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Grupă: 232

Examen la probabilități și statistică

$\boxed{i=44}$

Probabilități

① ~~A~~ $(0,i] \cap N = (0,44] \cap N = \{1,2,3,\dots 44\}$

a) Nr. cazuri favorabile $= \left[\frac{44-1}{3}\right] = 14$

Nr. cazuri totale $= 44$

$Pb = \frac{Nr.\ c.f.}{Nr.\ c.t.} = \frac{14}{44} = \frac{7}{22} \sim 0,318$

b) Fie $A = \{1,4,9,16,25,36\}$

$Nr.\ c.f. = |A| = 6$

$Pb = \frac{Nr.\ c.f.}{Nr.\ c.t.} = \frac{6}{44} = \frac{3}{22} \sim 0,136$

c) Fie $B = \{2,3,5,7,11,13,17,19,23,29,31,37,41,43\}$

$Nr.\ c.f. = |B| = 14$

$Pb = \frac{Nr.\ c.f.}{Nr.\ c.t.} = \frac{14}{44} \sim 0,318$

② $X \sim \begin{pmatrix} -1 & 0 & 1 \\ i/1000 & 1/100 & c \end{pmatrix} \Rightarrow X \sim \begin{pmatrix} -1 & 0 & 1 \\ 44/1000 & 1/100 & c \end{pmatrix}$

a) $\begin{cases} \frac{44}{1000} + {}^{10)}\frac{1}{100} + {}^{1000)}c = 1 \Rightarrow \frac{1000c + 10 + 44}{1000} = 1 \Rightarrow 1000c + 54 = 1000 \\ c \in (0,1) \end{cases}$

$c = \frac{946}{1000} \in (0,1)$

$\Rightarrow X \sim \begin{pmatrix} -1 & 0 & 1 \\ 44/1000 & 1/100 & 946/1000 \end{pmatrix}$

b) ~~Var(X) =~~ $E(X) = \sum_{j=1}^{\infty} x_j\, p(x_j) = (-1)\cdot\frac{44}{1000} + 0\cdot\frac{1}{100} + 1\cdot\frac{946}{1000} =$

$= \frac{-44+946}{1000} = \frac{902}{1000} = 0,902$

$$Var(x) = E((x - 0{,}902)^2)$$

x	-1	0	1
$p(x)$	$\frac{44}{1000}$	$1/100$	$\frac{946}{100}$
$\left(x-\frac{902}{1000}\right)^2$	$\left(\frac{1902}{1000}\right)^2$	$\left(\frac{902}{1000}\right)^2$	$\left(\frac{98}{1000}\right)^2$

$$Var(x) = \left(\frac{1902}{1000}\right)^2 \cdot \frac{44}{1000} + \left(\frac{902}{1000}\right)^2 \cdot \overset{10}{\frac{1}{100}} + \left(\frac{98}{1000}\right)^2 \cdot \overset{10}{\frac{946}{100}} =$$

$$= \frac{159\,174\,576 + 8136040 + 90\;853840}{10^6} = \frac{258\;164\;456}{10^6} =$$

$$= 258{,}164\,456$$

c) $$Var(2X) = 2^2 Var(x) = 4 Var(x) = 4 \cdot \frac{258164456}{10^6} =$$

$$= \frac{1\,032\,657\,824}{10^6} = 1032{,}657\,824$$

$= \frac{1}{44^2 \cdot 484} \cdot \frac{44^2}{2} \cdot x = \frac{x}{968}$ $\quad P(X \le 3) = \int_0^3 f_X(x)\,dx = \int_0^3 \frac{x}{968}\,dx =$

$= \frac{1}{968} \cdot \frac{x^2}{2} \Big|_0^3 = \frac{9}{2} \cdot \frac{1}{968} =$

$= 0{,}004$

$= \frac{9}{1936}$

c) ~~$P(X=0, Y=44) = P(X=0)\,P(Y=44)$~~

X și Y independente $\Leftrightarrow F(x,y) = F_X(x) \cdot F_Y(y)$

$F_X(x) = F(x, 44), \quad F_Y(y) = F(44, y)$

~~$F(x,y) =$~~

(6) $\frac{i}{200} = \frac{44}{200} = 0{,}22 \Rightarrow$ Interval simetric $= [20.39, 20.61]$

$[20{,}17, 20{,}83]$

Intervalul este simetric deoarece cantitatea de probabilitate rămasă în afara lui, în oricare din cele 2 părți, este aceeași, și anume 0,39.

(3) c) X și Y independente $\Leftrightarrow f(x,y) = f_X(x) \cdot f_Y(y)$

$f_X(x) = \int_0^{44} f(x,y)\,dy = \int_0^{44} cxy\,dy = cx \int_0^{44} y\,dy = cx \frac{y^2}{2} \Big|_0^{44} =$

$= c \cdot x \cdot \frac{44^2}{2} = \frac{1}{484 \cdot 44^2} \cdot x \cdot \frac{44^2}{2} = \frac{x}{968}$

$f_Y(y) = \int_0^{44} f(x,y)\,dx = \int_0^{44} cxy\,dx = cy \int_0^{44} x\,dx = cy \frac{x^2}{2} \Big|_0^{44} =$

$= cy \frac{44^2}{2} = \frac{1}{484 \cdot 44^2} \cdot \frac{44^2}{2} \cdot y = \frac{y}{968}$

$$\left.\begin{array}{l} f(x,y) = cxy = \frac{xy}{484 \cdot 44^2} \\ f_X(x) \cdot f_Y(y) = \frac{x}{968} \cdot \frac{y}{968} \end{array}\right] \Rightarrow f(x,y) = f_X(x) \cdot f_Y(y)$$

4) Statistică

$P(\Delta_H / A) = 0,5$ $\qquad P(A) = \frac{44}{200} = 0,22$

$P(\Delta_H / B) = 0,8$ $\qquad P(B) = \frac{156}{200} = 0,78$

Probabilitatea predictivă a priori:

$P(\Delta_H) = P(\Delta_H / A) P(A) + P(\Delta_H / B)\, P(B) = 0,5 \cdot 0,22 + 0,8 \cdot 0,78 =$
$= 0,11 + 0,624 = 0,734$

Probabilitatea predictivă a posteriori:

$P(\Delta_H / \Delta) = P(\Delta_H / A) \cdot P(A/\Delta) + P(\Delta_H / B) \cdot P(B/\Delta)$
$= 0,5 \cdot 0,149 + 0,8 \cdot 0,85 = 0,0745 + 0,68 = 0,7545$

Tabel Bayes:

H	P(H)	P(Δ/H)	P(Δ/H)·P(H)	P(H/Δ)
A	0,22	0,5	0,11	$\frac{0,11}{0,734} = 0,149$
B	0,78	0,8	0,624	$\frac{0,624}{0,734} = 0,850$
total	1		~~[illegible]~~ 0,734	1

3) a) $X : m \to [0, 44]$
$X : m \to [0, 44]$

$\int_0^{44} \left(\int_0^{44} cxy\, dx \right) dy = 1 \Rightarrow \int_0^{44} \left(c \frac{x^2}{2} y \Big/_0^{44} \right) dy = 1 \Rightarrow \int_0^{44} \left(\frac{c}{2} \cdot 1936\, y \right) dy = 1$

$\Rightarrow \int_0^{44} (968 c y) dy = 1 \Rightarrow 968 c \int_0^{44} y\, dy = 1 \Rightarrow 968 c \cdot \frac{44^2}{2} = 1$

$c = \frac{2}{968 \cdot 44^2} = \frac{1}{484 \cdot 44^2}$

b) $f_X(x) = \int_0^{44} f(x,y)\, dy = \frac{1}{44^2 \cdot 484} \int_0^{44} xy\, dy = \frac{1}{44^2 \cdot 484} \cdot x \cdot \frac{y^2}{2} \Big/_0^{44} =$

(5) 44 averse

$110 - 44 = 66$ reverse

$(110, \theta)$ repartiția binomială

verosimilitatea $P(X/\theta) = C_{110}^{44}\, \theta^{44} (1-\theta)^{66}$

Tabel

hypothesis	prior	likelihood	Bayes num.	posterior
θ	$1 \cdot d\theta$	$\binom{110}{44} \theta^{44} (1-\theta)^{66}$	$\binom{110}{44} \theta^{44} (1-\theta)^{66} d\theta$	$C_2\, \theta^{44} (1-\theta)^{66} d\theta$
			$T = \binom{110}{44} \int_0^1 \theta^{44} (1-\theta)^{66} d\theta$	1

pdf a posteriori $f(\theta/x) = c\,\theta^{44} (1-\theta)^{66}$

const. de normalizare $c = \dfrac{(45+67-1)!}{(45-1)!\,(67-1)!} = \dfrac{111!}{44!\,66!}$