

1a) Unusual

3'v's, 1'n, 1'o, 1'1', 1'5"

1 v, 1

$$2v, \frac{4!}{1! \cdot 3!} = \frac{4 \cdot 3!}{1! \cdot 3!} \Rightarrow 4$$

$1+4+6 \Rightarrow 11$ unique subsets

$$3v, \frac{4!}{2! \cdot 2!} \Rightarrow \frac{4 \cdot 3 \cdot 2!}{2! \cdot 2!} = \frac{12}{2} \Rightarrow 6$$

1b. 1 u, 5!

$$2v, \frac{6!}{2! \cdot 4!} \cdot \frac{4!}{1! \cdot 3!} \cdot \frac{3!}{2! \cdot 1!} \cdot \frac{2!}{1! \cdot 1!} \cdot \frac{6!}{1! \cdot 2!} \Rightarrow \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2!}{2!}$$

$$3v, \frac{7!}{3! \cdot 4!} \cdot \frac{4!}{1! \cdot 3!} \cdot \frac{3!}{1! \cdot 2!} \Rightarrow \frac{7!}{3! \cdot 2!} \Rightarrow \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3!}{3! \cdot 2!} \Rightarrow \frac{7 \cdot 6 \cdot 5 \cdot 4}{2!} \Rightarrow 7 \cdot 6 \cdot 5 \cdot 2$$

$$5! + (6 \cdot 5 \cdot 4 \cdot 3) + (7 \cdot 6 \cdot 5 \cdot 2) \text{ or } 120 + 420 + 360$$

900

$$\frac{52}{44}$$

$$2. \quad \frac{13!}{1! \cdot 12!} \cdot \frac{4!}{2! \cdot 2!} \cdot \frac{12!}{1! \cdot 11!} \cdot \frac{4!}{2! \cdot 2!} \cdot \frac{44!}{1! \cdot 43!}$$

↑ first face
(2) combs
↑ second face
(2) combs
↑ random pick

$$\frac{13!}{1! \cdot 12!} \cdot \frac{4!}{2! \cdot 2!} \cdot \frac{12!}{1! \cdot 11!} \cdot \frac{4!}{2! \cdot 2!} \cdot \frac{44!}{1! \cdot 43!} \rightarrow \frac{13! \cdot 2! \cdot 12! \cdot 2! \cdot 44!}{1! \cdot 12! \cdot 1! \cdot 11! \cdot 2! \cdot 2! \cdot 1! \cdot 43!}$$

$$13 \cdot 12 \cdot 6 \cdot 6 \cdot 44$$

$$247,104$$

3. 16 songs 7 couples.

$$0 \quad 1 \quad 3 \quad 5 \quad 7 \quad 9 \quad 11 \quad 13 \quad 15$$

$$2 \quad 4 \quad 6 \quad 8 \quad 10 \quad 12 \quad 14 \quad 16$$

0 at fewest, 8 at most
9 options

$$\frac{22!}{7! \cdot 15!} - \text{all cases where couple has 9 or more songs}$$

$$1+6+7+28+84+210+462+924=1722$$

$$16 \text{ songs} \rightarrow 1$$

$$15 \text{ songs} \rightarrow \frac{6!}{1! \cdot 5!} \rightarrow 6$$

$$14 \text{ songs} \rightarrow \frac{7!}{6! \cdot 1!} \rightarrow 7$$

$$13 \text{ songs} \rightarrow \frac{8!}{6! \cdot 2!} \rightarrow 56/2 \rightarrow 28$$

$$12 \text{ songs} \rightarrow \frac{9!}{6! \cdot 3!} \rightarrow 504/6 \rightarrow 84$$

$$11 \text{ songs} \rightarrow \frac{10!}{6! \cdot 4!} \rightarrow 5040/24 \rightarrow 210$$

$$10 \text{ songs} \rightarrow \frac{11!}{6! \cdot 5!} \rightarrow 55440/120 \rightarrow 462$$

$$9 \text{ songs} \rightarrow \frac{12!}{6! \cdot 6!} \rightarrow 665,280/720 \rightarrow 924$$

$$22 \cdot 21 \cdot 20 \cdot 19 \cdot 18 \cdot 17 \cdot 16 \cdot 15!$$

$$7! \cdot 18!$$

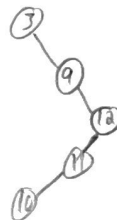
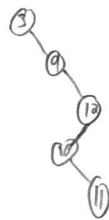
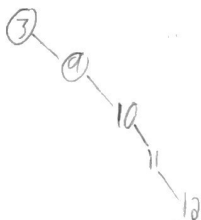
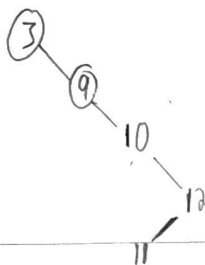
$$859,541,760$$

$$5040$$

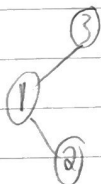
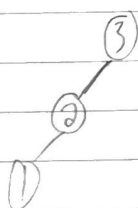
$$170,544 - 1722$$

$$168,822 \text{ ways}$$

4.



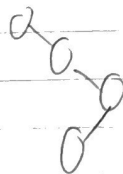
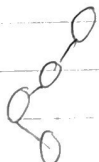
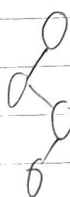
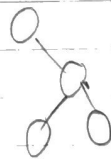
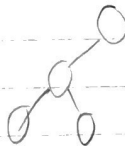
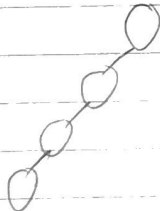
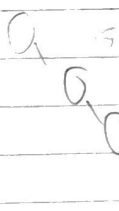
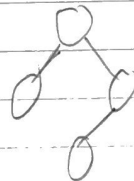
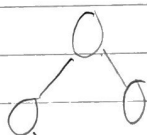
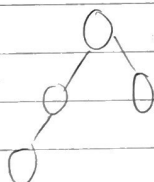
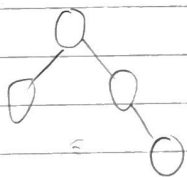
5 permutations after 9



2 permutations before 3

2 4 5
1 3 7 4 nodes

1 2 5 12 42
1 node 2 nodes 3 nodes



10!

2 · 42 · 5

420

5.

3 noise distribution + 4 noise distribution
stars & bars

$$\binom{r-1}{r-n} \quad \begin{array}{l} 10 \text{ stars} \\ 3 \text{ bars} \end{array} \quad \begin{array}{l} 4 \text{ bars} \\ 1 \end{array}$$

$$\binom{9}{7} \rightarrow \frac{9!}{2!7!} \rightarrow \frac{9 \cdot 8 \cdot 7!}{2! \cdot 7!} \rightarrow \frac{72}{2} = 36$$

$$\binom{9}{6} \rightarrow \frac{9!}{3!6!} \rightarrow \frac{9 \cdot 8 \cdot 7 \cdot 6!}{3! \cdot 6!} \rightarrow \frac{504}{6} = 84$$

$$\begin{array}{r} 36 \\ + 84 \\ \hline 120 \end{array}$$