

Tarefa básica - Regra de Cramer

01) a)
$$\begin{cases} 2x - y = 2 \\ -x + 3y = -3 \end{cases}$$

$$D = \begin{vmatrix} 2 & -1 \\ -1 & 3 \end{vmatrix} = 6 - 1 = 5$$

$$D_x = \begin{vmatrix} 2 & -1 \\ -3 & 3 \end{vmatrix} = 6 - 3 = 3$$

$$x = \frac{D_x}{D} = \frac{3}{5}$$

$$D_y = \begin{vmatrix} 2 & 2 \\ -1 & -3 \end{vmatrix} = -6 - (-2) = -4$$

$$y = \frac{D_y}{D} = \frac{-4}{5}$$

$$V = \left\{ \left(\frac{3}{5}, -\frac{4}{5} \right) \right\}$$

b)
$$\begin{cases} 3x - y + z = 1 \\ 2x + 3z = -1 \\ 4x + y - 2z = 7 \end{cases}$$

$$D = \begin{vmatrix} 3 & -1 & 1 \\ 2 & 0 & 3 \\ 4 & 1 & -2 \end{vmatrix} = -10 - 13 = -23$$

$$D_x = \begin{vmatrix} -1 & 1 & 1 \\ -1 & 0 & 3 \\ 7 & 1 & -2 \end{vmatrix} = -22 - 1 = -23$$

$$x = \frac{D_x}{D} = \frac{-23}{-23} = 1$$

-4 -3 -4

$$Dy = \begin{vmatrix} 3 & 1 & 1 & 3 & 1 \\ 2 & -1 & 3 & 2 & -1 \\ 4 & 7 & 2 & 4 & 7 \end{vmatrix} \quad 32 - 55 = -23 \quad y = \frac{Dy}{D} = \frac{-23}{-23} = 1 //$$

6 12 14
-3 -14

$$Dz = \begin{vmatrix} 3 & -1 & 1 & 3 & -1 \\ 2 & 0 & -1 & 2 & 0 \\ 4 & 1 & 7 & 4 & 1 \end{vmatrix} \quad 6 - (-17) = 23 \quad z = \frac{Dz}{D} = \frac{23}{-23} = -1 //$$

0 4 2

$$V = \{(1, -1, -1)\}$$

Q2) $\begin{cases} 3x + 4y - 2z = 1 & \text{(I)} \\ 4x + 5y + 2z = 12 & \text{(II)} \\ x - 2y + 3z = 8 & \text{(III)} \end{cases} \Rightarrow \begin{cases} 4x + 2y + 2z = 9 \\ 4x + 5y + 2z = 12 & \text{(I')} \end{cases}$

(I) + (III)

$$3x + 4y - 2z = 1$$

$$x - 2y + 3z = 8$$

$$4x + 2y + 2z = 9$$

$$\begin{cases} 4x + 2y + 2z = 9 \\ -4x - 5y - 2z = 12 \end{cases}$$

$$-3y = -3$$

$$y = \frac{-3}{-3} = 1$$

R.A //

$$y = 1 //$$

04)

$$\begin{cases} x + 2y - 3z = 29 \\ x + 3y + 2z = 4 \\ x - y - 2z = 8 \end{cases}$$

D =

$$\begin{vmatrix} 1 & 2 & -3 \\ 1 & 3 & 2 \\ 1 & -1 & -2 \end{vmatrix} \begin{vmatrix} 1 & 2 \\ 1 & 3 \\ 1 & -1 \end{vmatrix}$$

$$-9 - 2 - 4 = -15$$

$$1 + 15 = 16$$

$$-6 + 4 + 3 = 1$$

$$-12 - 58 - 16 = -146$$

$$D_x = \begin{vmatrix} 29 & 2 & -3 \\ 4 & 9 & 2 \\ 8 & -1 & -2 \end{vmatrix} \begin{vmatrix} 29 & 2 \\ 4 & 9 \\ 8 & -1 \end{vmatrix}$$

$$-130 - (-146) = 16$$

$$x = \frac{D_x}{D} = \frac{16}{16} = 1$$

$$-134 + 32 + 12 = -130$$

$$-12 + 16 - 58 = -54$$

$$D_y = \begin{vmatrix} 1 & 29 & -3 \\ 1 & 4 & 2 \\ 1 & 8 & -2 \end{vmatrix} \begin{vmatrix} 1 & 29 \\ 1 & 4 \\ 1 & 8 \end{vmatrix}$$

$$= 26 - (-54) = 80$$

$$y = \frac{D_y}{D} = \frac{80}{16} = 5$$

$$-8 - 58 - 24 = -90$$

$$22 - 4 + 16 = 34$$

$$D_z = \begin{vmatrix} 1 & 2 & 29 \\ 1 & 3 & 4 \\ 1 & -1 & 8 \end{vmatrix} \begin{vmatrix} 1 & 2 \\ 1 & 3 \\ 1 & -1 \end{vmatrix}$$

$$3 - 99 = -96$$

$$z = \frac{D_z}{D} = \frac{-96}{16} = -6$$

$$24 + 8 - 29 = 3$$

$$1 + 5 - 6 = 0 \quad \text{R.A}$$

$$0 + 4 + 0 = 4$$

05)

$$\begin{cases} 2x + y = 5 \\ 2y + z = 3 \\ 3x + 2y + z = 7 \end{cases}$$

D =

$$\begin{vmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 3 & 2 & 1 \end{vmatrix} \begin{vmatrix} 2 & 1 \\ 0 & 2 \\ 3 & 2 \end{vmatrix}$$

$$7 - 4 = 3$$

$$14 - 10 + 3 = 7$$

$$4 - 3 + 0 = 1$$

$$D_x = \begin{vmatrix} 5 & 1 & 0 \\ 3 & 2 & 1 \\ 7 & 2 & 1 \end{vmatrix} \begin{vmatrix} 5 & 1 \\ 3 & 2 \\ 7 & 2 \end{vmatrix}$$

$$23 - 27 = -4$$

D_y =

$$\begin{vmatrix} 2 & 0 & 1 \\ 0 & 2 & 1 \\ 3 & 2 & 1 \end{vmatrix} \begin{vmatrix} 2 & 0 \\ 0 & 2 \\ 3 & 2 \end{vmatrix}$$

$$21 - 14 = 7$$

$$10 - 9 + 6 = 7$$

$$x = \frac{-4}{3}$$

$$6 - 15 + 0 = -9$$

$$y = \frac{7}{3}$$

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S T Q Q S S D

30 12 0

2	1	5	2	1
0	2	3	0	2
3	2	2	2	2

20 9 0

$37 - 42 = -5$

$z = -\frac{5}{3}$

R.D.

Tarefa básica - Escalonamento

$$01) \begin{cases} 2x - y - 3z = -5 \text{ I} \\ x + 3y - z = 11 \text{ II} \\ x - 5z = 3 \text{ III} \rightarrow x = 3 + 5z \end{cases}$$

$$\text{I} - 2(3 + 5z) - y - 3z = -5$$

$$6 + 10z - y - 3z = -5$$

$$6 + 7z - y = -5$$

$$7z - y = -5 - 6$$

$$7z - y = -11$$

$$-y = -11 - 7z \quad (-1)$$

$$y = 11 + 7z - 6y$$

$$\text{II} - 3 + 5z + 3(7z + 11) - z = 11$$

$$3 + 5z + 21z + 33 - z = 11$$

$$3 + 25z + 33 = 11$$

$$25z = 11 - 36$$

$$25z = -25$$

$$z = \frac{-25}{25}$$

$$z = -1$$

$$\boxed{z = -1}$$

$$\text{III} - x - 5z = 3$$

$$x - 5(-1) = 3$$

$$x = 3 - 5$$

$$\boxed{x = -2}$$

$$\text{II} - x + 3y - z = 11$$

$$-2 + 3y - z = 11$$

$$-2 + 3y + 1 = 11$$

$$3y = 11 + 2 - 1$$

$$3y = 12$$

$$y = \frac{12}{3}$$

$$y = 4$$

$$\boxed{y = 4}$$

$$x = -2, y = 4, z = -1$$

$$02) \begin{cases} x = 2y \\ 2y = 3z \rightarrow z = \frac{2y}{3} \\ x + y + z = 11 \end{cases}$$

$$x + y + z = 11$$

$$\frac{2y}{1} + \frac{y}{1} + \frac{2y}{3} = 11$$

$$\frac{6y + 3y + 2y}{3} = 33$$

$$3$$

$$6y + 3y + 2y = 33$$

$$x = 2y \quad x + y + z = 11$$

$$x = 23 \quad 6 + 3 + z = 11$$

$$\boxed{x = 6}$$

$$9 + z = 11$$

$$z = 11 - 9$$

$$\boxed{z = 2}$$

$$x + 2y + 3z = ?$$

$$6 + 6 + 6 = 18 //$$

$$R. 8 //$$

$$11y = 33$$

$$y = \frac{33}{11}$$

$$\boxed{y = 3}$$

$$03) \begin{cases} x + y + z = 0 \\ 2x - y - 2z = 1 \\ 6y + 3z = -12 \end{cases}$$

$$\begin{cases} 2x + 2y + 2z = 0 \quad (-1) \\ 2x - y - 2z = 1 \quad + \\ \hline 6y + 3z = -12 \end{cases}$$

$$\begin{cases} -3y - 4z = 1 \quad (-2) \\ 6y + 3z = -12 \end{cases}$$

$$\begin{cases} -6y - 8z = 2 \\ 6y + 3z = -12 \end{cases}$$

$$-5z = -10$$

$$z = \frac{-10}{-5}$$

$$z = 2$$

R.D //

$$05) \begin{matrix} 134,00 & 115,00 & 48,00 \\ A = \begin{bmatrix} 0 & 3 & 4 \\ 1 & 0 & 5 \\ 2 & 1 & 0 \end{bmatrix} & X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \end{matrix}$$

$$\begin{cases} 0x + 3y + 4z = 135 \\ x + 0y + 5z = 115 \\ 2x + y + 0z = 48 \end{cases}$$

$$DX = \begin{bmatrix} 134 & 3 & 4 \\ 115 & 0 & 5 \\ 48 & 1 & 0 \end{bmatrix} \begin{bmatrix} 135 \\ 115 \\ 48 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 3 & 4 & 0 & 3 \\ 1 & 0 & 5 & 1 & 0 \\ 2 & 1 & 0 & 2 & 1 \end{bmatrix} = 34$$

$$1150 - 675 = 510$$

$$x = \frac{510}{34} = 15$$

S T Q Q S S D

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$$D_4 = \begin{array}{c|ccc} & 0 & 134 & 4 \\ \hline 1 & 1 & 115 & 5 \\ 2 & 2 & 48 & 0 \end{array} \quad \begin{array}{c|ccc} & 0 & 134 & \\ \hline 1 & 1 & 115 & \\ 2 & 2 & 48 & \end{array} \quad = 1532 - 920 = 612$$

$$0 \quad 1340 \quad 192$$

$$Y = \frac{612}{34} = 18$$

$$0 \quad 0 \quad 144$$

$$D_2 = \begin{array}{c|ccc} & 0 & 3 & 134 \\ \hline 1 & 1 & 0 & 115 \\ 2 & 2 & 1 & 48 \end{array} \quad \begin{array}{c|ccc} & 0 & 3 & \\ \hline 1 & 1 & 0 & \\ 2 & 2 & 1 & \end{array} \quad = 824 - 144 = 680$$

$$0 \quad 690 \quad 135$$

$$Z = \frac{680}{34} = 20 //$$

$$15 + 18 + 20 = 53,00$$

R.A