1. 15 students $\quad$ (uniform, independent) $\quad$ 8 questions,

$P(\text{only once}) \to \frac{15}{15} \cdot \frac{14}{15} \cdot \frac{13}{15} \cdot \frac{12}{15} \cdot \frac{11}{15} \cdot \frac{10}{15} \cdot \frac{9}{15} \cdot \frac{8}{15}$

$$\frac{\frac{15!}{7!}}{15^8} \Rightarrow \frac{259{,}459{,}200}{2{,}562{,}890{,}625} = .1012 = 10.12\%$$

2. 00000 - 99999 $\quad$ random, independent

10^5 possible combs per num $\quad$ unique nums

each 5-dig num $\left\{ \frac{5}{10} \cdot \frac{4}{10} \cdot \frac{7}{10} \cdot \frac{6}{10} \cdot \frac{5}{10} \right\} \to \frac{4200}{10^5} = \frac{42}{10^3} = \underbrace{.042}_{\text{prob of 1 five dig num under constraints}}$

odd $\quad$ different odd $\quad$ even

10-1 $\quad$ 10-2 $\quad$ 10-3 $\quad$ 10-4 $\quad$ 10-5

Bernoulli trials

$\binom{n}{k} p^k (1-p)^{n-k} \Rightarrow \frac{8!}{3!5!} (.042)^5 (1-.042)^3$

$n = 8$ trials, 8 numbers

$k = 5$ nums meet criteria

$$\frac{8 \cdot 7 \cdot \cancel{6} \cdot \cancel{5!}}{\cancel{6} \cdot \cancel{5!}} (.042)^5 (.958)^3 = .00000643474$$

3. $\quad n=3$ Bernoulli $\quad k=2 \quad\quad n=3, k=3$

$p(A) = \binom{3}{2}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^1 + \binom{3}{3}\left(\frac{1}{2}\right)^3$

$3\left(\frac{1}{4}\right)\left(\frac{1}{2}\right) + 1\left(\frac{1}{8}\right) \Rightarrow \frac{3}{8} + \frac{1}{8} = \frac{1}{2}$

for all 6 rolls

$p(B) = \frac{6}{6}\left(\frac{1}{6}\right)^2 = (1)\frac{1}{36} = \frac{1}{36}$

$\begin{array}{r} 36 \\ \times 6 \\ \hline 216 \end{array}$

$p(A \cap B) = \frac{3 \leftarrow 4,5,6}{6^3} = \frac{3}{216} = \frac{1}{72}$

$P(A) \cdot P(B) = \frac{1}{2} \cdot \frac{1}{36} = \frac{1}{72}$

Because $P(A \cap B) == P(A) \cdot P(B)$ they are independent.

4. 4 suits 13 cards

$\diamondsuit, \spadesuit, \heartsuit, \clubsuit$ A,2,3,4,5,6,7,8,9,10,J,Q,K

E $\left(\binom{4}{1} = \frac{4}{1!3!} = 4\right) \left(\binom{13}{5} = \frac{13!}{8!5!} = \frac{13\cdot12\cdot11\cdot10\cdot9\cdot\cancel{8!}}{5!\cdot\cancel{8!}}\right)$

E $\frac{4\cdot13\cdot12\cdot11\cdot10\cdot9}{5!} = \frac{617760}{120}$

S: $\binom{52}{5}$ $\frac{52!}{5!47!} = \frac{52\cdot51\cdot50\cdot49\cdot48\cdot\cancel{47!}}{5!\,\cancel{47!}}$ $\frac{E}{S} = \frac{5148}{2598960}$

$S = \frac{52\cdot51\cdot50\cdot49\cdot48}{5!} = \frac{311875200}{120}$

$P(A) = \frac{5148}{2598960}$ $\frac{\text{winning hands}}{\text{possible hands}}$ $\Rightarrow \frac{1}{P(A)} = \frac{2598960}{5148}$ hands per match

504.84 hands/ per 1 match

5. 70% with ☆ $\left(\frac{7}{10}\right)$ 50% without ☆ $\left(\frac{1}{2}\right)$ 5 games

not equal prob

(All or none ⇒ Bernoulli

75% $\left(\frac{3}{4}\right)$ chance of play

4/5 games won

n = 5

k = 4

$\binom{5}{4}\left(\frac{7}{10}\right)^4\left(1-\frac{7}{10}\right)^1\underbrace{\left(\frac{3}{4}\right)}_{\text{chance of play}} + $ n = 5 k = 4 $\binom{5}{4}\left(\frac{1}{2}\right)^4\left(1-\frac{1}{2}\right)^1\underbrace{\left(\frac{1}{4}\right)}_{\text{doesn't play}}$

$5\cdot\frac{7^4}{10^4}\cdot\left(\frac{3}{10}\right)\left(\frac{3}{4}\right) + 5\left(\frac{1}{24}\right)\left(\frac{1}{2}\right)\left(\frac{1}{4}\right)$

S is chance of winning all 4,

E: chance of winning with ☆

E: Chances he plays & wins

$\frac{108045}{400000} + \frac{5}{128}$

$.2701 + .0390$

S: .309

$\frac{E}{S} = \frac{.2701}{.3090} = 0.8741$