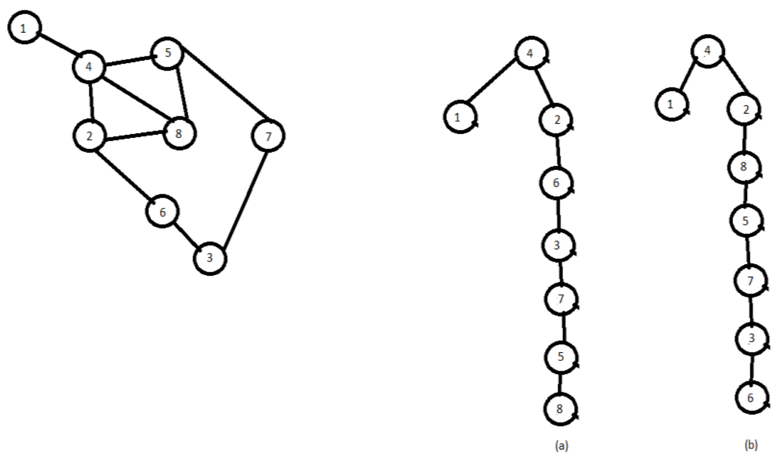
All classroom material is allowed. Answers must be justified. Algorithms can be written in pseudo-code, or be just described in natural languages. Algorithms that are correct but are suboptimal will be taken into consideration (up to half of the points).

1) For a graph $G=(V,E)$, a *proper k-coloring* is a function $c : V \rightarrow \{1,2,\ldots,k\}$ such that $c(u) \neq c(v)$ for every edge uv of G. The *chromatic number* of G, denoted by $\chi(G)$, is the least integer k such that G admits a proper k-coloring.
    a. What is the chromatic number of a clique? Of a cycle? **/1**
    b. Let G be the disjoint union of two smaller graphs G_1, G_2. Express $\chi(G)$ as a function of $\chi(G_1)$ and $\chi(G_2)$. **/1**
    c. Let G be the join of two smaller graphs G_1, G_2. Express $\chi(G)$ as a function of $\chi(G_1)$ and $\chi(G_2)$. **/1**
    d. Propose a linear-time algorithm for computing the chromatic number of a cograph. **/1**

2) Consider the following graph and its two spanning trees:



    a. Which spanning trees can be generated with DFS? **/1**
    b. Which spanning trees can be generated with LexDFS? **/1**

3) For a graph $G=(V,E)$, we denote by $p(G)$ the largest order (number of vertices) of a *prime induced subgraph* of G.
    a. What is $p(G)$ if G is a cograph? If G is a path? **/1**
    b. Recall that two sets *overlap* if they have a nonempty intersection but none of them is contained in the other. Let H and M be a prime induced subgraph and a module of G, respectively. Show that if H and M overlap, then they have exactly one vertex in common. **/1**
    c. Let H and C be a prime induced subgraph and a co-connected component of G, respectively. Show that H and C do not overlap. **/1**
    d. Propose a linear-time algorithm in order to computer $p(G)$. **/1**