Adrián Lima García — PX 2011 - Hoja 6.

$$S = \left\{ \begin{pmatrix} 2 & 2 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 4 \\ 2 & -4 \end{pmatrix} \right\} \quad T = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} / a = b = 2d \right\}$$

a)

<u>Paso 1</u> Tenemos un SG y comprobaremos si es libre.

$$A = \begin{pmatrix} 2 & 1 & 1 \\ 2 & 0 & 4 \\ 1 & 0 & 2 \\ 1 & 2 & -4 \end{pmatrix} \xrightarrow{F_4 = F_4 - F_3} \begin{pmatrix} 2 & 1 & 1 \\ 2 & 0 & 4 \\ 1 & 0 & 2 \\ 0 & 2 & -6 \end{pmatrix} \xrightarrow{F_2 = F_2 - F_3} \begin{pmatrix} 2 & 1 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 2 \\ 0 & 2 & -6 \end{pmatrix} \xrightarrow{F_1 = F_1 - 2F_3} \begin{pmatrix} 0 & 1 & -3 \\ 0 & 0 & 0 \\ 1 & 0 & 2 \\ 0 & 2 & -6 \end{pmatrix}$$

$$\xrightarrow{F_4 = F_4 - 2F_1} \begin{pmatrix} 0 & 1 & -3 \\ 0 & 0 & 0 \\ 1 & 0 & 2 \\ 0 & 0 & 0 \end{pmatrix} \xrightarrow{F_3 \leftrightarrow F_2} \begin{pmatrix} 0 & 1 & -3 \\ 1 & 0 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \xrightarrow{F_1 \leftrightarrow F_2} \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$\text{Rang}(A) = \text{Rang}(A^*) = 2$

Hay 2 vectores linealmente indep. hemos de quitar el restante.

$$S = \left\{ \begin{pmatrix} 2 & 2 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} \right\} \text{ tq } \text{ es base (libre?)}$$

$$A = \begin{pmatrix} 2 & 1 \\ 2 & 0 \\ 1 & 0 \\ 1 & 2 \end{pmatrix} \xrightarrow{F_1 = F_1 - F_2} \begin{pmatrix} 0 & 1 \\ 2 & 0 \\ 1 & 0 \\ 1 & 2 \end{pmatrix} \xrightarrow{F_4 = F_4 - F_3} \begin{pmatrix} 0 & 1 \\ 2 & 0 \\ 1 & 0 \\ 0 & 2 \end{pmatrix} \xrightarrow{F_4 = F_4 - 2F_1} \begin{pmatrix} 0 & 1 \\ 2 & 0 \\ 1 & 0 \\ 0 & 0 \end{pmatrix} \rightarrow$$

$$\xrightarrow{F_2 = F_2 - 2F_3} \begin{pmatrix} 0 & 1 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \end{pmatrix} \xrightarrow{F_1 \leftrightarrow F_3} \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} \xrightarrow{F_2 \leftrightarrow F_3} \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$\text{Rang}(A) = 2$.

Es libre.

<u>Es base</u>

$$S = \left\{ \begin{pmatrix} 2 & 2 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} \right\}$$

Ecuaciones paramétricas:

$$S = \left\{ a \begin{pmatrix} 2 & 2 \\ 1 & 1 \end{pmatrix} + b \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} / a, b \in \mathbb{R} \right\} \boxed{\dim S = 2}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 2\alpha + \beta & 2\alpha \\ \alpha & \alpha + \beta \end{pmatrix} \longrightarrow \begin{cases} a = \alpha + b \\ b = 2\alpha \\ c = \alpha \\ d = \alpha + \beta \end{cases}$$

$$\left(\begin{array}{cc|c} 2 & 1 & a \\ 2 & 0 & b \\ 1 & 0 & c \\ 1 & 2 & d \end{array}\right) \xrightarrow{F_4 = F_4 - F_3} \left(\begin{array}{cc|c} 2 & 1 & a \\ 2 & 0 & b \\ 0 & 2 & c \\ & & d \end{array}\right) \xrightarrow{F_1 = F_1 - F_2} \left(\begin{array}{cc|c} 0 & 1 & a-b \\ 2 & 0 & b \\ 1 & 0 & c \\ 0 & 2 & d \end{array}\right)$$

$$\xrightarrow{F_2 = F_2 - 2F_3} \left(\begin{array}{cc|c} 0 & 1 & a-b \\ 0 & 0 & \boxed{b-2c} \\ 1 & 0 & c \\ 0 & 0 & \boxed{c-d-2(a-b)} \end{array}\right)$$

~~Ec. paramétricas~~

Ec. implícitas:

$$S = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} / \begin{array}{l} a - 2c = 0 \\ c - d - 2a + 2b = 0 \end{array} \right\} M_{2\times2}$$

$T = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} / a = b = 2d \right\} \rightarrow \begin{matrix} a = 2d \\ b = 2d \end{matrix}$

$A = \left(\begin{array}{cccc|c} 1 & 0 & 0 & -2 & 0 \\ 0 & 1 & 0 & -2 & 0 \end{array} \right) \rightarrow Rang(A) = Rang(A^*) = 2$

Nº parámetros = dim $M_{2\times2}$ - Rang $(N) = 4 - 2 = 2$

$\begin{cases} a = 2\lambda \\ b = 2\lambda \\ c = \beta \\ d = \lambda \end{cases} \quad \begin{pmatrix} 2\lambda & 2\lambda \\ \beta & \lambda \end{pmatrix} \rightarrow T = \underbrace{\left\{ \lambda \begin{pmatrix} 2 & 2 \\ 0 & 1 \end{pmatrix} + \beta \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} / \lambda, \beta \in \mathbb{R} \right\}}_{\text{ec. paramétricas}}$

Ahora tenemos el sig. sist. gdor:

$G = \left\{ \begin{pmatrix} 2 & 2 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \right\}$ pero... ¿es base? vemos si es libre:

$\begin{pmatrix} 2 & 0 \\ 2 & 0 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} \xrightarrow{F_1 = F_2 - F_1} \begin{pmatrix} 0 & 0 \\ 2 & 0 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} \xrightarrow{F_2 = 2F_4 - F_2} \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 1 & 0 \end{pmatrix}$ Reordenamos y vemos que es escalonada de rango 2.

Entonces podemos decir que es base.

(c), (d)

Calculamos $S+T$

$$\left.\begin{array}{l} B_S = \left\{ \begin{pmatrix} 2 & 2 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} \right\} \\ \\ B_T = \left\{ \begin{pmatrix} 2 & 2 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \right\} \end{array}\right\} B_{S+T} = B_S \cup B_T$$

$$B_S \cup B_T = \left\{ \begin{pmatrix} 2 & 2 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}, \begin{pmatrix} 2 & 2 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \right\}$$ es sist. gdor de $S+T$

Vemos como $S \wedge T$

Ec. implícitas de S:

$$S = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} / b - 2c = c - d - 2a + 2b = 0 \right\}$$

Ec implícita de T: $\longrightarrow \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} / a = b = 2d \right\}$

$$S\cap T = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \Big/ b-2c = c-d-2a+2b = 0 \atop a = b = 2d \right\} \begin{matrix} a = 2d \\ b = 2d \end{matrix}$$

Calculemos una base de $S\cap T$, pasamos a ec. paramétricas.

$$\begin{cases} b-2c=0 \\ c-d-2a+2b=0 \\ a-2d=0 \\ b-2d=0 \end{cases} \longrightarrow A = \left(\begin{array}{cccc|c} 0 & 1 & -2 & 0 & 0 \\ -2 & 2 & 1 & -1 & 0 \\ 1 & 0 & 0 & -2 & 0 \\ 0 & 1 & 0 & -2 & 0 \end{array}\right) \longrightarrow$$

$$\xrightarrow{F_4 = F_4 - F_1} \left(\begin{array}{cccc|c} 0 & 1 & -2 & 0 & 0 \\ -2 & 2 & 1 & -1 & 0 \\ 1 & 0 & 0 & -2 & 0 \\ 0 & 0 & 2 & -2 & 0 \end{array}\right) \xrightarrow{F_2 = 2F_3 + F_2} \left(\begin{array}{cccc|c} 0 & 1 & -2 & 0 & 0 \\ 0 & 2 & 0 & -5 & 0 \\ 1 & 0 & 0 & -2 & 0 \\ 0 & 0 & 2 & -2 & 0 \end{array}\right) \longrightarrow$$

$$\xrightarrow{F_1 = F_2 - 2F_1} \left(\begin{array}{cccc|c} 0 & 0 & 4 & -5 & 0 \\ 0 & 2 & 0 & -5 & 0 \\ 1 & 0 & 0 & -2 & 0 \\ 0 & 0 & 2 & -2 & 0 \end{array}\right) \xrightarrow{F_4 = F_1 - 2F_4} \left(\begin{array}{cccc|c} 0 & 0 & 4 & -5 & 0 \\ 0 & 2 & 0 & -5 & 0 \\ 1 & 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & -1 & 0 \end{array}\right) \longrightarrow$$

$$\xrightarrow{F_1 \leftrightarrow F_3} \left(\begin{array}{cccc|c} 1 & 0 & 0 & -2 & 0 \\ 0 & 2 & 0 & -5 & 0 \\ 0 & 0 & 4 & -5 & 0 \\ 0 & 0 & 0 & -1 & 0 \end{array}\right) \quad \text{Rang}(A) = \text{Rang}(A^*) = \text{nº incógnitas}$$

SCD → una solución.

$$\text{nº parámetros} = \dim M_{2\times 2} - \text{Rang}(A) = 4 - 4 = 0$$

$$\begin{cases} a-2d=0 \rightarrow a=0 \\ 2b-5d=0 \rightarrow 2b=0;\ b=0 \\ 4c-5d=0 \rightarrow c=0 \\ -d=0 \rightarrow d=0 \end{cases}$$

Luego $\dim(S+T) = \dim S + \dim T - \dim S\cap T$

$2+2-0=0$ libre.

$$B_{S+T} = \left\{ \begin{pmatrix} 2 & 2 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}, \begin{pmatrix} 2 & 2 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \right\}$$

$S + T$ son suplementarios pues $S \cap T = 0$