

# Problem Set 3 Solutions

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## Question 1

To capture the life-cycle pattern of wages, consider the following multiple linear regression model

$$wage = \beta_0 + \beta_1 educ + \beta_2 exper + \beta_3 exper^2 + \epsilon$$

where *wage* is the wage rate measured in dollar per hour, *educ* is years of education, *exper* is years of work experience, *exper*<sup>2</sup> is squared years of experience (in the dataset is called *expersq*).

- (1) Estimate the equation and report the results. (For full credit give your answer as a formula and a word explanation)

**Solution:**

$$wage = -3.9649 + 0.5953educ + 0.2683exper - 0.0046exper^2$$

Standard interpretation. For example take the coefficient on *educ*. The estimated coefficient for *educ* ( $\beta_1$ ) is 0.5953. *pvalue* < 0.05, thus at 0.05 level of significance, the coefficient  $\beta_1$  is statistically significant. It says that on average, one extra year of education increases the wage of a person by 0.5963 dollar per hour, holding other variables constant.

- (2) What is the marginal effect of a year increase in the work experience for a person with 18 years of work experience?

**Solution:**

$$\frac{\partial wage}{\partial exper} \Big|_{educ} = 0.2683 - 2 * 0.0046 * exper = 0.2683 - 2 * 0.0046 * 18 = 0.1027$$

## Question 2

Load data via link below and store in dataframe *fare*, where *year* is 1997, 1998, 1999, 2000, *id* is route identifier, *dist* is the distance of the route measured in miles, *passen* is the average number of passengers per day, *fare* is the average one-way fare of the route measured in dollar per day, *conc* is the percent of market controlled by the biggest carrier on route

- (1) Generate a new variable called *yr00* which takes on values 1 if the observation is in 2000 and 0 otherwise (a dummy variable).

**Solution:**

skipped...

- (2) Estimate the model 1 using OLS and print the summary results.

**Solution:**

Model 1):  $\text{fare} = 46.1894 + 0.0888\text{dist} + 0.7336\text{conc} + \epsilon$

- (3) Consider the null hypothesis that  $\beta_2 = 0$  and  $\beta_3 = 0$  in Model 1. What is the alternative hypothesis? Conduct the test and state your decision.

**Solution:**

$F(2, 4593) = 1633$

$\text{Prob} > F = 0.0000$

The alternative hypothesis is that either of the coefficients is nonzero. The  $p$  value is very low so we reject the null in favor of the alternative implying that these coefficients are nonzero.

- (4) State in words and numbers how we should interpret the estimated coefficients in Model 1. (for full credit explain how you know if it is statistically significant)

**Solution:**

Standard interpretation. For example take the coefficient on *conc*.  $\beta_3 = 0.7336$ ,  $p\text{value} < 0.05$ , thus at 0.05 level of significance, the coefficient for *conc* ( $\beta_3$ ) is statistically significant. It refers to market concentration and thus on average, an 1% increase in the market concentration increases the fare by 0.7336 dollar, holding other variables constant.

- (5) Generate an interaction term called *yr00dist* by multiplying the variable *dist* by the variable you created in part (1),  $\text{yr00dist} = \text{yr00} * \text{dist}$

**Solution:**

skipped...

- (6) Estimate the model 2 using OLS and print the summary results.

**Solution:**

Model 2):  $\text{fare} = 41.6998 + 0.0893\text{dist} + 0.7438\text{conc} + 14.7778\text{yr00} - 0.0016\text{yr00dist} + \epsilon$

- (7) What is the base group for Model 2?

**Solution:**

Where the dummy variables take the 0 values - in this case, where  $\text{yr00} = 0$ , that is in all other years than 2000

- (8) State in words and numbers how we should interpret the estimated coefficients respectively in Model 2. (for full credit explain how you know if it is statistically significant)

**Solution:**

The coefficient of *yr00dist* is -0.0016. We can interpret that on average fare in 2000 decreases by 0.1% less than it does in other years than 2000 when distance increases by

one mile, holding other variables constant. However, the  $p$  value is 0.604 which is larger than 0.05, thus at the level of 0.05 significance, the interaction term is not statistically significant.