University of Bern
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The effects of ageing on intergenerational wealth distribution in politico-economic equilibrium

Master Thesis

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Abstract

We examine the effects of population ageing on intergenerational wealth distribution in politico-economic equilibrium. To this purpose, we introduce a probabilistic voting setting à la Lindbeck and Weibull (1987) into a standard three-period OLG. We find that a decrease in population growth rates raises social security taxes and benefits due to the shift in the political power towards the elderly. Besides, ageing changes prices because of a decline in the workforce. Young and middle-aged workers react to these developments by decreasing their savings. Since the savings of the young decrease disproportionately compared to the savings of the middle-aged, the intergenerational wealth inequality rises slightly. In a model economy without government sector, population ageing increases wealth inequality as well, but much more strongly than in the politico-economic model. In this setting, workers are affected by population ageing only through prices. We conclude that the introduction of a politico-economic pay-as-you-go setting dampens the rise in inequality because of the effects that are induced by the rise in social security taxes.

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Chapter 1

Introduction

Wealth inequality is an important global challenge that will shape the economic, political and social future. The literature treating threats posed by wealth (and income) inequality is enormous. In an article of the World Economic Forum, Pickett (2015) clusters the risks of inequality into five key categories: Health problems, worsened social relationships, hampered economic growth and stability, impairment of the human capital development and the undermining of strategies towards a more sustainable economy. The World Economic Forum even declared inequality as one of the top five risks for the global economy in 2017 (Elliott (2017)).
Wealth inequality can arise from economic factors, such as different endowments, incomes or propensities to save. However, another important source of wealth inequality is age. In fact, the link between age and wealth is very strong (Vandenbroucke and Zhu (2017)). Households soon to be retired carry three to four times more wealth than young households, as is apparent in Figure 1.1. This follows from the fact that wealth is accumulated over the life cycle, reaching its peak before retirement, from where on it is slowly consumed.

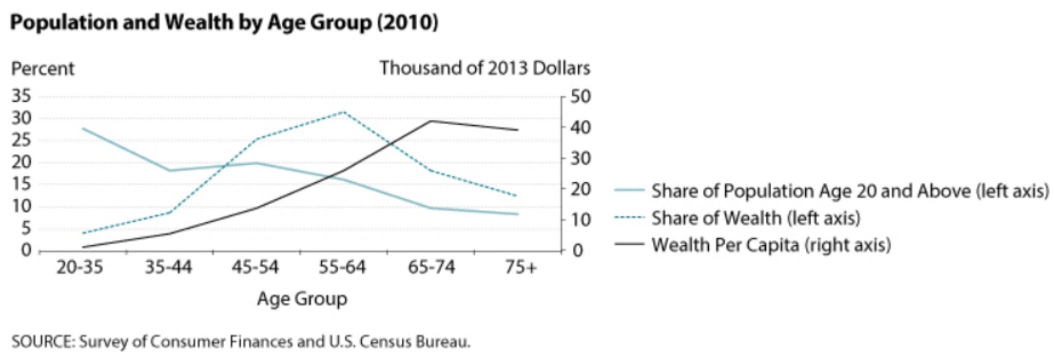

Figure 1.1: Population and wealth by age group (Vandenbroucke and Zhu (2017))

At the same time, the population of the industrialized world is rapidly ageing, what makes the issue of wealth inequality even more delicate. Present thesis works closely on these issues by posing the following questions: What are the implications of population ageing on intergenerational wealth distribution? Will economies with public pension programs perform better regarding wealth inequality during this demographic transition compared to economies without such systems? What are the effects of ageing on the social security tax rate[1] of said systems through the induced shift in the political power? And will a shift in the tax rate, in turn, affect wealth inequality? The goal of this thesis is to address these questions within a theoretical framework designed to analyze the link between ageing, social security and wealth distribution in politico-economic equilibrium. Our setting thus tries to capture the different channels through which population ageing enters the agents' decisions and hence, the intergenerational wealth distribution. In order to quantify wealth inequality, we will calculate the Gini coefficient[2].

Several authors have analyzed the effects of the demographic transition towards an older population on wealth distribution. In a short essay, Vandenbroucke and Zhu (2017) show that a shift in the demographic structure can help improve the wealth equality between generations. They demonstrate that the number of older and thus wealthier agents increases with population ageing whereas the number of young and thus poorer agents decreases. This can result in a decline in the Gini coefficient and hence, a decrease in wealth inequality. The drawback of this approach is the implicit assumption that the number of assets that the different households hold remain constant during the demographic transition. Mierau and Turnovsky (2014) relax this assumption and find the converse. By using an endogenous growth model where they allow for a general demographic structure, they discover that a drop in the mortality rate will push up savings for life-cycle purposes and increases the number of persons who are at the top of their life-cycle savings. This results in a large increase in wealth inequality. Other contributions that examine this topic with respect to comparing different pension programs in OLG economies are Krueger and Ludwig (2007) and Bielecki et al. (2017). Bielecki et al. (2017) find that longevity increases wealth inequality, regardless of the exact form of the pension system. Contrarily, Krueger and Ludwig (2007) disclose that ageing does not increase wealth inequality, using the same arguments as Vandenbroucke and Zhu (2017). The research is, therefore, marked by mixed findings. However, all these contributions do not include politico-economic aspects and therefore do not take into consideration repercussions of ageing on the support for social security. In this respect, our work will differ from the named contributions. Actually, to our knowledge, our model is the first that addresses this question in a politico-economic context. Another aspect in which our work will differ from afore-named contributions is that we will not talk about

[1] The corresponding rate in the U.S. is the Old Age and Survivor's Insurance tax rate (OASI rate), which is currently equal to 12.4%.

[2] A Gini coefficient of 0 indicates no inequality, meaning that everyone in the economy holds the same amount of wealth, whereas a value of 1 symbolizes perfect inequality, meaning that one person holds all the wealth.

fertility, longevity or mortal rates, but only about changes in the population growth rates.

The economic environment of the model we use incorporates a three-period OLG with young workers, middle-aged workers and retirees. Retirees receive social security benefits from a pay-as-you-go system that is financed by an income tax on the two working cohorts. Following the work of Gonzalez-Eiras and Niepelt (2008), the social security tax rate is determined in a probabilistic voting setup. This setup introduces two victory-seeking political candidates that each propose a tax rate. Voters do not only care about the proposed tax rate, but form also ideological preferences, which the candidates, however, cannot observe. The political candidates respond to this uncertainty by proposing a policy platform that maximizes a weighted average welfare of all current voters. Households, therefore, have two different roles: In their role as consumers, they take prices and taxes as given and optimize their lifetime utility. In their role as voters, however, they think through all effects that each proposed tax rate will trigger on future economic and political variables. Based on this they form their policy preferences. A shift in the characteristics of the household may change the political support for the social security system. The probabilistic voting setup thus adequately captures the mutual dependence between social security tax rates and the properties of the households. We will focus entirely on Markov perfect equilibria, suppressing the possibility for more sophisticated forms of equilibria[3]. The solution will be unique since we compute the limit of a finite-horizon economy. We will compare this setting to an economy without government sector in order to quantify the effect of the politico-economic pay-as-you-go system.

We find that intergenerational wealth inequality increases with population ageing, both in the model economy with politico-economic pay-as-you-go and in the model economy without government sector. In the model economy without government sector, the only channel through which population ageing enters the economy is through prices. Lower population growth rates, implying a higher proportion of old people, increase the capital-labor ratio, which pushes wages up and interests down. This price channel disproportionately decreases the savings of the young relative to the savings of the middle-aged workers. This outweighs the induced shift in the population shares and therefore increases the intergenerational wealth inequality. In fact, the Gini coefficient of the population growth rate of $n_{2040} = 1.10$ is about 5.7% higher than the Gini coefficient of $n_{1960} = 1.26$.
Contrary to this, in the model with politico-economic pay-as-you-go system population ageing not only enters the economy through prices but also through equilibrium taxes

[3]See Gonzalez-Eiras and Niepelt (2008) for a more detailed discussion on this topic.

and expected social security benefits. Social security taxes increase with age due to the induced shift in the political power towards the elderly. Higher taxes, in turn, decrease savings further, due to the cut in disposable income as well as an increase in expected social security benefits. This decrease of savings affects all workers more or less proportionally. However, the savings of the young decrease more strongly than the savings of workers of middle age, such that wealth inequality is increased during population ageing. Nevertheless, the increase in inequality is much weaker than in the model without government sector, since the tax channel partly counterbalances this effect. In fact, we show that if the tax channel is sufficiently strong, wealth inequality can be kept constant despite population ageing. We conclude that economies with public pension programs will perform better regarding the rise in wealth inequality during population ageing than economies without such systems.
Our results are in line with Mierau and Turnovsky (2014), as we show that, in general, ageing increases intergenerational wealth inequality. The effects described by Vandenbroucke and Zhu (2017) or Krueger and Ludwig (2007) are generally not strong enough in our model to overweigh the shifts in the savings of the different cohorts. The main difference between our results and the results of Mierau and Turnovsky (2014) is, however, that in our setting steady-state savings of all cohorts decrease with population ageing instead of increasing.

There are two important limitations of these results. Firstly, the numerical results strongly depend on the parameter choice. A rise in the discount factor β not only decreases the Gini coefficient for a given population growth rate but also reduces the increase in inequality during population ageing. On the contrary, an increase in the intergenerational income inequality has the opposite effects. It encourages workers to postpone building up savings to middle age when labor productivity and thus wages are high. This drives a wedge between the savings of the young and the savings of the middle-aged workers, which boosts wealth inequality substantially and also increases the growth in inequality during population ageing. Similarly, a decline in the capital share α deteriorates wealth inequality and speeds up the growth of inequality during population ageing.

The second limitation is that the numerical predictions regarding the Gini coefficient cannot be applied to the actual Gini coefficient of the United States. The way we calculate the Gini coefficient is highly stylized, since on the one hand it omits all other factors affecting wealth inequality, and on the other hand it only includes two age cohorts[4].

[4]Note that we have 3 cohorts in the model but only two of them hold savings. Retirees do not build up savings because they die at the end of the period.

Our work is part of a growing literature treating politico-economic equilibria, where dynamic interactions between the characteristics of the households and the policy outcome arise. Our model is very strongly based on the work of Gonzalez-Eiras and Niepelt (2008), who introduce the probabilistic voting setup of Lindbeck and Weibull (1987) in a standard two-period OLG. By assuming log-utility and a Cobb-Douglas production function, they gain a closed-form solution for social security tax rates in politico-economic equilibrium. Song (2011) adopts this model and introduces within-cohort heterogeneity in order to examine the interactions between social security and intragenerational wealth inequality, still by relying on analytical results. Unfortunately, in this thesis, we do not obtain closed-form results due to the complexity of the politico-economic layer. We will, therefore, as most research in this field, resort to numerical methods in order to compute the policy outcome. A prominent contribution is the one of Krusell et al. (1997). They offer a very practical numeric solution method for solving Markov perfect equilibria, which we will apply in order to compute the policy outcome function.

The remainder of the paper is structured as follows: In chapter 2 the model is presented in detail, whereas in chapter 3 we calculate the optimality conditions. The politico-economic equilibrium is then derived in chapter 4. We introduce the economy without government sector in chapter 5. The quantitative exercise is performed in chapter 6, where the models are first calibrated in section 6.1 and then simulated and discussed in section 6.2. Chapter 7 concludes.

2 Chapter 2 Model

2.1 Households

The economy is populated by an infinite sequence of three overlapping generations. Agents enter the economy as young workers, grow into workers of middle age after one period and retire after another period.
Young workers inelastically supply labor (normalized to unity) at wage $a_y w_t$, where a_y equals their labor productivity. They pay a social security tax τ_t on their labor income. Disposable income $(1-\tau_t)a_y w_t$ is allocated to consumption $c_{y,t}$ and savings for the next period $s_{y,t+1}$.
Equivalently, middle-aged workers inelastically supply labor (normalized to unity) at wage $a_m w_t$, with $a_m > a_y$. This reflects the fact that income is increasing in age. Note that the ratio $\frac{a_m}{a_y}$ is also a measure for intergenerational income distribution. Exactly as young workers, workers of middle age pay social security taxes τ_t on their labor income. Disposable income equals $(1-\tau_t)a_m w_t + R_t s_{y,t}$, where R_t denotes the interest rate. It is divided into consumption $c_{m,t}$ and savings for retirement $s_{m,t+1}$.
Retirees consume their savings plus interest $R_t s_{m,t}$ as well as the social security benefit b_t.
N_t denotes the number of agents born in period t. Therefore, the gross cohort growth rate is equal to $\frac{N_t}{N_{t-1}} = n_t$. Note that the ratio of young workers relative to retirees is equal to $\frac{N_t}{N_{t-2}} = n_t n_{t-1}$. The sequence $\{n_{t+i}\}_{i=0}^{\infty}$ is deterministic and therefore perfectly predictable.
As regards the notation, the first part of the subscript indicates the cohort, $i \in \{y, m, r\}$, where y stands for "young worker", m for "middle-aged worker" and r for "retiree", whereas the second part denotes the time index, t.
Young workers solve

$$\max_{c_{y,t}, c_{m,t+1}, c_{r,t+2}} u(c_{y,t}) + \beta u(c_{m,t+1}) + \beta^2 u(c_{r,t+2}), \tag{2.1}$$

subject to

$$c_{y,t} = w_t a_y(1-\tau_t) - s_{y,t+1}, \tag{2.2a}$$

$$c_{m,t+1} = w_{t+1} a_m(1-\tau_{t+1}) + R_{t+1}s_{y,t+1} - s_{m,t+2}, \tag{2.2b}$$

$$c_{r,t+2} = R_{t+2}s_{m,t+2} + b_{t+2}, \tag{2.2c}$$

where β is the discount factor, for which holds $\beta \in (0,1)$. For simplicity we assume $u(c) = log(c)$. Middle-aged workers in turn solve the optimization problem

$$\max_{c_{m,t},c_{r,t+1}} u(c_{m,t}) + \beta u(c_{r,t+1}), \tag{2.3}$$

subject to

$$c_{m,t} = w_t a_m(1-\tau_t) + R_t s_{y,t} - s_{m,t+1}, \tag{2.4a}$$

$$c_{r,t+1} = R_{t+1}s_{m,t+1} + b_{t+1}. \tag{2.4b}$$

The consumption decision of a middle-aged worker is consistent with what she decided as a young worker. This follows from the fact, that the optimality condition between consumption during middle age and consumption during retirement is equal for both a young worker and a worker of middle age, i.e. $u'(c_{m,t}) = \beta R_{t+1}u'(c_{r,t+1})$.

2.2 Firms

Output is determined by equation (2.5), a standard Cobb-Douglas production function with constant returns to scale. A is equal to total factor productivity while α denotes the output elasticity of capital. We assume perfect competition in the factor markets, therefore prices coincide with marginal products. By clearance of the factor markets, labor and capital are given by equations (2.6) and (2.7). Note that the ratio between capital and labor can be written as $\frac{K_t}{L_t} = \frac{s_{y,t}+\frac{s_{m,t}}{n_{t-1}}}{a_y n_t + a_m}$.

$$Y_t = AK_t^{\alpha}L_t^{1-\alpha} \tag{2.5}$$

$$L_t = a_y N_t + a_m N_{t-1} \tag{2.6}$$

$$K_t = N_{t-1}s_{y,t} + N_{t-2}s_{m,t} \tag{2.7}$$

2.3 Government

The government runs a pay-as-you-go system: A tax rate τ_t is levied on the labor income of the two working cohorts in order to finance the social security benefits of the retirees. This tax rate is determined through a political process which will be specified below. We assume that the pay-as-you-go program is self-financing, meaning that at any time, the contributions of the workers equal the payments to the retirees. This balanced budget assumption is expressed in equation (2.8).

$$b_t = (a_m n_{t-1} + a_y n_t n_{t-1})\tau_t w_t \tag{2.8}$$

2.4 Timing of the events

At the beginning of every period, two victory-oriented political candidates propose a social security tax rate. The households turn into their role as voters and evaluate how the proposed tax rates will affect future economic variables as well as future political decisions. Based on these considerations, households vote for a political candidate. The candidate with the most votes wins the election.

After the election has taken place, the proposed tax rate is implemented and determines the disposable income of the working cohorts as well as income for the retirees. Households turn into their role as consumers, take prices and taxes as given and decide, how much they want to save.

3

Chapter 3

Optimality

3.1 Households' saving choice

When turning to their role of consumers, young workers maximize utility over lifetime taking as given the sequences $\{w_{t+i}\}_{i=0}^{\infty}$, $\{R_{t+i}\}_{i=0}^{\infty}$ and $\{\tau_{t+i}\}_{i=0}^{\infty}$. The first order conditions for the young household are given by equations (3.1a) and (3.1b).

$$\frac{1}{c_{y,t}} = \beta \frac{R_{t+1}}{c_{m,t+1}} \tag{3.1a}$$

$$\frac{1}{c_{m,t+1}} = \beta \frac{R_{t+2}}{c_{r,t+2}} \tag{3.1b}$$

By inserting the households' budget constraints (2.2) and the governments' balanced budget assumption (2.8), $s_{y,t+1}$ is given by equation (3.2) and $s_{m,t+2}$ by equation (3.3).

$$\begin{aligned} s_{y,t+1} = & \frac{(\beta+\beta^2) w_t a_y (1-\tau_t)}{(1+\beta+\beta^2)} - \frac{w_{t+1} a_m (1-\tau_{t+1})}{(1+\beta+\beta^2) R_{t+1}} \\ & - \frac{(a_m n_{t+1} + a_y n_{t+2} n_{t+1}) \tau_{t+2} w_{t+2}}{(1+\beta+\beta^2) R_{t+1} R_{t+2}} \end{aligned} \tag{3.2}$$

$$s_{m,t+2} = \frac{\beta w_{t+1} a_m (1-\tau_{t+1})}{1+\beta} + \frac{\beta R_{t+1} s_{y,t+1}}{1+\beta} - \frac{(a_m n_{t+1} + a_y n_{t+2} n_{t+1}) \tau_{t+2} w_{t+2}}{(1+\beta) R_{t+2}} \tag{3.3}$$

As we stated earlier, the savings decision is time-consistent. Therefore we iterate equation (3.3) one period back to gain the optimal savings decision from today's middle aged workers, namely $s_{m,t+1}$:

$$s_{m,t+1} = \frac{\beta w_t a_m (1-\tau_t)}{1+\beta} + \frac{\beta R_t s_{y,t}}{1+\beta} - \frac{(a_m n_t + a_y n_{t+1} n_t) \tau_{t+1} w_{t+1}}{(1+\beta) R_{t+1}}. \tag{3.4}$$

The savings of the young $s_{y,t+1}$ increase with higher income w_t and decrease in today's taxes τ_t, since they curtail disposable income. If workers of young age anticipate a higher income tomorrow w_{t+1} they reduce their savings, obeying the principles of consumption smoothing. By the same logic, an increase in tomorrow's taxes τ_{t+1} increases savings. High anticipated social security benefits during retirement cut savings. Anticipated benefits are high when there will be a vast workforce that supports the retirees as well as abundant tax revenues, i.e. high wages paired with high taxes on labor income. Hence, when the population grows slower, workers of young age build up private savings because they will be supported by fewer workers during old age. High interest rates R_{t+1} and R_{t+2} encourage the accumulation of savings by yielding a higher return. The effect of young workers' productivity a_y is not perfectly clear, since it increases today's income but also expected retirement benefits. However, the impact of a_m is unambiguous: It cuts the private savings of the young since all future sources of income expand.

The effects for the savings of the middle-aged workers $s_{m,t+1}$ are similar: They do increase in today's income w_t and decrease in today's taxes τ_t. They also increase in the savings that are brought from the young period $s_{y,t}$ and in the interest on these savings R_t. High anticipated retirement benefits b_{t+1} discourage middle-aged workers to save for retirement. Consequently, high expected population growth provides the incentive to rely on tomorrow's pay-as-you-go benefits. An increased productivity of young workers a_y has the same effect, whereas with a_m we observe both an increased disposable income and higher retirement benefits.

Note that population growth enters the savings decisions both through expected retirement benefits and through prices since the capital-labor-ratio $\frac{K_t}{L_t} = \frac{s_{y,t} + \frac{s_{m,t}}{n_{t-1}}}{a_y n_t + a_m}$ depends on the cohort growth rate. Moreover, we will later see that the social security taxes depend on the demographic structure, too.

Even though the population grows, the key variables of this economy only depend on growth rates. With constant population growth rates the economy will thus converge into a steady state, and not into a balanced growth path.

3.2 Optimality conditions of the firms

In optimum, prices are equal to the marginal products:

$$R_t = A\alpha \left(\frac{K_t}{L_t}\right)^{\alpha-1}, \tag{3.5}$$

$$w_t = A(1-\alpha) \left(\frac{K_t}{L_t}\right)^{\alpha}. \tag{3.6}$$

Chapter 4

4 Politico-economic equilibrium

The social security tax rate τ_t is determined in a political process at the beginning of every period. There are different ways to model afore-said political process. In this thesis, we adopt the probabilistic voting setting of Lindbeck and Weibull (1987). This setting is characterized by two political candidates whose only goal is to win the election, namely to acquire the majority of all votes. Voters care not only about the proposed economic program of the political candidates but also about the candidates' ideology. The political candidates, however, cannot observe the voters' ideological preferences. In order to respond to this uncertainty, the political candidates will suggest a social security tax rate that maximizes a weighted-average welfare of all current voters. The weights are set such that the welfare of voters with weak ideological preferences (so-called swing voters) will be favored the most since these voters are the most sensitive to policy changes. The maximization problem of the political candidate is equal to

$$\max_{\tau_t \in [0,1]} U_{y,t} + \omega_m \frac{1}{n_t} U_{m,t} + \omega_r \frac{1}{n_t n_{t-1}} U_{r,t}, \tag{4.1}$$

where ω_m denotes the political weight of the middle-aged households relative to the young and ω_r is the political weight of the retirees relative to the young. The utility functions are defined as $U_{y,t} \equiv u(c_{y,t}) + \beta u(c_{m,t+1}) + \beta^2 u(c_{r,t+2})$, $U_{m,t} \equiv u(c_{m,t}) + \beta u(c_{r,t+1})$ and $U_{r,t} \equiv u(c_{r,t})$.

We concentrate on Markov perfect equilibria, where the state of the economy is summarized only by current values of the state variables, and not by their past histories[5]. Following the work of Krusell et al. (1997), the first step of computing a Markov perfect equilibrium is to postulate a policy outcome function Ψ that maps the state variables of the economy into a policy outcome. The states of the economy are savings $s_{y,t}$ and $s_{m,t}$ as well as all parameters, including cohort growth n. We summarize the states to a variable S. Thus we conjecture that the Markov perfect tax rate τ_t^* is defined by the function $\tau_t^* = \Psi(S)$.

In their role as voters, households take this policy outcome function Ψ as given when assessing their optimal tax rate. By means of Ψ they know, how the future economic

[5] See Gonzalez-Eiras and Niepelt (2008) for a more detailed discussion about equilibria that might arise if the Markov assumption is relaxed.

and political variables are affected by the policy decision.
The next step of computing a Markov perfect equilibrium is to calculate a competitive equilibrium for every possible policy outcome function Ψ. This means that for every Ψ, we compute the equilibrium between optimal savings of the young (3.2), optimal savings of the middle-aged workers (3.4), factor prices (3.5) and (3.6) as well as the policy outcome function $\tau_t^* = \Psi(S)$. This yields the following solution:

$$s_y^* = f_y(S; \Psi) \tag{4.2a}$$
$$s_m^* = f_m(S; \Psi) \tag{4.2b}$$
$$w^* = f_w(S) \tag{4.2c}$$
$$R^* = f_r(S) \tag{4.2d}$$
$$\tau^* = \Psi(S) \tag{4.2e}$$

Based on these solution functions, the different voters decide, which tax rate maximizes their lifetime utility. The probabilistic voting setting aggregates these individual policy preferences and delivers an optimal tax rate $\tau_t = p(S)$. The last step of computing the Markov perfect equilibrium is to make sure that the policy coming from the electoral process $p(S)$ corresponds to $\Psi(S)$.
The formal definition of the Markov perfect equilibrium is given as follows: Given any Ψ, the political decision solves (4.1) subject to equations (4.2) being in equilibrium and a non-negativity constraint of τ_t. The policy outcome function resulting from this political decision is denoted as $\tau_t = p(S)$. In Markov perfect equilibrium, $\Psi(S) = p(S)$ holds. Thus if agents correctly predict the policy outcome function Ψ, it is optimal in the political process to choose the tax rate according to Ψ.[6]

4.1 The equilibrium policy rule

The first order condition of the political candidate is equal to equation (4.3).

$$\frac{\partial U_{y,t}}{\partial \tau_t} + \frac{\omega_m}{n_t}\frac{\partial U_{m,t}}{\partial \tau_t} + \frac{\omega_r}{n_t n_{t-1}}\frac{\partial U_{r,t}}{\partial \tau_t} = 0 \tag{4.3}$$

The optimality condition states that the marginal costs of the social security taxes carried by the working cohorts have to equal the marginal benefits that favor the retirees. The partial derivative of the utility of the young with respect to the current tax rate is given by equation (4.4).

[6] See Krusell et al. (1997) for an extensive definition of Markov perfect equilibria.

$$
\begin{aligned}
\frac{\partial U_{y,t}}{\partial \tau_t} =& - w_t a_y u'(c_{y,t}) \\
&+ \beta u'(c_{m,t+1}) \left(a_m (1-\tau_{t+1}) \frac{\partial w_{t+1}}{\partial K_{t+1}} + s_{y,t+1} \frac{\partial R_{t+1}}{\partial K_{t+1}} \right) \frac{\partial K^*_{t+1}}{\partial \tau_t} \\
&- \beta u'(c_{m,t+1}) w_{t+1} a_m \frac{\partial \tau_{t+1}}{\partial \tau_t} \\
&+ \beta^2 u'(c_{r,t+2}) \left((a_m n_{t+1} + a_y n_{t+2} n_{t+1}) \tau_{t+2} \frac{\partial w_{t+2}}{\partial K_{t+2}} + s_{m,t+2} \frac{\partial R_{t+2}}{\partial K_{t+2}} \right) \frac{\partial K^*_{t+2}}{\partial \tau_t} \\
&+ \beta^2 u'(c_{r,t+2}) (a_m n_{t+1} + a_y n_{t+2} n_{t+1}) w_{t+2} \frac{\partial \tau_{t+2}}{\partial \tau_t}
\end{aligned}
\tag{4.4}
$$

The first line of this partial derivative states that higher current taxes τ_t lead to a lower current disposable income of a young worker. The second line captures the general equilibrium effect working through capital accumulation K^*_{t+1}. On the one hand, higher current taxes τ_t diminish savings and thus deteriorate tomorrow's wage, which in turn worsen tomorrow's tax base. On the other hand, lower savings decrease capital accumulation, which boosts the interest rate of tomorrow. In line with Song (2011), the third line is called the strategic effect $\frac{\partial \tau_{t+1}}{\partial \tau_t}$. It involves the effect of τ_t on τ_{t+1} via the state variables. Line four and five continue this plot by looking at the effects on economic and political variables in two periods. Note that the derivative of savings $s_{y,t+1}$ and $s_{m,t+2}$ with respect to taxes τ_t cancel due to the envelope theorem. Figure 4.1 visualizes this partial derivative $\frac{\partial U_{y,t}}{\partial \tau_t}$ by pointing out the different interactions in this chain of effects, which young voters have to think through. The yellow arrows display the effects on economic variables, whereas the red arrows indicate how policy outcome is affected.

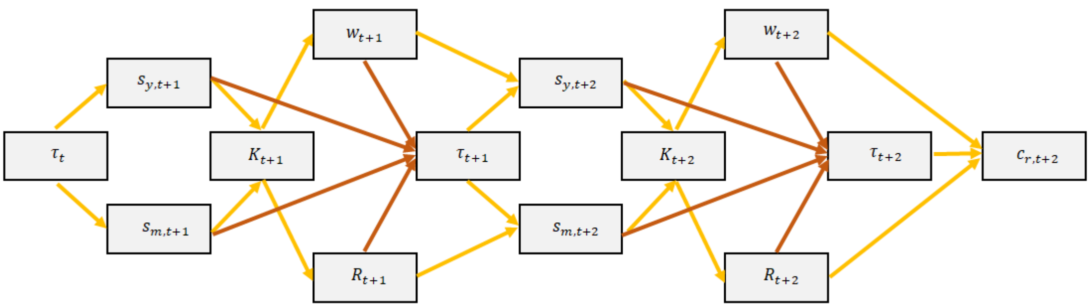

Figure 4.1: The chain of effects young voters think through

The partial derivative of utility of a middle-aged worker with respect to current taxes is given by equation (4.5). The effects are similar to those discussed for young workers.

$$
\begin{aligned}
\frac{\partial U_{m,t}}{\partial \tau_t} = &- w_t a_m u'(c_{m,t}) \\
&+ \beta u'(c_{r,t+1}) \left((a_m n_t + a_y n_{t+1} n_t) \tau_{t+1} \frac{\partial w_{t+1}}{\partial K_{t+1}} + s_{m,t+1} \frac{\partial R_{t+1}}{\partial K_{t+1}} \right) \frac{\partial K^*_{t+1}}{\partial \tau_t} \\
&+ \beta u'(c_{r,t+1})(a_m n_t + a_y n_{t+1} n_t) w_{t+1} \frac{\partial \tau_{t+1}}{\partial \tau_t}
\end{aligned}
\tag{4.5}
$$

The effect on retirees' utility is much simpler, as shown in equation (4.6): Higher current taxes increase the pay-as-you-go benefits for the retirees, enhancing their utility.

$$
\frac{\partial U_{r,t}}{\partial \tau_t} = (a_m n_{t-1} + a_y n_t n_{t-1}) w_t u'(c_{r,t}) \tag{4.6}
$$

An important insight of Gonzalez-Eiras and Niepelt (2008), that applies here as well, is that workers do not dislike taxes as much as retirees like them. The reason is that taxes do not only have negative effects on workers. Higher taxes, for example, depress capital accumulation, which in turn increase future interest rates. Gonzalez-Eiras and Niepelt (2008) reason that the introduction of the probabilistic voting setting enables intergenerational transfers without having to impose altruism, commitment or trigger strategies.

Since we do not know the exact form of p and cannot obtain it analytically on account of the complexity of the model, we resort to a numerical procedure for the sake of calculating the policy outcome function in Markov perfect equilibrium. In order to do so, we rely on backward induction: We start in the last period of a finite-horizon economy, where the political policy function can be computed analytically. In the last period, consumption levels are given by

$$
c_{y,T} = w_T a_y (1 - \tau_T), \tag{4.7a}
$$

$$
c_{m,T} = w_T a_m (1 - \tau_T) + R_T s_{y,T}, \tag{4.7b}
$$

$$
c_{r,T} = (a_m n_{T-1} + a_y n_T n_{T-1}) \tau_T w_T + R_T s_{m,T}. \tag{4.7c}
$$

The political policy function (4.8) which maps the states into a policy outcome τ_T is much simpler compared to equation (4.3), since effects of the tax rate on future states do not exist.

$$
\begin{aligned}
\frac{1}{(1-\tau_T)} + \frac{\omega_m}{n_T} \frac{w_T a_m}{w_T a_m (1-\tau_T) + R_T s_{y,T}} \\
- \frac{\omega_r}{n_T n_{T-1}} \frac{(a_m n_{T-1} + a_y n_T n_{T-1}) w_T}{R_T s_{M,T} + (a_m n_{T-1} + a_y n_T n_{T-1}) \tau_T w_T} = 0
\end{aligned} \tag{4.8}
$$

As can be seen in equation (4.8), an increase in ω_r strengthens the political power of the old and thus pushes social security tax rates up. Contrarily, an increase in ω_m reduces tax rates.

The state variables $s_{y,T}$ and $s_{m,T}$ enter the equation both directly and indirectly through prices w_T and R_T. The higher $s_{y,T}$, the lower is the marginal utility of consumption of the middle-aged workers. Higher taxes are imposed because their marginal loss is lower. Intuitively this means that middle-aged agents are richer and are thus hurt less by higher taxes. On the other hand, a high $s_{y,T}$ increases the equilibrium wage w_T and curtails the equilibrium interest rate R_T. We impose that the overall effect through prices cancels more or less.

The higher $s_{m,T}$, the lower the marginal utility of consumption of the retirees is. The marginal benefit of an increase in taxes is very low, therefore equilibrium taxes decrease. In other words, retirees do not need high benefits because they are rich anyway, thus equilibrium taxes are lower.

If population growth decreases, retirees have higher political power whereas young and middle-aged agents suffer from a loss of political influence. This shifts taxes upwards. Secondly, the workforce is smaller, meaning that retirees receive fewer benefits. Their marginal utility of consumption is higher, which pushes taxes upwards as well. Thirdly, the capital-labor ratio $\frac{K_t}{L_t} = \frac{s_{y,t} + \frac{s_{m,t}}{n_{t-1}}}{a_y n_t + a_m}$ increases with an ageing population, which raises the wage w_T and cuts the interest rate R_T.

We conclude that the social security tax rate in the last period is an increasing function of the savings of the young, a decreasing function of the savings of the middle-aged workers and a decreasing function in cohort growth rates.

The numeric procedure is as follows: The policy outcome function $\tau_T = p_T(S)$ is used to compute the equilibrium of the set of equations (4.2) in time $T-1$ for every combination of states S and the tax rate τ_{T-1}. For every S, the tax rate τ_{T-1} which yields the highest welfare is extracted. This generates the political policy function $\tau_{T-1} = p_{T-1}(S)$. The procedure is repeated until the political policy function converges to $\tau = p(S)$.

4.2 Equilibrium effects of population ageing

We expect population ageing to enter the equilibrium allocation through three channels. First of all, ageing enters the allocation via expected benefits. If the workforce decreases over time, agents expect lower social security benefits and react by building up savings. Secondly, ageing affects prices since the capital-labor ratio increases with a shrinking workforce. Thirdly, ageing pushes equilibrium taxes up because of the induced shift in the political power towards the elderly. From now on, we will call the first channel the benefit channel, the second the price channel and the third the tax channel.

Chapter 5

5 Economy without government sector

In order to separate the three discussed channels through which population ageing enters the equilibrium, we compare our model to an economy without government sector. In this setting, the only channel being present is the price channel. This follows from the fact that we omit the whole mechanism of collecting taxes and redistributing those in the form of social security benefits. Retirees have to fully rely on their private savings that they have accumulated over their lifetime.

The main model of chapter (2) is adapted accordingly: Taxes $\{\tau_{t+i}\}_{i=0}^{\infty}$ and benefits $\{b_{t+i}\}_{i=0}^{\infty}$ are set to zero. Consequently, no political process takes place. The model is presented in Appendix A. The competitive equilibrium law of motions for $s^*_{y,t+1}$ and $s^*_{m,t+1}$ are given by equations (5.1) and (5.2).

$$s^*_{y,t+1} = \frac{\frac{\beta(1+\beta)a_y}{1+\beta+\beta^2} w_t - \frac{a_m(1-\alpha)}{\alpha(1+\beta+\beta^2)(n_{t+1}a_y+a_m)} \frac{1}{n_t} \frac{\beta(a_m w_t + R_t s_{y,t})}{(1+\beta)}}{1 + \frac{a_m(1-\alpha)}{\alpha(1+\beta+\beta^2)(n_{t+1}a_y+a_m)}} \tag{5.1}$$

$$s^*_{m,t+1} = \frac{\beta(a_m w_t + R_t s_{y,t})}{1+\beta} \tag{5.2}$$

The savings of the young (5.1) increase with higher disposable income today and decrease with higher expected income tomorrow. An increase in the interest rates decreases their savings because it pays less to transfer wealth to the future. The savings of the middle-aged workers (5.2) increase with higher disposable income today. An increase in the interest rate pushes disposable income and thus the savings of the middle-aged up.

We expect population ageing to decrease the workforce, such that the capital-labor ratio $\frac{K}{L}$ increases. This, in turn, has positive effects on wages as well as negative effects on interest rates.

Chapter 6

6 Quantitative exercise

6.1 Calibration

The period length is set to 20 years in order to match the life expectancy of approximately 80 years[7]. It is therefore assumed that young workers enter the economy being 20 years old and turn into middle-aged workers by the age of 40. Workers of middle age are retired by the age of 60 and spend the last 20 years as pensioners.
We will simulate the economy for different population growth rates in order to extract the link between ageing and wealth distribution. Note that in steady state the population growth rates are equal to the cohort growth rates, henceforth, we will only speak of population growth rates. According to the US Bureau of Census, the 20-years population growth rate for the year 1960 was approximately equal to $n_{1960} = 1.26$. The projected 20-years population growth rate for the year 2040 is estimated at $n_{2040} = 1.1$.[8] We will simulate the model for different values from n_{1960} to n_{2040} in order to grasp the big picture. Note that we will compare the steady states of the different population growth rates. The exercise thus cannot be regarded as a transition from n_{1960} to n_{2040}, where one period exactly corresponds to 20 years.
The model will be calibrated according to the economic variables of the period 1960-1980. Total factor productivity A is normalized to unity whereas output elasticity of capital α is set to the standard value in the literature, i.e. 0.3. In order to calibrate the cohort-specific productivity levels a_y and a_m we look at data of total income per age group. Unfortunately, we do not find such data for the desired period, therefore we rely on the total money income in 2016 by the age of the householder[9]. We calculate the mean income for the 15-39 and the 40-59 years-old by taking a weighted average of the different underlying age categories. By taking the quotient between these two age cohorts, we find that middle-aged workers earn around 1.28 times more than their young coworkers. Thus, a_m is set to 1.3 while a_y is normalized to unity. We will later

[7] The current estimate for life expectancy in the US is 78.74 years according to the Worldbank's estimate from 2015.

[8] Source: U.S. Census Bureau, Projections of the Population and Components of Change for the United States: 2015 to 2060, Table 1.

[9] Source: U.S. Census Bureau, Current Population Survey, 2018 Annual Social and Economic Supplement.

see how an increase in this intergenerational income inequality will affect the results. The discount factor β is used to calibrate the steady-state interest rate that should equal the average real return of 2.24% of ten-years U.S. treasury bills from 1962 to 1969, as it is done in Song (2011). Finally, the political weights ω_m and ω_r are chosen such that the steady-state social security tax rate approximately equals the mean OASI rate of the period 1960-1980, which is 8.5%.
By trial-and-error we find that the values $\beta = 0.85$, $\omega_m = 1$ and $\omega_r = 1.25$ yield quite satisfying results. The discount factor $\beta = 0.85$ corresponds to an annual discount rate of 0.9919. The political weight $\omega_r = 1.25$ reflects the fact that in general, older people are better coordinated in votes concerning social security topics. Therefore, they should be weighted more strongly.

For every simulation we calculate the Gini coefficient for the sake of quantifying the wealth distribution between the young and the middle-aged[10]. This Gini coefficient therefore only indicates the degree of inequality that is attributed to age composition and omits all other factors affecting the wealth distribution. The Gini coefficient takes values between 0, meaning that everyone in the economy holds the same amount of wealth, and 1, meaning that one person holds all the wealth. Hence, the coefficient increases, if a smaller share of the population holds a greater share of the wealth.
Note that it is not our goal to replicate the Gini coefficient of the United States, but to show how it changes with population ageing. Thus, the direction is more important than the absolute value.
We denote the steady-state population share of the young as $p_y = \frac{n}{1+n}$ and the population share of the middle-aged workers as $p_m = \frac{1}{1+n}$. The wealth share of the young is denoted as $\frac{p_y s_y^*}{p_y s_y^* + p_m s_m^*}$. The same principle applies for computing the wealth share of the middle-aged workers.
In our case, the highest Gini coefficient arises when the middle-aged workers hold all the wealth. This translates however only in a value equal to p_y. Thus, a coefficient of 1 cannot be reached unless the population share of the young approaches 100%.

[10] To be precise, we compare the wealth of the young and the middle-aged agents at the end of the period when savings are built and agents are about to enter the next period.

6.2 Results

6.2.1 Economy without government sector

The results are displayed in Table 6.1 and in Figure 6.1. Population ageing leads to the growth in the capital-labor ratio due to the reduction of the workforce. This pushes wages up and interest rates down. For example, a decrease in the population growth rate from $n_{1960} = 1.26$ to $n_{2040} = 1.10$ leads to an increase in the steady-state wage from 0.3690 to 0.3835 and a decline in the steady-state interest rate from 1.3363 to 1.2218.

n	β	a_m	α	s_y^*	s_m^*	w^*	R^*	Gini
1.26	0.85	1.3	0.3	0.0861	0.2733	0.3690	1.3363	0.273
1.10	0.85	1.3	0.3	0.0758	0.2717	0.3835	1.2218	0.289

Table 6.1: Steady-state allocations for different population growth rates (economy without government sector)

The savings of the middle-aged agents decrease only minimally because the negative effect on disposable income through lower interest rates and the positive effect on disposable income through higher wages balance each other out. However, the savings of the young decrease more visibly, as it is apparent in Figure 6.1. On the one hand, the young expect a higher income during middle age. On the other hand, this higher income is discounted with a lower interest rate, which makes it more valuable from today's point of view. Thus, young workers reduce their savings.

When calculating the Gini coefficient for every steady-state allocation we find that with ageing, the population share of the elderly with respect to the young increases. Hence, their wealth share grows as well. The problem is, however, that it grows disproportionately because of the reduction in the savings of the young. This results in higher wealth inequality, which is visualized in an increasing Gini coefficient. In fact, the Gini coefficient of the population growth rate of $n_{2040} = 1.10$ is about 5.7% higher than the Gini coefficient of $n_{1960} = 1.26$. If the savings, however, stayed constant during the demographic transition, the Gini coefficient would fall because of the induced effects through the population shares. This is what is explained by Vandenbroucke and Zhu (2017) or Krueger and Ludwig (2007).

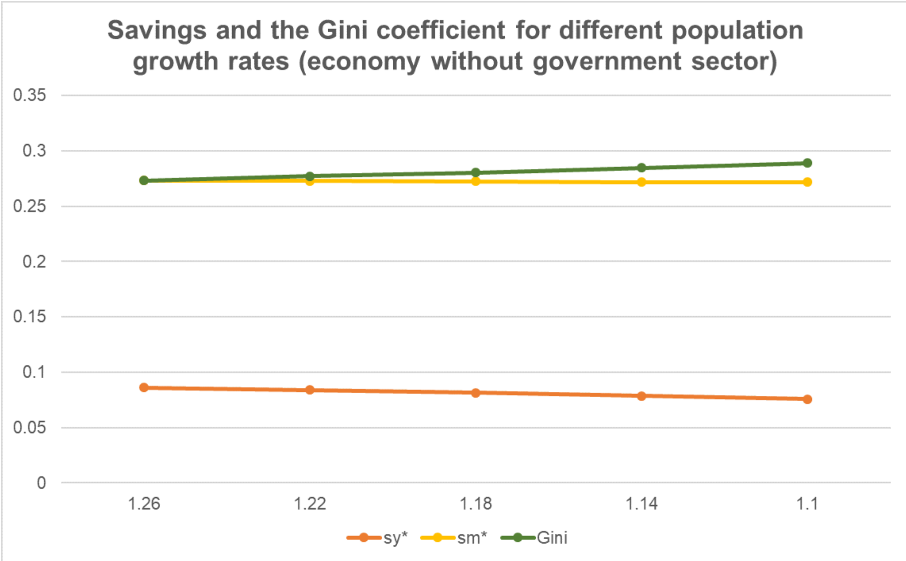

Figure 6.1: Savings and the Gini coefficient for different population growth rates (economy without government sector)

6.2.2 Main model

The results of the simulation of the model economy with politico-economic pay-as-you-go are presented in Table (6.2) and Figure (6.2). What stands out is that steady-state savings both of the young and the middle-aged workers fall with lower population growth rates: A decrease in the population growth rate from $n_{1960} = 1.26$ to $n_{2040} = 1.10$ leads to a reduction in s_y^* from 0.0756 to 0.0664 and a decrease in s_m^* from 0.2094 to 0.1926 (see Table 6.2). Further, we observe an increase in the steady-state tax rate from 8.53% to 11.52% because population ageing leads to the higher political power of the elderly, which increases the political benefit of taxes. Figure 6.3 shows the comparison between the tax rate that our model predicts and the actual OASI contribution rate that occurs during periods with similar population growth rates. Our model can explain part of the increase in the OASI rates, however not to the full extent. Gonzalez-Eiras and Niepelt (2008) for example predict tax rates up to 15.53% for the year 2040.

n	β	a_m	α	ω_r	s_y^*	s_m^*	τ^*	w^*	R^*	b^*	Gini
1.26	0.85	1.3	0.3	1.25	0.0756	0.2094	0.0853	0.3448	1.5664	0.0949	0.245
1.10	0.85	1.3	0.3	1.25	0.0664	0.1926	0.1152	0.3515	1.4976	0.1069	0.249

Table 6.2: Steady-state allocations for different population growth rates (main model)

It is interesting that benefits increase as well. The positive effect on benefits due to the increased tax is stronger than the negative effect on benefits due to the reduced workforce. The Gini coefficient increases slightly from 0.245 to 0.249, suggesting that wealth inequality is insignificantly higher.

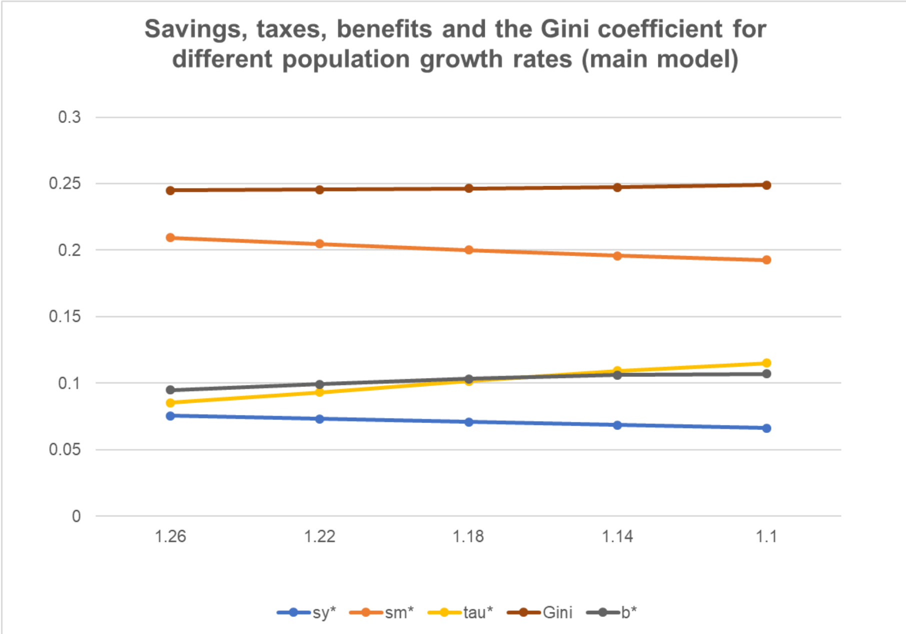

Figure 6.2: Savings, taxes, benefits and the Gini coefficient for different population growth rates (main model)

We explain the decrease in private savings as follows: Earlier we mentioned the three channels of population ageing. The benefit channel suggests that ageing translates into lower expected social security benefits which in turn boosts private savings. The price channel projects that ageing changes the capital-labor ratio and thus increases wages and decreases interest rates. This is what we observed in the economy without government sector where the price channel alone decreases savings. The tax channel involves the equilibrium tax rate, which grows over time. Higher taxes will decrease steady-state savings[11]. What we observe is, first of all, a very strong tax channel. It is indeed so strong, that it completely reverses the benefit channel: Workers save even less because they expect higher social security benefits. Second, the wage and the interest rate do not move as much as in the model without government sector. This is due to the fact that here the savings decrease more strongly and, therefore, the capital-labor ratio is pushed down harder than in the model without government

[11] In order to see this in an isolated context, we take equations (3.2) and (3.4), assume that prices are given by some constants w and R, and derive the partial derivative of steady-state savings with respect to steady-state taxes. The partial derivative $\frac{\partial s_m^*}{\partial \tau}$ is always negative, because higher taxes translate into a lower disposable income as well as higher social security benefits, which both reduce the savings of middle-aged workers in steady state. The partial derivative $\frac{\partial s_y^*}{\partial \tau}$ is negative only if $a_y R^2(\beta+\beta^2)+(a_m n+a_y n^2) > Ra_m$. This means that the incentive of decreasing savings because of lower disposable income and higher anticipated benefits is stronger than the incentive to increase savings because of higher anticipated labor income during middle age.

sector. It follows that the equilibrium price channel is not strong and that savings are mostly reduced because of higher taxes. The savings of the young however still decrease slightly more compared to the savings of older workers. This effect prevails the change in the population shares, which leads to a modest increase in the Gini coefficient, albeit a much smaller increase than in the model without government sector.

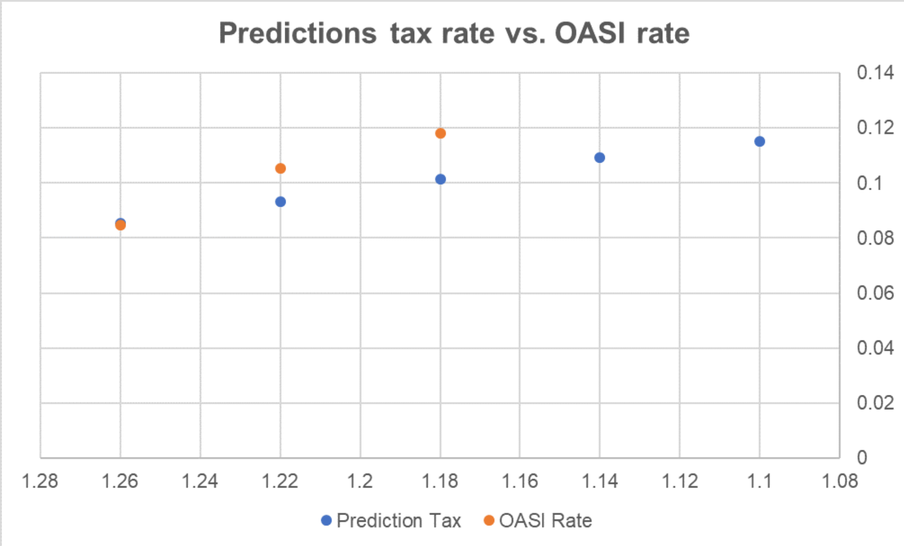

Figure 6.3: Predictions for tax rate vs. OASI rate

The question is, whether the increase in taxes is attributed to a change in the input arguments or also a change in the policy outcome function. By comparing the policy outcome function for $n_{1960} = 1.26$ (Figure 6.4)[12] and $n = 1.10$ (Figure 6.5) it is apparent that a lower population growth rate increases the equilibrium tax rate for every combination of states.

Let's record the most important differences between the model without government sector and the main model: By introducing a government sector, s_y^* decreases slightly, for example for $n = 1.26$ from 0.0861 to 0.0756, whereas s_m^* decreases more drastically from 0.2733 to 0.2094. Workers, especially middle-aged ones, are willing to reduce private savings because they will benefit from social security payments during old age. The introduction of a politico-economic pay-as-you-go setting thus reduces wealth inequality significantly. This is reflected in the Gini coefficient that falls from around 0.273 to 0.245. The demographic transition in the model without government sector even exacerbates wealth inequality, because the savings of the young disproportionately decrease. The Gini coefficient increases by around 0.016 in absolute terms or 5.71%.

[12]Note that the state variables are defined as sum of savings and distribution of the savings instead of s_y and s_m. Sum of savings is the sum of the savings of the young and the savings of the middle-aged workers, whereas distribution denotes the share of the sum of the savings that is held by the young. We can always write $s_y = sum * distribution$ and $s_m = sum * (1 - distribution)$.

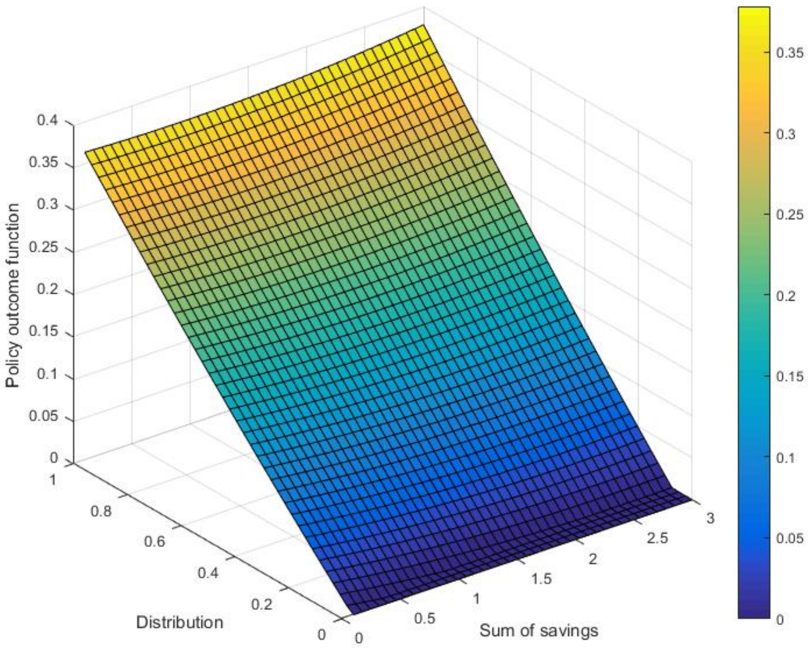

Figure 6.4: Policy outcome function for $n = 1.26$

Although this effect appears in the model with government sector as well, it is much weaker: The Gini coefficient only increases by 0.004 in absolute value or 1.63%.

6.2.3 Robustness with respect to parameter choice

The results are not necessarily robust since they depend very strongly on the parameter choice. We will show how the choice of different values of the income inequality $\frac{a_m}{a_y}$, the discount factor β, the output elasticity of capital α and the political weight ω_r will affect the outcome. The results are showed in Table 6.3 and 6.4, where the altered parameters, compared to the preceding calibration, are underlined.

The effects of intergenerational income inequality The results for the model economy without government sector are given by row three and four in Table 6.3. At this point, we want to stress that intergenerational income inequality $\frac{a_m}{a_y}$ is not an income inequality that is perceived as unfair since every young worker knows that she will earn the same amount later in her life.

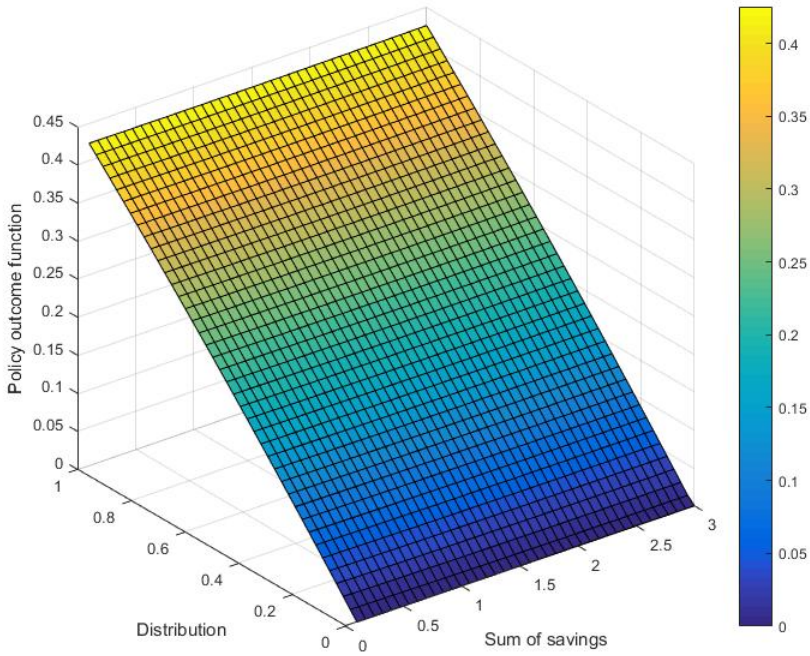

Figure 6.5: Policy outcome function for $n = 1.10$

For a given population growth rate, an increase in a_m affects the young workers' savings decision both directly, by boosting the expected income during middle age, and indirectly through prices (an increase in a_m increases productive labor and thus lowers the capital-labor ratio, which pushes wages down and interests up). The direct effect is much stronger. Thus, on the whole, young workers reduce private savings. The savings decision of workers of middle age are affected as well, both directly, by increasing disposable income and indirectly through prices. Again, the direct effect is very strong, which encourages the middle-aged workers to build up savings. The essence of an increase in the intergenerational income inequality is therefore that the collection of private savings is delayed into middle age. The wealth inequality significantly deteriorates, hence the Gini coefficient for $n_{1960} = 1.26$ rises from 0.273 to 0.325.

As always, the demographic transition works through the price channel and decreases savings further. However, we note that the evolution of the fraction $\frac{s_y^*}{s_m^*}$ is different than before. It decreases more strongly over time with $a_m = 1.5$ than with $a_m = 1.3$ both in absolute value and in percentage terms. In fact, equation (5.1) makes this interaction visible[13]. It means that young workers expect higher wages with population

[13] The interaction between a_m and n in equation (5.1) is highlighted in red color:

$$s^*_{y,t+1} = \frac{\frac{\beta(1+\beta)a_y}{1+\beta+\beta^2} w_t - \frac{a_m(1-\alpha)}{\alpha(1+\beta+\beta^2)(n_{t+1}a_y+a_m)} \frac{1}{n_t} \frac{\beta(a_m w_t + R_t s_{y,t})}{(1+\beta)}}{1+\frac{a_m(1-\alpha)}{\alpha(1+\beta+\beta^2)(n_{t+1}a_y+a_m)}}$$

n	β	a_m	α	s_y^*	s_m^*	w^*	R^*	Gini
1.26	0.85	1.3	0.3	0.0861	0.2733	0.3690	1.3363	0.273
1.10	0.85	1.3	0.3	0.0758	0.2717	0.3835	1.2218	0.289
1.26	0.85	<u>1.5</u>	0.3	0.0708	0.2947	0.3614	1.4030	0.325
1.10	0.85	<u>1.5</u>	0.3	0.0590	0.2936	0.3754	1.2836	0.343
1.26	<u>0.95</u>	1.3	0.3	0.1053	0.3059	0.3847	1.2126	0.255
1.10	<u>0.95</u>	1.3	0.3	0.0951	0.3047	0.4001	1.1062	0.268
1.26	0.85	1.3	<u>0.25</u>	0.0804	0.3111	0.4485	1.1693	0.312
1.10	0.85	1.3	<u>0.25</u>	0.0647	0.3077	0.4616	1.0723	0.336

Table 6.3: Steady-state allocations for different population growth rates and for different parameter combinations (economy without government sector).

ageing on the one hand, and with increased labor productivity on the other hand. The Gini coefficient thus decreases more in absolute value, but not in percentage value ($+5.42\%$ for $a_m = 1.5$ versus $+5.71\%$ for $a_m = 1.3$). This can be explained as follows: The percentage growth of the Gini coefficient depends on how the wealth shares evolve, but also on the initial level of inequality. Since we start already at a very high wealth inequality (remember that the maximum inequality is reached at p_y, which is here equal to 0.558) the percentage growth of the Gini coefficient is not that strong anymore.

The results for the model with government sector are given by row three and four in Table 6.4. An increase in intergenerational income distribution enters the youngs' savings decision through different channels: Firstly, an increase in a_m leads to higher expected income during middle-age. Secondly, we observe the same effect through prices as in the model economy without government sector, namely lower wages paired with higher interest rates. Thirdly, a higher a_m enhances tax earnings and thus expected social security benefits. Fourthly, a change in a_m may affect the policy outcome function. However, when comparing the political policy functions for different values of a_m, we do not observe significant discrepancies in the policy outcome function (see Appendix C). On the whole, young workers reduce savings, because the effect of a higher income during middle age paired with higher expected social security benefits predominate all other effects. For middle-aged agents, the argumentation is similar: An increase in a_m means an increase in disposable income, an increase in expected retirement benefits as well as the effects through prices. The first effect however obviously outweighs all other effects. A decrease in s_y paired with an increase in s_m has negative repercussions on taxes. For example, for $n_{1960} = 1.26$ the tax rate falls from 8.53% to 6.85%. Since we argued that the policy outcome functions stay more or less identical for an increase in a_m, the cut in taxes is solely attributed to the change in inputs. The conclusion is the same as in the model economy without government sector: For a given population growth rate, an increase in the intergenerational income inequality shifts the buildup of savings to middle age. The Gini coefficient increases significantly, namely for $n_{1960} = 1.26$ from 0.245 to 0.302.

n	β	a_m	α	ω_r	s_y^*	s_m^*	τ^*	w^*	R^*	b^*	Gini
1.26	0.85	1.3	0.3	1.25	0.0756	0.2094	0.0853	0.3448	1.5664	0.0949	0.245
1.10	0.85	1.3	0.3	1.25	0.0664	0.1926	0.1152	0.3515	1.4976	0.1069	0.249
1.26	0.85	<u>1.5</u>	0.3	1.25	0.0652	0.2393	0.0685	0.3426	1.5897	0.0816	0.302
1.10	0.85	<u>1.5</u>	0.3	1.25	0.0555	0.2557	0.0893	0.3510	1.5023	0.0896	0.311
1.26	<u>0.95</u>	1.3	0.3	1.25	0.0910	0.2364	0.0838	0.3599	1.4172	0.0973	0.231
1.10	<u>0.95</u>	1.3	0.3	1.25	0.0807	0.2174	0.1131	0.3667	1.3563	0.1095	0.234
1.26	0.85	1.3	<u>0.25</u>	1.25	0.0719	0.2159	0.108	0.4163	1.4620	0.1450	0.262
1.14	0.85	1.3	<u>0.25</u>	1.25	0.0638	0.2022	0.1278	0.4207	1.4170	0.1496	0.268
1.26	0.85	1.3	0.3	<u>1.5</u>	0.0716	0.1851	0.1276	0.3345	1.6809	0.1377	0.230
1.10	0.85	1.3	0.3	<u>1.5</u>	0.0633	0.1672	0.1637	0.3395	1.6247	0.1467	0.230

Table 6.4: Steady-state allocations for different population growth rates and for different parameter combinations (main model)

During the demographic transition, we again observe an interaction between n and a_m: The fraction $\frac{s_y^*}{s_m^*}$ decreases more strongly than it did with $a_m = 1.3$, which is translated into a faster-growing Gini coefficient.
We conclude that higher intergenerational income inequality increases intergenerational wealth distribution for a given population growth rate. Further, it intensifies the increase in inequality during population ageing.

The effects of the discount factor What are the effects of an increase in β in the model economy without government sector for a given population growth rate? The results are displayed in row five and six in Table 6.3. Workers save more because they consider the future as more important. Correspondingly, wages increase and interest rates fall. Since the young save disproportionately more with a higher discount factor than the middle-aged workers, the Gini coefficient will fall accordingly, for example for $n_{1960} = 1.26$ from 0.273 to 0.255. The reason for this is that young workers have to take into account their entire remaining life cycle, for which they care more. They increase private savings already in young age in order to be able to increase savings also during middle age.

For the sake of grasping the mechanism during the demographic transition, we compare the numbers in row five with the numbers in row six. When calculating the ratio $\frac{s_y^*}{s_m^*}$ over time, it is evident that this ratio decreases less strongly in terms of percentages with a high β than with a low β. We deduce that there must be a small interaction between β and n in the competitive equilibrium savings of the young, which is visible in equation (5.1): During the demographic transition, the young save less because of the price channel, i.e. higher wages and lower interest rates. But, since this expected income is also perceived as less valuable with a higher discount factor, the young save disproportionately more. This translates into a Gini coefficient growing less strongly during the demographic transition with higher discount factors both in absolute terms ($+0.013$ instead of $+0.016$) and percentage terms (5.19% instead of 5.71%).

The effects in the model economy with government sector are again similar, as can be seen in row five and six in Table 6.4. The channels through which β increases the savings decisions are twofold: On the one hand, as explained earlier, workers build up more private savings since they value the future as more important. On the other hand, they experience a change in the policy outcome function. The higher β, the more patient voters are as well. Thus, for young and middle-aged voters the effects of today's taxes on tomorrow's economic and political variables become more important. The welfare cost of taxes increases. The political candidates take this into consideration and propose a lower equilibrium tax rate (see Appendix C). This is in line with the findings in Gonzalez-Eiras and Niepelt (2008). Both these channels increase the incentives to build up savings. Again the rise in the savings of the young is greater than the rise in the savings of the middle-aged for a given population growth rate. The Gini coefficient, therefore, decreases for $n = 1.3$ from 0.251 to 0.225.
When looking at the behavior of the variables during the demographic transition, we observe similar effects as in the economy without government sector: The fraction $\frac{s_y^*}{s_m^*}$ decreases less strongly during the demographic transition with $\beta = 0.95$, compared to the case with $\beta = 0.85$. The growth in the Gini coefficient is thus weaker. We conclude that a higher β decreases the rise in inequality during population ageing.

The effects of the output elasticity of capital The results of a decrease in α from 0.3 to 0.25 for the model economy without government sector are shown in row seven and eight in Table 6.3. The reduction in α increases the wage and decreases the interest rate. This, in turn, reduces private savings of the young and raises those of workers of middle age. Young workers have a higher disposable income today, but also expect a higher income as middle-aged workers which is additionally discounted by a

lower interest rate, which makes it more valuable from today's point of view. Thus, young workers decrease their savings. The middle-aged suffer from lower interest rates but benefit from higher wages. Obviously, the effect of the increased wage outweighs the effect of the reduced interest rate. Wealth inequality is higher, therefore the Gini coefficient increases from 0.273 to 0.319 for $n_{1960} = 1.26$.
When looking at the results during the demographic transition, we realize that the ratio $\frac{s_y^*}{s_m^*}$ disproportionately decreases over time, due to the interaction $\frac{(1-\alpha)}{\alpha}$ and n in equation (5.1). This is the reason for a strong rise in wealth inequality with population ageing, as the Gini coefficient grows from 0.319 to 0.336.

The results for the main model are given by row seven and eight in Table 6.4[14]. The effect of an increase in the capital share on prices is qualitatively the same as in the model without government sector. Another noticeable fact is the increase in the tax rate: For $n = 1.26$, the steady-state tax rate increases from 8.53% to 12.76%. The rise in the tax rate is attributed to a change in the policy outcome function (see Appendix C). This is consistent with the findings in Gonzalez-Eiras and Niepelt (2008). They explain this increase by stating that a decrease in the output elasticity of capital α leads to an upsurge in the marginal benefit of transfers for the retirees whereas it reduces the political cost of taxation for the working cohorts. This will, in turn, result in a rise in the equilibrium tax rate.
The effects on savings s_y and s_m are similar to the model without government sector, with the exception that in this model, additionally the effect of higher taxes arises. This effect decreases both the savings of the young and the middle-aged workers. This is the reason that the Gini coefficient does not increase as much as in the model without government sector (both for given population growth rates and during the transition), although it still increases compared to the reference case. The transition is similar to the simulations before: The ratio $\frac{s_y^*}{s_m^*}$ decreases over time and wealth inequality deteriorates with population ageing.
We conclude that a decrease in α deteriorates the wealth inequality and speeds up the growth of inequality during population ageing. However, this effect is worse for the model without government sector.

The effects of the political weights The results are given by row nine and ten in Table 6.4. An increase in the political weight of the elderly ω_r goes hand in hand with an increase in the policy outcome function: For every combination of state variables, equilibrium taxes rise (see Appendix C). For $n_{1960} = 1.26$ steady-state taxes increase

[14]Note that the model did not converge for the parameter combination $\alpha = 0.25$ and $n_{2040} = 1.10$. We therefore analyze the effects of the demographic transition by comparing the simulations for $n_{1960} = 1.26$ and $n_{2020} = 1.14$.

from 8.53% for $a_m = 1.25$ to 12.76% for $a_m = 1.5$.
The demographic transition towards an older population further increases equilibrium taxes. This increase is disproportionately strong since there is a clear interaction between the population growth rate n and the political weight of the retirees ω_r. Taxes rise up to 16.37% for $n_{2040} = 1.10$.
The tax channel is very powerful in this setting. As discussed above, higher taxes decrease equilibrium savings. During the demographic transition, the savings of the young and the middle-aged agents are reduced more or less equally, which means that the ratio $\frac{s_y^*}{s_m^*}$ falls only slightly with ageing. Wealth inequality does not increase, as in the other settings, since the evolution of the population shares between young and middle-aged predominates the effect on the Gini coefficient.

We conclude that the evolution of the Gini coefficient during population ageing is influenced by two competing forces: The first force is the evolution of the population shares, which for given savings increase wealth equality over time. The reason is that with ageing, there are more wealthy and fewer poor agents in the economy. This is the effect described by Vandenbroucke and Zhu (2017) or Krueger and Ludwig (2007). The other force comes from the different channels: Due to the price channel, for example, the savings of the young disproportionately decrease with ageing compared to the savings of middle-aged workers. If this second effect predominates the first effect, wealth inequality increases with population ageing. Whether the first or second effect predominates depends completely on the choice of parameters. Therefore, the results are not necessarily robust. However, we observe that a stronger tax channel dampens the decrease of the ratio $\frac{s_y^*}{s_m^*}$ during population ageing. Simulations where steady-state taxes were particularly high, as for example the case with $\omega_r = 1.5$ generate results where inequality does not increase with ageing.
What we didn't observe at all are increasing savings during population ageing like in Mierau and Turnovsky (2014), although this is a very intuitive relation: Workers prepare for a retirement period with reduced benefits, because of the reduction in the workforce. In all our simulations, however, taxes increase so strongly with ageing, that social security benefits even rise. Workers, therefore, reduce their private savings. In order to mimic higher savings with ageing, it may help to include longevity in the model, so that workers have to prepare for a longer retirement period.

Chapter 7
Conclusion

We have constructed a model environment that aims to examine the effects of population ageing on intergenerational wealth distribution by taking into account various channels of ageing, including the effects through social security tax rates. To that end, we introduced a probabilistic voting setup into a 3-period OLG, in line with Gonzalez-Eiras and Niepelt (2008).

We found that intergenerational wealth inequality increases with an ageing population. This is true both for the model economy with a politico-economic pay-as-you-go system as well as for the model economy without government sector. The main effect that causes this result in both models is that the ratio $\frac{s_y^*}{s_m^*}$ decreases with ageing, meaning that the savings of the young disproportionately decrease compared to the savings of the middle-aged workers. However, we observe that the increase in wealth inequality is much stronger in the model without government sector than in the model with politico-economic pay-as-you-go. The reason for this result is that ageing boosts social security taxes through the induced shift in the political power. Higher taxes, in turn, decrease the savings of all workers more or less proportionally, which dampens the decrease in the ratio $\frac{s_y^*}{s_m^*}$. We conclude that economies with public pension programs will perform better regarding wealth equality during population ageing than economies without such systems. Nevertheless, we find that ageing worsens inequality in both settings, suggesting that countries that suffer from inequality issues need to oppose these matters by introducing appropriate policy measures.

There are a number of caveats that have to be considered. Firstly, the numerical results strongly depend on the values chosen for the parameters and are thus not necessarily robust. In fact, we can show that if the tax channel is particularly strong, we even observe an increase in wealth equality in the model economy with politico-economic pay-as-you-go. Secondly, the numerical predictions regarding the Gini coefficient are not really applicable to the actual Gini coefficient of the United States, since our measure of inequality is highly stylized and omits all other factors affecting wealth inequality. Thirdly, in the numerical procedure, we relied on smoothing techniques for the policy outcome function, which implies that the function may not be perfectly accurate at every point in the grid.

Regarding future work, it would be very interesting to compare the outcome of the politico-economic model with a Ramsey allocation chosen by a benevolent planner under commitment, in order to quantify how much of the results can be attributed purely to the probabilistic voting setup and how much of the results are associated with pay-as-you-go systems in general. Further, the quality of the model may be increased by introducing a larger number of cohorts to imitate the actual age structure. In addition, it may be promising to insert further aspects of population ageing, as for example increased longevity. We expect that increased longevity may intensify the incentive to build up savings for old age because agents have to equip for a longer retirement period. We missed such effects in our results, as the benefit channel did not become evident at all.

Another possible extension is the introduction of endogenous labor supply. This would measure the effects that higher social security taxes have on labor supply, disposable income and hence, savings. Besides, by introducing government borrowing we'd have a tool to analyze the effects on the fiscal budget. It would be imaginable to include the possibility that the government is not able to pay out the social security benefits in times of great distress, which is a current fear of young agents when looking into the vast future. An expected sovereign default would definitely affect the savings of the different agents and thus the wealth distribution. We leave these ideas for future research.

Appendix A

Model economy without government sector

Young workers solve

$$\max_{c_{y,t}, c_{m,t+1}, c_{r,t+2}} u(c_{y,t}) + \beta u(c_{m,t+1}) + \beta^2 u(c_{r,t+2}), \tag{A.1}$$

subject to

$$c_{y,t} = w_t a_y - s_{y,t+1}, \tag{A.2a}$$
$$c_{m,t+1} = w_{t+1} a_m + R_{t+1} s_{y,t+1} - s_{m,t+2}, \tag{A.2b}$$
$$c_{r,t+2} = R_{t+2} s_{m,t+2}. \tag{A.2c}$$

Middle-aged workers solve

$$\max_{c_{m,t}, c_{r,t+1}} u(c_{m,t}) + \beta u(c_{r,t+1}), \tag{A.3}$$

subject to

$$c_{m,t} = w_t a_m + R_t s_{y,t} - s_{m,t+1}, \tag{A.4a}$$
$$c_{r,t+1} = R_{t+1} s_{m,t+1}. \tag{A.4b}$$

Output is determined by a standard Cobb-Douglas production function:

$$Y_t = A K_t^{\alpha} L_t^{1-\alpha} \tag{A.5}$$

By clearance of the factor markets, labor and capital are defined as

$$L_t = a_y N_t + a_m N_{t-1}, \tag{A.6a}$$

$$K_t = N_{t-1} s_{y,t} + N_{t-2} s_{m,t}. \tag{A.6b}$$

Appendix B

B Results

beta	am	alpha	n	sy*	sm*	sy/sm	w*	R*	Py	Pm	Total Wealth	sw_y	sw_m	Gini
0.85	1.3	0.3	1.26	0.0861	0.2733	0.3150384	0.369	1.3363	0.5575221	0.4424779	0.168931858	0.284154	0.715846	0.2733682
0.85	1.3	0.3	1.22	0.0838	0.2729	0.3070722	0.3724	1.3081	0.5495495	0.4504505	0.16898018	0.2725305	0.7274695	0.2770191
0.85	1.3	0.3	1.18	0.0814	0.2724	0.2988253	0.3759	1.2796	0.5412844	0.4587156	0.169014679	0.2606907	0.7393093	0.2805937
0.85	1.3	0.3	1.14	0.0787	0.272	0.2893382	0.3796	1.2508	0.5327103	0.4672897	0.169027103	0.248033	0.751967	0.2846773
0.85	1.3	0.3	1.1	0.0758	0.2717	0.2789842	0.3835	1.2218	0.5238095	0.4761905	0.169085714	0.2348203	0.7651797	0.2889892
Effekte innerhalb Transition				-11.96%	-0.59%	-11.44%	3.93%	-8.57%	-6.05%	7.62%	0.09%	-17.36%	6.89%	5.71%
0.85	1.5	0.3	1.26	0.0708	0.2947	0.2402443	0.3614	1.403	0.5575221	0.4424779	0.169870796	0.2323682	0.7676318	0.325154
0.85	1.5	0.3	1.22	0.0682	0.2944	0.2316576	0.3647	1.3736	0.5495495	0.4504505	0.170091892	0.2203472	0.7796528	0.3292023
0.85	1.5	0.3	1.18	0.0654	0.2941	0.2223733	0.3681	1.3439	0.5412844	0.4587156	0.170308257	0.2078584	0.7921416	0.333426
0.85	1.5	0.3	1.14	0.0623	0.2938	0.212049	0.3717	1.314	0.5327103	0.4672897	0.17047757	0.1946758	0.8053242	0.3380345
0.85	1.5	0.3	1.1	0.059	0.2936	0.2009537	0.3754	1.2836	0.5238095	0.4761905	0.170714286	0.1810321	0.8189679	0.3427774
Effekte innerhalb Transition				-16.67%	-0.37%	-16.35%	3.87%	-8.51%	-6.05%	7.62%	0.50%	-22.09%	6.69%	5.42%
Effekte zwischen Modellen				-17.77%	7.83%	-23.74%	-2.06%	4.99%	0.00%	0.00%	0.56%	-18.22%	7.23%	18.94%
0.95	1.3	0.3	1.26	0.1053	0.3059	0.3442301	0.3847	1.2126	0.5575221	0.4424779	0.194061062	0.3025186	0.6974814	0.2550035
0.95	1.3	0.3	1.22	0.1031	0.3056	0.3373691	0.3883	1.1864	0.5495495	0.4504505	0.194316216	0.2915792	0.7084208	0.2579704
0.95	1.3	0.3	1.18	0.1006	0.3052	0.3296199	0.3921	1.1599	0.5412844	0.4587156	0.194453211	0.2800325	0.7199675	0.2612519
0.95	1.3	0.3	1.14	0.098	0.305	0.3213115	0.396	1.1332	0.5327103	0.4672897	0.194728972	0.2680937	0.7319063	0.2646166
0.95	1.3	0.3	1.1	0.0951	0.3047	0.3121103	0.4001	1.1062	0.5238095	0.4761905	0.194909524	0.2555765	0.7444235	0.2682331
Effekte innerhalb Transition				-9.69%	-0.39%	-9.33%	4.00%	-8.77%	-6.05%	7.62%	0.44%	-15.52%	6.73%	5.19%
Effekte zwischen Modellen				22.30%	11.93%	9.27%	4.25%	-9.26%	0.00%	0.00%	14.88%	6.46%	-2.57%	-6.72%
0.85	1.3	0.25	1.26	0.0804	0.3111	0.2584378	0.4485	1.1693	0.5575221	0.4424779	0.182479646	0.2456426	0.7543574	0.3118795
0.85	1.3	0.25	1.22	0.0769	0.3102	0.2479046	0.4516	1.1454	0.5495495	0.4504505	0.18199009	0.2322124	0.7677876	0.3173371
0.85	1.3	0.25	1.18	0.0731	0.3094	0.2362637	0.4548	1.1213	0.5412844	0.4587156	0.181494495	0.2180115	0.7819885	0.3232729
0.85	1.3	0.25	1.14	0.0691	0.3085	0.223987	0.4581	1.097	0.5327103	0.4672897	0.180969159	0.2034064	0.7965936	0.3293039
0.85	1.3	0.25	1.1	0.0647	0.3077	0.2102697	0.4616	1.0723	0.5238095	0.4761905	0.180414286	0.1878481	0.8121519	0.3359614
Effekte innerhalb Transition				-19.53%	-1.09%	-18.64%	2.92%	-8.30%	-6.05%	7.62%	-1.13%	-23.53%	7.66%	7.72%
Effekte zwischen Modellen				-6.62%	13.83%	-17.97%	21.54%	-12.50%	0.00%	0.00%	8.02%	-13.55%	5.38%	14.09%

Figure B.1: All results (economy without government sector)

beta	am	alpha	omega_r	n	sy*	sm*	sy/sm	tau*	w*	R*	b*	Py	Pm	Total Wealth	sw_y	sw_m	Gini
0.85	1.3	0.3	1.25	1.26	0.0756	0.2094	0.3610315	0.0853	0.3448	1.5664	0.0949	0.5575221	0.442477876	0.13480354	0.3126674	0.6873326	0.2448547
0.85	1.3	0.3	1.25	1.22	0.0733	0.2047	0.358085	0.0932	0.3461	1.5521	0.0992	0.5495495	0.45045045	0.132489189	0.3040398	0.6959602	0.2455098
0.85	1.3	0.3	1.25	1.18	0.0709	0.2	0.3545	0.1015	0.3475	1.5379	0.1032	0.5412844	0.458715596	0.130120183	0.2949355	0.7050645	0.2463489
0.85	1.3	0.3	1.25	1.14	0.0686	0.1958	0.3503575	0.1092	0.3492	1.5202	0.1061	0.5327103	0.46728972	0.128039252	0.2854119	0.7145881	0.2472984
0.85	1.3	0.3	1.25	1.1	0.0664	0.1926	0.344756	0.1152	0.3515	1.4976	0.1069	0.5238095	0.476190476	0.126495238	0.2749586	0.7250414	0.2488509
Effekte innerhalb Transition					-12.17%	-8.02%	-4.51%	35.05%	1.94%	-4.39%	12.68%	-6.05%	7.62%	-6.16%	-12.06%	5.49%	1.63%
0.85	1.5	0.3	1.25	1.26	0.0652	0.2393	0.2724613	0.0685	0.3426	1.5897	0.0816	0.5575221	0.442477876	0.142235398	0.2555654	0.7444346	0.3019568
0.85	1.5	0.3	1.25	1.22	0.0629	0.2357	0.2668647	0.0737	0.3445	1.5697	0.0843	0.5495495	0.45045045	0.140737838	0.2456103	0.7543897	0.3039392
0.85	1.5	0.3	1.25	1.18	0.0604	0.2311	0.2613587	0.0807	0.3451	1.5532	0.0881	0.5412844	0.458715596	0.138702752	0.2357097	0.7642903	0.3055747
0.85	1.5	0.3	1.25	1.14	0.058	0.2274	0.2550572	0.0863	0.3481	1.5318	0.0904	0.5327103	0.46728972	0.137158879	0.2252657	0.7747343	0.3074445
0.85	1.5	0.3	1.25	1.1	0.0555	0.2257	0.2459016	0.0893	0.351	1.5023	0.0896	0.5238095	0.476190476	0.136547619	0.2129032	0.7870968	0.3109063
Effekte innerhalb Transition					-14.88%	-5.68%	-9.75%	30.36%	2.45%	-5.50%	9.84%	-6.05%	7.62%	-4.00%	-16.69%	5.73%	2.96%
Effekte zwischen Modellen					-13.76%	14.28%	-24.53%	-19.70%	-0.64%	1.49%	-13.97%	0.00%	0.00%	5.51%	-18.26%	8.31%	23.32%
0.95	1.3	0.3	1.25	1.26	0.091	0.2364	0.3849408	0.0838	0.3599	1.4172	0.0973	0.5575221	0.442477876	0.155336283	0.3266108	0.6733892	0.2309113
0.95	1.3	0.3	1.25	1.22	0.0886	0.2311	0.3833838	0.0913	0.3614	1.4033	0.1014	0.5495495	0.45045045	0.152789189	0.318675	0.681325	0.2308746
0.95	1.3	0.3	1.25	1.18	0.086	0.2262	0.3801945	0.0987	0.3629	1.3895	0.1048	0.5412844	0.458715596	0.150311927	0.3096924	0.6903076	0.231592
0.95	1.3	0.3	1.25	1.14	0.0833	0.2213	0.3764121	0.1065	0.3646	1.3751	0.1080	0.5327103	0.46728972	0.147785981	0.3002637	0.6997363	0.2324466
0.95	1.3	0.3	1.25	1.1	0.0807	0.2174	0.3712052	0.1131	0.3667	1.3563	0.1095	0.5238095	0.476190476	0.145795238	0.289937	0.710063	0.2338726
Effekte innerhalb Transition					-11.32%	-8.04%	-3.57%	34.96%	1.89%	-4.30%	12.55%	-6.05%	7.62%	-6.14%	-11.23%	5.45%	1.28%
Effekte zwischen Modellen					20.37%	12.89%	6.62%	-1.76%	4.38%	-9.53%	2.54%	0.00%	0.00%	15.23%	4.46%	-2.03%	-5.69%
0.85	1.3	0.25	1.25	1.26	0.0719	0.2159	0.3330245	0.108	0.4163	1.462	0.1450	0.5575221	0.442477876	0.135616814	0.2955816	0.7044184	0.2619405
0.85	1.3	0.25	1.25	1.22	0.0693	0.2126	0.3259643	0.1128	0.4182	1.4429	0.1450	0.5495495	0.45045045	0.13384955	0.2845268	0.7154732	0.2650228
0.85	1.3	0.25	1.25	1.18	0.0667	0.2089	0.3192915	0.1186	0.4199	1.4251	0.1457	0.5412844	0.458715596	0.131929358	0.2736591	0.7263409	0.2676253
0.85	1.3	0.25	1.25	1.14	0.0638	0.2022	0.3155292	0.1278	0.4207	1.417	0.1496	0.5327103	0.46728972	0.128472897	0.2645454	0.7354546	0.2681649
Effekte innerhalb Transition					-11.27%	-6.35%	-5.25%	18.33%	1.06%	-3.08%	3.12%	-4.45%	5.61%	-5.27%	-10.50%	4.41%	2.38%
Effekte zwischen Modellen					-4.89%	3.10%	-7.76%	26.61%	20.74%	-6.66%	52.87%	0.00%	0.00%	0.60%	-5.46%	2.49%	6.98%
0.85	1.3	0.3	1.5	1.26	0.0716	0.1851	0.3868179	0.1276	0.3345	1.6809	0.1377	0.5575221	0.442477876	0.121821239	0.3276816	0.6723184	0.2298405
0.85	1.3	0.3	1.5	1.22	0.0694	0.1801	0.3853415	0.1376	0.3355	1.6692	0.1419	0.5495495	0.45045045	0.119264865	0.3197818	0.6802182	0.2297677
0.85	1.3	0.3	1.5	1.18	0.0675	0.1753	0.3850542	0.1463	0.3366	1.6563	0.1441	0.5412844	0.458715596	0.116949541	0.3124142	0.6875858	0.2288702
0.85	1.3	0.3	1.5	1.14	0.0657	0.173	0.3797688	0.1513	0.3389	1.6313	0.1426	0.5327103	0.46728972	0.115840187	0.3021323	0.6978677	0.230578
0.85	1.3	0.3	1.5	1.1	0.0633	0.1672	0.3785885	0.1637	0.3395	1.6247	0.1467	0.5238095	0.476190476	0.11277619	0.2940084	0.7059916	0.2298012
Effekte innerhalb Transition					-11.59%	-9.67%	-2.13%	28.29%	1.49%	-3.34%	6.57%	-6.05%	7.62%	-7.42%	-10.28%	5.01%	-0.02%
Effekte zwischen Modellen					-5.29%	-11.60%	7.14%	49.59%	-2.99%	7.31%	45.12%	0.00%	0.00%	-9.63%	4.80%	-2.18%	-6.13%

Figure B.2: All results (main model)

Appendix C

C Policy outcome function

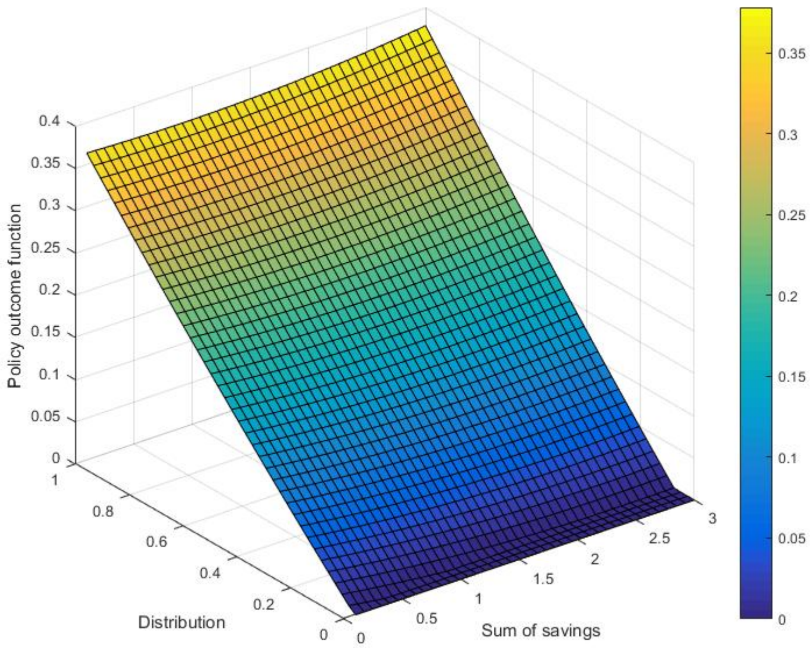

Figure C.1: Policy outcome function for $n = 1.26$ for reference case, i.e. $a_m = 1.3$, $\beta = 0.85$, $\alpha = 0.3$ and $\omega_r = 1.25$

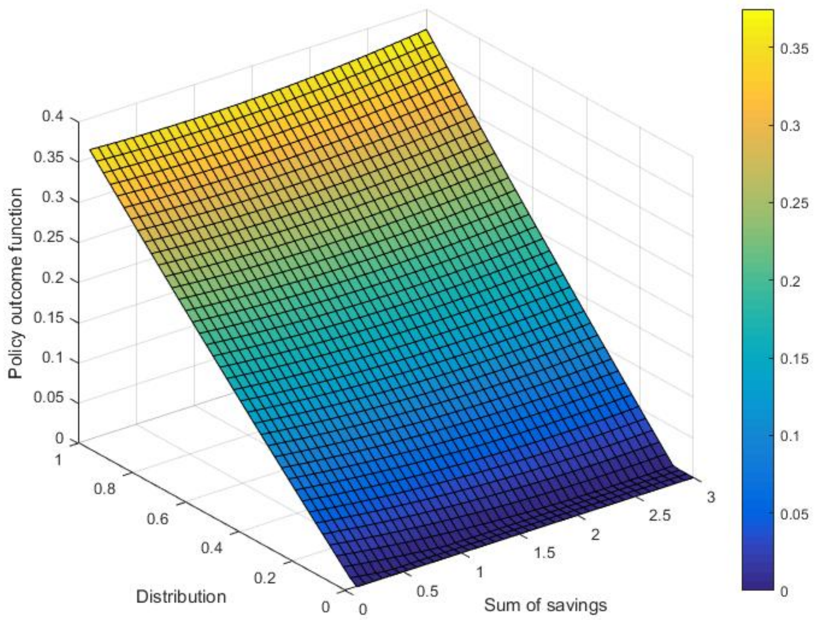

Figure C.2: Policy outcome function for $n = 1.26$ and $a_m = 1.5$, $\beta = 0.85$, $\alpha = 0.3$ and $\omega_r = 1.25$

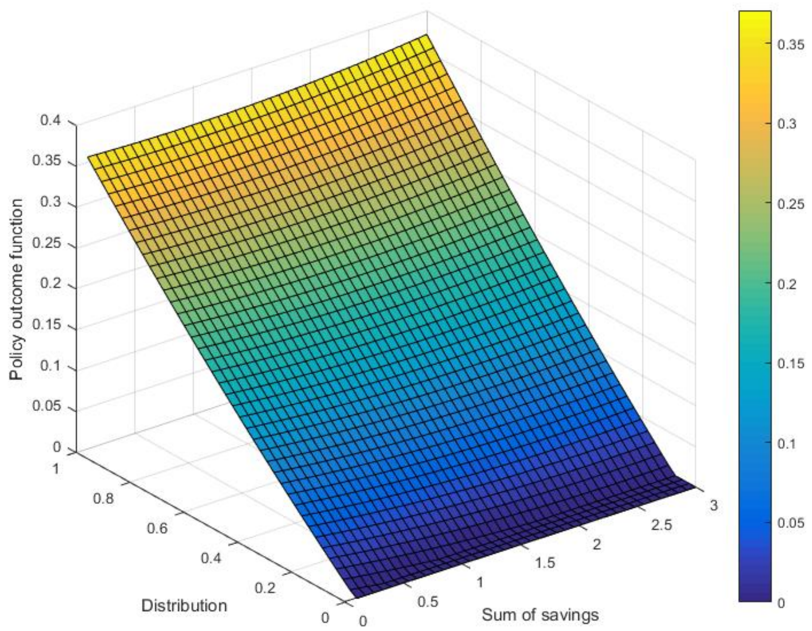

Figure C.3: Policy outcome function for $n = 1.26$ and $a_m = 1.3$, $\beta = 0.95$, $\alpha = 0.3$ and $\omega_r = 1.25$

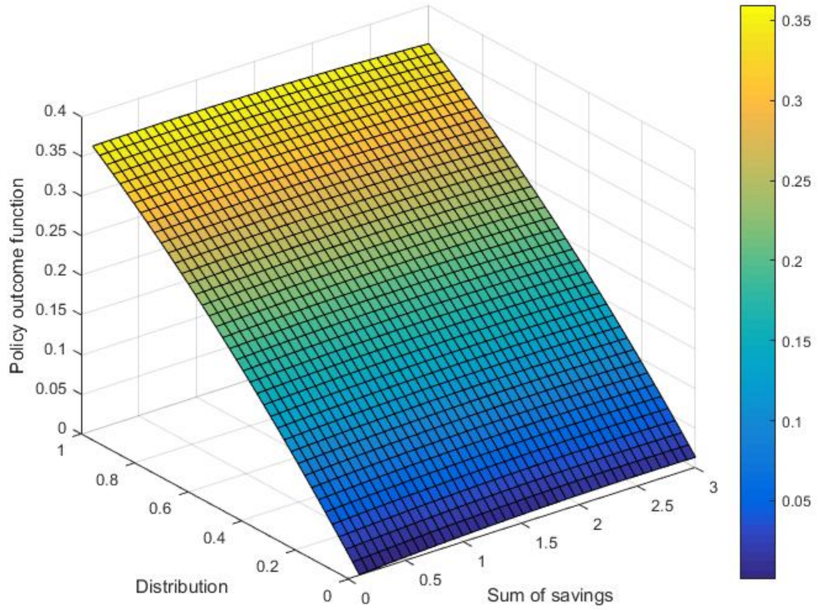

Figure C.4: Policy outcome function for $n = 1.26$ and $a_m = 1.3$, $\beta = 0.85$, $\alpha = 0.25$ and $\omega_r = 1.25$

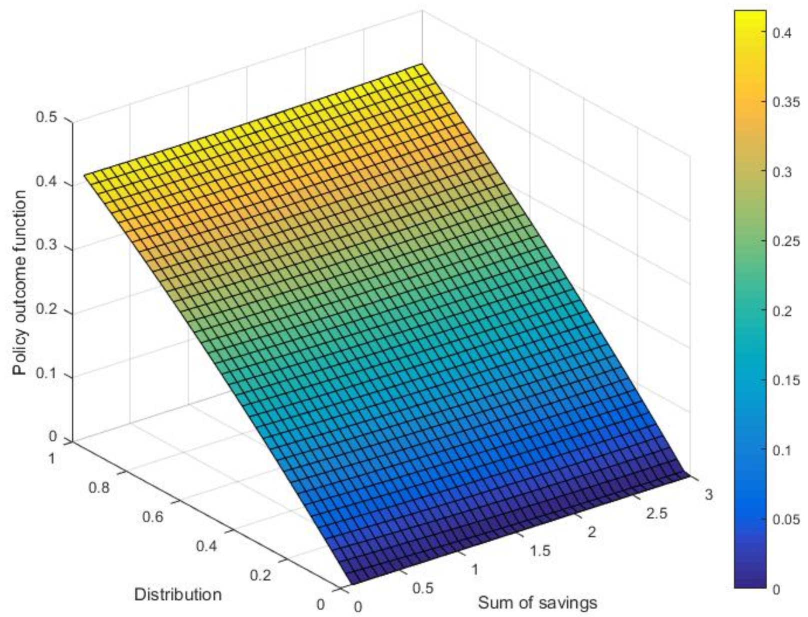

Figure C.5: Policy outcome function for $n = 1.26$ and $a_m = 1.3$, $\beta = 0.85$, $\alpha = 0.3$ and $\omega_r = 1.5$

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