2)

Adrián Lima García

a)

$M_{B_1B_1}(f)$ siendo: $f: \mathbb{R}^3[x]$ $f(p(x)) = x\,p'(x) + 3p(x)$

$$\boxed{f(p(x)) = 3a + 4bx + 5cx^2 + 6dx^3}$$

$B_1 = \{\underset{u_1}{1}, \underset{u_2}{1+x}, \underset{u_3}{1+x^2}, \underset{u_4}{1+x^3}\}$

$u_1 = 1$

$a = 1$ $\quad f(u_1) = (3,0,0,0)_{B_C}$

$b = 0$

$c = 0$ $\quad$ Pasamos a B_1

$d = 0$

$) = (\lambda_1, \lambda_2, \lambda_3, \lambda_4)$

$= \lambda_1 + \lambda_2 + \lambda_2 x + \lambda_3 + \lambda_3 x^2 + \lambda_4 + \lambda_4 x^3$

$= (\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4) + \lambda_2 x + \lambda_3 x^2 + \lambda_4 x^3$

$(3,0,0,0) = (\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4, \lambda_2; \lambda_3, \lambda_4)$

$3 = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4$; $\boxed{3 = \lambda_1}$

$\boxed{0 = \lambda_2}$

$\boxed{0 = \lambda_3}$

$\boxed{0 = \lambda_4}$

$$\boxed{f(u_1) = (3,0,0,0)_{B_1}}$$

$u_2 = 1+x$

$a=1$
$b=1$
$c=0$
$d=0$

$f(u_2) = (3,4,0,0)_{Bc}$

Cambiamos a B_1

$3 = \lambda_1+\lambda_2+\lambda_3+\lambda_4$; $\boxed{\lambda_1=-1}$
$\boxed{4=\lambda_2}$
$\boxed{0=\lambda_3}$
$\boxed{0=\lambda_4}$

$\boxed{f(u_2) = (-1,4,0,0)_{B_1}}$

$u_3 = 1+x^2$ $\quad f(u_3) = (3,0,5,0)_{Bc}$

$a=1$
$b=0$
$c=1$
$d=0$

Paso a B_1

$3 = \lambda_1+\lambda_2+\lambda_3+\lambda_4$; $\boxed{\lambda_1=-2}$
$\boxed{0=\lambda_2}$
$\boxed{5=\lambda_3}$
$\boxed{0=\lambda_4}$

$f(u_3) = (-2,0,5,0)_{B_1}$

$u_4 = 1+x^3$

$a=1$
$b=0$
$c=0$
$d=1$

$f(u_4) = (3,0,0,6)$

Cambio base

$3 = \lambda_1+\lambda_2+\lambda_3+\lambda_4$; $\boxed{\lambda_1=-3}$
$\boxed{0=\lambda_2}$
$\boxed{0=\lambda_3}$
$\boxed{6=\lambda_4}$

$f(u_4) = (-3,0,0,6)_{B_1}$

Entonces:

$$M_{B_1B_1}(f) = \begin{pmatrix} 3 & -1 & -2 & -3 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 6 \end{pmatrix}$$

2

Adrián Lima García PX204

b)

$B_2 = \{ \underbrace{1+x^3}_{u_1}, \underbrace{2+x^2}_{u_2}, \underbrace{3+x}_{u_3}, \underbrace{1}_{u_4} \}$

Con la f siendo la hallada en el apartado anterior...

$f(p(x)) = 3a + 4bx + 5cx^2 + 6dx^3$

Entonces sacamos los componentes de la base y los ponemos respecto a la base canónica

$B_c = \{1 + x + x^2 + x^3\}$

$u_1 = 1 + x^3 \begin{cases} a = 1 \\ b = 0 \\ c = 0 \\ d = 1 \end{cases}$ $f(p(u_1)) = (3 + 6x^3)_{Bc}$

$= (3, 0, 0, 6)_{Bc}$

Ahora debemos pasarla a una base de B_2.

$(3,0,0,6) = (\lambda_1, \lambda_2, \lambda_3, \lambda_4)$

$\hookrightarrow = \lambda_1(1+x^3) + \lambda_2(2+x^2) + \lambda_3(3+x) + \lambda_4(1)$

$= (\lambda_1 + \lambda_1 x^3 + 2\lambda_2 + \lambda_2 x^2 + 3\lambda_3 + \lambda_3 x + \lambda_4$

$(3,0,0,6) = (\lambda_1 + 2\lambda_2 + 3\lambda_3 + \lambda_4) + \lambda_3 x + \lambda_2 x^2 + \lambda_1 x^3$

Sacamos los valores de los escalares:

$3 = \lambda_1 + 2\lambda_2 + 3\lambda_3 + \lambda_4$; $3 = \lambda_4 + 6$; $\boxed{\lambda_4 = -3}$

$\boxed{0 = \lambda_3}$

$\boxed{0 = \lambda_2}$

$\boxed{6 = \lambda_1}$

$f(u_1) = (6, 0, 0, -3)$

Adrián Lima García

$u_2 = 2 + x^2$

$a = 2$
$b = 0$
$c = 1$
$d = 0$

$f(u_2) = (6, 0, 5, 0)_{Bc}$

$(6, 0, 5, 0) = \lambda_1(1 + x^3) + \lambda_2(2 + x^2) + \lambda_3(3 + x) + \lambda_4$

$6 = \lambda_1 + 2\lambda_2 + 3\lambda_3 + \lambda_4$; $6 = 10 + \lambda_4$; $\boxed{\lambda_4 = -4}$

$\boxed{0 = \lambda_3}$
$\boxed{5 = \lambda_2}$
$\boxed{0 = \lambda_1}$

$f(u_2) = (0, 5, 0, -4)_{B_2}$

$u_3 = (3 + x)$

$a = 3$
$b = 1$
$c = 0$
$d = 0$

$f(u_3) = (9, 4, 0, 0)_{Bc}$

$(3, 4, 0, 0) = \lambda_1(1 + x^3) + \lambda_2(2 + x^2) + \lambda_3(3 + x) + \lambda_4$

$9 = \lambda_1 + 2\lambda_2 + 3\lambda_3 + \lambda_4$; $9 = 12 + \lambda_4$; $\boxed{\lambda_4 = -3}$

$4 = \lambda_3$
$0 = \lambda_2$
$0 = \lambda_1$

$f(u_3) = (0, 0, 4, -3)$

$u_4 = (1)$

$f(u_4) = (3, 0, 0, 0)_{Bc}$

$a = 1$
$b = 0$
$c = 0$
$d = 0$

$(3, 0, 0, 0) = (\lambda_1 + 2\lambda_2 + 3\lambda_3 + \lambda_4, \lambda_3, \lambda_4)$

$3 = \lambda_1 + 2\lambda_2 + 3\lambda_3 + \lambda_4$; $\boxed{\lambda_4 = 3}$

$\boxed{0 = \lambda_3}$
$\boxed{0 = \lambda_2}$
$\boxed{0 = \lambda_1}$

$f(u_4) = (0, 0, 0, 3)$

$$M_{B_2B_2} f = \begin{pmatrix} 6 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ -3 & -4 & -3 & 3 \end{pmatrix}$$

c)

$B_3 = \{1+x,\ 1-x,\ x+x^2,\ x^2+x^3\}$ base de $\mathbb{R}_3[x]$

$u_1 = 1+x$

$a = 1$
$b = 1$
$c = 0$
$d = 0$

$f(u_1) = (3,4,0,0)_{B_c}$

Pasamos $f(u_1)$ de B_c a B_3.

$$3+4x = \lambda_1(1+x)+\lambda_2(1-x)+\lambda_3(x+x^2)+\lambda_4(x^2+x^3)$$
$$3+4x = \lambda_1+\lambda_1 x+\lambda_2-\lambda_2 x+\lambda_3 x+\lambda_3 x^2+\lambda_4 x^2+\lambda_4 x^3$$
$$3+4x = \lambda_1+\lambda_2+(-\lambda_2+\lambda_3)x+(\lambda_3+\lambda_4)x^2+\lambda_4 x^3$$
$$(3,4,0,0) = (\lambda_1+\lambda_2, \lambda_1-\lambda_2+\lambda_3, \lambda_3+\lambda_4, \lambda_4)$$

$3 = \lambda_1+\lambda_2$; $3 = \lambda_1+\lambda_1$; $3 = 2\lambda_1$; $\boxed{\lambda_1 = \frac{7}{2}}$
$4 = \lambda_1-\lambda_2+\lambda_3$; $4 = \lambda_1-\lambda_2$; ; $\boxed{\lambda_2 = \frac{-1}{2}}$
$0 = \lambda_3+\lambda_4$; $\boxed{\lambda_3 = 0}$
$\boxed{0 = \lambda_4}$

$$\boxed{f(u_1) = \left(\frac{7}{2}, \frac{-1}{2}, 0, 0\right)}$$

Adrián Lima García

$u_2 = 1 - x$

$a = 1$
$b = -1$
$c = 0$
$d = 0$

$f(u_2) = (3, -4, 0, 0)_{Bc}$

Cambio de base

$(3,-4,0,0) = (\lambda_1+\lambda_2,\ \lambda_1-\lambda_2+\lambda_3,\ \lambda_3+\lambda_4,\ \lambda_4)$

$3 = \lambda_1 + \lambda_2$; $3 = \lambda_1 + \lambda_1 + 4$; $\boxed{\lambda_1 = \frac{-1}{2}}$

$-4 = \lambda_1 - \lambda_2 + \lambda_3$; $-4 = \lambda_1 - \lambda_2$; $\lambda_2 = \lambda_1 + 4$; $\boxed{\lambda_2 = \frac{7}{2}}$

$0 = \lambda_3 + \lambda_4$; $\boxed{\lambda_3 = 0}$

$0 = \lambda_4$

$$\boxed{f(u_2) = \left(\frac{-1}{2}, \frac{7}{2}, 0, 0\right)_{B3}}$$

$u_3 = x + x^2$

$a = 0$
$b = 1$
$c = 1$
$d = 0$

$f(u_3) = (0, 4, 5, 0)_{Bc}$

Cambio de base

$(0,4,5,0) = (\lambda_1+\lambda_2,\ \lambda_1-\lambda_2+\lambda_3,\ \lambda_3+\lambda_4,\ \lambda_4)$

$0 = \lambda_1 + \lambda_2$; $0 = \lambda_1 + \lambda_1 - 1$; $\boxed{\lambda_1 = \frac{1}{2}}$

$4 = \lambda_1 - \lambda_2 + \lambda_3$; $4 = \lambda_1 - \lambda_2 + 5$; $-\lambda_2 = 1 - \lambda_1$

$5 = \lambda_3 + \lambda_4$; $\boxed{\lambda_3 = 5}$

$\boxed{0 = \lambda_4}$

$\boxed{\lambda_2 = \lambda_1 - 1}$

$\lambda_2 = \frac{1}{2} - 1$;

$\boxed{\lambda_2 = \frac{-1}{2}}$

$f(u_3) = \left(\frac{1}{2}, \frac{-1}{2}, 5, 0\right)$

Adrián Lima García

1

$u_4 = x^2 + x^3$

$a = 0$

$b = 0$

$c = 1$

$d = 1$

$f(u_4) = (0, 0, 5, 6)_{Bc}$

Pasamos a B_3

$(0,0,5,6) = (\lambda_1+\lambda_2,\ \lambda_1-\lambda_2+\lambda_3,\ \lambda_3+\lambda_4,\ \lambda_4)$

$0 = \lambda_1+\lambda_2$; $0 = \lambda_1+\lambda_1+1$; $\boxed{\lambda_1 = \frac{-1}{2}}$

$0 = \lambda_1-\lambda_2+\lambda_3$; $0 = \lambda_1-\lambda_2+1$; $\lambda_2 = \lambda_1+1$; $\boxed{\lambda_2 = \frac{1}{2}}$

$5 = \lambda_3+\lambda_4$; $\lambda_3 = 5-4$; $\boxed{\lambda_3 = +1}$

$\boxed{6 = \lambda_4}$

$$\boxed{f(u_3) = \left(\frac{-1}{2}, \frac{1}{2}, 1, 6\right)}$$

$$\boxed{M_{B_3B_3} = \begin{pmatrix} \frac{7}{2} & \frac{-1}{2} & \frac{1}{2} & \frac{-1}{2} \\ \frac{-1}{2} & \frac{7}{2} & \frac{-1}{2} & \frac{1}{2} \\ 0 & 0 & 5 & 1 \\ 0 & 0 & 0 & 6 \end{pmatrix}}$$