Quantitative Finance

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The Master's degree: Applied data science for banking and finance

Contents:

Part A)

1. Financial Derivatives

Financial Derivatives definition:

Derivatives are financial assets in the form of contracts between two parties and their value is based on the value movements on one or more underlying asset.

Underlying asset :

The underlying asset is the yardstick for the value of the derivatives (on which the derivative's value is calculated). Thus, the value of derivatives is not fixed, but it is derived from the underlying asset that the parties have decided to indicate in the contract.

The underlying asset could be: stock/s, bonds, currencies.

The price of the derivative is a function of the price of the underlying asset.

$$D_j(t) = f[S_j(t)]$$

$D_j(t)$ The price of derivative j at time t.

$S_j(t)$ Represents the underlying asset of derivative *j* at time *t*.

Financial Derivatives Classification

Derivatives can be classified according to different factors:

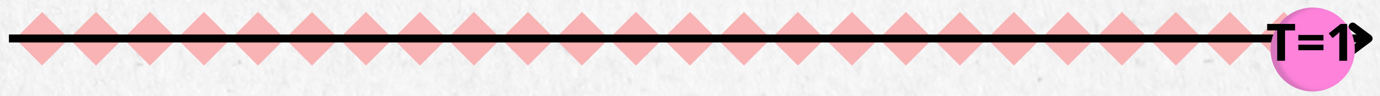
By Maturity:

European derivatives:

Contracts that the holder may exercise only at maturity T=1.

American derivatives:

American derivatives (depending on the type of the derivative) may be exercised at any time within T<=1



By consideration of the path of the underlying asset's price during the option's life:

Path independent derivatives:

Path-independent derivatives, also known as **European-style** derivatives, have ***payoffs that depend only on the price of the underlying asset at the expiration date.***

Path Dependent:

Derivatives with payoffs that ***depend on the entire price path of the underlying asset*** over the life of the derivative, not just its value at the expiration date.

Example: Barrier Options

Complexity and Standardization :

Plain Vanilla Derivatives:

Standardized financial instruments with terms and conditions that are ***well-defined and straightforward structures***. **Examples**: forwards, futures, swaps, options

(Exotic)NO plain vanilla derivatives:

Highly customized contracts designed to meet specific needs or hedge particular risks of investors and a more complex payoff structures. **Examples:** basket options, rainbow options.

Plain Vanilla Derivatives: Forwards

Forwards are agreements to buy or sell an asset at a future prearranged date for a price agreed upon today (delivery price).

- **Long forward contract---->** agreement to buy a financial asset in the future (pre-set t=1) at a certain price (said to be delivery price) fixed today (t=0).
- **Short forward contract---->** agreement to sell a financial asset in a future date (t=1) for the (delivery) price that we set today (t=0).
- **The execution of the forward contract** is mandatory (distinguishing it from options).
- **Highly customizable** and traded over-the-counter (distinguishes them from the futures)

Long position:

fj(0) → **fj(1)**

Today t=0 → **Future data t=1**

where:

fj(0): price of the forward today

and

fj(1):=Sj(1)-E

(final payoff)

Short position:

fj(0) → **fj(1)**

Today t=0 → **Future data t=1**

where:

fj(0): price of the forward today

and

fj(1):=E-Sj(1)

(final payoff)

Plain Vanilla Derivatives: Futures

- **A future has similar features of the forwards and is an agreement (obligation) to buy or sell a specified quantity of an asset at a specified price with delivery at a specified date in the future (used to hedge the price movement of an underlying asset to help prevent losses from unfavourable price changes). However there are some specificities that distinguish the futures from the other derivatives.**
- Futures contracts have standardized features (size, expiration, delivery specification) , while the forwards are tailored to meet the needs of the counterparties.
- Futures contracts are settled through a clearing house, while forwards are settled between the counterparties .
- Traded on organized exchanges (with clearing house), providing standardized contracts with set terms and conditions
- Finally, because of being exchange-traded, futures are regulated, whereas forwards, which are mostly over-the-counter (OTC) contracts, are loosely regulated (at least until the global financial crisis 2008).

Two examples of futures:

- **Energy Futures**: These provide exposure to the most common fuels and energy products, such as crude oil and natural gas.
- **Currency Futures:** provide exposure to changes in the exchange rates and interest rates of different national currencies.

Plain Vanilla Derivatives: Swaps

A swap is a derivative agreement where two parties exchange cash flows or obligations associated with two distinct financial instruments for a specified period .

- At the time the contract is initiated, **the value of at least one of the assets being swapped is determined by a random or uncertain variable**, such as an interest rate, a commodity price
- Swaps are customized contracts **traded in the over-the-counter market privately,** versus options and futures traded on a public exchange.

Three Examples of swaps:

1. **Equity swaps** are contracts where two counterparties agree to exchange the total return on a stock in exchange of a floating rate of interest. Equity swaps can be used to hedge the positions in equity without giving up ownership of his share. At the same time, the party receiving equity return enjoys exposure without actually taking ownerships of shares.

2. Interest rate swaps, are used to hedge against the risk of a floating interest rate on a loan. In this arrangement, a borrower enters into a derivative with a counterparty, agreeing to pay a fixed rate (for example) and receive the floating rate.

3. Credit default swaps, can be categorized as a distinct type of swaps. In these contracts, parties **don't hedge or speculate on the fluctuation in the value of an underlying asset**; rather, they focus on the **creditworthiness of an entity.** This contract serves to protect the buyer from losses stemming from the default of the issuer (of a bond) or a lender (loan). The contract involves payment of a periodic premium from the buyer to the seller

Plain Vanilla Derivatives: Options

An option is a contract between two parties , the *holder* and the *writer*, which gives the former <u>the right (but not the obligation)</u>, to buy from or sell to the writer a particular underlying security at a predetermined strike price (K).

- This specific feature of the options **limits the investor's possible loss to the cost of the option**
- The *holder* of the option is said to be in **long position** and the *writer* is said to be in *short position*
- Options are usually traded on public exchange.

There are **two** basic types of options and each comes in two styles:

1) Call Option

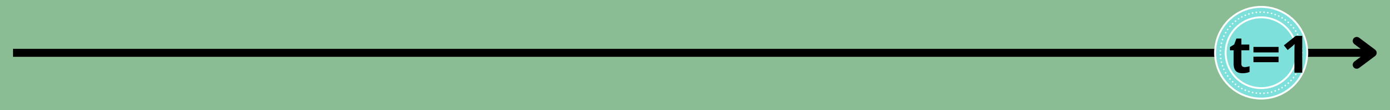
A call option gives the holder the right, but not the obligation, to buy the underlying security at the strike price on or before expiration. A call option will therefore become more valuable as the underlying security rises in price.

2) Put Option

Opposite to call options, a put gives the holder the right, but not the obligation, to instead sell the underlying stock at the strike price on or before expiration. **A long put**, therefore, **is a short position** in the underlying security (the put gains value as the underlying's price falls).

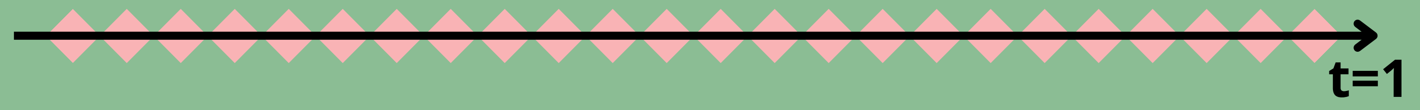
European Style:

The holder of the option (put and call) can buy or sell **on a given data <u>*(t=1)*</u> only (usually maturity)**.



American Style:

The holder of the option (put and call) **can buy or sell up to (till and including <u>*t<=1*</u>)** a given date



European Call Options

The option holder has the right to purchase a security (j) from the writer at a predetermined strike price (E) at the contract's maturity (*T*). The holder of a call option will exercise the call option if at maturity, the price of the security is greater than the strike price.

Payoff at Maturity can be expressed as max (*Sj(t=1)*−E, 0)

- **If Sj (t=1)≥E**, the holder will exercise the option and receive a payoff of Sj (t=1)−E.
- **If Sj (t=1)<E**, the holder will not exercise the option because he/she will sustain a greater cost.

We can say that at time t, the option is:

- **In-the-Money** if Sj (t)>E, so the option has intrinsic value.
- **At-the-Money** if Sj (t)=E, so the option's strike price equals the current asset price.
- **Out-of-the-Money** if Sj (t)<E, so the option has no intrinsic value.

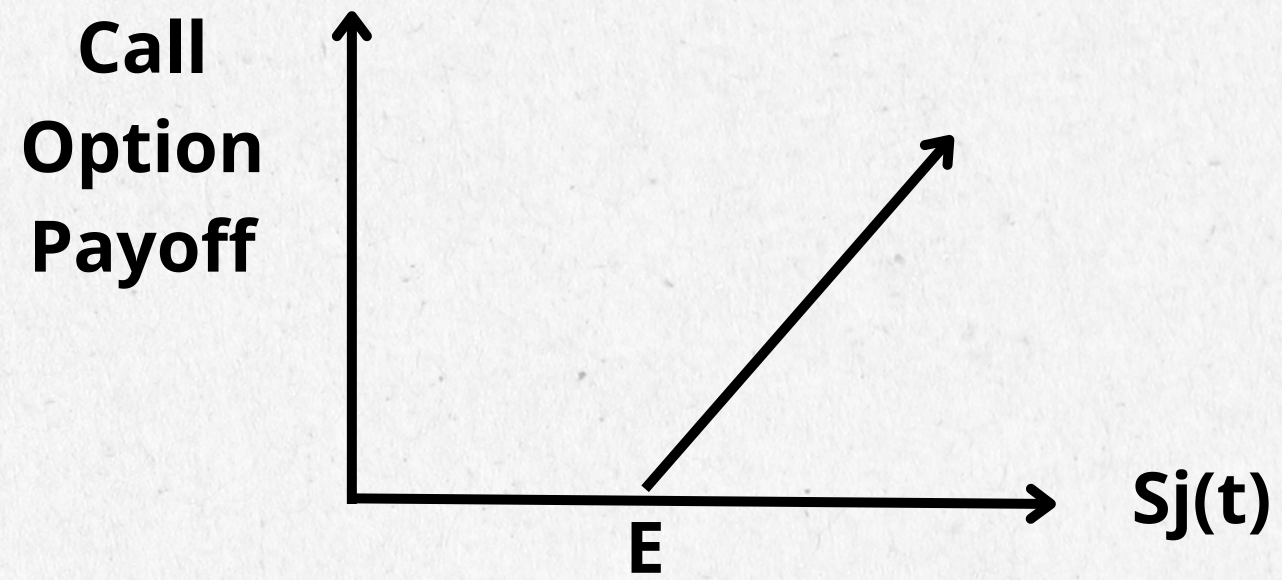
t=0 → **t=1**

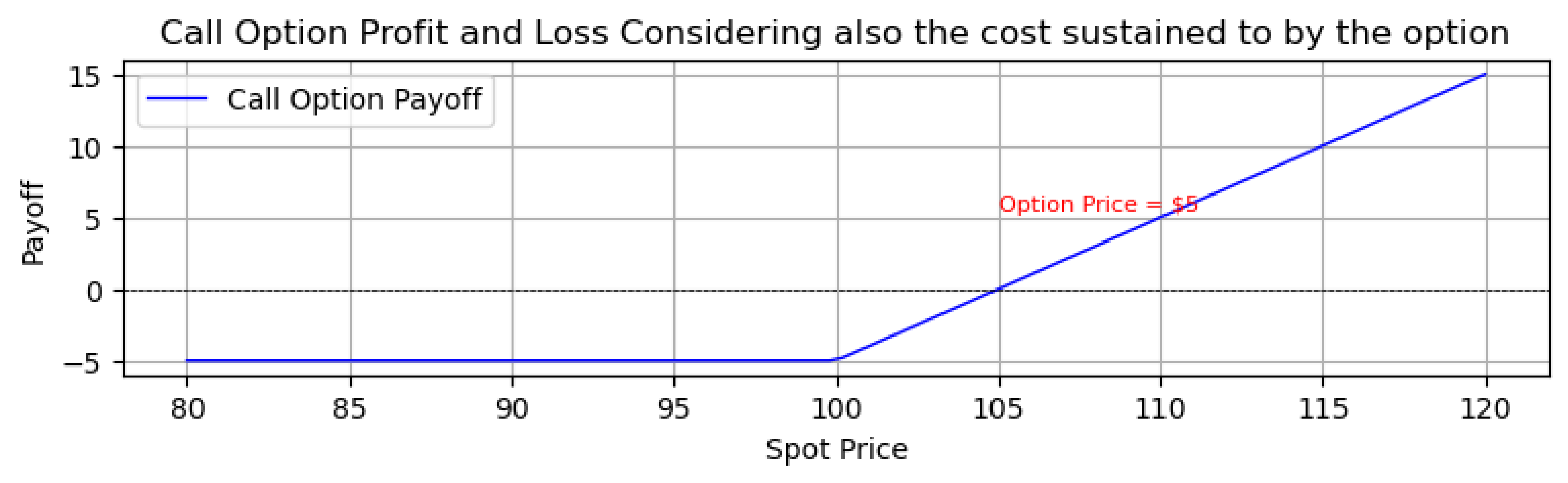
$C_j(t=0)$

$S_j(1) - E$

The final payoff at $t = 1$ will be:

$$C_{j(1)} = \max\left[S_j(1) - E, 0\right]$$





Assuming option execution

Since the **holder** of the option ***has a right with no obligation,*** and the **writer** instead ***has a potential obligation***, the option has a value and therefore a price, called a *premium*.

To find the initial price (premium) of a call option we could follow two strategies **two (A and B) strategies:** :

A) The initial cost (price) of the call option (Cj(0)) will be ***equal to the initial cost (VΘ (0)) of the replication portfolio (Θ) by the Law of One Price:***

Assuming:

1)The final payoff of the option=final payoff of the replicating portfolio (we replicating the payoff)

$$\widetilde{C}_j(1) = V\theta_C(1)$$

2)The payoff of the replicating portfolio

$$V\theta_C(1)$$

The existing replicatiing portfolio θ_C

3 We can find the replicating portfolio (M(1) shall be invertible (det. different than 0)

$$\theta_C = \left[\widetilde{M}(1)\right]^{-1} V\theta_C(1)$$

From the law of one price:

$$C_j(0) = V\theta_C(0)$$

We find the price of the put option

$$C_j(0) = [-M(0)]\theta_C$$

B) The initial cost of the call option Cj(0) is e***qual to the expected value Cj (1) under the risk probability measure (Q)***

$$C_j(0) = \frac{1}{1+r} \cdot \mathbb{E}^Q[\widetilde{C}_j(1)]$$

European Put Options

The option holder has the right to sell a security (j) to the writer at a predetermined strike price (E) at the contract's maturity (T). The holder of a put option is hoping the asset price will fall (the opposite of the call option case).

Payoff at Maturity can be expressed as E-Sj (t=1) :

- **If Sj (t=1)≤E,** the holder will exercise the option and receive a payoff of E-Sj (t=1)
- **If Sj (t=1)>E,** the holder will not exercise the option because they would sell the security for a smaller amount of money than the current price of the security.

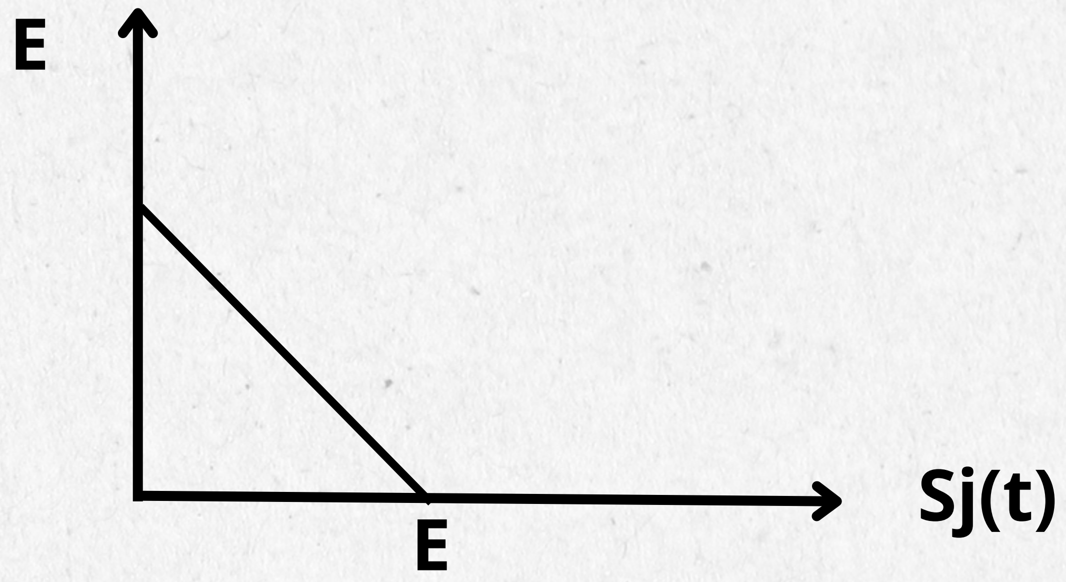
We can say that at time t, the option is:

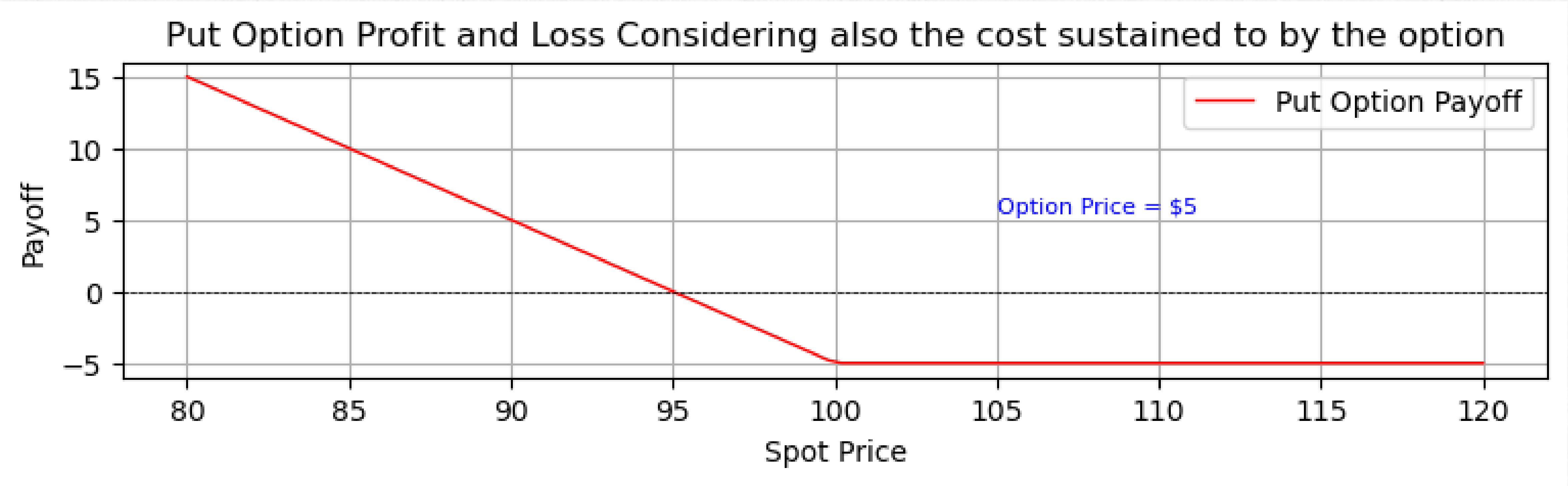
- **In-the-Money if Sj (t)<E,** so the option has intrinsic value.
- **At-the-Money if Sj (t)=E,** so the option's strike price equals the current asset price.
- **Out-of-the-Money if Sj (t)>E,** so the option has no intrinsic value.

t=0 t=1

$P_j(0)$ $P_j(1)$

The final payoff at t=1 will be
$P_j(1) = max[E - S_j\ (1), 0]$





Assuming option execution

Since the **holder** of the option ***has a right with no obligation,*** and the **writer** instead ***has a potential obligation***, the option has a value and therefore a price, called a *premium*.

To find the initial price (premium) of a put option we could follow **two (A and B)** strategies:

A) The initial cost of the put option (Pj(0)) will be equal to the initial cost (VΘ (0)) of the replication portfolio (Θ):

Assuming:

1 The final payoff of the option=final payoff of the replicating portfolio

$$\widetilde{P}_j(1) = V\theta_p(1)$$

2 The payoff of the replicating portfolio

$$V\theta_p(1)$$

The existing replicatiing portfolio θ_p

3 We can find the replicating portfolio

$$\theta_p = \left[\widetilde{M}(1)\right]^{-1} V\theta_p(1)$$

From the law of one price:

$$P_j(0) = V\theta_p(0)$$

We find the price of the put option

$$P_j(0) = [-M(0)]\theta_p$$

B) The initial cost of the put option Cj(0) is equal to the expected value Pj(1) under the risk probability measure (Q)

$$P_j(0) = \frac{1}{1+r} \cdot \mathbb{E}^Q[\widetilde{P}_j(1)]$$

Hedging and Speculation with Options

***Hedging*: refers to strategic activities relating to attempts to reduce the amount of risk, or volatility, associated with a security's price change.**

E.g. an airline company could take a long position in a call option that gives the company the right to buy fuel at a pre stablished price K. The call option reduces the risk associated with the price of fuel for the company.

***Speculation*: refers to a strategy attempting to make a profit from a security's price change and is more vulnerable to market fluctuations.**

E.g. If the speculator believes that an asset will decrease in value, they would instead purchase put options with a strike price that is higher than the anticipated price level. If the price of the asset does fall below the put option's strike price, the speculator can sell the put options for a price that is equal to the difference between the strike price and the market price, plus any remaining time value, to realize a gain.

It is important to note that while the holder of the option has only the price of the option to lose, the writer stands a significantly greater loss .

To hedge against this loss, the writer could could use the money acquired from selling the call option and start a portfolio ***(hedging strategy)*** with a maturity sufficiently large to settle the claim of the holder.

E.g. **The writer of a call option** could take a long position in one share of the security, resulting in a portfolio called a ***covered call.*** It involves selling a call option while simultaneously holding a long position in the underlying security to hedge against potential losses. If the call option is exercised, the writer uses the held shares to fulfill the obligation at the agreed-upon strike price. At time T the writer's net profit is ***S(T)-ce^rT,*** the value of of the security less the loan. If the option is exercised, the holder can use the portfolio to settle the obligation of ***S(T)-K .***

Put and Call Combination Portfolio

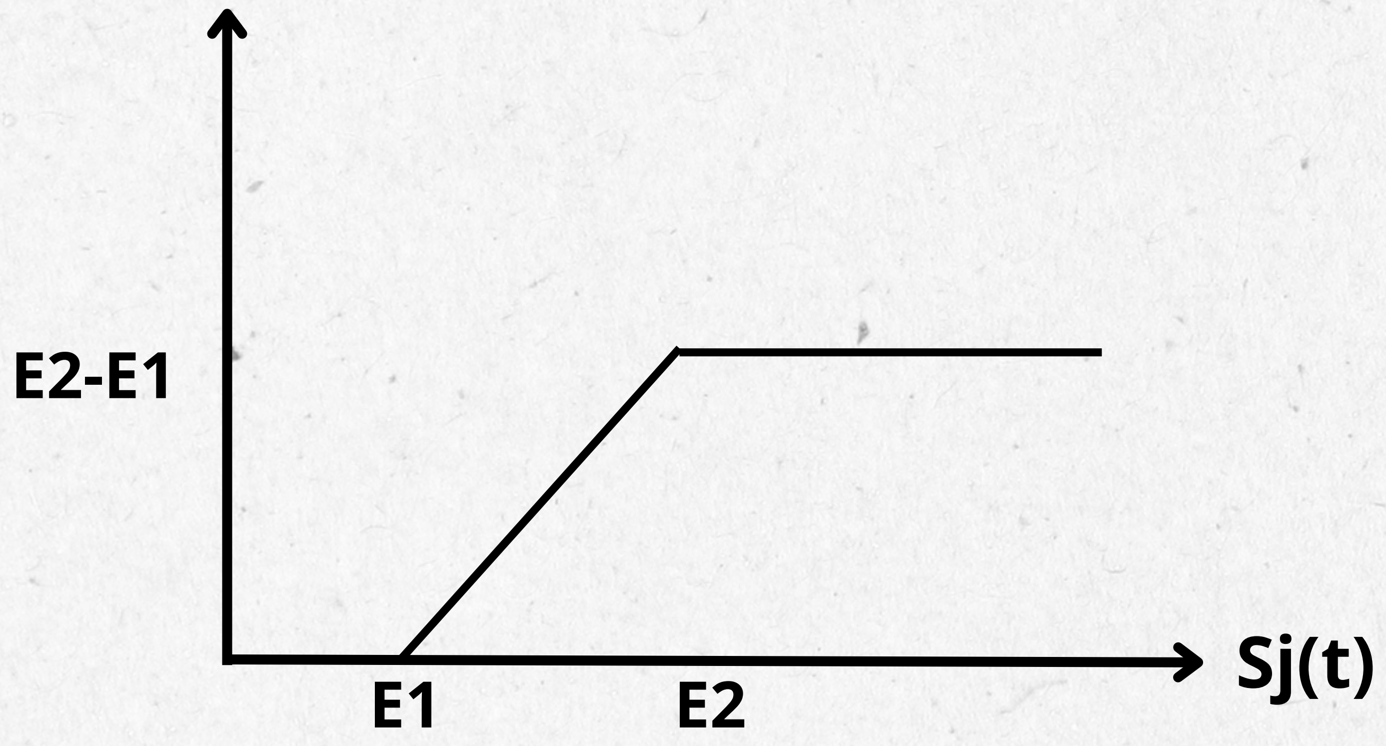
Put and call ***combination portfolios involve using both put and call options in various combinations to achieve specific investment objective, payoffs and hedging effects.*** For example, consider a portfolio that consists of **a long position in a call option with strike price E_1 and a short position in a call option with strike price E2> E1** each with underlying security j and maturity T. Such a portfolio is called a **bull call spread**.

A bull call spread is an options trading strategy that involves **purchasing a call option while simultaneously selling another call option with the same expiration date but a higher strike price.** This strategy is also known as a "debit call spread" or "call vertical spread."

How a bull call spread portfolio works:

1. The investor buys a call option, giving them the right to purchase the underlying asset at a specific strike price (lower strike).
2. Simultaneously, the investor sells a call option with the same expiration date but a higher strike price (higher strike).

This call option sold helps offset the cost of buying the initial call option.



Payoff of bull call spread

$$(S_j(t) - E_1)^+ - (S_j(t) - E_2)^+ \begin{cases} 0 & if S_j(t) \le E_1 \\ S_j(t) - E_1 & if E_1 \le S_j(t) \le E_2 \\ E_2 - E_1 & if E_2 < S_j(t) \end{cases}$$

A **bear call spread** is an options trading strategy that involves selling a call option (c1) and simultaneously buying another call option (c2) with the same expiration date but a higher strike price. This strategy is also referred to as a “credit call spread” or “call vertical spread.”

How a bear call spread portfolio works:

1. The investor sells a call option, granting someone else the right to buy the underlying asset at a specific strike price (lower strike).
2. Simultaneously, the investor buys a call option (c2) with the same expiration date but a higher strike price (higher strike).

This purchased call option ***limits*** the potential losses from the obligation created by selling the initial call.

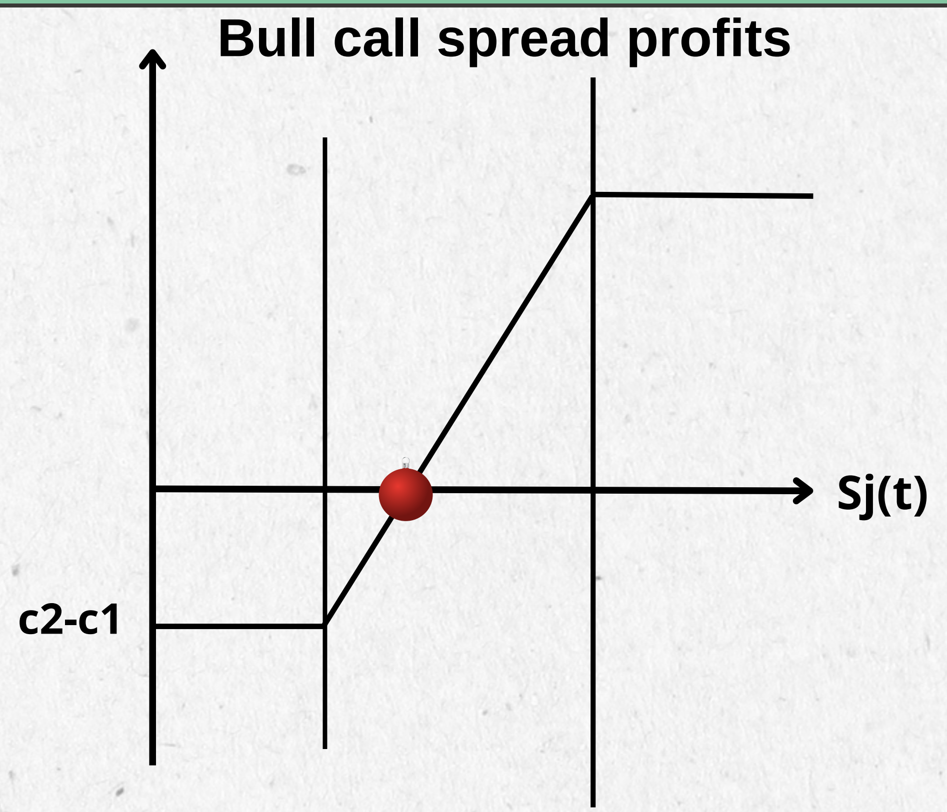
The payoff in this case is simply the negative of that of the bull call spread, hence the payoff diagram is the reflection in the Sj(t) axis of that of the bull call spread.

cj: denotes the cost of the Kj-call.

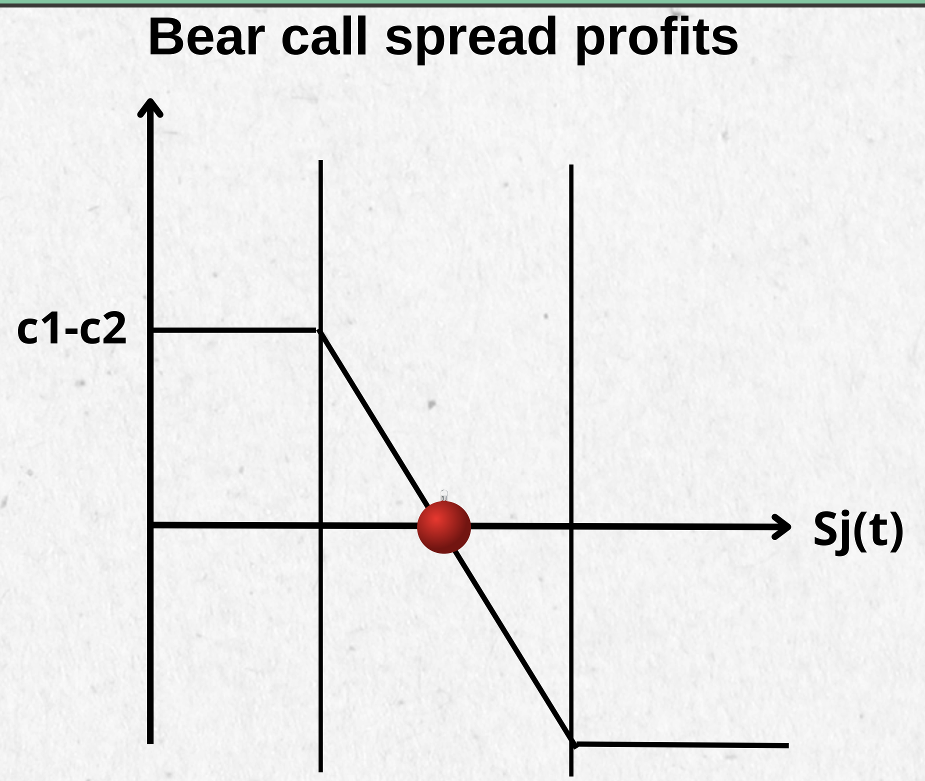
: Breakeven: the profit and loss are zero

(bull) : The spread profits **as the underlying asset's price moves above the higher strike** price (the strike price E2 of the call option that the investor buys)**. The maximum profit is** capped at the difference between the strike prices minus the net premium paid for the options.

(bear): Profits in a bear call spread occur when **the price of the underlying asset remains below the lower strike price.** The maximum profit is capped at the net premium received for the options.



Considering the fact that the option with a lower strike price produces a greater payoff, we see that in the “bull” case we start with a **positive cost** which is the difference between premiums of the two options (c2-c1) negative value.



The difference in premiums is often referred to as the **"credit"** when you receive **more premium from the options you sell than you pay for the options you buy.** Considering the fact that the option with a lower strike price produces a greater payoff, we see that in the case of bear initially we have **credit** (c1-c2)

Put and Call Parity Formula

The put-call parity is a financial principle that defines a relationship between the price of European call options and European put options of the same class (with the same strike prices and expiration dates).

Put-call parity states that simultaneously holding **a short European put and long European call of the same class** will deliver the same return **as holding one forward contract on the same underlying asset**, with the same expiration, **and a forward price equal to the option's strike price .**

The payoff from the call option

$$\widetilde{C}_j(1) = \max[\widetilde{S}_1(1) - E, 0]$$ **for j=1,...N**

The payoff from the put option

$$\widetilde{P}_j(1) = \max[E - \widetilde{S}_1(1), 0]$$ **for j=1,...N**

So our financial position is:

$$\widetilde{C}_j(1) - \widetilde{P}_j(1) = \max[\widetilde{S}_j(1) - E, 0] - \max[E - \widetilde{S}_j(1), 0]$$

Considering the property we can do the following:

$$\widetilde{C}_j(1) - \widetilde{P}_j(1) = \max[\widetilde{S}_j(1) - E, 0] + \min[S_j(1) - E, 0]$$

If $\widetilde{S}_j(1) \geq E$ **then** $\widetilde{C}_j(1) - \widetilde{P}_j(1) = \widetilde{S}_j(1) - E + 0$ $\quad \forall w_k \in \Omega$

If $\widetilde{S}_j(1) < E$ **then** $\widetilde{C}_j(1) - \widetilde{P}_j(1) = 0 + \widetilde{S}_j(1) - E,$ $\quad \forall w_k \in \Omega$

We move from our financial position: we buy 1 unit of the call option and we sell 1 unit of the put option

We see that in both cases the initial financial position **is equivalent (has the same final payoff) to buying** the risky security j (or holding a forward contract on j) and selling a riskless security (***assuming riskless security's price=E (strike price)*** *if not a forward price (selling) equal to the option's strike price)*.

From the Law of One price, the two financial positions have (today the same financial price.

$$C_j(0) - P_j(0) = S_j(0) - \mathbb{E}^{\mathbb{Q}}[E] \cdot \frac{1}{(1+r)}$$

This formula is said to be the put call parity

Put and Call Parity Formula

Moving from this one $\widetilde{C}_j(1) - \widetilde{P}_j(1) = \widetilde{S}_j(1) - E$

$$\widetilde{C}_j(1) = \widetilde{P}_j(1) + \widetilde{S}_j(1) - E$$

A long position on a call option can be replicated ***by buying 1 unit of a put option, 1 unit of a risky security j and selling one unit of the riskless security***

$$\widetilde{P}_j(1) = \widetilde{C}_j(1) - \widetilde{S}_j(1) + E$$

A long position on a put option can be replicated ***by buying 1 unit of a call option, 1 unit of a riskless security and selling one unit of the risky security j.***

$$\widetilde{S}_j(1) = \widetilde{C}_j(1) - \widetilde{P}_j(1) + E$$

A long position on risky security j can be replicated by buying ***1 unit of a call option, 1 unit of a riskless security and selling one unit of put option on j.***

$$E = \widetilde{S}_j(1) - \widetilde{C}_j(1) + \widetilde{P}_j(1)$$

A long position on riskless security j can be replicated ***by buying 1 unit of risky security j , 1 unit of put option on j and selling one unit of call option on j.***

$$\widetilde{C}_j(1) - \widetilde{P}_j(1) = \widetilde{S}_j(1) - E$$

$$\tilde{f}_j(1) = \widetilde{S}_j(1) - E \quad \text{for every } w_k \in \Omega$$

This implies that ***our financial position*** (long position on a call option and short position on a put option ***is equivalent to a forward contract*** as stated in the previous slide.

Theoretical implications of the put-call parity formula

Price Relationships: Put-call parity establishes a clear relationship between the prices of European call and put options with the same strike price and expiration date. It ensures that the prices of these options are interrelated in a specific way.

Arbitrage-Free Pricing: When one side of the put-call parity equation is greater than the other, this represents an arbitrage opportunity. You can sell the more expensive side of the equation and buy the cheaper side to make, for all intents and purposes, a risk-free profit.

Synthetic Positions: Put-call parity allows for the creation of synthetic positions. For example, by holding a long call an investor can replicate the payoff of a portfolio with a long position on one unit of put option and risky security and short position on a riskless security.

Market Efficiency: Put-call parity contributes to the efficiency of financial markets. If there were discrepancies in option prices that violate put-call parity, sophisticated investors could exploit these discrepancies, leading to market corrections.

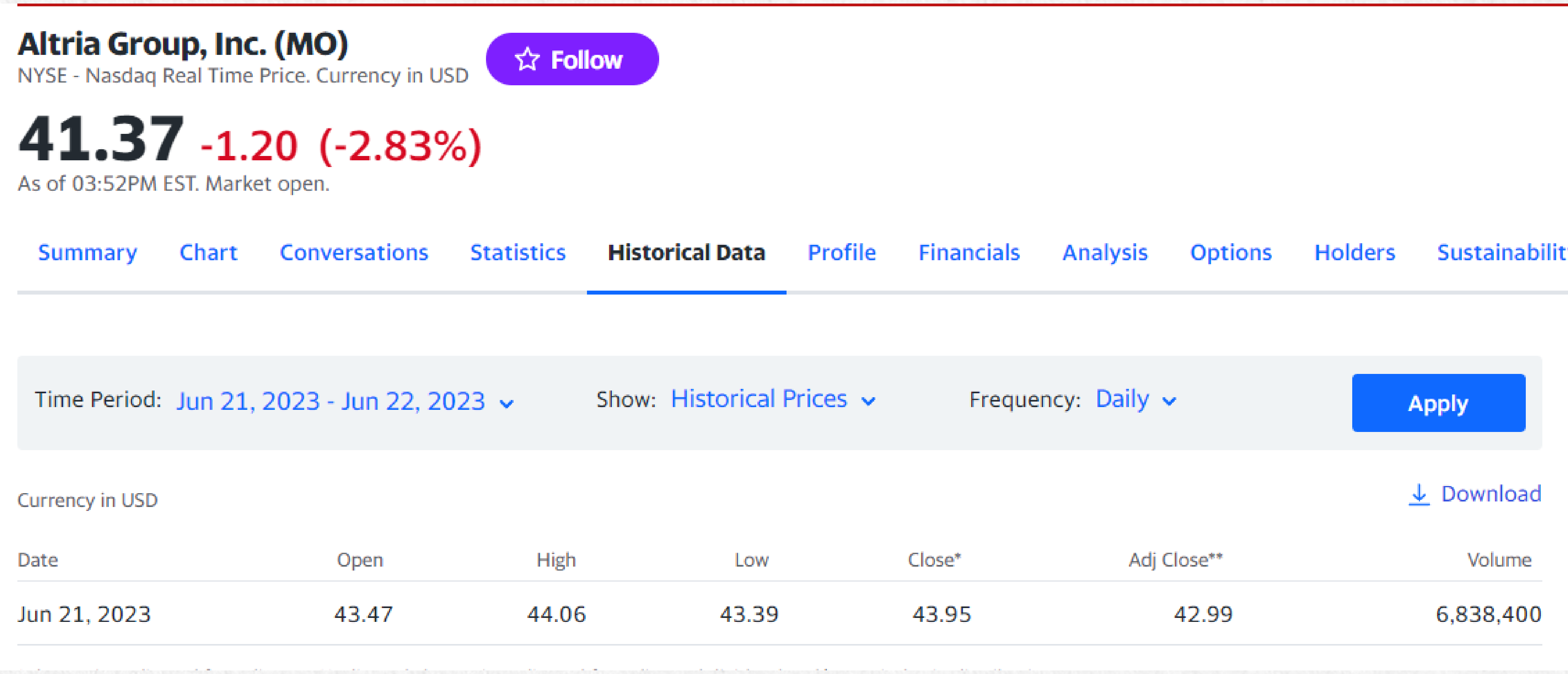
Part B) Pricing of contingent claims: Data

We are using the treasury bill rates from U.S. DEPARTMENT OF THE TREASURY

52 WEEKS Bill rate June 2023: COUPON EQUIVALENT: 5.26% (you can click on the text to check the rates)

Contract Name: MO240621C00040000 (you can click on the text to check the contract)
Strike Price: 40
Implied Volatility: 14.49%
Expiration Date of the call option : 21/06/2024

The Closing of the underlying asset Price 21/06/2023 : 43.95 $



Throughout the project we use the continuous interest rate that is derived from the one-year treasury bill (coupon equivalent interest rate).

The implied volatility is taken directly from the option contract.

!Considering the lack of put options for the company, we assumed the same strike for a put option

Legend for the Excel File:
S: price risk security
c: call option
p: put option
pi and **(1-pi):** synthetic probabilities

Note that for each period we have created ***different sheets within the Excel file***

Pricing of contingent claims: 1 periods

For the one period pricing we use two strategies

1) The Binomial Model
2) Replicating Portfolio Strategy

We see that in both cases the result is the same

Click here to see the manual resolution of the exercise

Click here to access the Excel file with the formulas

Strike Price	**Implied Volatility**	**Price underlyimg asset**	**Simple interest Rate**
40,00	0,1449	43,95	5,26%
Continous Interest			
0,05126329394			
Up Factor	**Down Factor**	**pi**	**1-p**
1,216725573	0,9106135225	0,4638382489	0,5361617511
	S1		
S0	53,47508893		
43,95			
	40,02146432		
Using the Binomial Model			
	c1		
c0	13,48		
5,948859966			
	0,02		
Using Replicating Portfolio			
Delta (amount of risky asset)	**Amount of riskless asset**		
1	-38,00114003		
	we are taking a loan	**c0=delta*S0+b**	**5,948859966**

The result from the replicating portfolio

Pricing of contingent claims: 2 periods

When pricing the contingent claim while considering two steps we use the Binomial model to find both the call option and the put option

Click here to see the manual resolution of the exercise

Click here to access the Excel file with the formulas

Strike Price	**Implied Volatility**	**Price underlyin**	**Simple interest Rate**			
40,00	0,1449	43,95	5,26%			
Continous Interest						
0,05126329394						
		Discount Factor				
		0,974694055				
Up Factor	**Down Factor**	**Pi**	**1-Pi**	Check		
1,136656911	0,926049004	0,4744074422	0,5255925578	1		
		S1				
	S1	56,78291358				
S0	49,95607122					
43,95		46,26177				
	40,69985373					
		37,69005901				
		c2				p2
	c1	16,78			p1	0,00
c0	10,96830902			p0	0	
6,555087887		6,26		0,6062279208		0,00
	2,895455682				1,183364156	
		0,00				2,31

Pricing of contingent claims: 4 periods

As for the previous ones we used the Binomial model for the option pricing

Even though our option is American, we decided to check the put-call parity formula.

Unfortunately the parity is absent

Put Call Parity Formula	
5,948859966	c(0)-p(0)
4,46	S(0)-Strike*Discount Factor

We remind that we assumed a put option considering the lack of such. Therefore such an option does not exist in reality and it could be that assuming such a put option we are violating the 'no-arbitrage'condition.

We also checked the put-call parity formula for an easier example (2 periods) and we saw that neither the parity is present.

Click here to access the Excel file with the formulas

Click here to see the manual resolution of the exercise

				S4
			S3	61,813137
		S2	56,76103544	
	S1	52,12185127		53,475089
S0	47,86183619		49,10447124	
43,95		45,09107219		46,261770
	41,40569566		42,48071017	
		39,00868334		40,021464
			36,75043619	
				34,622921
				c4
			c3	21,813137
		c2	17,27039746	
	c1	13,13408907		13,475089
c0	9,370546124		9,61383326	
6,316948921		6,103309993		6,261770
	3,634022226		2,990072193	
		1,427776998		0,021464
			0,01021183975	
				0,000000
				p4
			p3	0,000000
		p2	0	
	p1	0		0,000000
p0	0		0	
0,3680889547		0		0,000000
	0,7196166356		0	
		1,406855858		0,000000
			2,750413633	
				5,377079

Pricing of contingent claims: 4 periods

January 19, 2024 | In The Money | Show: **List** | Straddle | Option Lookup

Calls for January 19, 2024

Contract Name	Last Trade Date	Strike	Last Price	Bid	Ask	Change	% Change	Volume	Open Interest	Implied Volatility
E240119C00030000	2023-12-08 10:33AM EST	30.00	2.53	2.50	2.85	+0.21	+9.05%	1	2	33.11%
E240119C00035000	2023-11-29 9:30AM EST	35.00	0.25	0.00	0.35	0.00	-	-	3	27.54%

Puts for January 19, 2024

Contract Name	Last Trade Date	Strike	Last Price	Bid	Ask	Change	% Change	Volume	Open Interest	Implied Volatility
E240119P00032500	2023-12-06 11:08AM EST	32.50	0.95	0.55	2.35	0.00	-	-	17	50.73%

To make sure that we have exhausted all possible options at hand, we took an **European option contract.** E240119C00030000

What we immediately saw is that **none of the options with the same maturity (put and call) were of the same class (same strike price)** and as expected (considering that the condition for parity is violated) the put-call parity is absent even in the case of the European option (You can see in the Excel file).

We adjusted all the input values according to the maturity and the contract (volatility, interest rate, strike price, continuous interest rate).

Click here to access the Excel file with the formulas

				S4
			S3	94,82398516
		S2	72,44180838	
	S1	55,34270251		68,09626632
S0	42,27965576		52,02287868	
32,3		39,74344047		48,9022
	30,36243091		37,35936426	
		28,54109013		35,11830081
			26,82900549	
				25,21962308

				c4
			c3	64,82398516
		c2	45,396634	
	c1	30,96132061		38,09626632
c0	20,29969826		24,97770431	
12,75600132		15,36205857		18,9022
	8,937949708		10,31418988	
		5,298028312		5,118300812
			2,11655464	
				0

				p4
			p3	0
		p2	0	
	p1	0		0
p0	0		0	
0,271060764		0		0
	0,5554762932		0	
		1,138320086		0
			2,33272353	
				4,780376917

Pricing of contingent claims: 8 periods

Here you can see the pricing for call and put options for 8 periods.

To access the entire data with all the estimations you can use the following link.

We have used the Recombinant Binomial model (for simplicity)

[Click here to access the Excel file with the formulas](#)

								c8^0
							c7^0	29,6968
						c6^0	26,04874442	
					c5^0	22,61764817		22,9093
				c4^0	19,39131841		19,64143342	
			c3^0	16,35820609		16,56919911		16,7829
		c2^0	13,6648309		13,6816321		13,85810186	
	c1^0	11,17388014		11,27726367		11,10978154		11,2530
c0	8,871369562		8,947401892		9,134339873		8,637983183	
6,819960507		6,795056733		6,846056911		7,372058784		6,2617
	4,956492913		4,835381569		4,75787198		3,926228584	
	c1^1	3,271880806		2,985734903		2,333881416		1,7565
		c2^2	1,827476985		1,339526907		0,8503130722	
			c3^3	0,7499713233		0,4116224475		0,0000
				c4^4	0,1992595961		0	
					c5^5	0		0,0000
						c6^6	0	
							c7^7	0,0000
								c8^8

								p8^0
							p7^0	0,000000
						p6^0	0,00	
					p5^0	0,00		0,000000
				p4^0	0,00		0,00	
			p3^0	0,00		0,00		0,000000
		p2^0	0,00		0,00		0,00	
	p1^0	0,04		0,00		0,00		0,000000
p0	0,19		0,08		0,00		0,00	
0,5139474926		0,34		0,16		0,00		0,000000
	0,83		0,58		0,31		0,00	
	p1^1	1,31		0,99		0,60		0,000000
		p2^2	2,01		1,66		1,18	
			p3^3	3,00		2,69		2,309941
				p4^4	4,31		4,17	
					p5^5	5,90		5,980408
						p6^6	7,63	
							p7^7	9,293424
								p8^8

Pricing of contingent claims: 16 periods

<u>**We have used the Recombinant Binomial model (for simplicity) . (Months are considered as non discrete).**</u>

Here you can see the prices of the underlying asset (a stock) of the option throughout the 16 periods.

																S16^0
															S15^0	82,59225558
														S14^0	79,399094	
													S13^0	76,329386		76,820069
												S12^0	73,378359		73,850070	
											S11^0	70,541423		70,994897		71,451287
										S10^0	67,814168		68,250110		68,688855	
									S9^0	65,192353		65,611442		66,033224		66,457717
								S8^0	62,67190286		63,07478833		63,48026375		63,88834575	
							S7^0	60,248897		60,636207		61,026006		61,418310		61,813137
						S6^0	57,91956949		58,291905		58,666633		59,043771		59,423333	
					S5^0	55,68029752		56,038238		56,398479		56,761035		57,125923		57,493156
				S4^0	53,52759972		53,87170116		54,218015		54,566554		54,917335		55,270370	
			S3^0	51,45812899		51,78892687		52,121851		52,456916		52,794134		53,133521		53,475089
		S2^0	49,46866762		49,78667627		50,10672924		50,428840		50,753021		51,079286		51,407648	
	S1^0	47,55612231		47,86183619		48,16951535		48,479172		48,790820		49,104471		49,420139		49,737835
S0	45,71751935		46,0114138		46,30719754		46,60488273		46,904482		47,206006		47,509470		47,814884	
43,95		44,23253197		44,5168802		44,80305636		45,091072		45,380940		45,672670		45,966276		46,261770
	42,52242484		42,79577966		43,07089175		43,34777239		43,626433		43,906885		44,189140		44,473209	
	S1^1	41,14121989		41,40569566		41,67187161		41,939759		42,209368		42,480710		42,753797		43,028639
		S2^2	39,80487896		40,06076408		40,31829416		40,577480		40,838332		41,100860		41,365077	
			S3^3	38,5119448		38,75951831		39,008683		39,259450		39,511829		39,765830		40,021464
				S4^4	37,26100746		37,50053934		37,741611		37,984232		38,228414		38,474164	
					S5^5	36,05070282		36,282454		36,515696		36,750436		36,986686		37,224454
						S6^6	34,87971106		35,103935		35,329600		35,556716		35,785292	
							S7^7	33,746755		33,963696		34,182031		34,401770		34,622921
								S8^8	32,65059983		32,86049376		33,071737		33,28433821	
									S9^9	31,590050		31,793126		31,997507		32,203203
										S10^10	30,563948		30,760428		30,958171	
											S11^11	29,571176		29,761274		29,952594
												S12^12	28,610651		28,794574	
													S13^13	27,681325		27,859274
														S14^14	26,782186	
															S15^15	25,912252
																S16^16

Click here to access the Excel file with the formulas

Pricing of contingent claims: 16 periods

By using the Backward Induction Strategy we get the Present Value of the Call Option.

In the Excel file you can see the values also for the Put option.

Click here to access a PDF with the Call Option Tree

Click here to access the Excel file with the formulas

																c16^0
															c15^0	42,592256
														c14^0	39,527048	
													c13^0	36,584883		36,820069
												c12^0	33,760991		33,978024	
											c11^0	31,050785		31,250394		31,451287
										c10^0	28,449854		28,632743		28,816808	
									c9^0	25,953959		26,120804		26,288721		26,457717
								c8^0	23,559025		23,710474		23,862897		24,016299	
							c7^0	21,261135		21,397812		21,535368		21,673807		21,813137
						c6^0	19,056522		19,179027		19,302319		19,426404		19,551286	
					c5^0	16,941567		17,050475		17,160084		17,270397		17,381420		17,493156
				c4^0	14,914381		15,008654		15,105137		15,202240		15,299968		15,398323	
			c3^0	12,977766		13,053337		13,134089		13,218521		13,303496		13,389018		13,475089
		c2^0	11,141399		11,191861		11,249872		11,315962		11,388707		11,461919		11,535602	
	c1^0	9,421487		9,440600		9,467163		9,503610		9,552426		9,613833		9,675636		9,737835
c0	7,837751		7,822158		7,811157		7,807559		7,815646		7,841692		7,892102		7,942837	
6,409043		6,359769		6,310605		6,263308		6,221066		6,189924		6,182032		6,221773		6,261770
	5,071566		4,989499		4,903219		4,813482		4,722435		4,635942		4,571773		4,601162	
	c1^1	3,861164		3,746848		3,622427		3,486319		3,336924		3,174083		3,009294		3,028639
		c2^2	2,797312		2,655238		2,496581		4,722435		2,105157		1,846127		1,493030	
			c3^3	1,899191		1,739197		1,557794		1,346308		1,087531		0,735980		0,021464
				c4^4	1,182015		1,019721		0,836230		0,622943		0,362778		0,010504	
					c5^5	0,652109		0,507220		0,349572		0,178811		0,005140		0,000000
						c6^6	0,301685		0,193111		0,088130		0,002516		0,000000	
							c7^7	0,105365		0,043434		0,001231		0,000000		0,000000
								c8^8	0,021405		0,000602		0,000000		0,000000	
									c9^9	0,000295		0,000000		0,000000		0,000000
										c10^10	0,000000		0,000000		0,000000	
											c11^11	0,000000		0,000000		0,000000
												c12^12	0,000000		0,000000	
													c13^13	0,000000		0,000000
														c14^14	0,000000	
															c15^15	0,000000
																c16^16

The trees are on the right in the Excel (after the estimation of the values) file

Pricing of contingent claims: 16 & 32 periods

By using the Backward Induction Strategy we get the Present Value of the Call Option.

In the Excel file you can see the values also for the Put option.

By using Python we managed to estimate the present value for the Call Option for 16 and for 32 periods.

16 periods

```
5th period: [14.914381367505914, 11.191860835331351, 7.811157483425823, 4.9032188366447595, 2.6552375814553564, 1.1820154748547105]
4th period: [12.977765840122519, 9.440600238444546, 6.310604696315053, 3.746848166719318, 1.89919143536481]
3th period: [11.141399287964733, 7.822157505696496, 4.989499178106789, 2.797311952266522]
2th period: [9.421487361396304, 6.359768788833327, 3.861163902342425]
1th period: [7.837751337057636, 5.071565698052871]
0th period: [6.409042767121809]
```

we see that the result (PV of call option) for 16 periods is the same as the one we received by using Excel.

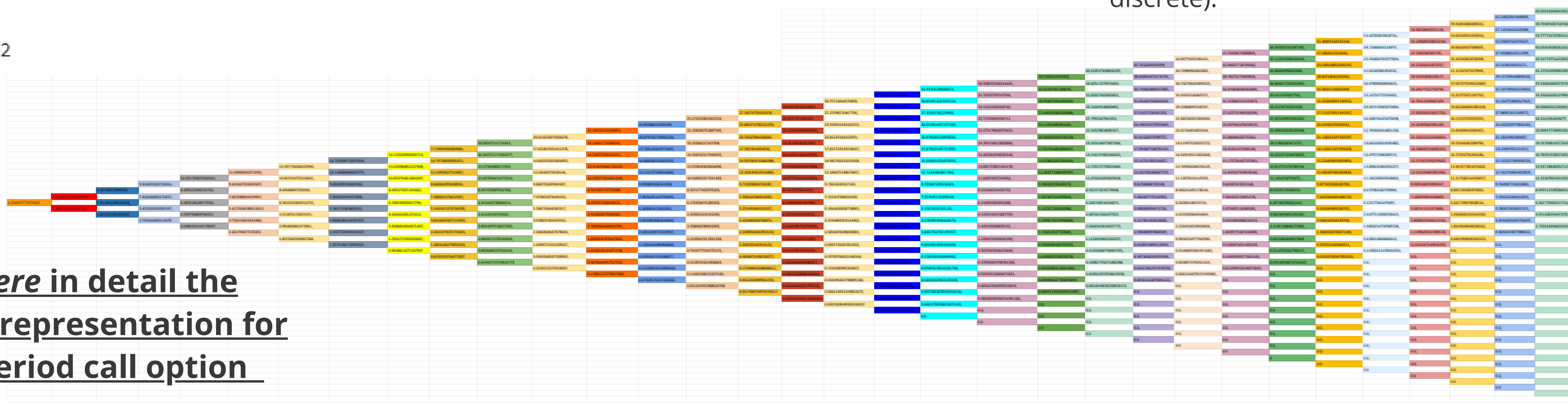
32 periods

```
27      [12.08996050572092, 9.643647254200197, 7.403588064139947, 5.4175648286611615, 3.738153833233484, 2.40479467727025
28      [10.833769823560301, 8.495652940525702, 6.387618149575594, 4.559794869766553, 3.0580334142178107]
29      [9.634293223716341, 7.416246800170237, 5.453259250095197, 3.793216265513278]
30      [8.49744337899105, 6.411903299233168, 4.605224481905445]
31      [7.429407536437494, 5.488196014362396]
32      [6.436052771879402]

The Present Value of the option is: 6.436052771879402
```

We have used the Recombinant Binomial model. (Months are considered as non discrete).

Click here to access and view the Python Code with Google Colab and the results



Click here **in detail the graphical representation for the 32 period call option**

A) Sources

- Investopedia. (n.d.). Investopedia. https://www.investopedia.com/
- (2023) U.S. Department of the Treasury. Available at: https://home.treasury.gov/resource-center/data-chart-center/interest-rates/TextView?type=daily_treasury_bill_rates&field_tdr_date_value=2023 (Accessed: 07 December 2023).
- Yahoo Finance - Stock Market Live, Quotes, Business & Finance News. (n.d.). Yahoo Finance - Stock Market Live, Quotes, Business & Finance News. https://finance.yahoo.com/

B) Instruments

- Python Programming language
- Google Sheets (Excel)
- Excel
- Google Colab

Thank you very much!