Negative binomial as gamma Poisson mixture

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The negative binomial distribution can be understood as a mixture distribution

If our data are Poisson distributed, we defined the likelihood for each datum Y_i as:

$$P(Y = y_i|\mu_i = \theta) = \frac{e^{-\theta}\theta^{y_i}}{y_i!}$$

and μ_i can of course be a function of predictors:

$$log(\mu_i) = \beta_0 + \beta_1 X$$

The probability density function for μ_i under a Gamma distribution would be:

$$g(\theta) = \frac{\alpha^\beta}{\Gamma(\beta)}\theta^{\beta-1}e^{-\alpha\theta}$$

where $\Gamma(\beta) = (\beta - 1)!$

If you multiply the two and do a bunch of calculus to solve for $P(Y = y_i)$ (not shown), you get the negative binomial distribution:

$$P(Y = y_i) = \binom{n + \beta - 1}{n}\left(\frac{\alpha}{\alpha + 1}\right)^\beta\left(\frac{1}{\alpha + 1}\right)^n$$

With all the formulas above, it may still be obscure to you as to what exactly is going on. If you work with Bayesian statistics and have to specify the likelihood of your model, you would specify the negative binomial simply as:

```
# NUMBER OF FLOWERING STALKS (fs) LIKELIHOOD

for ( t in 1:NtotalFS ) { # go through all stalks in my data
  #each response (stalk) has a poisson distribution
  fs[t] ~ dpois( mustar.fs[t] )
  #But the lambda in the Poission (mu.fs) is multiplied by rho
  mustar.fs[t] <- rho.fs[t]*mu.fs[t]
  #the mu.fs is simply the likelihood I get from a glm
  #where the log of mu.fs is a function of time since fire (TSF)
  log(mu.fs[t]) <- a0.fs + a1.fs[TSF[t]]
  # This is the gamma mixture
  rho.fs[t] ~ dgamma( alpha.fs , alpha.fs )
}
```

With the parameterization above, we get this:

$$Y \sim P(\rho\mu) where \rho \sim Gamma(\alpha\alpha)$$

This makes:

$$X \sim NB(\mu\alpha)$$