

# Optimization methods for the memory allocation problems in embedded systems

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# Outline

## 1 Introduction

- Embedded systems
- Memory allocation
- Conflict graph

## 2 Memory allocation problems

- Unconstrained memory allocation problem
- Allocation with constraint on the number of memory banks
- General memory allocation problem
- Dynamic memory allocation problem

## 3 Conclusions and future work



# Outline

1

## Introduction

- Embedded systems
- Memory allocation
- Conflict graph

2

## Memory allocation problems

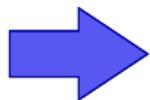
3

## Conclusions and future work



# What is an embedded system?

## Embedded system



Minicomputer designed  
to satisfy specific  
requirements

# Embedded Systems

Daily activities



Industry



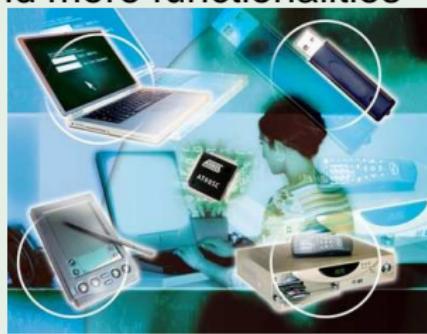
Embedded systems



# Design challenge



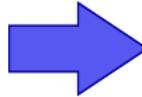
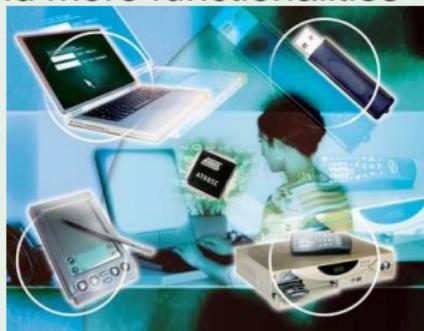
Technology offers more  
and more functionalities



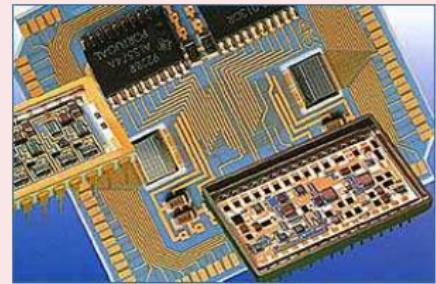
# Design challenge



Technology offers more and more functionalities



Design of embedded systems becomes more and more complex

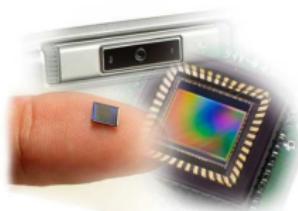


# Designer objectives

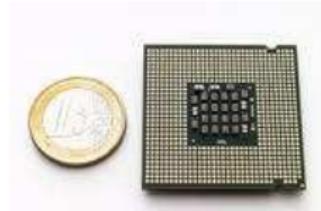
Designers must take into account:



Power  
consumption (W)



Area (mm<sup>2</sup>)



Cost (\$)

# Architecture

## Embedded system



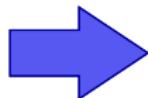
# Architecture

## Embedded system

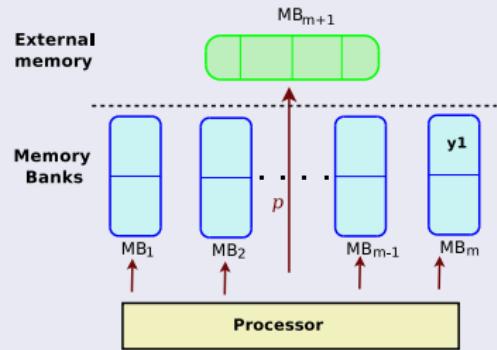


# Architecture

## Embedded system



## Simplified architecture



# Application

## Embedded system



### Functionalities

- phoning
- texting
- digital camera
- web browsing
- etc...

# Application

## Embedded system



### Functionalities

- phoning
- texting
- digital camera
- web browsing
- etc...

## C source code

```
1 /* LMS dual-channel filter */

11 void main() {
12     int y11, y12, y21, y22, e1, e2;
13     for(int k=0;k<10;k++)
14     {
15         int n = (k+10)%L;
16         y11=0;
17         for(int i=0;i<10;i++)
18             { y11 = X1[(i+k)%L]*H11[(L-1+k-i)%L]+y11; }
19         y12=0;
20         for(int i=0;i<10;i++)
21             { y12 = X2[(i+k)%L]*H12[(L-1+k-i)%L]+y12; }
22         H11[(n+1)%L] = H11[n]+mu11*X1[n]*e1;
23         H12[(n+1)%L] = H12[n]+mu12*X2[n]*e1;
24         y21=0;
25         for(int i=0;i<10;i++)
26             { y21 = X1[(i+k)%L]*H21[(L-1+k-i)%L]+y21; }
27         y22=0;
28         for(int i=0;i<10;i++)
29             { y22 = X2[(i+k)%L]*H22[(L-1+k-i)%L]+y22; }
30         e2 = y2[n]-y21-y22;
31         H21[(n+1)%L] = H21[n]+mu21*X1[n]*e2;
32         H22[(n+1)%L] = H22[n]+mu22*X2[n]*e2; }
```

# Data structures

## Application

### Source code

```
1 /* LMS dual-channel filter */

11 void main() {
12     int y11, y12, y21, y22, e1, e2;
13     for(int k=0;k<10;k++)
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15         int n = (k+10)%L;
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18             { y11 = X1[(i+k)%L]*H11[(L-1+k-i)%L]+y11; }
19         y12=0;
20         for(int i=0;i<10;i++)
21             { y12 = X2[(i+k)%L]*H12[(L-1+k-i)%L]+y12; }
22         e1 = y1[n]-y11-y12; /* error */
23         H11[(n+1)%L] = H11[n]+mul1*X1[n]*e1;
24         H12[(n+1)%L] = H12[n]+mul2*X2[n]*e1;
25         y21=0;
26         for(int i=0;i<10;i++)
27             { y21 = X1[(i+k)%L]*H21[(L-1+k-i)%L]+y21; }
28         y22=0;
29         for(int i=0;i<10;i++)
30             { y22 = X2[(i+k)%L]*H22[(L-1+k-i)%L]+y22; }
31         e2 = y2[n]-y21-y22;
32         H21[(n+1)%L] = H21[n]+mu21*X1[n]*e2;
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```



# Data structures

## Application

### Source code

```
1 /* LMS dual-channel filter */

11 void main() {
12     int y11, y12, y21, y22, e1, e2;
13     for(int k=0;k<10;k++)
14     {
15         int n = (k+10)%L;
16         y11=0;
17         for(int i=0;i<10;i++)
18             { y11 = X1[(i+k)%L]*H11[(L-1+k-i)%L]+y11; }
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21             { y12 = X2[(i+k)%L]*H12[(L-1+k-i)%L]+y12; }
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28         y22=0;
29         for(int i=0;i<10;i++)
30             { y22 = X2[(i+k)%L]*H22[(L-1+k-i)%L]+y22; }
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```



# Data structures

## Application

### Source code

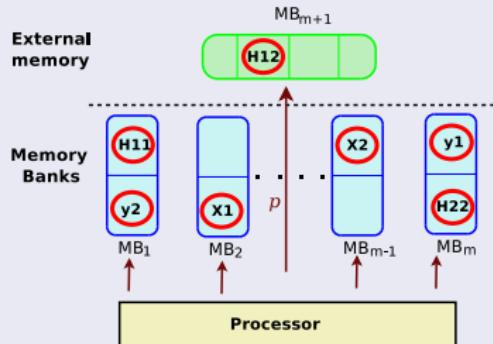
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29             { y22 = X2[(i+k)%L]*H22[(L-1+k-i)%L]+y22; }
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```



## Memory architecture



# Data structures

## Application

### Source code

```

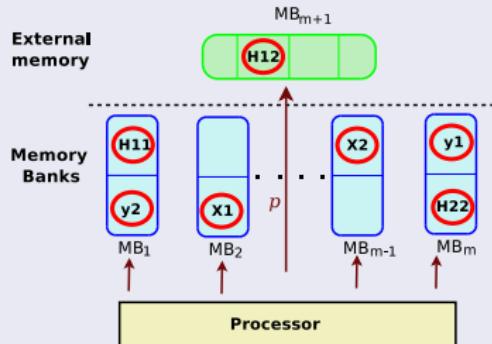
1 /* LMS dual-channel filter */

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32         H21[(n+1)%L] = H21[n]+mu21*X1[n]*e2;
33         H22[(n+1)%L] = H22[n]+mu22*X2[n]*e2; }

```



## Memory architecture



Memory allocation impacts power consumption, area and cost

# Processor

## Application

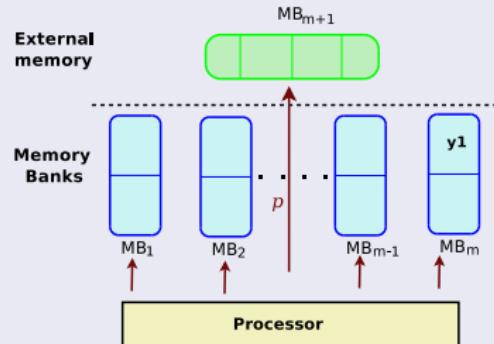
### Source code

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12     int y11, y12, y21, y22, e1, e2;
13     for(int k=0;k<10;k++) {
14         int n = (k+10)%L;
15         y11=0;
16         for(int i=0;i<10;i++) {
17             { y11 = X1[(i+k)%L]*H11[(L-1+k-i)%L]-y11; }
18             y12=0;
19             for(int i=0;i<10;i++) {
20                 { y12 = X2[(i+k)%L]*H12[(L-1+k-i)%L]+y12; }
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24                 y21=0;
25                 for(int i=0;i<10;i++) {
26                     { y21 = X1[(i+k)%L]*H21[(L-1+k-i)%L]+y21; }
27                     y22=0;
28                     for(int i=0;i<10;i++) {
29                         { y22 = X2[(i+k)%L]*H22[(L-1+k-i)%L]+y22; }
30                         e2 = y2[n]-y21-y22;
31                         H21[(n+1)%L] = H21[n]+mu21*X1[n]*e2;
32                         H22[(n+1)%L] = H22[n]+mu22*X2[n]*e2; } }
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## Memory architecture



# Processor

## Application

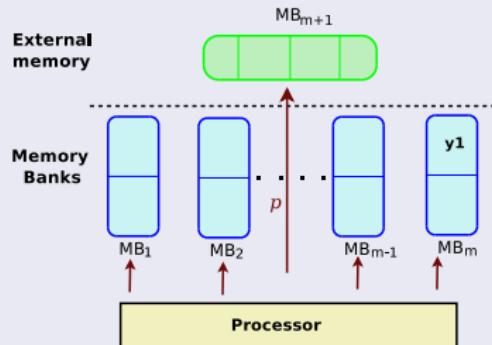
### Source code

```

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15         y11=0;
16         for(int i=0;i<10;i++)
17             { y11 = X1 * H11 y11; } y11
18         y12=0;
19         for(int i=0;i<10;i++)
20             { y12 = X2[(i+k)%L]*H12[(L-1+k-i)%L]+y12; }
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28         for(int i=0;i<10;i++)
29             { y22 = X2[(i+k)%L]*H22[(L-1+k-i)%L]+y22; }
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32         H22[(n+1)%L] = H22[n]+mu22*X2[n]*e2; } }
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## Memory architecture



# Processor

## Application

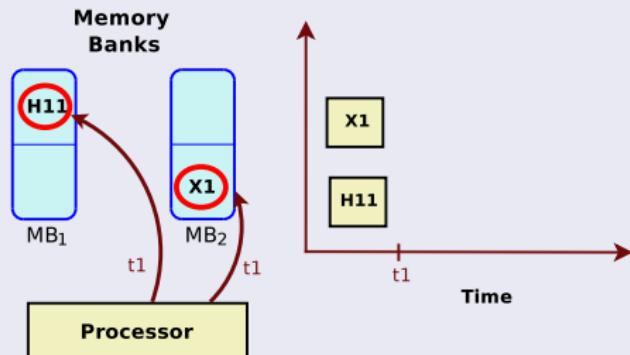
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17         for(int i=0;i<10;i++)
18             { y11 = X1 * H11 y11; } y11;
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20         for(int i=0;i<10;i++)
21             { y12 = X2[(i+k)%L]*H12[(L-1+k-i)%L]+y12; }
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23         H11[(n+1)%L] = H11[n]+mu11*X1[n]*e1;
24         H12[(n+1)%L] = H12[n]+mu12*X2[n]*e1;
25         y21=0;
26         for(int i=0;i<10;i++)
27             { y21 = X1[(i+k)%L]*H21[(L-1+k-i)%L]+y21; }
28         y22=0;
29         for(int i=0;i<10;i++)
30             { y22 = X2[(i+k)%L]*H22[(L-1+k-i)%L]+y22; }
31         e2 = y2[n]-y21-y22;
32         H21[(n+1)%L] = H21[n]+mu21*X1[n]*e2;
33         H22[(n+1)%L] = H22[n]+mu22*X2[n]*e2; }
```



## Loading process



# Processor

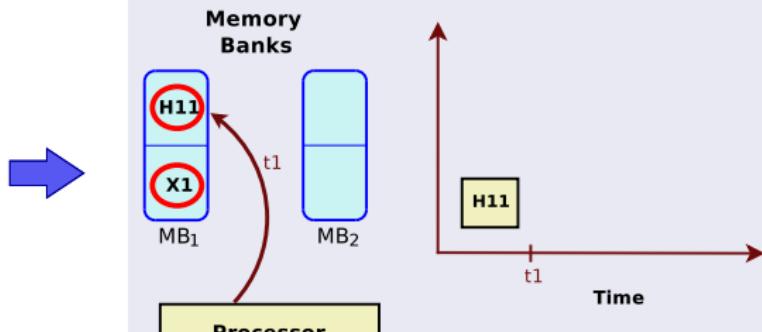
## Application

### Source code

```
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11 void main() {
12     int y11, y12, y21, y22, e1, e2;
13     for(int k=0;k<10;k++)
14     {
15         int n = (k+10)%L;
16         y11=0;
17         for(int i=0;i<10;i++)
18             { y11 = X1 * H11 y11; } y11;
19         y12=0;
20         for(int i=0;i<10;i++)
21             { y12 = X2[(i+k)%L]*H12[(L-1+k-i)%L]+y12; }
22         e1 = y1[n]-y11-y12; /* error */
23         H11[(n+1)%L] = H11[n]+mu11*X1[n]*e1;
24         H12[(n+1)%L] = H12[n]+mu12*X2[n]*e1;
25         y21=0;
26         for(int i=0;i<10;i++)
27             { y21 = X1[(i+k)%L]*H21[(L-1+k-i)%L]+y21; }
28         y22=0;
29         for(int i=0;i<10;i++)
30             { y22 = X2[(i+k)%L]*H22[(L-1+k-i)%L]+y22; }
31         e2 = y2[n]-y21-y22;
32         H21[(n+1)%L] = H21[n]+mu21*X1[n]*e2;
33         H22[(n+1)%L] = H22[n]+mu22*X2[n]*e2; }
```

## Loading process



# Processor

## Application

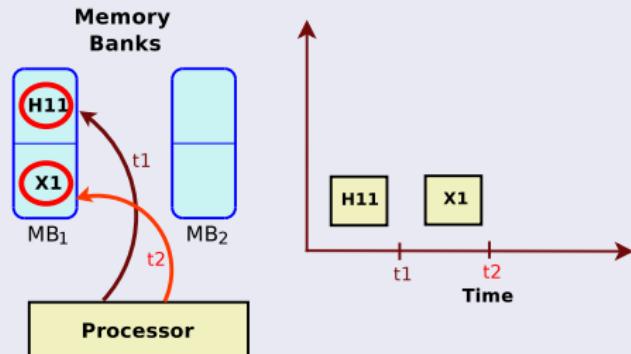
### Source code

```
1 /* LMS dual-channel filter */

11 void main() {
12     int y11, y12, y21, y22, e1, e2;
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14     {
15         int n = (k+10)%L;
16         y11=0;
17         for(int i=0;i<10;i++)
18             { y11 = X1 * H11 y11; } y11;
19         y12=0;
20         for(int i=0;i<10;i++)
21             { y12 = X2[(i+k)%L]*H12[(L-1+k-i)%L]+y12; }
22         e1 = y1[n]-y11-y12; /* error */
23         H11[(n+1)%L] = H11[n]+mu11*X1[n]*e1;
24         H12[(n+1)%L] = H12[n]+mu12*X2[n]*e1;
25         y21=0;
26         for(int i=0;i<10;i++)
27             { y21 = X1[(i+k)%L]*H21[(L-1+k-i)%L]+y21; }
28         y22=0;
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30             { y22 = X2[(i+k)%L]*H22[(L-1+k-i)%L]+y22; }
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32         H21[(n+1)%L] = H21[n]+mu21*X1[n]*e2;
33         H22[(n+1)%L] = H22[n]+mu22*X2[n]*e2; }
```



## Loading process



# Conflicts

## Application

### Source code

```
1 /* LMS dual-channel filter */

11 void main() {
12     int y11, y12, y21, y22, e1, e2;
13     for(int k=0;k<10;k++)
14     {
15         int n = (k+10)%L;
16         y11=0;
17         for(int i=0;i<10;i++)
18             { y11 = X1 * H11 y11; } y12=0;
19         for(int i=0;i<10;i++)
20             { y12 = X2[(i+k)%L]*H12[(L-1+k-i)%L]+y12; }
21         e1 = y1[n]-y11-y12; /* error */
22         H11[(n+1)%L] = H11[n]+mu11*X1[n]*e1;
23         H12[(n+1)%L] = H12[n]+mu12*X2[n]*e1;
24         y21=0;
25         for(int i=0;i<10;i++)
26             { y21 = X1[(i+k)%L]*H21[(L-1+k-i)%L]+y21; }
27         y22=0;
28         for(int i=0;i<10;i++)
29             { y22 = X2[(i+k)%L]*H22[(L-1+k-i)%L]+y22; }
30         e2 = y2[n]-y21-y22;
31         H21[(n+1)%L] = H21[n]+mu21*X1[n]*e2;
32         H22[(n+1)%L] = H22[n]+mu22*X2[n]*e2; }
```



## Conflict

- Two structures are said to be conflicting if they are required at the same time (e.g.  $X1 * H11$ )

# Conflicts

## Application

### Source code

```
1 /* LMS dual-channel filter */

11 void main() {
12     int y11, y12, y21, y22, e1, e2;
13     for(int k=0; k<10; k++)
14     {
15         int n = (k+10)%L;
16         y11=0;
17         for(int i=0; i<10; i++)
18             { y11 = X1[i][i+1] X1 * H11 y11; } y12=0;
19         for(int i=0; i<10; i++)
20             { y12 = X2[(i+k)%L]*H12[(L-1+k-i)%L]+y12; }
21         e1 = y1[n]-y11-y12; /* error */
22         H11[(n+1)%L] = H11[n]+mu11*X1[n]*e1;
23         H12[(n+1)%L] = H12[n]+mu12*X2[n]*e1;
24         y21=0;
25         for(int i=0; i<10; i++)
26             { y21 = X1[(i+k)%L]*H21[(L-1+k-i)%L]+y21; }
27         y22=0;
28         for(int i=0; i<10; i++)
29             { y22 = X2[(i+k)%L]*H22[(L-1+k-i)%L]+y22; }
30         e2 = y2[n]-y21-y22;
31         H21[(n+1)%L] = H21[n]+mu21*X1[n]*e2;
32         H22[(n+1)%L] = H22[n]+mu22*X2[n]*e2; }
```



## Cost conflict

- Conflicts have a cost in milliseconds (ms)  
(e.g.  $X1 * H11 \rightarrow$  cost of 100 ms)
- Automatically computed by *SoftExplorer*

<http://www.softexplorer.fr>

# Conflicts

## Application

### Source code

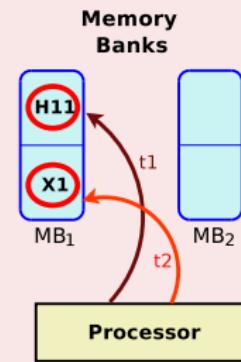
```
1 /* LMS dual-channel filter */

11 void main() {
12     int y11, y12, y21, y22, e1, e2;
13     for(int k=0;k<10;k++)
14     {
15         int n = (k+10)%L;
16         y11=0;
17         for(int i=0;i<10;++)
18             { y11 = X1[i]*H11[y11]; } X1 * H11
19         y12=0;
20         for(int i=0;i<10;++)
21             { y12 = X2[(i+k)%L]*H12[(L-1+k-i)%L]+y12; }
22         e1 = y1[n]-y11-y12; /* error */
23         H11[(n+1)%L] = H11[n]+mu11*X1[n]*e1;
24         H12[(n+1)%L] = H12[n]+mu12*X2[n]*e1;
25         y21=0;
26         for(int i=0;i<10;i++)
27             { y21 = X1[(i+k)%L]*H21[(L-1+k-i)%L]+y21; }
28         y22=0;
29         for(int i=0;i<10;i++)
30             { y22 = X2[(i+k)%L]*H22[(L-1+k-i)%L]+y22; }
31         e2 = y2[n]-y21-y22;
32         H21[(n+1)%L] = H21[n]+mu21*X1[n]*e2;
33         H22[(n+1)%L] = H22[n]+mu22*X2[n]*e2; }
```



## Open conflict

**X1** and **H11** are allocated in the same memory bank → cost is paid



# Conflicts

## Application

### Source code

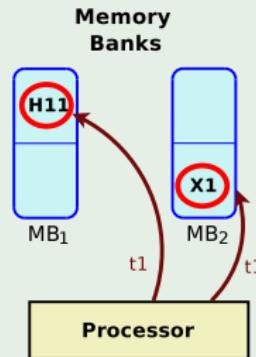
```
1 /* LMS dual-channel filter */

11 void main() {
12     int y11, y12, y21, y22, e1, e2;
13     for(int k=0;k<10;k++)
14     {
15         int n = (k+10)%L;
16         y11=0;
17         for(int i=0;i<10;++)
18             { y11 = X1[i](i+1) * H11[y11]; }
19         y12=0;
20         for(int i=0;i<10;i++)
21             { y12 = X2[(i+k)%L]*H12[(L-1+k-i)%L]+y12; }
22         e1 = y1[n]-y11-y12; /* error */
23         H11[(n+1)%L] = H11[n]+mu11*X1[n]*e1;
24         H12[(n+1)%L] = H12[n]+mu12*X2[n]*e1;
25         y21=0;
26         for(int i=0;i<10;i++)
27             { y21 = X1[(i+k)%L]*H21[(L-1+k-i)%L]+y21; }
28         y22=0;
29         for(int i=0;i<10;i++)
30             { y22 = X2[(i+k)%L]*H22[(L-1+k-i)%L]+y22; }
31         e2 = y2[n]-y21-y22;
32         H21[(n+1)%L] = H21[n]+mu21*X1[n]*e2;
33         H22[(n+1)%L] = H22[n]+mu22*X2[n]*e2; }
```



## Closed conflict

**x1** and **h11** are assigned to different memory banks  
→ no cost



# Conflicts

## Application

### Source code

```
1 /* LMS dual-channel filter */

11 void main() {
12     int y11, y12, y21, y22, e1, e2;
13     for(int k=0;k<10;k++)
14     {
15         int n = (k+10)%L;
16         y11=0;
17         for(int i=0;i<10;i++)
18             { y11 = X1[(i+k)%L]*H11[(L-1+k-i)%L]+y11; }
19         y12=0;
20         for(int i=0;i<10;i++)
21             { y12 = X2[(i+k)%L]*H12[(L-1+k-i)%L]+y12; }
22         e1 = y1[n]-y11-y12; /* error */
23         H11[(n+1)%L] = H11[n]+mu11*X1[n]*e1;
24         H12[(n+1)%L] = H12[n]+mu12*X2[n]*e1;
25         y21=0;
26         for(int i=0;i<10;i++)
27             { y21 = X1[(i+k)%L]*H21[(L-1+k-i)%L]+y21; }
28         y22=0;
29         for(int i=0;i<10;i++)
30             { y22 = X2[(i+k)%L]*H22[(L-1+k-i)%L]+y22; }
31         H21[n+1] = H21[n] mu21*X1[n]*e2;
32         H22[(n+1)%L] = H22[n]+mu22*X2[n]*e2; }
```



## Special cases

- *Auto-conflict.* a data structure is in conflict with itself (**always open**)  
For example **H21[n+1] = H21[n]**
- Isolated data structure: a data structure is not in conflict with any other one

# Conflict Graph $G = (X, U)$

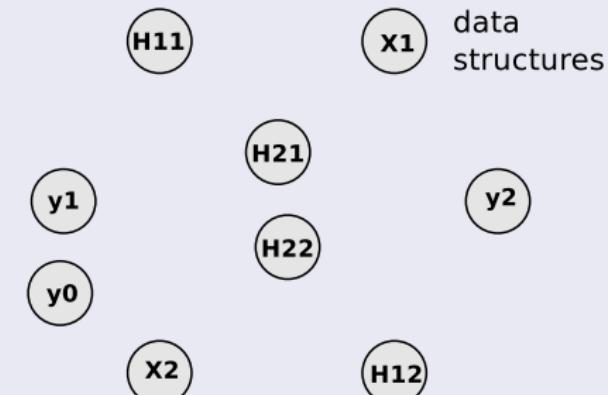
## Application

### Source code

```
1 /* LMS dual-channel filter */

11 void main() {
12     int y11, y12, y21, y22, e1, e2;
13     for(int k=0;k<10;k++)
14     {
15         int n = (k+10)%L;
16         y11=0;
17         for(int i=0;i<10;i++)
18             { y11 = X1[(i+k)%L]*H11[(L-1+k-i)%L]+y11; }
19         y12=0;
20         for(int i=0;i<10;i++)
21             { y12 = X2[(i+k)%L]*H12[(L-1+k-i)%L]+y12; }
22         e1 = y1[n]-y11-y12; /* error */
23         H11[(n+1)%L] = H11[n]+mu11*X1[n]*e1;
24         H12[(n+1)%L] = H12[n]+mu12*X2[n]*e1;
25         y21=0;
26         for(int i=0;i<10;i++)
27             { y21 = X1[(i+k)%L]*H21[(L-1+k-i)%L]+y21; }
28         y22=0;
29         for(int i=0;i<10;i++)
30             { y22 = X2[(i+k)%L]*H22[(L-1+k-i)%L]+y22; }
31         e2 = y2[n]-y21-y22;
32         H21[(n+1)%L] = H21[n]+mu21*X1[n]*e2;
33         H22[(n+1)%L] = H22[n]+mu22*X2[n]*e2; }
```

## Conflict graph



# Conflict Graph $G = (X, U)$

## Application

### Source code

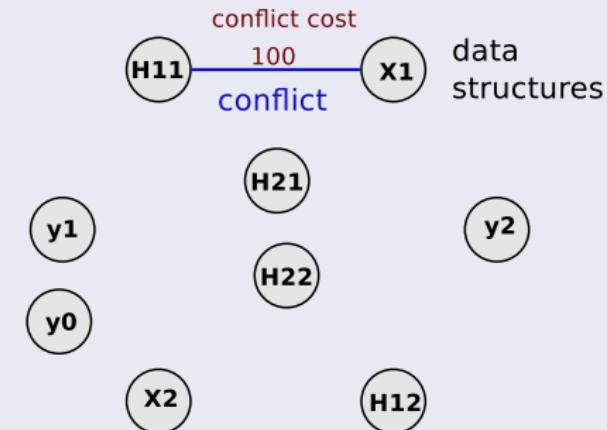
```

1 /* LMS dual-channel filter */

11 void main() {
12     int y11, y12, y21, y22, e1, e2;
13     for(int k=0;k<10;k++) {
14         int n = (k+10)%L;
15         y11=0;
16         for(int i=0;i<10;i++) {
17             { y11 = X1[(i+k)%L]*H11[(L-1+k-i)%L]-y11; }
18             y12=0;
19             for(int i=0;i<10;i++) {
20                 { y12 = X2[(i+k)%L]*H12[(L-1+k-i)%L]+y12; }
21                 e1 = y1[n]-y11-y12; /* error */
22                 H11[(n+1)%L] = H11[n]+mu11*X1[n]*e1;
23                 H12[(n+1)%L] = H12[n]+mu12*X2[n]*e1;
24                 y21=0;
25                 for(int i=0;i<10;i++) {
26                     { y21 = X1[(i+k)%L]*H21[(L-1+k-i)%L]+y21; }
27                     y22=0;
28                     for(int i=0;i<10;i++) {
29                         { y22 = X2[(i+k)%L]*H22[(L-1+k-i)%L]+y22; }
30                         e2 = y2[n]-y21-y22;
31                         H21[(n+1)%L] = H21[n]+mu21*X1[n]*e2;
32                         H22[(n+1)%L] = H22[n]+mu22*X2[n]*e2; } }
```



## Conflict graph



# Conflict Graph $G = (X, U)$

## Application

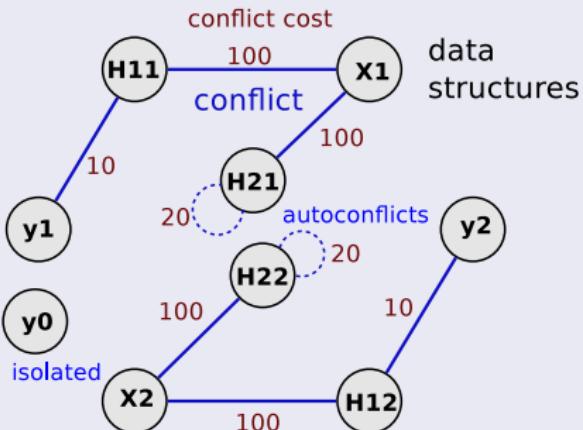
### Source code

```
1 /* LMS dual-channel filter */

11 void main() {
12     int y11, y12, y21, y22, e1, e2;
13     for(int k=0; k<10; k++) {
14         int n = (k+10)%L;
15         y11=0;
16         for(int i=0; i<10; i++) {
17             { y11 = X1[(i+k)%L]*H11[(L-1+k-i)%L]-y11; }
18         y12=0;
19         for(int i=0; i<10; i++) {
20             { y12 = X2[(i+k)%L]*H12[(L-1+k-i)%L]-y12; }
21         e1 = y1[n]+y11+y12; // error /
22         H11[(n+1)%L] = H11[n]+mu11*X1[n]*e1;
23         H12[(n+1)%L] = H12[n]+mu12*X2[n]*e1;
24         y21=0;
25         for(int i=0; i<10; i++) {
26             { y21 = X1[(i+k)%L]*H21[(L-1+k-i)%L]-y21; }
27         y22=0;
28         for(int i=0; i<10; i++) {
29             { y22 = X2[(i+k)%L]*H22[(L-1+k-i)%L]-y22; }
30         e2 = y2[n]+y21+y22;
31         H21[(n+1)%L] = H21[n]+mu21*X1[n]*e2;
32         H22[(n+1)%L] = H22[n]+mu22*X2[n]*e2; } }
```

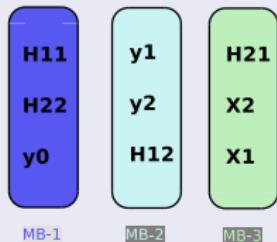


## Conflict graph



# Solution

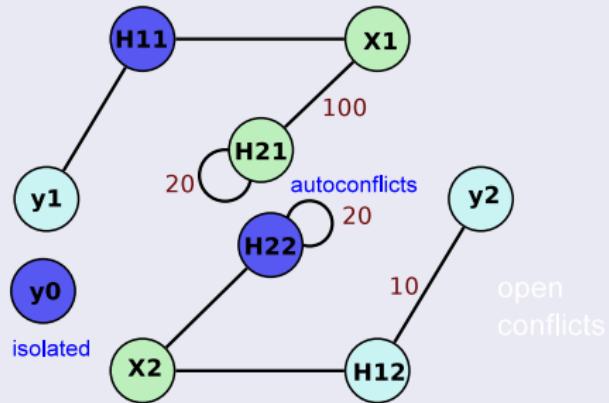
## Memory allocation



Memory banks → colors



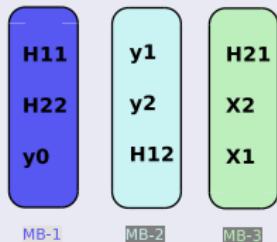
## Coloration



Total cost: 150 ms

# Solution

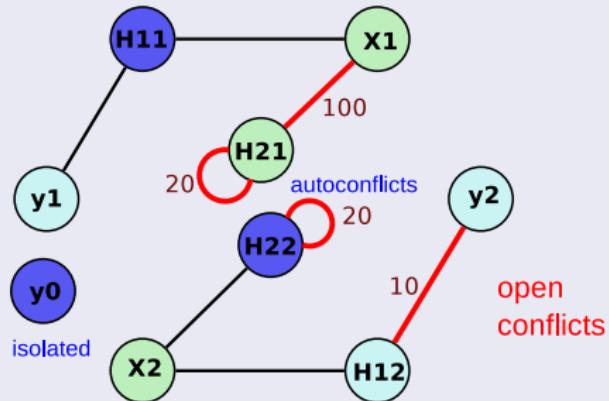
## Memory allocation



Memory banks → colors



## Coloration



Total cost: 150 ms

# Outline

## 1 Introduction

## 2 Memory allocation problems

- Unconstrained memory allocation problem
- Allocation with constraint on the number of memory banks
- General memory allocation problem
- Dynamic memory allocation problem

## 3 Conclusions and future work

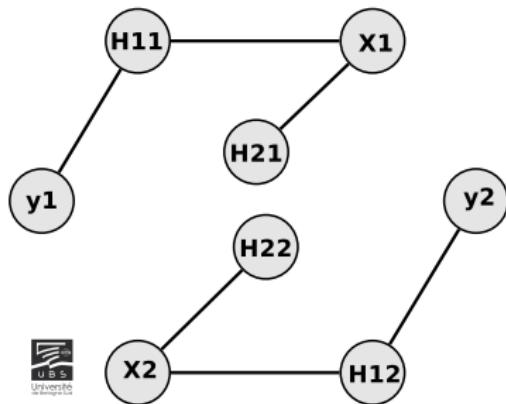


# Unconstrained memory allocation problem

- From now, auto-conflicts are ignored because they are always open

## Objective

Find the minimum number of memory banks for which all conflicts are closed



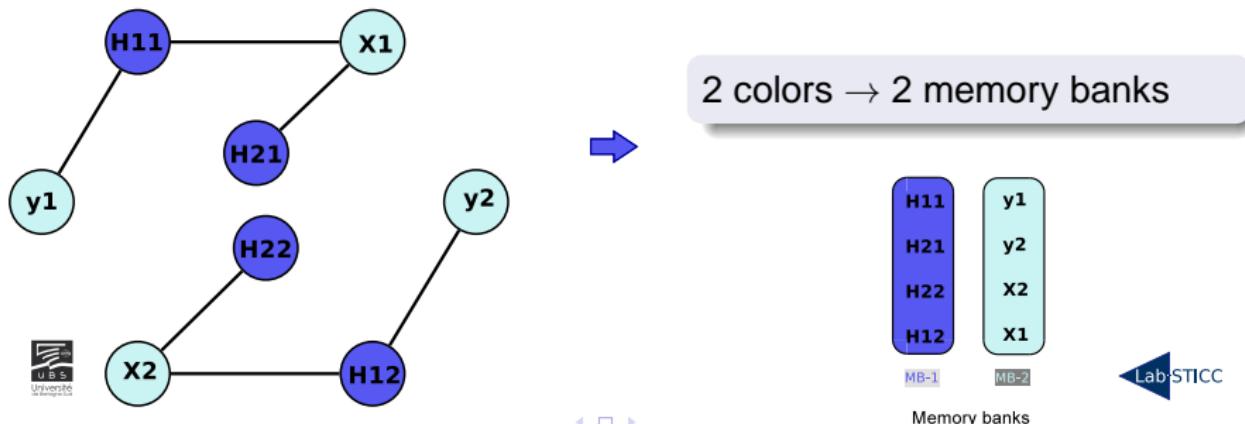
Finding the minimum number  
of colors to color graph  
→ **Chromatic number**  $\chi$

# Unconstrained memory allocation problem

- From now, auto-conflicts are ignored because they are always open

## Objective

Find the minimum number of memory banks for which all conflicts are closed



## New upper bounds

- Easily computable, even for large graphs
  - Outperform the upper bounds from literature
  - But far from the optimal solution
- 
- Sophisticated methods spend too much time
  - Problem solved repeatedly in CAD software
- 
- Upper bounds provide a satisfactory solution

## Allocation with constraint on the number of memory banks

- Fixed number of memory banks
- Cost of conflicts

### Objective

Find memory allocation for data structures such that total cost of open conflicts is minimized

- $k$ -weighted graph coloring problem

Proposed approach from operations research:  
→ Memetic Algorithm hybridized with Tabu Search



## Allocation with constraint on the number of memory banks

- Fixed number of memory banks
- Cost of conflicts

### Objective

Find memory allocation for data structures such that total cost of open conflicts is minimized

- $k$ -weighted graph coloring problem

Proposed approach from operations research:  
→ Memetic Algorithm hybridized with Tabu Search



## Allocation with constraint on the number of memory banks

- Fixed number of memory banks
- Cost of conflicts

### Objective

Find memory allocation for data structures such that total cost of open conflicts is minimized

- $k$ -weighted graph coloring problem

Proposed approach from operations research:  
→ Memetic Algorithm hybridized with Tabu Search

## Memetic Algorithm

### Population

### Solutions



## Memetic Algorithm

Population

Solutions



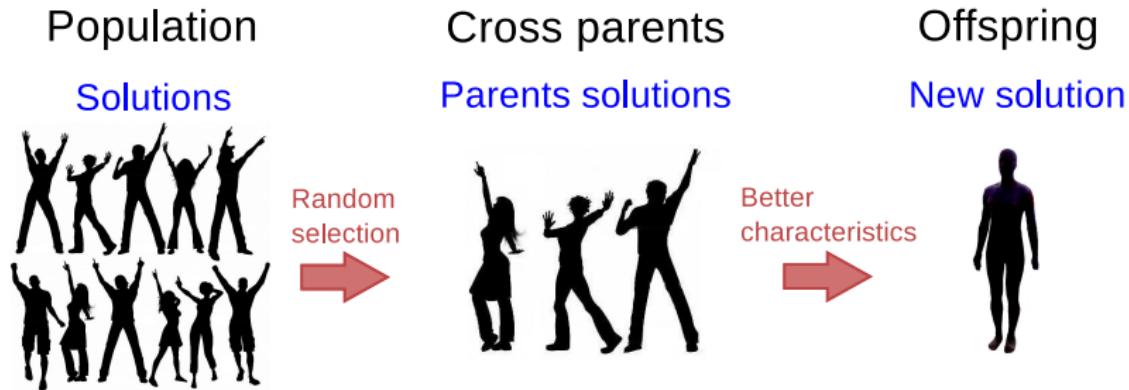
Random selection  
→

Cross parents

Parents solutions



## Memetic Algorithm



## Memetic Algorithm

Population

Solutions



Random selection  
→

Cross parents

Parents solutions



Better characteristics  
→

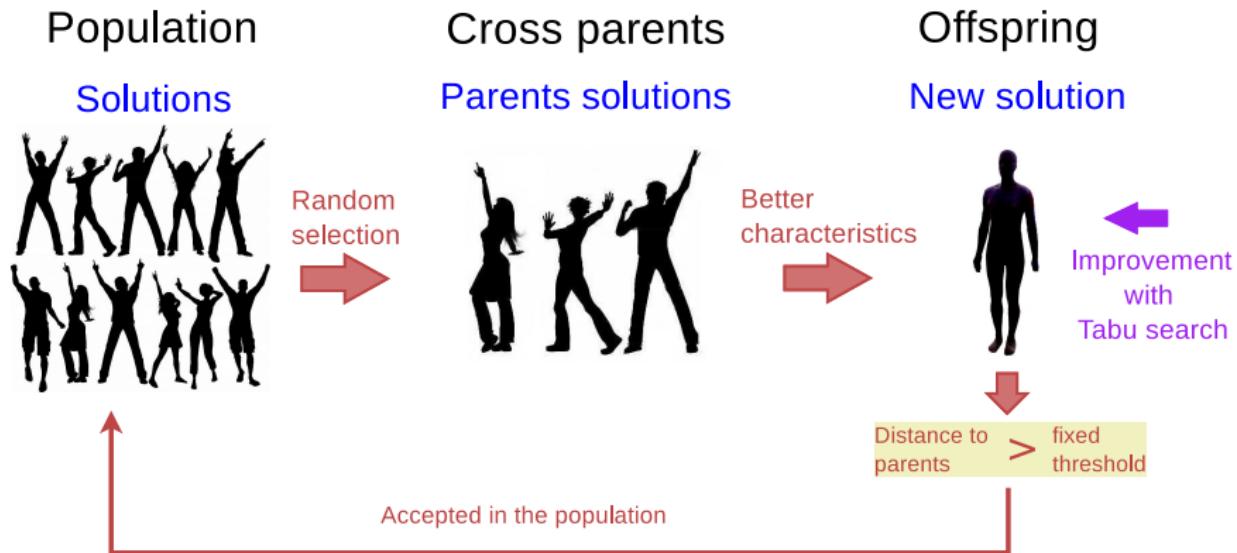
Offspring

New solution

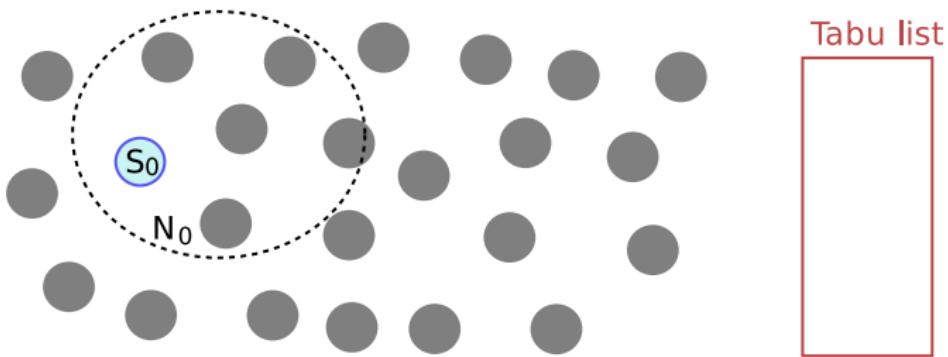


← Improvement with Tabu search

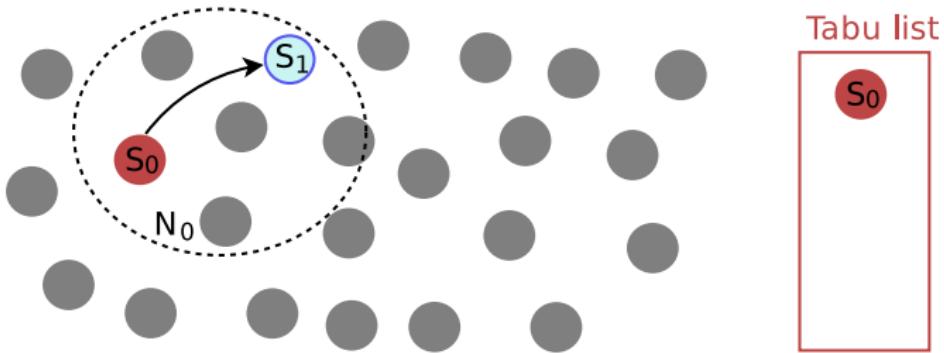
## Memetic Algorithm



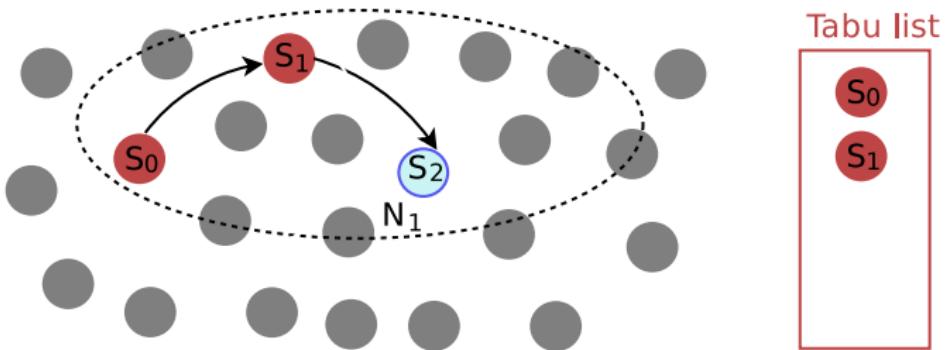
## Tabu Search



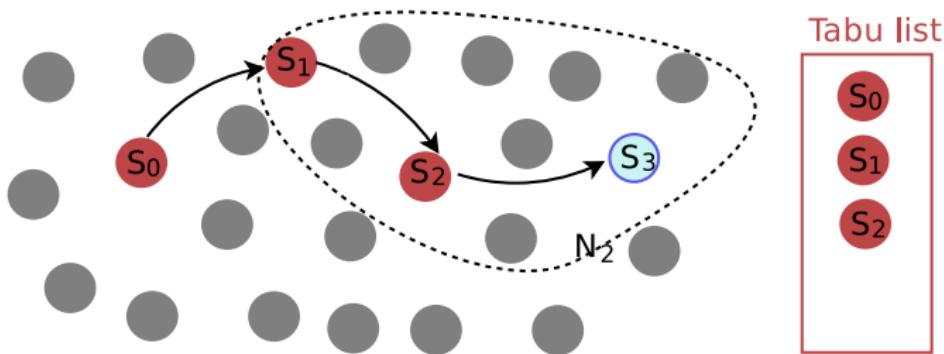
## Tabu Search



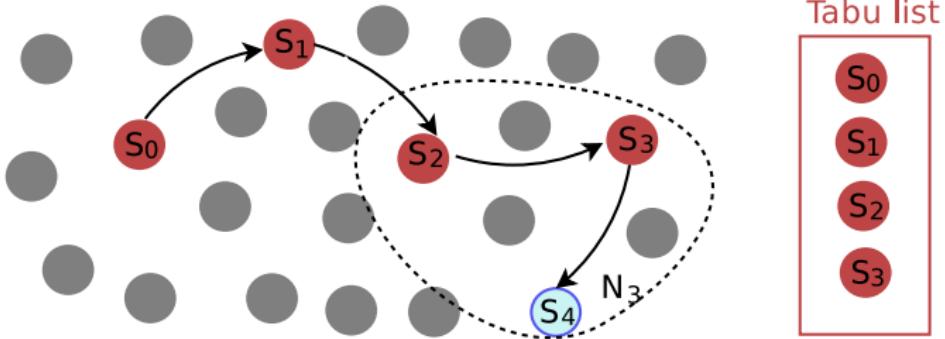
## Tabu Search



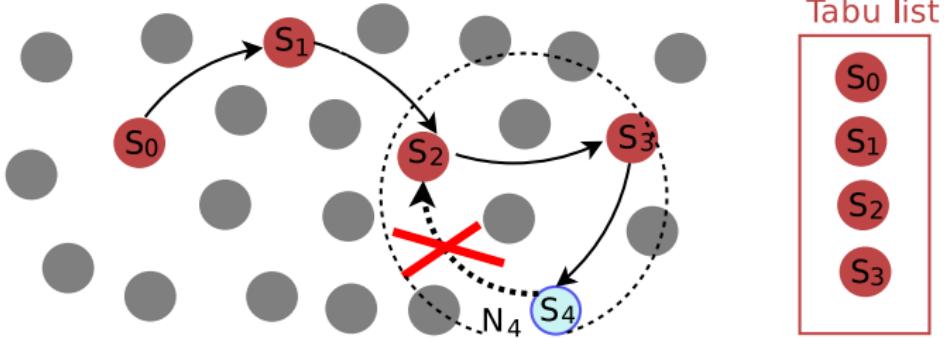
## Tabu Search



## Tabu Search



## Tabu Search



# General memory allocation problem

Same characteristics as previous problem, plus

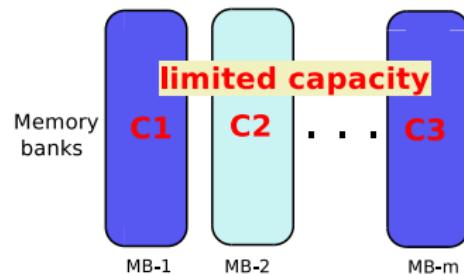
- Sizes of data structures
- Number of accesses of data structures



# General memory allocation problem

Same characteristics as previous problem, plus

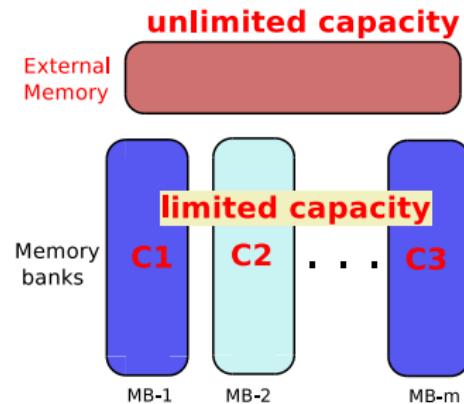
- Sizes of data structures
- Number of accesses of data structures
- Limited capacity of memory banks



# General memory allocation problem

Same characteristics as previous problem, plus

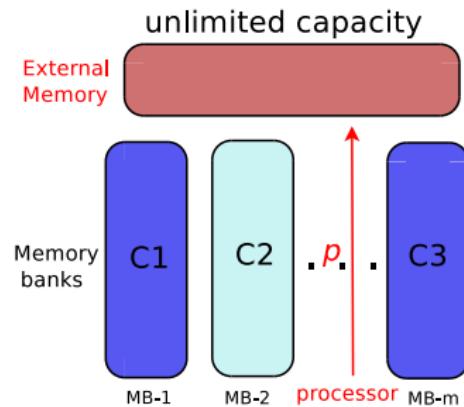
- Sizes of data structures
- Number of accesses of data structures
- Limited capacity of memory banks
- External memory  
→ unlimited capacity



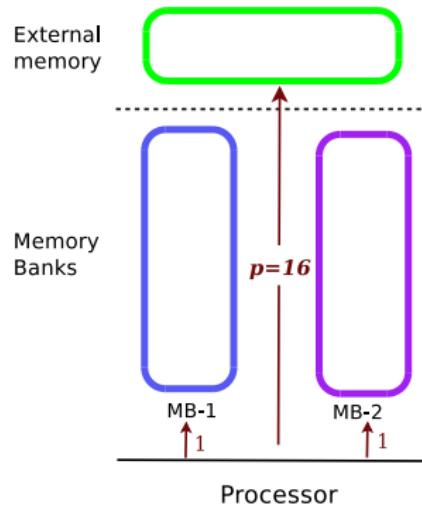
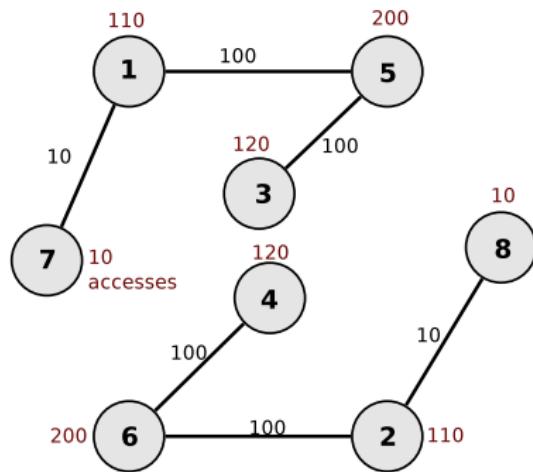
# General memory allocation problem

Same characteristics as previous problem, plus

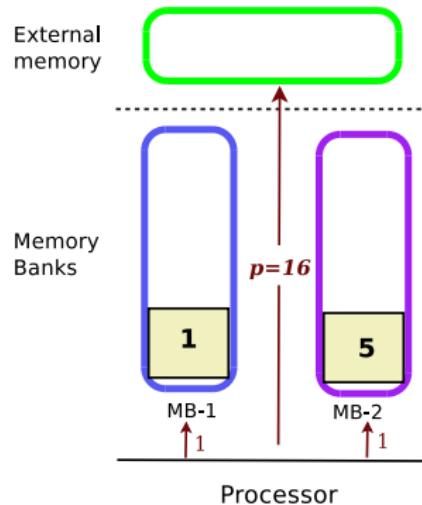
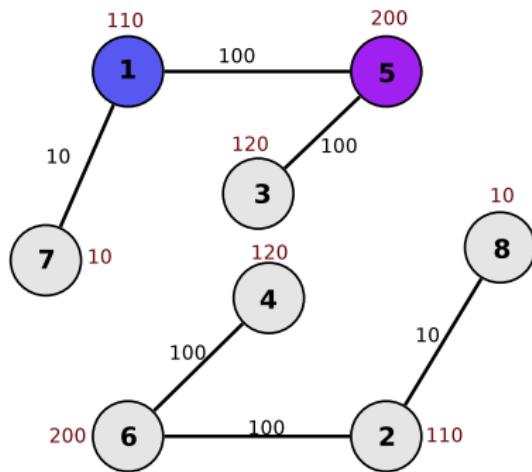
- Sizes of data structures
- Number of accesses of data structures
- Limited capacity of memory banks
- External memory → unlimited capacity
- Access to external memory →  $p$



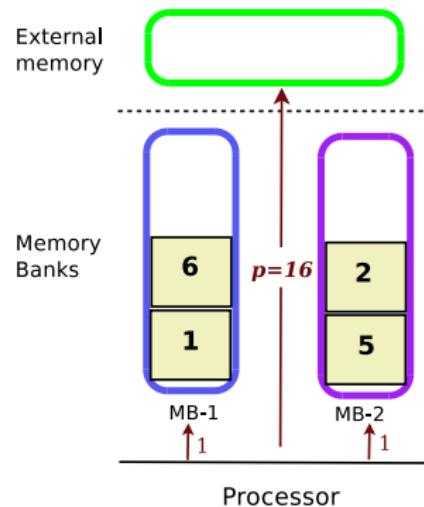
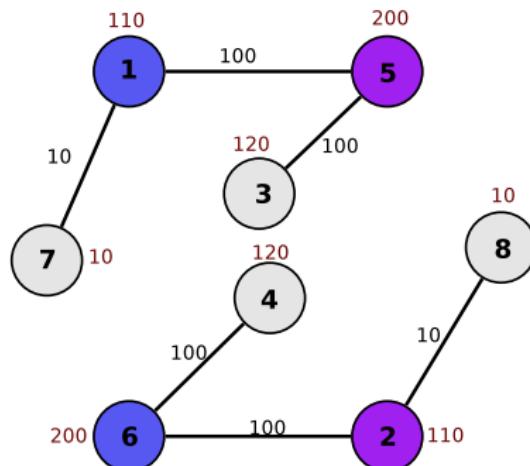
# Example



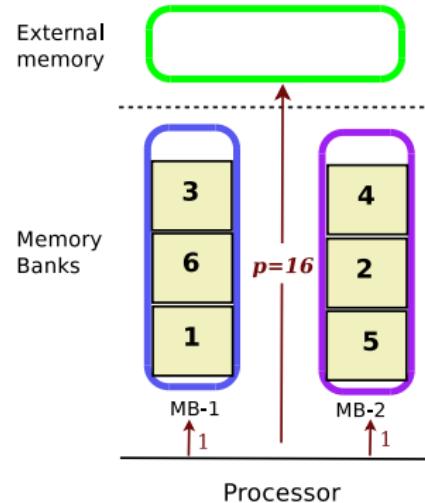
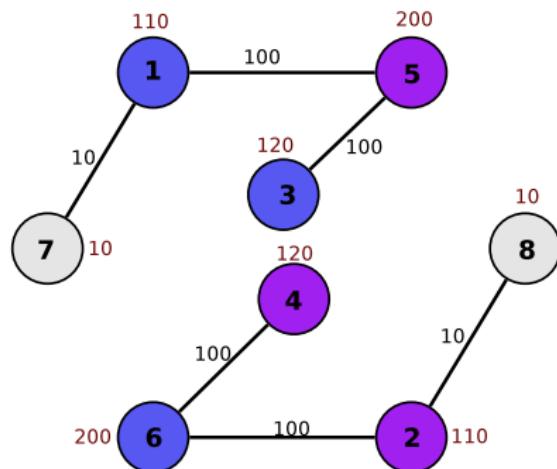
# Example



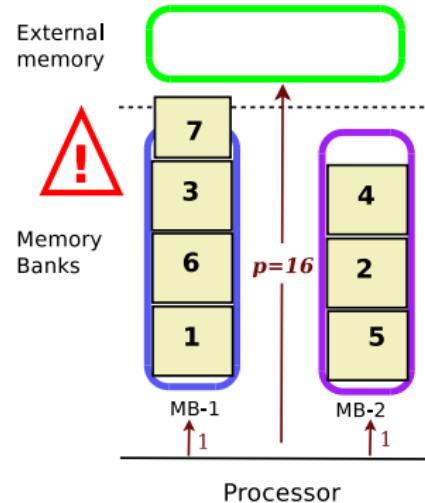
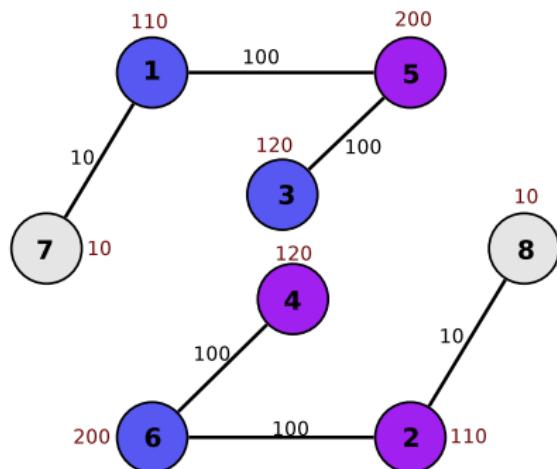
# Example



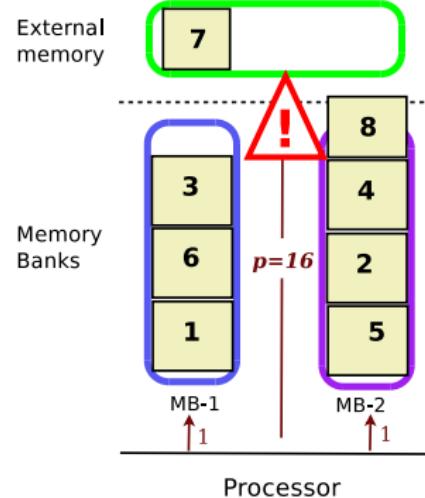
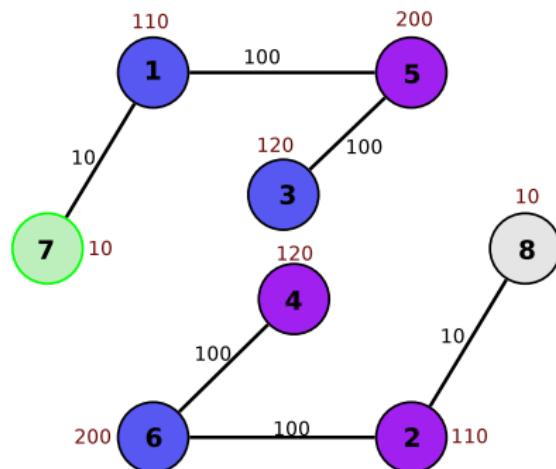
# Example



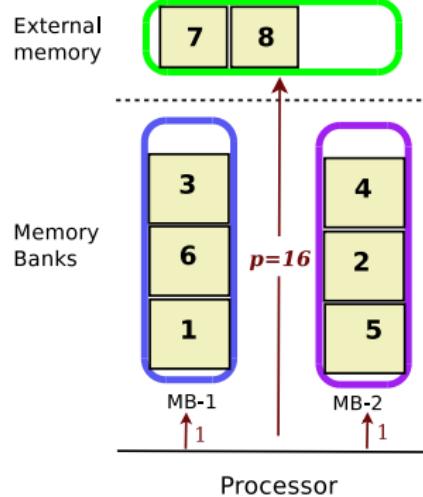
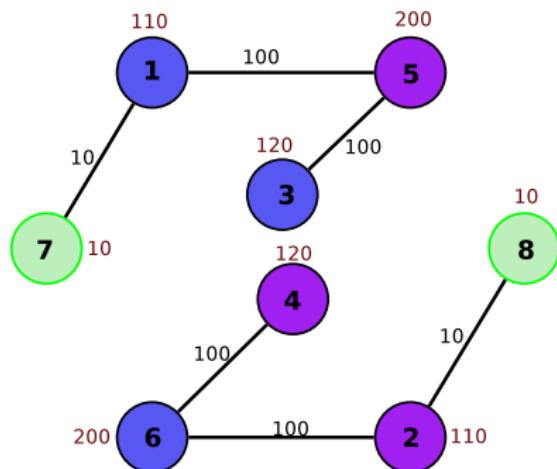
# Example



# Example



# Example



# General memory allocation problem

## Objective:

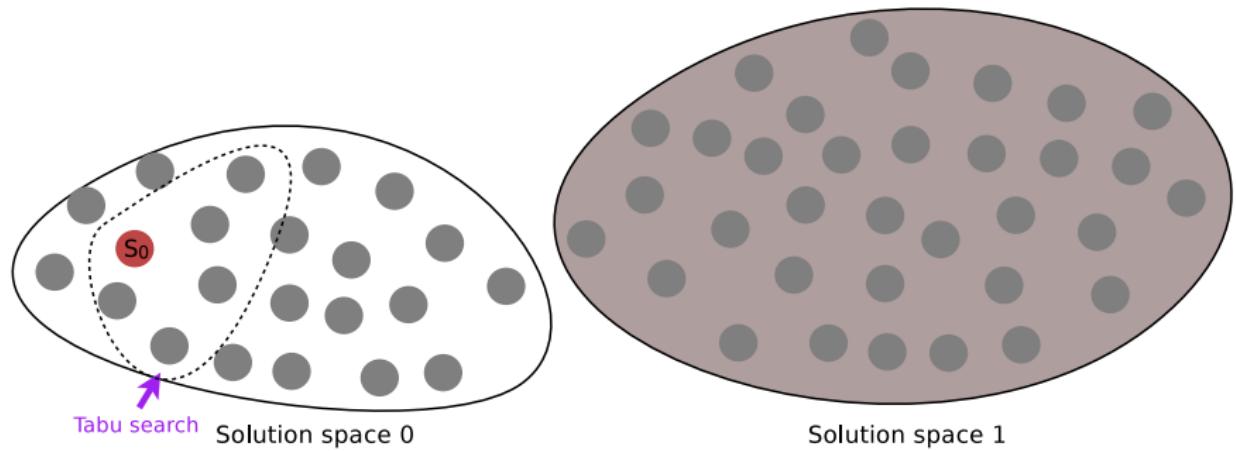
For a given number of capacitated memory banks and an external memory, find a memory allocation for data structures such that the time spent accessing these data is minimized.

## Proposed approach:

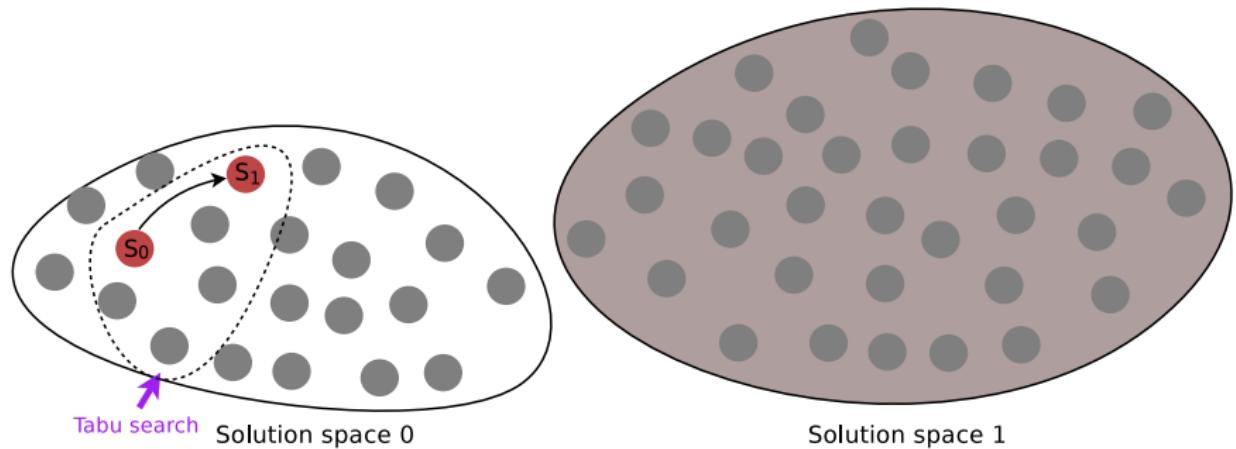
→ Variable Neighborhood Search-Tabu Search (VNS-TS)



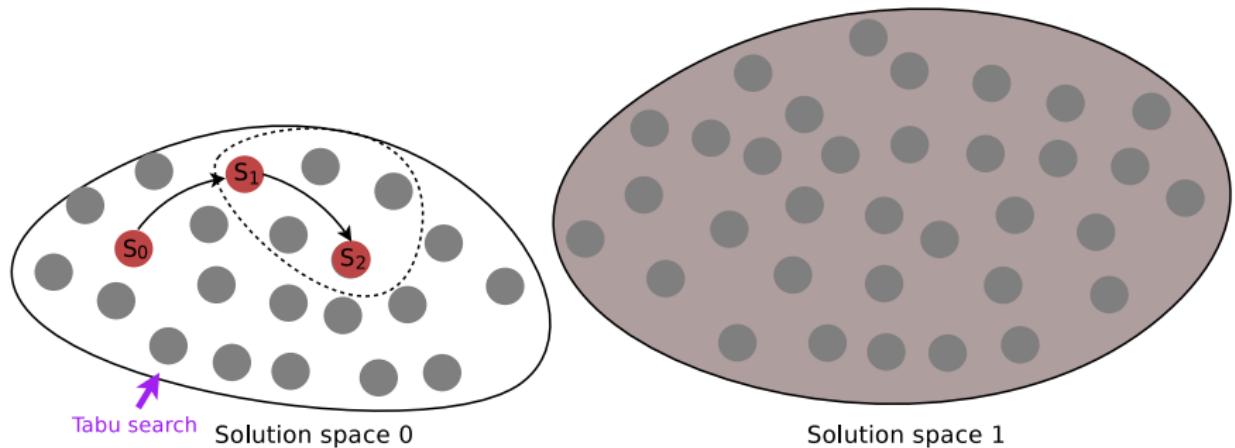
# Hybrid approach (VNS-TS)



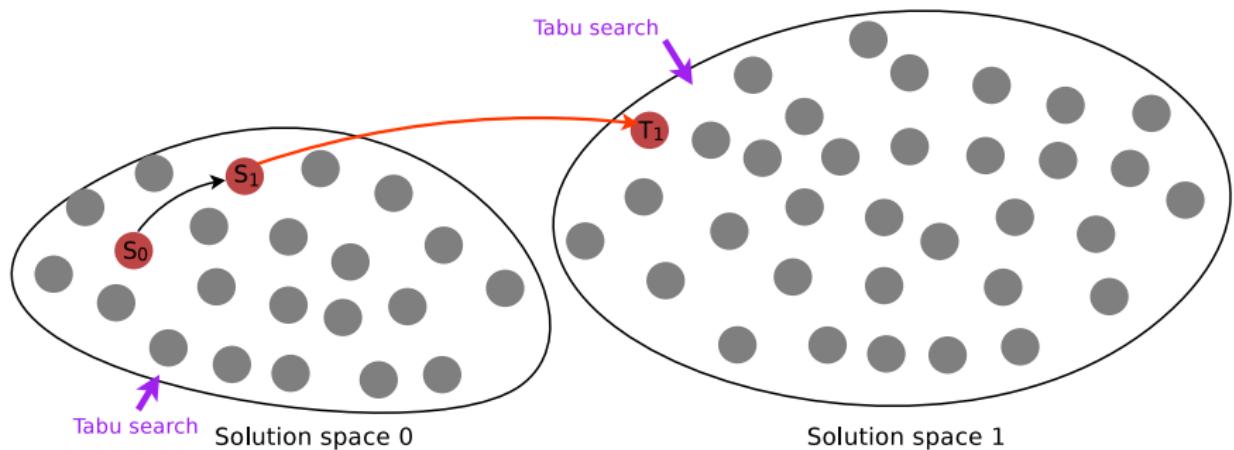
## Hybrid approach (VNS-TS)



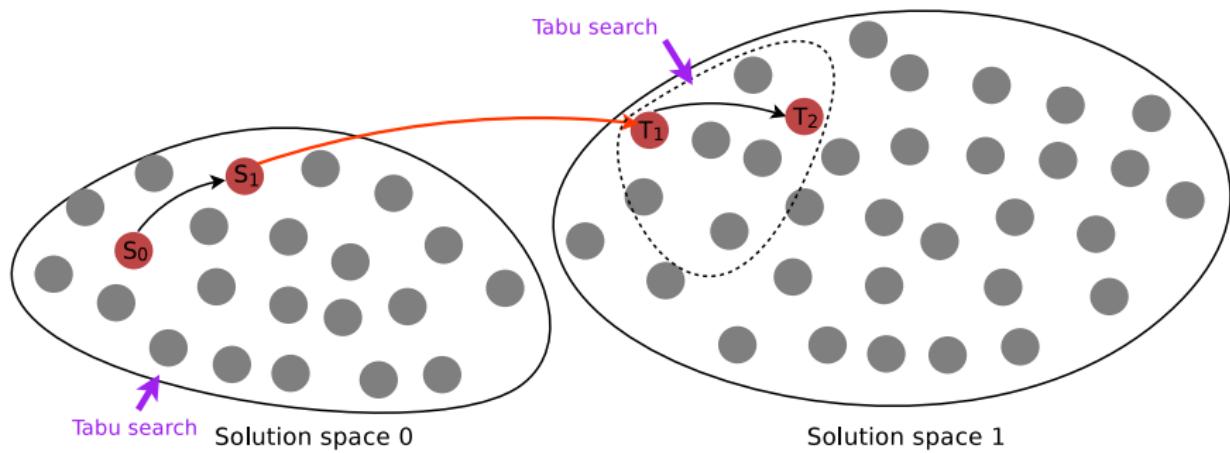
## Hybrid approach (VNS-TS)



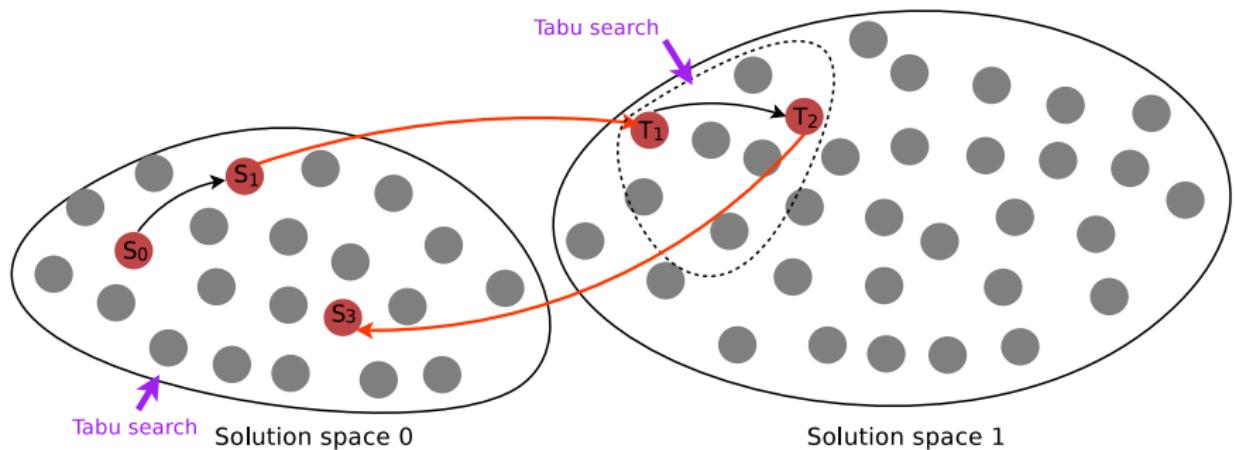
# Hybrid approach (VNS-TS)



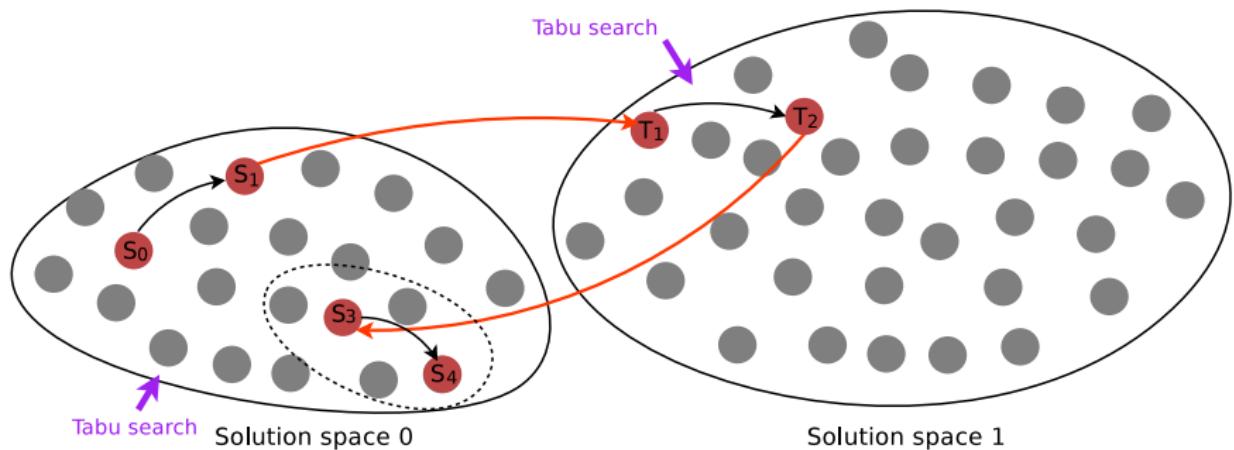
# Hybrid approach (VNS-TS)



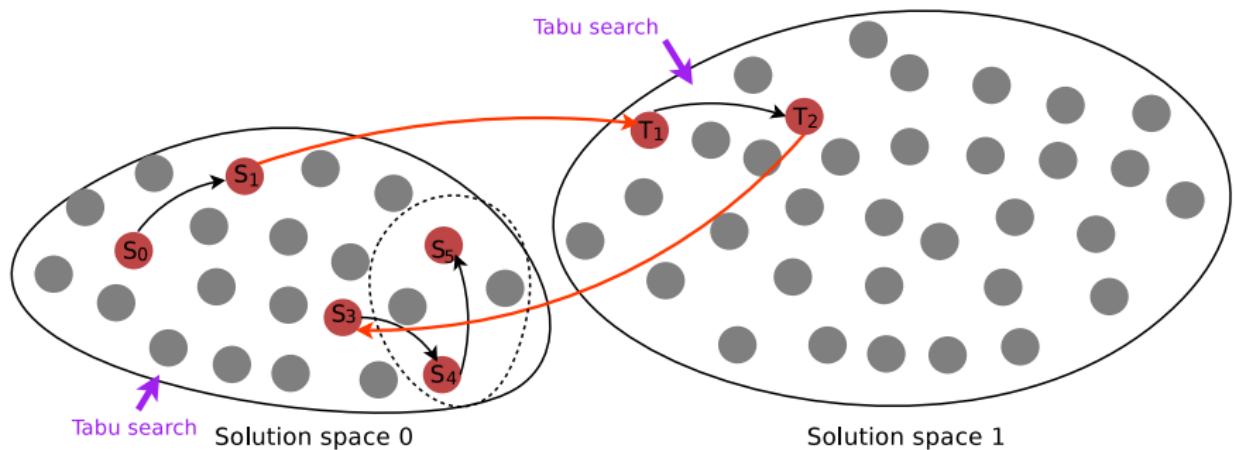
# Hybrid approach (VNS-TS)



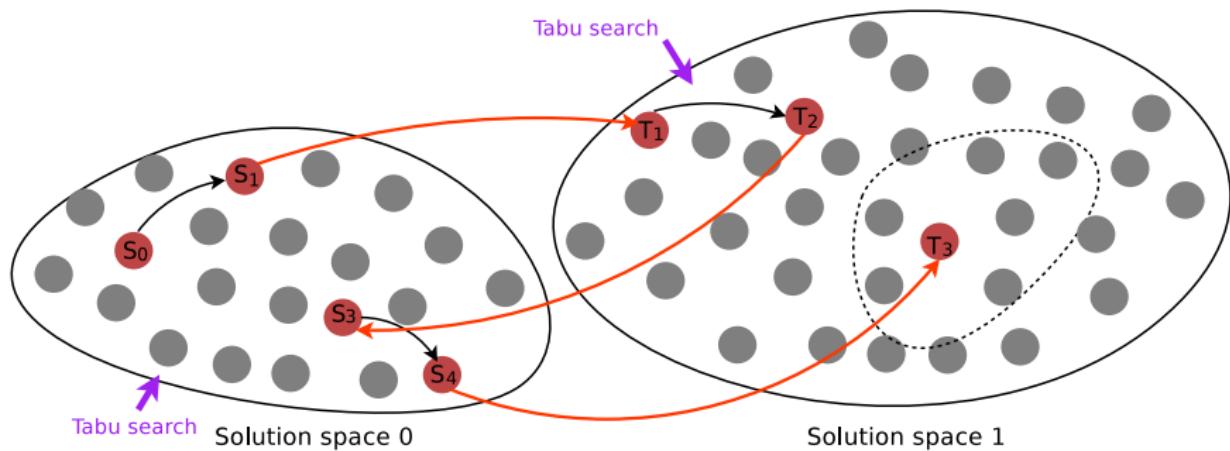
# Hybrid approach (VNS-TS)



# Hybrid approach (VNS-TS)



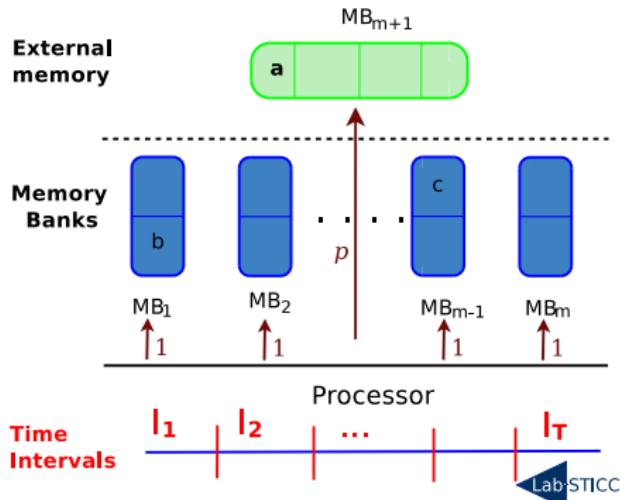
# Hybrid approach (VNS-TS)



# Dynamic memory allocation problem

Same characteristics as general memory allocation problem,  
plus

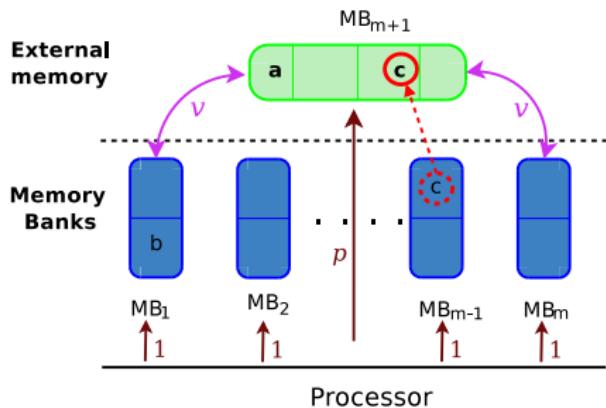
- Application time split into time intervals



# Dynamic memory allocation problem

Same characteristics as general memory allocation problem,  
plus

- Application time split into time intervals
- Transfer rates for data structures
  - from external memory to memory banks,  $v$



Transfer time:

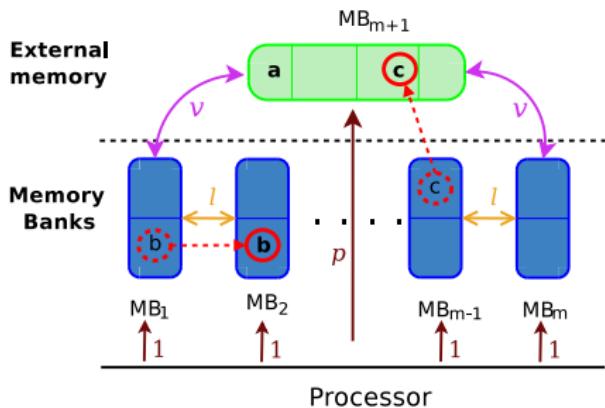
$$C \rightarrow S_C * v \dots$$



# Dynamic memory allocation problem

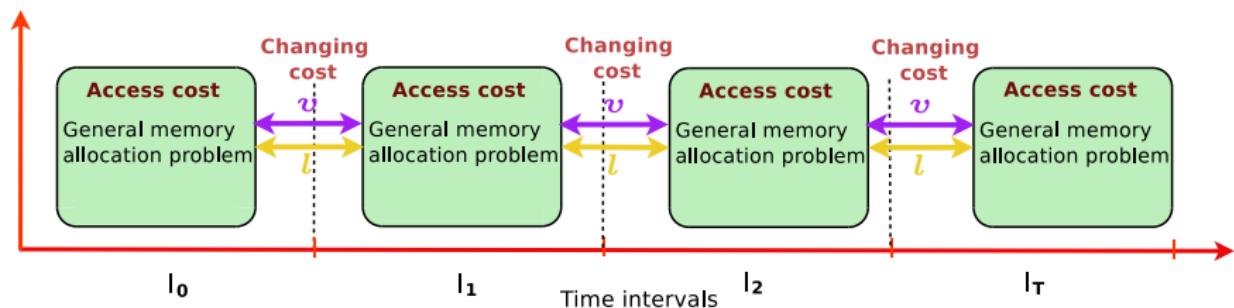
Same characteristics as general memory allocation problem,  
plus

- Application time split into time intervals
- Transfer rates for data structures
  - from external memory to memory banks,  $v$
  - between memory banks,  $l$



Transfer time:  $b \rightarrow s_b * l$ ,  
 $c \rightarrow s_c * v \dots$

## Dynamic memory allocation problem

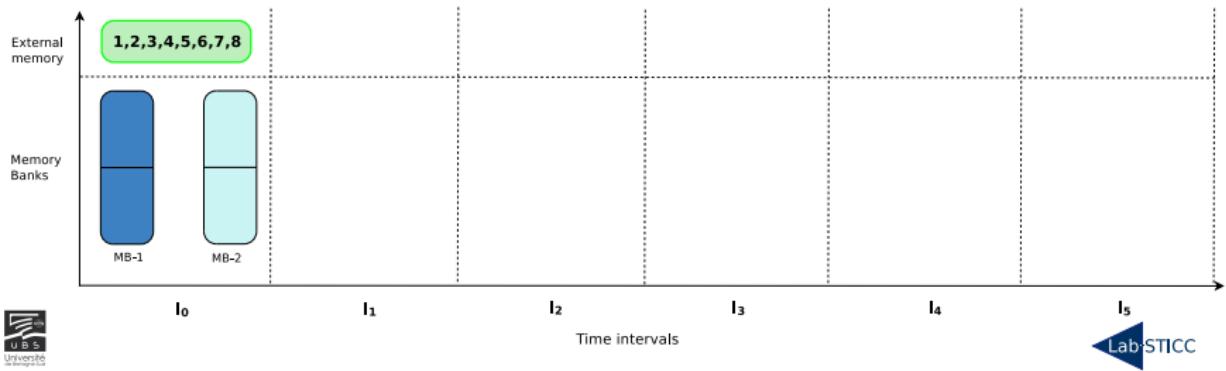


# Example

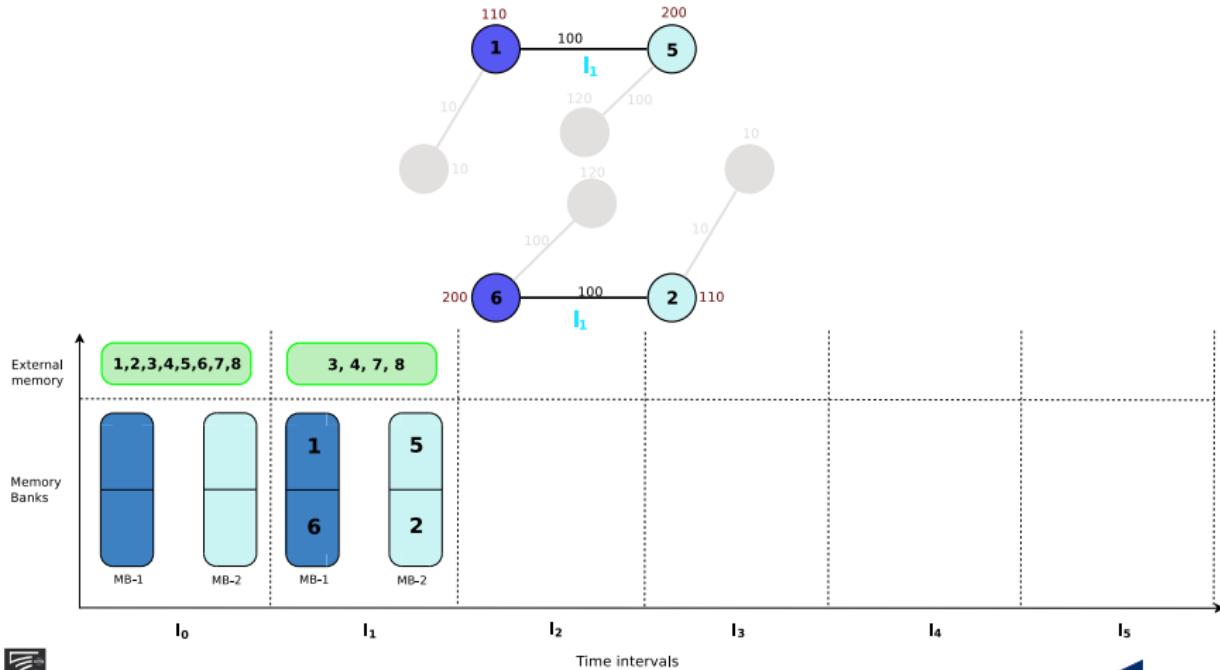
$p = 16$  ms, and  $l = v = 1$  ms/ko.

Intervals $t = 1, \dots, 5$	Data structures $\{a_{1,t}, \dots, a_{n_t,t}\}$	Conflicts $(a_{k_1,t}, a_{k_2,t})$	Cost $d_{k,t}$	Access time $e_{a_{i,t},t}$
1	{ 1, 5, 2, 6 }	(1;5) (2;6)	1,046,529 1,046,529	$e_{1,1} = e_{2,1} =$ $e_{5,1} = e_{6,1} = 1,046,529$
2	{ 3, 4, 5, 6 }	(3;5) (4;6)	1,046,529 1,046,529	$e_{3,2} = e_{5,2} =$ $e_{4,2} = e_{6,2} = 1,046,529$
3	{ 1,5,7 }	(1;7) (1;5)	1,023 1,023	$e_{1,3} = 2,046$ $e_{5,3} = e_{7,3} = 1,023$
4	{ 2,6,8 }	(2;6) (2;8)	1,023 1,023	$e_{2,4} = 2,046$ $e_{6,4} = e_{8,4} = 1,023$
5	{ 3,4 }	(3;3) (4;4)	2,046 2,046	$e_{3,5} = e_{4,5} = 2,046$

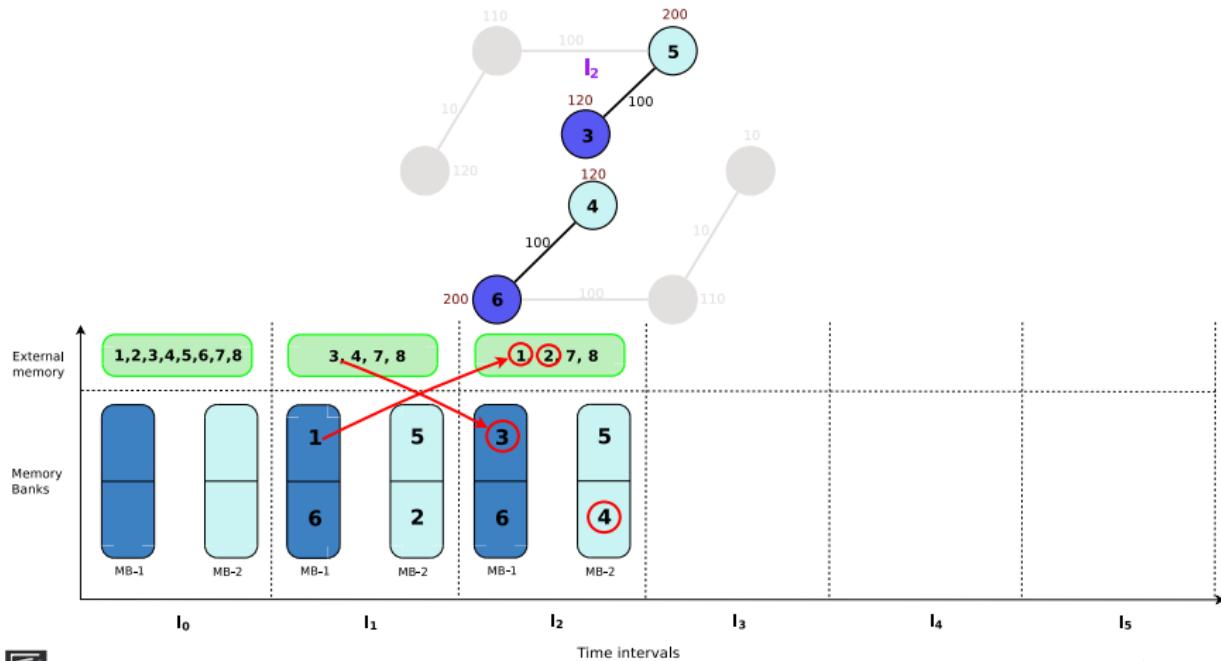
# Example



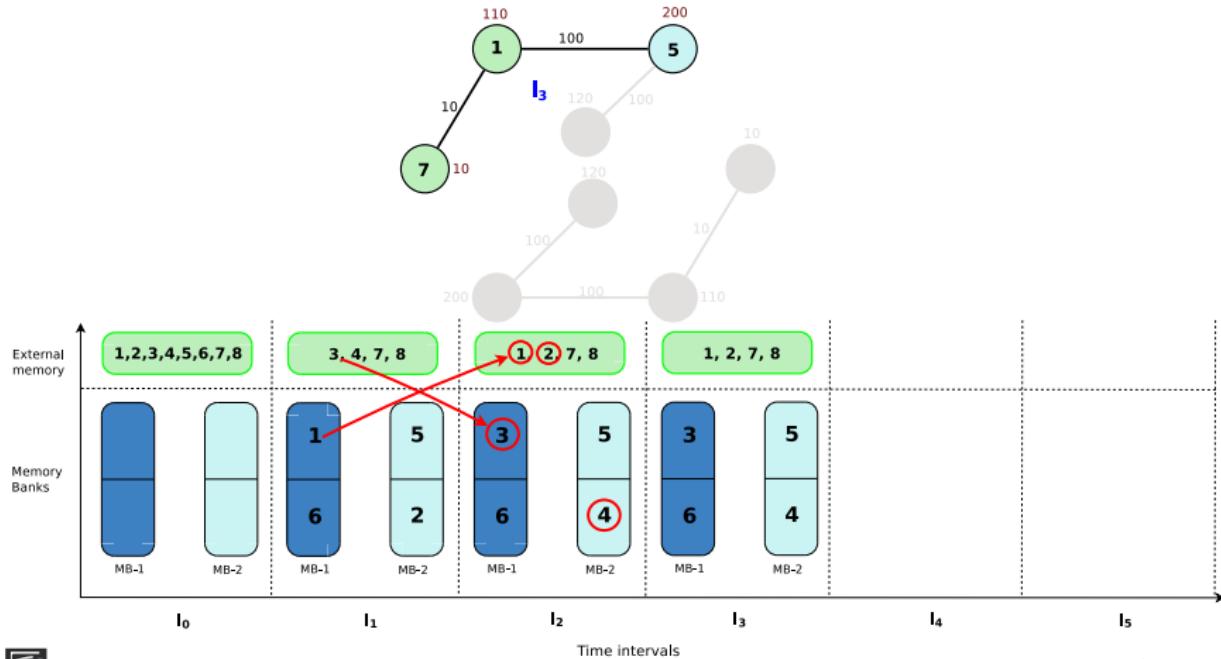
# Example



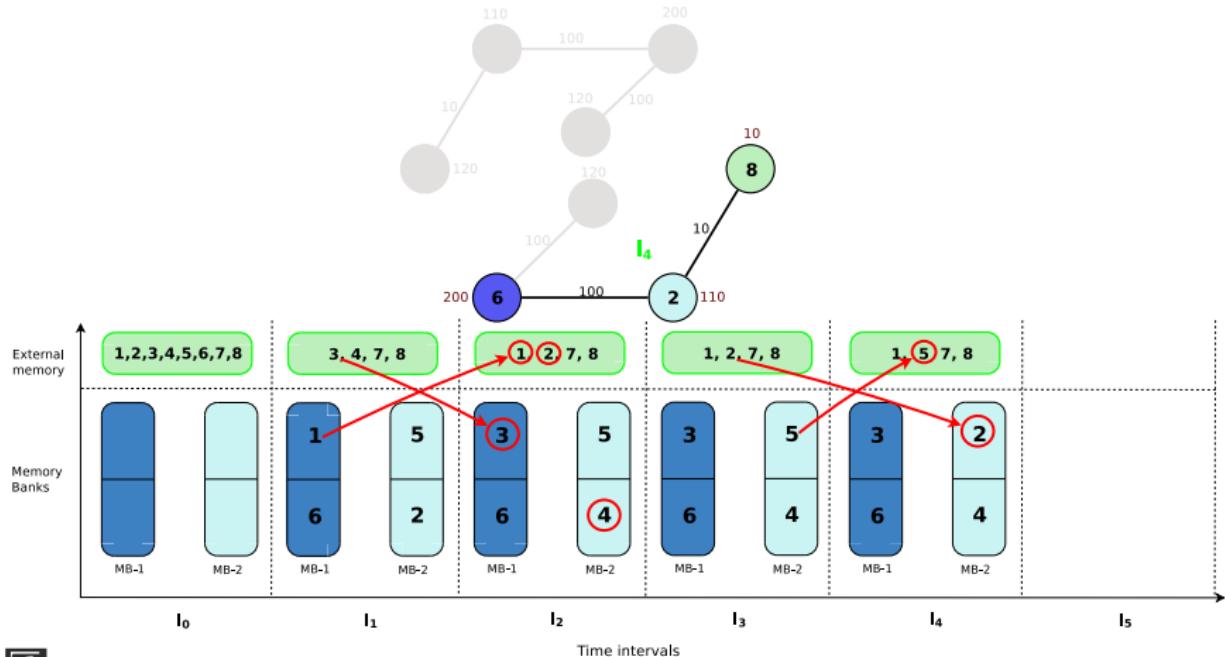
# Example



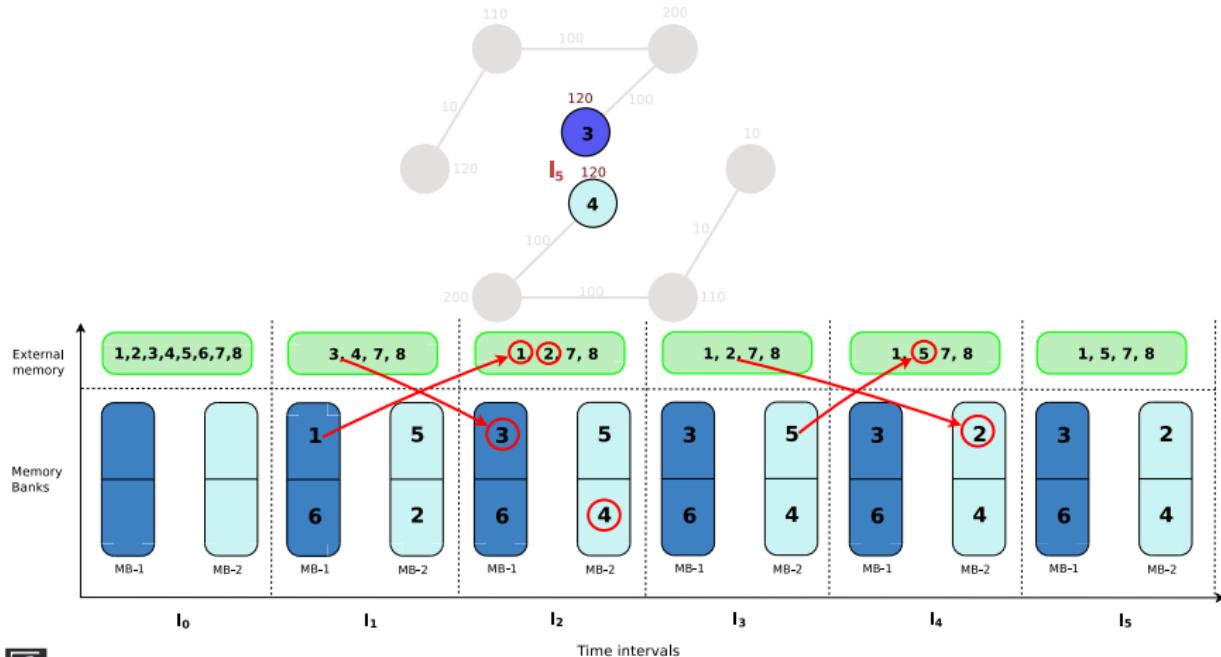
# Example



# Example



# Example



# MemExplorer Dynamic

## Objective

Allocate a memory bank or the external memory to any data structure of the application for **each time interval**

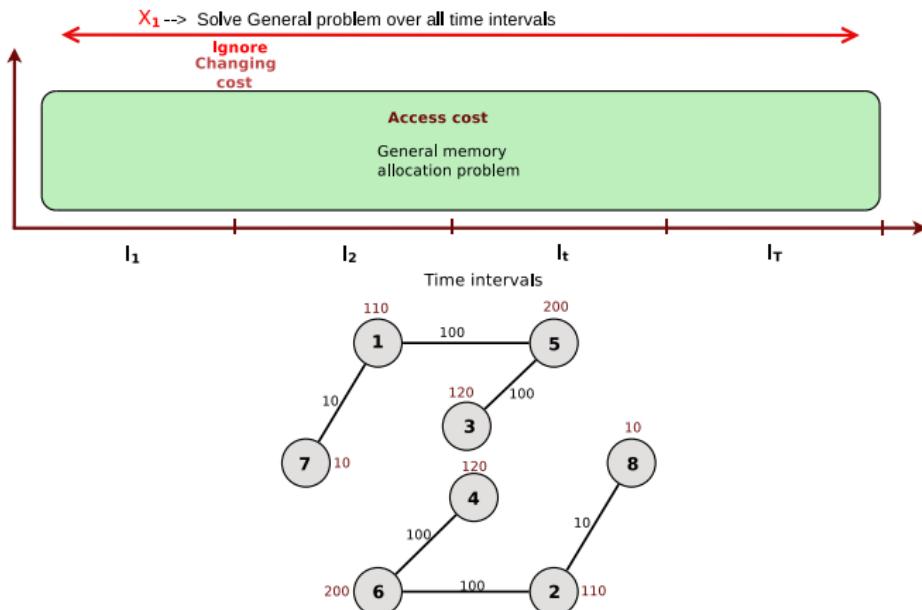
- minimize data access time and data moving time
- satisfy the memory banks' capacity.

Two proposed approaches:

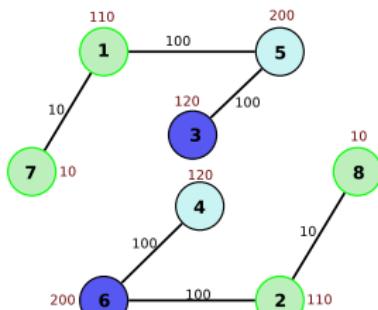
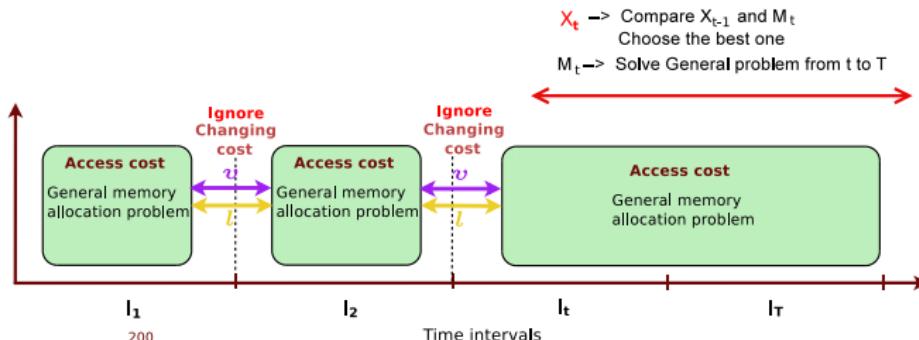
- Long-term approach
- Short-term approach



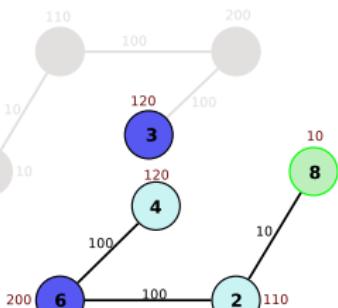
## Long-term approach



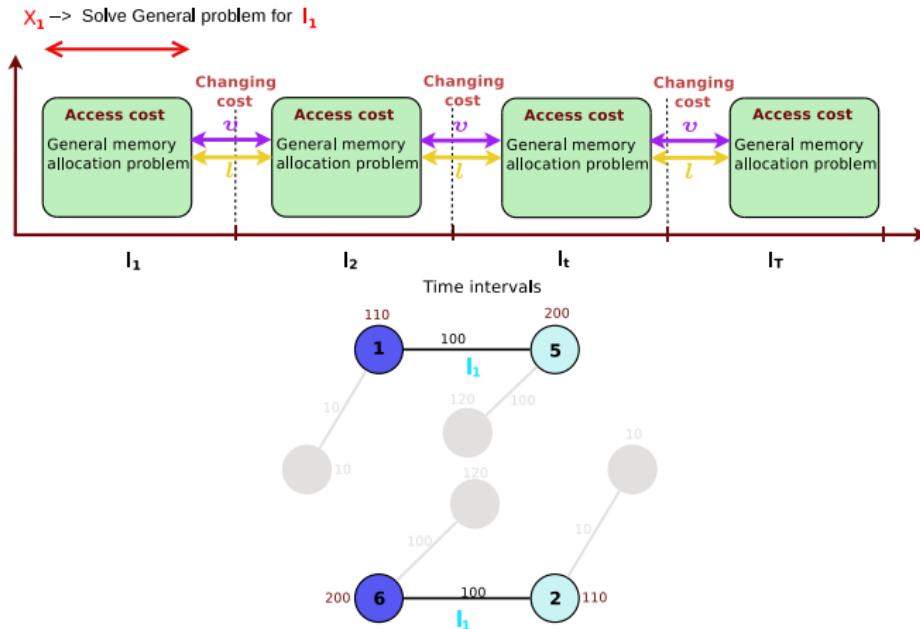
## Long-term approach



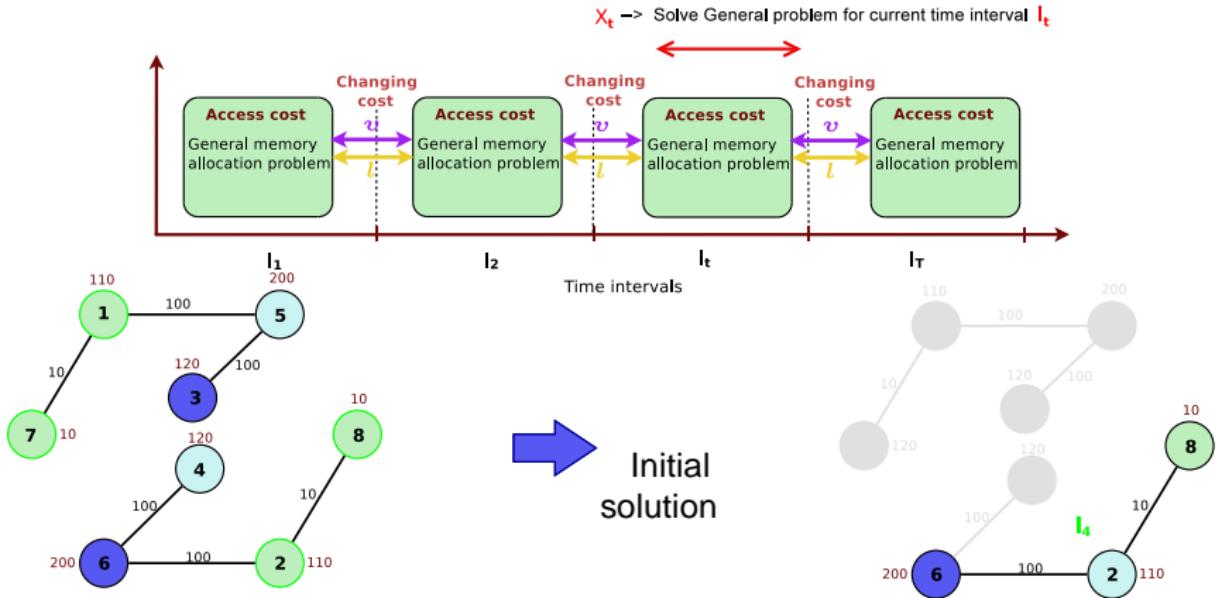
Compare  
two solutions



## Short-term approach



## Short-term approach



# Outline

- 1 Introduction
- 2 Memory allocation problems
- 3 Conclusions and future work



# Conclusions

- Four problems to memory allocation
- Approaches inspired from graph coloring algorithms
- Well balanced search in terms of intensification and diversification
- Results are better than ILP (Xpress-MP) and LocalSearch 1.0 (Bouygues e-lab. <http://e-lab.bouygues.com>)
- General memory allocation problem implemented in *SoftExplorer*



## Future work

- Adapt our approaches for the memory allocations with small granularity
  - Extend our approaches for other memory allocation problems
- 
- New upper bounds exploiting graph topology
  - Adapt algorithms for the Bin Packing to memory allocation problems
  - Use the idea of the Knapsack algorithm for our problems
  - Design matheuristics to memory allocation problems



## Future work

- Mid-term approach to combine the benefits of Short term and Long term approaches
- Global approach for the dynamic memory allocation
- Continue the implementation in *SoftExplorer*



# Thank you for your attention

Publications:

- Two journal articles:
  - Discrete Applied Mathematics
  - Journal of Heuristics
- Three international conferences:  
**Evocop, EU/Meeting and CTW09**
- Three national conferences:  
**Roadef 2011, MajecSTIC and Roadef 2010**

