

# LOW-PRIORITY QUEUE FLUCTUATIONS IN TANDEM OF QUEUING SYSTEMS UNDER CYCLIC CONTROL WITH PROLONGATIONS

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## Abstract

A tandem of queuing systems is considered. Each system has a high-priority input flow and a low-priority input flow which are conflicting. In the first system, the customers are serviced in the class of cyclic algorithms. The serviced high-priority customers are transferred from the first system to the second one with random delays and become the high-priority input flow of the second system. In the second system, customers are serviced in the class of cyclic algorithms with prolongations. Low-priority customers are serviced when their number exceeds a threshold. A mathematical model is constructed in form of a multidimensional denumerable discrete-time Markov chain. The recurrent relations for partial probability generating functions for the low-priority queue in the second system are found.

**Keywords:** tandem of controlling queuing systems, cyclic algorithm with prolongations, conflict flows, multidimensional denumerable discrete-time Markov chain

## 1. Introduction

Conflict traffic flows control at a crossroad is one of the classical problem of queuing theory. The problem has been solved for different classes of algorithms: the class of algorithms with a cyclic fixed rhythm, with renewals ("with a loop") with dynamic priorities, etc. However, several (two in our case) consecutive crossroads are of great interest, because in a real life after car passed one highway intersection it finds itself at another one. In other words, output flow of the first intersection forms input flow of the second intersection. Hence, the second input flow no longer has simple probabilistic structure known a priori (for example, non-ordinary Poisson process) and knowledge about service algorithm should be taken into account to deduce formation properties of the first output flow.

In [1] the problem of tandem of two crossroads was carried out and rigorous probabilistic model was built. At each of the intersections, in addition to the high priority flow, there was the traffic flow "in the perpendicular direction", with lower priority. The service of the flows on first intersection is supposed to be in the class of cyclic algorithms and in the of cyclic algorithms with prolongations on the second intersection. Over and above, vehicles could not turn from one conflict direction to another.

In this paper we continue the study of described problem and deepen in low priority flow of the second intersection investigation.

## 2. The setting of the problem

This section contains statement of the problem according to [1]. First of all the tandem is presented as queuing system to give generic formulation while in the end of the section system of two consecutive crossroads is brought as simple example. More rigorous mathematical model is built in next section.

Consider queuing system of the following type: There are four input flows of cus-

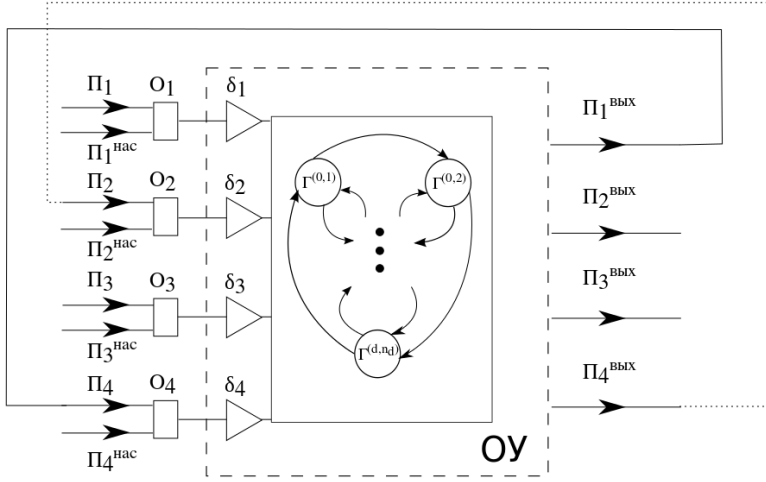


Figure 1: Structure scheme of queuing system

tomers coming into the system with one server. Customers from input flow  $\Pi_j$  arrive to corresponding queue with an unbounded capacity,  $j \in \{1, 2, 3, 4\}$ . For  $j \in \{1, 2, 3\}$  discipline of the queue  $O_j$  is FIFO (First In First Out). In other words, the earlier customer came to system the earlier it is serviced. Discipline of the queue  $O_4$  is described below. Input flows  $\Pi_1$  and  $\Pi_3$  are generated by external environment, which has only one state. Each of these flows is a nonordinary Poisson process. Denote  $\lambda_1$  and  $\lambda_3$  the intensity for the bulk arrival for the flows  $\Pi_1$  and  $\Pi_3$  respectively. Generating function of customer number in the bulk of flow  $\Pi_j$  is

$$f_j(z) = \sum_{v=1}^{\infty} p_v^{(j)} z^v, \quad j \in \{1, 3\}, \quad (1)$$

assuming it is finite for any  $z \in \mathbb{C}$  such as  $|z| < (1+\varepsilon)$ ,  $\varepsilon > 0$ . Quantity  $p_v^{(j)}$  is probability of event that customers number in a bulk of flow  $\Pi_j$  is exactly  $v$ . Being serviced  $\Pi_1$  customers come back to the system as  $\Pi_4$  customers.  $\Pi_4$  customers in turn after service

come back to the system as  $\Pi_2$  ones. Flows  $\Pi_2$  and  $\Pi_3$  are conflicting in the sense that customers arriving from different sources cannot be served simultaneously in the same queuing system. This means that the problem cannot be reduced to a corresponding problem with fewer input flows by merging the flows together.

Given positive integers  $d, n_0, n_1, \dots, n_d$ , we introduce a finite set  $\Gamma = \{\Gamma^{(k,r)} : k = 0, 1, \dots, d; r = 1, 2, \dots, n_k\}$  of states in which server can be. At each state  $\Gamma^{(k,r)}$  server stays during time  $T^{(k,r)}$ . Define sets  $\Gamma^I, \Gamma^{II}, \Gamma^{III}$  and  $\Gamma^{IV}$  as follows. In state  $\gamma \in \Gamma^I$  only customers of queues  $O_1, O_2$  and  $O_4$  are serviced. In state  $\gamma \in \Gamma^{II}$  only customers of queues  $O_2$  and  $O_4$  are serviced. In state  $\gamma \in \Gamma^{III}$  only customers of queues  $O_1, O_3$  and  $O_4$  are serviced. In state  $\gamma \in \Gamma^{IV}$  only customers of queues  $O_3$  and  $O_4$  are serviced. The set  $\Gamma$  defines as union  $\Gamma = \Gamma^I \cup \Gamma^{II} \cup \Gamma^{III} \cup \Gamma^{IV}$  disjoint sets. We will also need following sets in the future  $^1\Gamma = \Gamma^I \cup \Gamma^{III}, ^2\Gamma = \Gamma^I \cup \Gamma^{II}, ^3\Gamma = \Gamma^{III} \cup \Gamma^{IV}$ .

Server changes its state according following rule. We will call the set  $C_k = \{\Gamma^{(k,r)} : r = 1, 2, \dots, n_k\}$   $k$ th cycle,  $k = 1, 2, \dots, d$  (Pic. ??). When  $k = 0$  state of the form  $\Gamma^{(0,r)}$  we will call prolongation state,  $r = 1, 2, \dots, n_0$ . Denote  $r \oplus 1 = r + 1$  for  $r < n_k$  and  $r \oplus 1 = 1$  while  $r = n_k, k = 0, 1, \dots, d$ . In cycle  $C_k$  we select subsets  $C_k^O$  input,  $C_k^I$  output and  $C_k^N = C_k \setminus (C_k^O \cup C_k^I)$  neutral states. Then after server was in state  $\Gamma^{(k,r)} \in C_k \setminus C_k^O$  it switches to the state  $\Gamma^{(k,r \oplus 1)}$  of the same cycle  $C_k$ . After state  $\Gamma^{(k,r)}$  in  $C_k^O$  server switches to the state  $\Gamma^{(k,r \oplus 1)}$ , if number of customers in the queue  $O_3$  in switching moment greater than predetermined threshold  $L$ . In other case, that is if number of customers in the queue  $O_3$  lesser or equal  $L$ , then new state is prolongation state  $\Gamma^{(0,r_1)}$ , where  $r_1 = h_1(\Gamma^{(k,r)})$  and  $h_1(\cdot)$  — given mapping of the set  $\bigcup_{k=1}^d C_k^O$  on  $\{1, 2, \dots, n_0\}$ . After state  $\Gamma^{(0,r)}$  the state of the same type  $\Gamma^{(0,r_2)}$  is chosen, if number of customers in queue  $O_3$  is lesser or equal  $L$ , where  $r_2 = h_2(r)$  and  $h_2(\cdot)$  — given mapping of the set  $\{1, 2, \dots, n_0\}$  on itself; in other case state of the form  $\Gamma^{(k,r_3)} \in C_k^I$  is on, where  $\Gamma^{(k,r_3)} = h_3(r)$  and  $h_3(\cdot)$  — given mapping  $\{1, 2, \dots, n_0\}$  on the set  $\bigcup_{k=1}^d C_k^I$ . We assume that all prolongation states  $\Gamma^{(0,r)}$  belong to the set  $^2\Gamma$ , and relations  $C_k^O \subset ^2\Gamma$  and  $C_k^I \subset ^3\Gamma$  are hold. We also assume that all the cycles have exactly one input and output state. And last assumption is that all the prolongation vertices form one cycle, that is we can put  $h_2(r) = r \oplus 1$ .

More formally server changes its states according to the following rule:

$$h(\Gamma^{(k,r)}, y) = \begin{cases} \Gamma^{(k,r \oplus 1)}, & \text{if } (\Gamma^{(k,r)} \in C_k \setminus C_k^O) \text{ or } (\Gamma^{(k,r)} \in C_k^O \text{ \& } y > L); \\ \Gamma^{(0, h_1(\Gamma^{(k,r)}))}, & \text{if } \Gamma^{(k,r)} \in C_k^O \text{ and } y \leq L; \\ \Gamma^{(0, r \oplus 1)}, & \text{if } k = 0 \text{ and } y \leq L; \\ h_3(r), & \text{if } k = 0 \text{ and } y > L. \end{cases} \quad (2)$$

In general service durations of different customers can be dependent and have different distributions. That is why instead of classic approach which specifies distributions of every particular customer, saturation flows will be used. Such a flow  $\Pi_j^{\text{sat}}$ ,  $j \in \{1, 2, 3, 4\}$ , is defined as virtual output flow given maximum loading of the server,

and for  $j \in \{1, 2, 3\}$  given also maximum loading of the corresponding queue. Saturation flow  $\Pi_j^{\text{sat}}$ ,  $j \in \{1, 2, 3\}$  contains non-random number of customers  $\ell_{k,r,j}$ , serviced during  $T^{(k,r)}$ , if server's state is  $\Gamma^{(k,r)}$ . Let  $\mathbb{Z}_+$  — be the set of non-negative integer numbers. Then given the fact, that queue  $O_4$  contains  $x \in \mathbb{Z}_+$  customers, saturation flow  $\Pi_4^{\text{sat}}$  is determined to contain all the  $x$  customers. Finally in the state  $\Gamma^{(k,r)}$  every customer from queue  $O_4$  with probability  $p_{k,r}$  and independently of others ends servicing and come to queue  $O_2$  of the flow  $\Pi_2$ . The customer of queue  $O_4$  stay on up to the next round with probability  $1 - p_{k,r}$ . On next round the picture is the same.

As we mentioned earlier in the end of the section we introduce real-life example of defined queuing system: tandem of two consecutive crossroads (Fig. ??). Flows of

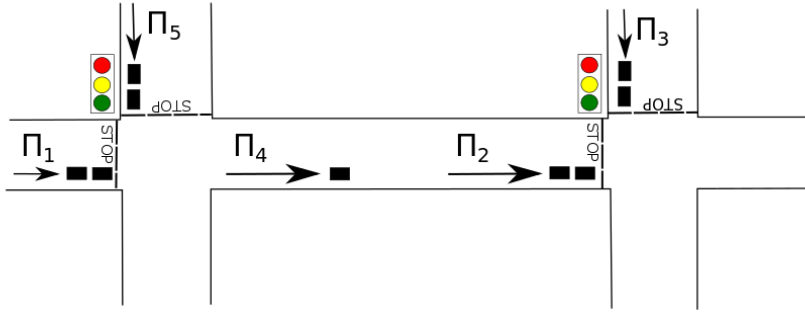


Figure 2: Example: crossroads tandem

arriving cars play role of customer flows formed by external environment: conflicting flows  $\Pi_1$  and  $\Pi_5$  on the first crossroad; flow  $\Pi_3$  on the second. Every car from flow  $\Pi_1$ , after passing first road intersection, comes to the queue  $O_4$  of the flow  $\Pi_4$ . Whereupon the car arrives to the next road intersection with some probability ( $p_{k,r}$  for the state  $\Gamma^{(k,r)}$ ) or does not have time to do so and stays for another round. In case it has time it comes in queue  $O_2$  and waits it's turn to pass it. Such system is an instance of more general queuing model described above.

The full source text using  $\text{\TeX}$  as well as all auxiliary materials, namely,

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$$P(n, t) = \frac{\partial^n B(t)}{\partial t^n} \quad (3)$$

can be referred as (3).

#### 5. Theorems and proofs

Theorems are defined like follows

**Theorem 1.** *Text of the theorem.*

*Proof.* Proof of Theorem 1. If using such reference, you need to recompile your paper with  $\text{\LaTeX}$  twice. ■

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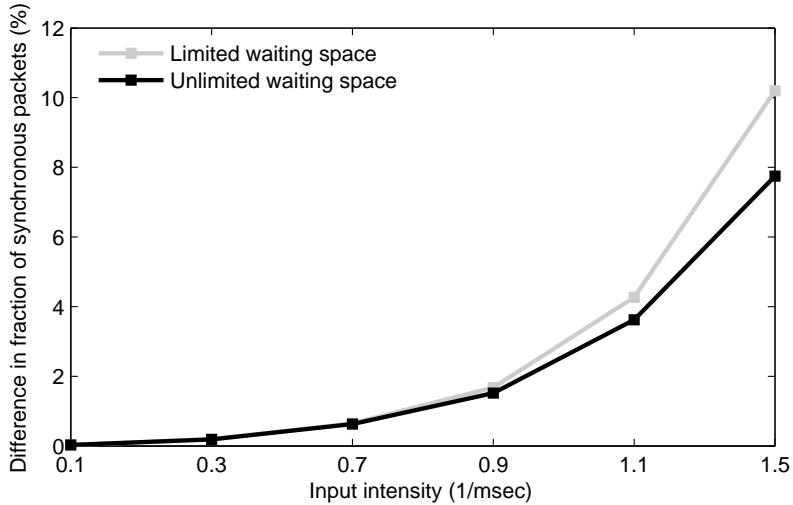


Figure 3: Difference in fractions of synchronous packets for  $W = 16$ .

Parameter	T	E	$\Delta$ , %	Parameter	T	E	$\Delta$ , %
$\rho_1^{(1)}$	0,187	0,194	3,7	$\rho_1^{(2)}$	0,127	0,120	5,6
$\rho_2^{(1)}$	0,073	0,072	1,4	$\rho_2^{(2)}$	0,052	0,053	1,9
$\rho_3^{(1)}$	0,148	0,147	0,7	$\rho_3^{(2)}$	0,103	0,103	0,0
$\rho_4^{(1)}$	0,036	0,036	0,0	$\rho_4^{(2)}$	0,026	0,027	3,7
$C^{(1)}$	0,479	0,476	0,6	$C^{(2)}$	0,656	0,640	2,5
$C_1^*$	0,341	0,339	0,6	$C_3^*$	0,323	0,329	1,8
$C_2^*$	0,296	0,298	0,7	$C_4^*$	0,286	0,286	0,0

Table 1: Comparison of system parameters.

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## 7. Conclusion

Place a full list of references [1, 2, 3, 4, 5] at the end of the paper. List the references according to the order of their appearance in the text.

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