

Tarefa básica - Regra de Cramer

01) a)
$$\begin{cases} 2x - y = 2 \\ -x + 3y = -3 \end{cases}$$

$$D = \begin{vmatrix} 2 & -1 \\ -1 & 3 \end{vmatrix} = 6 - 1 = 5$$

$$D_x = \begin{vmatrix} 2 & -1 \\ -3 & 3 \end{vmatrix} = 6 - 3 = 3$$

$$x = \frac{D_x}{D} = \frac{3}{5}$$

$$D_y = \begin{vmatrix} 2 & 2 \\ -1 & -3 \end{vmatrix} = -6 - (-2) = -4$$

$$y = \frac{D_y}{D} = \frac{-4}{5}$$

$$V = \left\{ \left(\frac{3}{5}, -\frac{4}{5} \right) \right\}$$

b)
$$\begin{cases} 3x - y + z = 1 \\ 2x + 3z = -1 \\ 4x + y - 2z = 7 \end{cases}$$

$$D = \begin{vmatrix} 3 & -1 & 1 \\ 2 & 0 & 3 \\ 4 & 1 & -2 \end{vmatrix} = -10 - 13 = -23$$

$$D_x = \begin{vmatrix} -1 & 1 & 1 \\ -1 & 0 & 3 \\ 7 & 1 & -2 \end{vmatrix} = -22 - 1 = -23$$

$$x = \frac{D_x}{D} = \frac{-23}{-23} = 1$$

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-4 6 3 -4

$$D_1 = \begin{vmatrix} 3 & 1 & 1 & 3 & 1 \\ 2 & -1 & 3 & 2 & -1 \\ 4 & 7 & 2 & 4 & 7 \end{vmatrix} \quad 32 - 55 = -23 \quad y = \frac{D_1}{D} = \frac{-23}{-23} = 1 //$$

$$D_2 = \begin{vmatrix} 3 & -1 & 1 & 3 & -1 \\ 2 & 0 & -1 & 2 & 0 \\ 4 & 1 & 2 & 4 & 1 \end{vmatrix} \quad 6 - (-17) = 23 \quad z = \frac{D_2}{D} = \frac{23}{-23} = -1 //$$

$$V = \{(1, -1, -1)\} //$$

$$02) \begin{cases} 3x + 4y - 2z = 1 \text{ (I)} \\ 4x + 5y + 2z = 12 \text{ (II)} \\ x - 2y + 3z = 8 \text{ (III)} \end{cases} \Rightarrow \begin{cases} 4x + 2y + 2z = 9 \\ 4x + 5y + 2z = 12 \text{ (iv)} \end{cases}$$

$$(I) + (III)$$

$$3x + 4y - 2z = 1$$

$$x - 2y + 3z = 8$$

$$4x + 2y + 2z = 9$$

$$\begin{cases} 4x + 2y + 2z = 9 \\ -4x - 5y - 2z = -12 \end{cases}$$

$$-3y = -3$$

$$y = \frac{-3}{-3} = 1$$

$$R \cdot A //$$

$$y = 1 //$$

$$234$$

04)
$$\begin{cases} x + 2y - 3z = 29 \\ x + 3y + 2z = 4 \\ x - y - 2z = 8 \end{cases} \quad D = \begin{vmatrix} 1 & 2 & -3 \\ 1 & 3 & 2 \\ 1 & -1 & -2 \end{vmatrix} \begin{vmatrix} 1 & 2 \\ 1 & 3 \\ 1 & -1 \end{vmatrix}$$

$-9 - 2 - 4 = -15$
 $-6 + 4 + 3 = 1$

$-12 - 58 - 16 = -146$

$Dx = \begin{vmatrix} 29 & 2 & -3 \\ 4 & 3 & 2 \\ 8 & -1 & -2 \end{vmatrix} \begin{vmatrix} 29 & 2 \\ 4 & 3 \\ 8 & -1 \end{vmatrix}$

$-130 - (-146) = 16$ $x = \frac{Dx}{D} = \frac{16}{16} = 1$

$-174 + 32 + 12 = -130$

$-12 + 16 - 58 = -54$

$Dy = \begin{vmatrix} 1 & 29 & -3 \\ 1 & 4 & 2 \\ 1 & 8 & -2 \end{vmatrix} \begin{vmatrix} 1 & 29 \\ 1 & 4 \\ 1 & 8 \end{vmatrix}$

$= 26 - (-54) = 80$ $y = \frac{Dy}{D} = \frac{80}{16} = 5$

$-8 - 58 - 24 = -90$

$82 - 4 - 16 = 99$

$Dz = \begin{vmatrix} 1 & 2 & 29 \\ 1 & 3 & 4 \\ 1 & -1 & 8 \end{vmatrix} \begin{vmatrix} 1 & 2 \\ 1 & 3 \\ 1 & -1 \end{vmatrix}$

$3 - 99 = -96$ $z = \frac{Dz}{D} = \frac{-96}{16} = -6$

$24 + 8 - 29 = 3$

$1 + 5 - 6 = 0$ $R: A //$

05)
$$\begin{cases} 2x + y = 5 \\ 2y + z = 3 \\ 3x + 2y + z = 7 \end{cases} \quad D = \begin{vmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 3 & 2 & 1 \end{vmatrix} \begin{vmatrix} 2 & 1 \\ 0 & 2 \\ 3 & 2 \end{vmatrix}$$

$7 - 4 = 3$

$14 - 10 - 3 = 1$

$Dx = \begin{vmatrix} 5 & 1 & 0 \\ 3 & 2 & 1 \\ 7 & 2 & 1 \end{vmatrix} \begin{vmatrix} 5 & 1 \\ 3 & 2 \\ 7 & 2 \end{vmatrix}$

$23 - 27 = -4$

$x = \frac{-4}{1} = -4$

$4 - 3 - 0 = 1$

$Dy = \begin{vmatrix} 2 & 0 & 1 \\ 0 & 2 & 1 \\ 3 & 2 & 1 \end{vmatrix} \begin{vmatrix} 2 & 0 \\ 0 & 2 \\ 3 & 2 \end{vmatrix}$

$21 - 14 = 7$

$y = \frac{7}{1} = 7$

$$Z = \begin{array}{ccc|cc} & 4 & 1 & 5 & 2 & 1 \\ 0 & 2 & 3 & 0 & 2 & \\ 3 & 2 & 2 & 2 & 2 & \end{array}$$

30 12 0

20 9 0

$$37 - 42 = -5$$

$$Z = -\frac{5}{3}$$

-R.D //

06) $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 2 & 2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{cases} x + 0 + 0 \\ 2x + y + 0 \\ -x + 2y + 2z \end{cases} \Rightarrow \begin{cases} x = 3 \\ 2x + y = 7 \\ -x + 2y + 2z = 1 \end{cases}$

$$D = \begin{array}{ccc|cc} & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & \\ 2 & 1 & 0 & 2 & 1 & = 2 \\ -1 & 2 & 2 & -1 & 2 & \end{array}$$

2 0 0

$$DZ = \begin{array}{ccc|cc} & -3 & 14 & 0 & 1 & 0 \\ 1 & 0 & 3 & 1 & 0 & \\ 2 & 1 & 7 & 2 & 1 & 11 - 11 = 0 \\ -1 & 2 & -1 & -1 & 2 & \end{array}$$

-1 0 12

$$DY = \begin{array}{ccc|cc} & 0 & 0 & 12 & 1 & 3 \\ 1 & 3 & 0 & 1 & 3 & \\ 2 & 7 & 0 & 2 & 7 & = 14 - 12 = 2 \\ -1 & -1 & 2 & -1 & -1 & \end{array}$$

14 0 0

$$Z = \frac{0}{2} = 0$$

$$x = 3$$

-R.E //

$$y = \frac{2}{2} = 1$$

Tarefa básica - Escalonamento

$$01) \begin{cases} 2x - y - 3z = -5 \text{ I} \\ x + 3y - z = 11 \text{ II} \\ x - 5z = 3 \text{ III} \rightarrow x = 3 + 5z \end{cases}$$

$$\text{I} - 2(3 + 5z) - y - 3z = -5$$

$$6 + 10z - y - 3z = -5$$

$$6 + 7z - y = -5$$

$$7z - y = -5 - 6$$

$$7z - y = -11$$

$$-y = -11 - 7z \quad (-1)$$

$$y = 11 + 7z \rightarrow y$$

$$\text{II} - 3 + 5z + 3(7z + 11) - z = 11$$

$$3 + 5z + 21z + 33 - z = 11$$

$$3 + 25z + 33 = 11$$

$$25z = 11 - 36$$

$$25z = -25$$

$$z = \frac{-25}{25}$$

$$z = -1$$

$$\boxed{z = -1}$$

$$\text{III} - x - 5z = 3$$

$$x - 5(-1) = 3$$

$$x = 3 - 5$$

$$\boxed{x = -2}$$

$$\text{II} - x + 3y - z = 11$$

$$-2 + 3y - z = 11$$

$$-2 + 3y + 1 = 11$$

$$3y = 11 + 2 - 1$$

$$3y = 12$$

$$y = \frac{12}{3}$$

$$y = 4$$

$$\boxed{y = 4}$$

$$x = -2, y = 4, z = -1$$

$$02) \begin{cases} x = 2y \\ 2y = 3z \rightarrow z = \frac{2y}{3} \\ x + y + z = 11 \end{cases}$$

$$x + y + z = 11$$

$$\frac{2y}{3} + y + \frac{2y}{3} = 11$$

$$\frac{6y + 3y + 2y}{3} = 33$$

$$6y + 3y + 2y = 33$$

$$11y = 33$$

$$y = \frac{33}{11}$$

$$\boxed{y = 3}$$

$$x = 2y \quad y + y + z = 11$$

$$x = 23 \quad 6 + 3 + z = 11$$

$$\boxed{x = 6}$$

$$9 + z = 11$$

$$z = 11 - 9$$

$$\boxed{z = 2}$$

$$x + 2y + 3z = ?$$

$$6 + 6 + 6 = 18$$

R B //

$$03) \begin{cases} x + y + z = 0 \\ 2x - y - 2z = 1 \\ 6y + 3z = -12 \end{cases}$$

$$\begin{cases} 2x + 2y + 2z = 0 \quad (-1) \\ 2x - y - 2z = 1 \quad + \\ \hline 6y + 3z = -12 \end{cases}$$

$$\begin{cases} -3y - 4z = 1 \quad (-2) \\ 6y + 3z = -12 \end{cases}$$

$$\begin{cases} -6y - 8z = 2 \\ 6y + 3z = -12 \\ \hline -5z = -10 \\ z = \frac{-10}{-5} \end{cases}$$

R.D //

$$\boxed{z = 2}$$

$$05) \begin{matrix} 134,00 & 115,00 & 48,00 \\ A = \begin{bmatrix} 0 & 3 & 4 \\ 1 & 0 & 5 \\ 2 & 1 & 0 \end{bmatrix} & X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \end{matrix}$$

$$\begin{cases} 0x + 3y + 4z = 135 \\ x + 0y + 5z = 115 \\ 2x + y + 0z = 48 \end{cases}$$

$$DX = \begin{bmatrix} 134 & 3 & 4 \\ 115 & 0 & 5 \\ 48 & 1 & 0 \end{bmatrix} \begin{bmatrix} 135 & 3 \\ 115 & 0 \\ 48 & 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 3 & 4 & 0 & 3 \\ 1 & 0 & 5 & 1 & 0 \\ 2 & 1 & 0 & 2 & 1 \end{bmatrix} = 34$$

$$1180 - 675 = 510$$

$$X = \frac{510}{34} = 15$$

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$$D_1 = \begin{array}{c|ccc} & 0 & 134 & 4 \\ \hline 0 & 0 & 134 & \\ 1 & 1 & 115 & 5 \\ 2 & 2 & 48 & 0 \end{array} \quad \begin{array}{c|cc} & 0 & 134 \\ \hline 0 & 0 & 134 \\ 1 & 1 & 115 \\ 2 & 2 & 48 \end{array} \quad = 1532 - 920 = 612$$

$$0 \quad 1340 \quad 192$$

$$Y = \frac{612}{34} = 18$$

$$0 \quad 0 \quad 144$$

$$D_2 = \begin{array}{c|ccc} & 0 & 3 & 134 \\ \hline 0 & 0 & 3 & \\ 1 & 1 & 0 & 115 \\ 2 & 2 & 1 & 48 \end{array} \quad \begin{array}{c|cc} & 0 & 3 \\ \hline 0 & 0 & 3 \\ 1 & 1 & 0 \\ 2 & 2 & 1 \end{array} \quad = 824 - 144 = 680$$

$$0 \quad 690 \quad 135$$

$$Z = \frac{680}{34} = 20$$

$$15 + 18 + 20 = 53,00$$

R.A