma$, $t = 1, 2, \ldots$ *the joint conditional probability distribution of vectors* η_i *and* ξ_i *has the form*

$$\mathbf{P}\Big(\big\{\omega\colon \eta_i(\omega) = a, \xi_i(\omega) = b\big\} \Big| \bigcap_{t=0}^{i} \big\{\omega\colon \Gamma_t(\omega) = \Gamma^{(k_t,r_t)}, \varkappa_t(\omega) = x^t\big\}\Big)$$
$$= \varphi(a, k_i, r_i, x^i) \cdot \zeta(b, k_i, r_i, x^i).$$

From now we focuse on low-priority customers in the queue O_3.

4. The low-priority queue

Here we will consider the stochastic sequence

$$\{(\Gamma_i(\omega), \varkappa_{3,i}(\omega)); i = 0, 1, \ldots\} \tag{7}$$

which includes the number of low-priority customers $\varkappa_{3,i}(\omega)$ in the queue O_3. In this section we will report several results concerning this stochastic sequence.

Theorem 2. *Let* $\Gamma_0(\omega) = \Gamma^{(k,r)} \in \Gamma$ *and* $\varkappa_{3,0}(\omega) = x_{3,0} \in \mathbb{Z}_+$ *be fixed. Then the stochastic sequence* (7) *is a homogeneous denumerable Markov chain.*

Theorem 3. *Let x_3, $\tilde{x}_3 \in \mathbb{Z}_+$ and $\Gamma^{(k,r)}$, $\Gamma^{(\tilde{k},\tilde{r})} = h(\Gamma^{(k,r)}, x_3) \in \Gamma$. Then the transition probabilities of the Markov chain* (7) *are*

$$\begin{aligned}\mathbf{P}(\{\omega\colon \Gamma_{i+1}(\omega) = \Gamma^{(\tilde{k},\tilde{r})}, \varkappa_{3,i+1}(\omega) = \tilde{x}\} \mid \{\omega\colon \Gamma_i(\omega) = \Gamma^{(k,r)}, \varkappa_{3,i}(\omega) = x\})\\ = (1 - \delta_{\tilde{x}_3,0}) \cdot \varphi\Big(\tilde{x}_3 + \ell(\tilde{k}, \tilde{r}, 3) - x_3, h_T(\Gamma^{(\tilde{k},\tilde{r})}, x_3)\Big)\\ + \delta_{\tilde{x}_3,0} \sum_{a=0}^{\ell(\tilde{k},\tilde{r},3)-x_3} \varphi_3\Big(a, h_T(\Gamma^{(\tilde{k},\tilde{r})}, x_3)\Big).\end{aligned}$$

The last theorem clarifies which states of the Markov chain $\{(\Gamma_i, \varkappa_{3,i}); i \geqslant 0\}$ are essential. To make a complete classification we introduce sets

$$S^{\,3}_{0,r} = \Big\{(\Gamma^{(0,r)}, x_3)\colon\; x_3 \in Z_+,\; L \geqslant x_3 > L - \max_{k=1,2,\ldots,d}\Big\{\sum_{t=0}^{n_k} \ell(k,t,3)\Big\}\Big\}, \qquad 1 \leqslant r \leqslant n_0,$$

$$S^{\,3}_{k,r} = \Big\{(\Gamma^{(k,r)}, x_3)\colon\; x_3 \in Z_+,\; x_3 > L - \sum_{t=0}^{r-1} \ell(k,t,3)\Big\}, \quad 1 \leqslant k \leqslant d, \quad 1 \leqslant r \leqslant n_k.$$

Theorem 4. *The set of essential states of the Markov chain $\{(\Gamma_i, \varkappa_{3,i}); i \geqslant 0\}$ consists of sets $\bigcup\limits_{1\leqslant r\leqslant n_0} S^{\,3}_{0,r}$ and $\bigcup\limits_{\substack{1\leqslant k\leqslant d\\ 1\leqslant r\leqslant n_k}} S^{\,3}_{k,r}$.*

As before, let $\Gamma^{(k,r)} \in \Gamma$ and $x_3 \in Z_+$. Denote by $H_{-1}(\Gamma^{(k,r)}, x_3)$ the set of all server states γ such that $h(\gamma, x_3) = \Gamma^{(k,r)}$ and put $r \ominus_k 1 = r - 1$ for $n_k \geqslant r > 0$, and $r \ominus_k 1 = n_k$ for $r = 0$ ($k = 0, 1, \ldots, d$). Then formula (2) makes it possible to define the mapping $H_{-1}(\Gamma^{(k,r)}, x_3)$ explicitly:

$$H_{-1}(\Gamma^{(k,r)}, x_3) = \begin{cases} \{\Gamma^{(k_1,r_1)}, \Gamma^{(0,r\ominus_0 1)}\} & \text{if } (k=0) \wedge (x_3 \leqslant L),\\ \{\Gamma^{(k,r\ominus_k 1)}, \Gamma^{(0,r_2)}\} & \text{if } (\Gamma^{(k,r)} \in C^{\mathrm{I}}_k) \wedge (x_3 > L),\\ \{\Gamma^{(k,r\ominus_k 1)}\} & \text{if } (\Gamma^{(k,r)} \in C^{\mathrm{O}}_k) \vee (\Gamma^{(k,r)} \in C^{\mathrm{N}}_k);\\ \varnothing & \text{if } (k=0) \wedge (x_3 > L)\\ & \quad\text{or } (\Gamma^{(k,r)} \in C^{\mathrm{I}}_k) \wedge (x_3 \leqslant L) \end{cases} \tag{8}$$

where $h_1(\Gamma^{(k_1,r_1)}) = r$ and $h_3(r_2) = \Gamma^{(k,r)}$.

Let's define for $\gamma \in \Gamma$ and $x_3 \in Z_+$ values

$$Q_{3,i}(\gamma, x) = \mathbf{P}(\{\omega\colon \Gamma_i(\omega) = \gamma, \varkappa_{3,i}(\omega) = x_3\}).$$

Suppose k and r are such that $\Gamma^{(k,r)} \in \Gamma$. Let's define the partial probability generating functions

$$\mathfrak{M}^{(i)}(k,r,v) = \sum_{w=0}^{\infty} Q_{3,i}(\Gamma^{(k,r)}, w) v^w, \qquad \Phi^{(i)}(k,r,v) = \sum_{x_3=0}^{\infty} \sum_{\gamma\in H_{-1}(\Gamma^{(k,r)},x_3)} Q_{3,i}(\gamma, x_3) v^{x_3},$$

$$q_{k,r}(v) = v^{-\ell(k,r,3)} \sum_{w=0}^{\infty} \varphi_3(w, T^{(k,r)}) v^w.$$

Theorem 5. *Let* $\tilde{\gamma} = \Gamma^{(\tilde{k},\tilde{r})} \in \Gamma$. *The following recurrent w.r.t.* $i \geqslant 0$ *relations take place for the partial probability generating functions:*

$$\mathfrak{M}^{(i+1)}(\tilde{k},\tilde{r},v) = q_{\tilde{k},\tilde{r}}(v)\Phi^{(i)}(\tilde{k},\tilde{r},v) + \sum_{x_3=0}^{\ell(\tilde{k},\tilde{r},3)} \sum_{\gamma\in H_{-1}(\tilde{\gamma},x_3)} Q_{3,i}(\gamma,x_3) \sum_{a=0}^{\ell(\tilde{k},\tilde{r},3)-x_3} \varphi_3(a,T^{(\tilde{k},\tilde{r})}) -$$
$$- \sum_{x_3=0}^{\ell(\tilde{k},\tilde{r},3)} \sum_{\gamma\in H_{-1}(\tilde{\gamma},x_3)} Q_{3,i}(\gamma,x_3) v^{x_3-\ell(\tilde{k},\tilde{r},3)} \sum_{w=0}^{\ell(\tilde{k},\tilde{r},3)+1-x_3} \varphi_3(w,T^{(\tilde{k},\tilde{r})}) v^w.$$

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