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A determination of the velocity of light using the twin superheterodyne principle

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ABSTRACT

Most of the newer electro-optical determinations of the velocity of light have been based on the modulation principle of Bergstrand. The author developed in 1952–54 the "Twin Superheterodyne Principle" for electro-optical distance measuring. This modulation principle is now incorporated in the AGA instruments and the author has used a Geodimeter 8 for a new determination of the velocity of light. The present study has been made in close cooperation with the Finnish Geodetic Institute which made its base line Niinisalo-Pihnari (22 219 848.3 $\pm$ 1.78 mm) available for the author. The final result from 2 weeks of observations was $c = 299\,792\,375 \pm 60$ m/sec. The final statistical analysis has been made with the use of Wiener-Hopf filtering technique. Hypothesis testing according to Fisher and Hart is included.

The French scientist Fizeau is famous for his invention of the cog-wheel as a device for modulating light and he was the first to make an adequate determination of the velocity of light with the use of terrestrial observations (1849). There is no technical difference between a distance measurement with modulated light and a determination of the velocity of light, and therefore we could consider Fizeau as a premature inventor of the device for distance measurement with modulated light. In Fizeau's device a cog-wheel interrupted a light bundle in such a way that the outgoing light was modulated with the wanted frequency. The light was reflected at a distant mirror and then forced to pass a second time through the cog-wheel. With this device Fizeau could verify that the incoming light was blocked by the cogs for selected frequencies. Thus it was proved that light has a finite velocity. Fizeau also gave the mechanical foundations for the first generation of light modulated distance measuring instruments all of which have used the single cog-wheel principle.

The geodetic application of modulated light measuring methods was first suggested by Colonel E. Lester Jones of the Coast and Geodetic Survey. This organization made the base measurements between Mount Wilson and San Antonio Peak, to be used for the determination of the velocity of light by Michelson and his collaborators. Mr W. Bowie was in charge of the classical base measurement and he wrote: "The director, realizing that the determination of the velocity of light, with great accuracy, might lead to the determination of distance in terms of light and then might furnish a means of measuring base lines in mountain regions or archipelagoes ..." However, the method of Michelson was never directly used for distance measuring.

Karolus and Mittelstaedt gave the basic principle of electro-optical light modulation with Kerr-cells and also showed that this new technique was useful for very short distances (cf. Mittelstaedt, 1929).

The legal inventor of the electro-optical distance measuring device was a member of the RCA staff, Irving Wolff. He designed an instrument that was described in detail in a patent application of 1939. In his first claim he writes:

"A distance measuring device including in combination, a light source, means for propagating light from said source toward a light reflecting object whose distance is to be measured, means coupled to said light source for modulating the outgoing, a receiver for receiving modulated light reflected from said object, a phase-meter for indicating said distance as a function of the relation between phases of the modulating means." Wolff introduced the technique of using electrical phase-measuring in an electro-optical distance measuring system.

Wolff also seems to be the first to use the photo-tube as a receiving system for this type of distance measuring. A careful study of the Wolff method indicates that it included a conversion of the "mechanical cog-wheel principle" to a modified "electronic cog-wheel principle".

New applications of the "simple cog-wheel principle" were given by Andersson (1937, 1940) and Hüttel (1940). Bergstrand (1950) introduced a modified technique for modulating the Kerr-cell. The high frequency signal was superimposed on a low frequency square wave in order to give better defined measuring points. Bergstrand (1942) originally outlined a method with three airborne signals (two optical and one electrical) and additive mixing of the two optical signals on each of the phototubes. A similar system was later used in the Tellurometer.

In this prototype Bergstrand varied the modulation frequency until the distance was an integral number of a quarter of the wavelength. The AGA company manufactured the first commercial instrument using fixed measuring frequencies. An optical delayer was used for checking a high frequency electrical delayer. (Patent of C. E. Granqvist.) The optical delayer in these instruments had a weight of approximately 10 kg. Further successful instruments by AGA were Geodimeters IV and VI. These last designs used exclusively high frequency electrical delayers. However, it was necessary to use a rather cumbersome individual calibration for each measuring frequency to overcome the large systematic errors of the delayers. The system was somewhat restricted because it made use of the squaring power of a Kerr-cell. The Geodimeter VI was the last instrument of the first generation of electro-optical distance measuring instruments. For this generation of instruments the basic principle can be traced back to the famous cog-wheel of Fizeau. It was a "straight" type of instrument with the phase measurement performed at the high frequency signal.

Besides the Bergstrand approach to distance measuring, an independent study had already started in 1940 at the Institute of Geodesy, KTH, Stockholm, with the original aim of using a purely electronic measuring system in combination with frequency transposition of the measuring signals. When the first successful electro-optical results of Bergstrand (1942) were announced we halted these studies of a purely electronic device. A pulse operated electro-optical system for surveying application was built 1950–1952 and the system for measurements with continuous waves from the early 1940 study was modified to an electro-optical system in 1952–1954. The basic problem was to find a practical method of converting the high frequency measuring signals to useful new signals of a low frequency. In the solution a modulated light signal was emitted from the instrument and after due reflection received in the same instrument where it was mixed in a multiplicative way with an auxiliary electrical signal of slightly different frequency. The primary modulating electrical signal was mixed in a similar way with the auxiliary signal and we obtained two low frequency signals. In this way we obtained a system of the double cog-wheel type. The auxiliary signal served like a second cog-wheel for the incoming signals and we got an output that varied in amplitude according to the beat frequency. This system works like an "electro-optical microscope" when reading the phase differences between the outgoing and incoming signals. The phase difference between the outgoing and incoming optical signals are converted to a corresponding phase difference at the actual low frequency. However, the time delay is multiplied by a number determined from the ratio between the high frequency and the low frequency. In this approach a type of twin superheterodyne principle was used which has a direct mechanical analogy in the double cog-wheel. Several electronic experts were rather sceptical concerning the possibility of maintaining the phase stability in the system. Not even the 1970 Nobel Prize winner H. Alfvén could promise that the system should work. However, using this method we finally could diminish some of the electronic internal errors in a drastic way. The new method made it possible to measure the actual phase difference between incoming and outgoing light in an indirect way by the low frequency as well as directly by the high frequency signals. The low frequency electronic phase measurement was found to be many times more accurate than the corresponding high frequency phase measurement and the transposition to the low frequency made it possible to filter out some of the noise from the daylight by simple integration methods (cf. Bjerhammar, 1954).[1] The experimental type of this instrument was called 'Terrameter'' and a number of slightly different instruments were built:

1952. Prototype (Grants from the Swedish Technical Research Council)
1953. AGA phase gonimeter for low frequencies
1953. Light modulating solid crystals
1954. Four electrode Kerr-cell
1956. Directly modulated zirkonium lamp
1959. Four phase capacitor for phase measurements
1960. Servo operated phase measuring system
1964. Gallium arsenide diode modulator
1966. Laser

Using the terminology from electronic science we can claim that this double cog-wheel principle gives a type of electro-optical twin superheterodyne method. In 1967 AGA started the production of instruments of this type marked "Geodimeter 6A/Modulation System Bjerhammar". Later followed the experimental instrument Geodimeter 7 and the laser instrument Geodimeter 8. In March this year the instrument Geodimeter 6B was introduced. The twin superheterodyne principle seems now to be used in all electro-optical distance measuring instruments on the market with the exception of the Mekometer.

In this report the results from a determination of the velocity of light with the AGA Geodimeter 8 will be given.

Historical notes

The early terrestrial determinations of the velocity of light were made by the French scientist Fizeau 1849 with the use of a rotating cog-wheel. Michelson devoted almost 40 years of his life for an accurate determination. His values are given below together with some later results.

Velocity of light in vacuum

Rotating mirror		
Michelson	1924	299 802 ± 30 km/sec
Michelson	1926	299 796 ± 4
Michelson	1935	299 774 ± 11
Electro-optical method		
Bergstrand	1949	299 796 ± 2
Bergstrand	1950	299 793 ± 0.26
Bergstrand	1950	299 793 ± 0.4
Cavity resonator		
Essen	1950	299 792.5 ± 1
Radio interferometry		
Froome	1951	299 792.6 ± 1
Froome	1951	299 792.5 ± 0.1
Froome	1954	299 793.0 ± 0.3
Simkin	1967	299 792.56 ± 0.11
Other studies (electro-optical)		
Schöldström	1955	299 792.4 ± 0.4
Edge	1956	299 792.2 ± 0.13[1]
Edge	1956	299 792.4 ± 0.11[1]
Karolus	1966	299 792.1 ± 0.2[2]
Karolus	1967	299 792.44 ± 0.2

[1] Only standard deviations
[2] Cancelled 1967

If we disregard the determinations of Michelson then we find that there is a systematic trend in the results. All determinations of Bergstrand give values above 299 793. The radio interferometric methods give rather consistent values around 299 792.5 and newer electro-optical determinations have all given values below 299 792.5 which is the present value accepted by IAG.

The calibration base line

The calibration base line has been built by the Finnish Geodetic Institute in collaboration with the Survey and Ranging Battalion (Mittaus pattery) in Niinisalo Finland. A complete description is given in a paper by Aimo Kiviniemi (1970) and we refer to this publication for further details. The base runs from Niinisalo ($\varphi = 61°51'.8$ N, $\lambda = 22°28'.5$ E and $h = 135.948$ m) to Pihnari ($\varphi = 61°57'.3$ N, $\lambda = 22°51'.0$ E and $h = 156.484$ m). The base line system includes five observation towers with heights above the

[1] Bjerhammar: Patent 161 172, 1954/1957, Sweden. Swiss Patent 333 562. Calim 1. Verfahren zur Entfernungsmessung mittels modulierten Licht, bei dem die Phase einer elektrischen Vergleichschwingung und der reflektierten Schwingung verglichen wird, daduch gekennzeichnet, dass die beiden zu vergleichenden Schwingungen durch Mischung mit einer Hilfsschwingung in ihrer Frequenz transportiert werden.

ground from 15.6 m to 41.5 m. The observation towers above the end marks are well centered and allow observations with very small excentricities. In Naurisjoki 4.5 km from Niinisalo is a break point in the base line which is properly marked. Observation towers in Peurala and Luojos make it possible to obtain meteorological observations approximately in the line of sight. The observation tower in Naurisjoki was used for meteorological and electro-optical observations. The length of the base line has been determined with the use of invar tapes which have been calibrated against an interferometric base line (Nummela). The following slope distances referred to the line of sight are given by Kiviniemi:

	D	H
Niinisalo–Pihnari	22 219 848.3 ± 1.78 mm	127.209 m
Niinisalo–Naurisjoki	4 510 704.0 ± 0.74 mm	126.234 m
Naurisjoki–Pihnari	17 709 176.3 ± 1.46 mm	127.457 m

D = distance, H = measuring elevation

Kiviniemi makes the following conclusion: "The observations at the Nummela standard base line and at the Niinisalo calibration base line are made symmetrically and directly above the used markers, and so they refer to the actual lengths of these base lines without any major projection errors. Further, the observations at the Nummela and Niinisalo are carried out with the same equipment by the same personnel and with the identical method and procedures. On the basis of these facts the total standard error can be considered a real error without significant systematic error."

All instruments were daily calibrated against the standards at the Niinisalo Meteorological station or at our own station.

Meteorological observations at the intermediate towers were made by personnel from Mittaus patteristo in Niinisalo and from the Finnish Geodetic Institute. All other observations were made by personnel from the Royal Institute of Technology Stockholm (KTH).

Eccentricities

The towers in Niinisalo, Naurisjoki and Pihnari were all very rigid and suitable for precise measurements. There was originally no direct line of sight from the upper platform to the marked point on the ground. In order to facilitate the centering we carved a hole through the wooden framework and only very small eccentricities were discovered. The maximum eccentricity was less than 5 mm which seems to prove that the towers have kept their position for a long time. Our determination of the eccentricities had a standard error of ±0.8 mm in Pihnari and ±0.5 mm in Naurisjoki. The error in Niinisalo does not contribute to the final errors of the last week.

Distance measurements

All distance measurements were made by a Geodimeter 8 that had been equipped with an installation for automatic measuring and printing. This modification was made by Mr Arne Forsberg of KTH. In this way approximately 200 measurements were made every day. However, the arithmetical mean of each set of 10 observations was recorded as a simple observation.

Table 1. *Distance measuring* (*Geodimeter mod. 8*)
Geodim. const. 0.226, refl. const. −0.030

Time	Temperature			Water	Ref.	Refractive beam		Rest		
(Reflector)	Dry	Wet	Press	vap.pr	coff.	Index	Curv.	length	Distance	Velocity
16.06.1970										
Excentr. = −0.0022,										
Stations weight for temp. and press 0.242, 0.0, 0.377, 0.258, 0.123										
10.04–10.08	17.0	9.6	748.3	5.3	0.14	1.0002780	−0.0029	1.892	22 002.566	299 792.200
10.12–10.16	17.0	9.3	748.3	5.0	0.14	1.0002780	−0.0029	1.880	22 002.555	299 792.353

Table 1 (*cont.*)

Time (Reflector)	Temperature Dry	Wet	Press		Ref. coff.	Refractive beam Index	Curv.	Rest length	Distance	Velocity
10.50–10.54	17.5	9.6	748.5	5.1	0.14	1.0002776	−0.0029	1.887	22 002.571	299 792.136
11.11–11.15	17.7	9.8	748.6	5.2	0.14	1.0002775	−0.0029	1.861	22 002.548	299 792.451
11.20–11.24	17.4	9.6	748.3	5.2	0.14	1.0002777	−0.0029	1.873	22 002.555	299 792.350
11.31–11.35	17.5	9.5	748.4	4.9	0.14	1.0002776	−0.0029	1.910	22 002.594	299 791.822
14.08–14.12	18.4	9.5	748.4	4.4	0.14	1.0002767	−0.0029	1.877	22 002.579	299 792.017
14.16–14.20	18.5	9.7	748.4	4.6	0.13	1.0002767	−0.0027	1.837	22 002.541	299 792.540
14.25–14.29	18.5	9.3	748.4	4.2	0.12	1.0002767	−0.0026	1.851	22 002.553	299 792.371
14.43–14.47	18.6	9.9	748.4	4.8	0.12	1.0002766	−0.0025	1.827	22 002.534	299 792.641
14.52–14.56	18.6	9.8	748.3	4.7	0.12	1.0002765	−0.0025	1.849	22 002.557	299 792.319
15.03–15.07	18.9	10.1	748.5	4.9	0.12	1.0002763	−0.0026	1.853	22 002.566	299 792.199
15.14–15.18	18.6	10.1	748.4	5.1	0.11	1.0002766	−0.0024	1.858	22 002.565	299 792.219
16.25–16.29	19.0	10.2	748.4	5.0	0.13	1.0002761	−0.0027	1.845	22 002.561	299 792.269
16.34–16.38	18.8	10.4	748.6	5.3	0.14	1.0002764	−0.0028	1.828	22 002.538	299 792.579
16.44–16.48	19.0	10.5	748.4	5.3	0.13	1.0002761	−0.0027	1.822	22 002.538	299 792.576
Arith. mean	18.19	9.81	748.40	4.94	0.130	1.00027694	−0.00272	1.8593	22 002.5576	299 792.3152
Stand. dev.	0.72	0.37	0.09	0.31	0.009	0.00000067	0.00017	0.0255	0.0161	0.2192
M. suc. dif.	0.25	0.18	0.09	0.19	0.004	0.00000022	0.00009	0.0151	0.0140	0.1905
							RMS	Error =	0.0040	0.0548
17.06.1970										
9.17– 9.22	18.2	10.3	749.3	5.5	0.10	1.0002772	−0.0021	1.854	22 002.547	299 792.456
9.33– 9.37	18.4	10.5	749.0	5.6	0.10	1.0002770	−0.0020	1.841	22 002.540	299 792.557
9.43– 9.46	18.6	10.6	749.1	5.6	0.11	1.0002768	−0.0023	1.830	22 002.533	299 792.654
9.51– 9.54	18.6	10.5	749.0	5.5	0.11	1.0002767	−0.0024	1.854	22 002.558	299 792.310
10.00–10.02	18.8	10.6	749.2	5.5	0.11	1.0002767	−0.0023	1.844	22 002.549	299 792.431
10.08–10.12	18.8	10.6	749.1	5.5	0.12	1.0002766	−0.0026	1.837	22 002.543	299 792.518
10.17–10.21	19.0	10.6	749.1	5.4	0.11	1.0002764	−0.0024	1.837	22 002.548	299 792.444
10.27–10.30	19.3	10.6	749.1	5.3	0.09	1.0002761	−0.0019	1.829	22 002.546	299 792.466
10.37–10.40	19.2	10.4	748.8	5.1	0.10	1.0002761	−0.0021	1.829	22 002.545	299 792.485
10.46–10.50	19.3	10.5	748.8	5.1	0.10	1.0002760	−0.0022	1.846	22 002.564	299 792.222
10.58–11.03	19.3	10.6	748.7	5.3	0.12	1.0002760	−0.0025	1.844	22 002.563	299 792.239
11.10–11.14	19.4	10.6	748.6	5.2	0.11	1.0002758	−0.0024	1.836	22 002.559	299 792.297
11.21–11.24	19.7	10.7	748.6	5.3	0.12	1.0002756	−0.0024	1.811	22 002.539	299 792.569
11.26–11.35	19.6	10.7	748.5	5.2	0.11	1.0002756	−0.0023	1.817	22 002.545	299 792.486
Arith. mean	19.01	10.56	748.92	5.37	0.108	1.00027633	−0.00229	1.8364	22 002.5485	299 792.4381
Stand. dev.	0.46	0.11	0.25	0.17	0.009	0.00000050	0.00019	0.0126	0.0094	0.1281
M. suc. dif.	0.11	0.08	0.10	0.07	0.008	0.00000011	0.00016	0.0091	0.0082	0.1112
							RMS	Error =	0.0025	0.0342
17.06.1970										
13.47–13.49	20.2	10.7	748.2	5.0	0.10	1.0002750	−0.0021	1.816	22 002.558	299 792.303
13.56–14.00	20.3	10.9	748.3	5.1	0.10	1.0002749	−0.0021	1.803	22 002.546	299 792.467
14.20–14.23	20.4	10.8	748.2	5.0	0.11	1.0002748	−0.0022	1.811	22 002.558	299 792.313
14.30–14.33	20.2	10.7	748.3	4.9	0.10	1.0002750	−0.0021	1.813	22 002.555	299 792.344
14.50–14.53	20.0	10.4	748.2	4.7	0.11	1.0002752	−0.0024	1.821	22 002.558	299 792.309
15.00–15.02	19.8	10.3	748.2	4.7	0.11	1.0002753	−0.0024	1.819	22 002.553	299 792.374
15.15–15.18	19.9	10.3	748.2	4.7	0.10	1.0002753	−0.0021	1.811	22 002.546	299 792.470
15.23–15.27	19.7	10.2	748.2	4.6	0.09	1.0002755	−0.0020	1.818	22 002.549	299 792.432
15.45–15.48	19.5	10.2	748.2	4.8	0.13	1.0002757	−0.0028	1.832	22 002.558	299 792.313
15.53–15.57	19.4	10.2	748.2	4.8	0.13	1.0002757	−0.0027	1.836	22 002.562	299 792.251
16.01–16.05	19.3	10.1	748.1	4.7	0.12	1.0002758	−0.0025	1.838	22 002.561	299 792.267
16.09–16.13	19.2	10.0	748.1	4.7	0.12	1.0002759	−0.0025	1.858	22 002.579	299 792.018
Arith. mean	19.82	10.42	748.20	4.81	0.110	1.00027533	−0.00233	1.8230	22 002.5571	299 792.3217
Stand. dev.	0.41	0.29	0.04	0.15	0.012	0.00000038	0.00024	0.0153	0.0088	0.1203
M. suc. dif.	0.12	0.09	0.03	0.07	0.010	0.00000011	0.00019	0.0069	0.0060	0.0818
							RMS error =		0.0025	0.0347

Table 1 (*cont.*)

Time (Reflector)	Temperature Dry	Wet	Press		Ref. coff.	Refractive beam Index	Curv.	Rest length	Distance	Velocity
18.06.1970										
9.00– 9.03	18.1	9.4	749.3	4.5	0.11	1.0002773	−0.0024	2.303	22 010.493	299 792.230
9.07– 9.10	18.2	9.4	749.3	4.5	0.11	1.0002773	−0.0024	2.294	22 010.485	299 792.337
9.44– 9.50	18.6	9.5	749.1	4.4	0.12	1.0002768	−0.0024	2.270	22 010.471	299 792.525
9.56–10.00	18.8	9.5	749.0	4.3	0.13	1.0002766	−0.0027	2.290	22 010.495	299 792.197
10.05–10.07	19.0	9.6	749.0	4.3	0.12	1.0002764	−0.0026	2.266	22 010.476	299 792.464
10.16–10.19	19.0	9.7	749.1	4.4	0.12	1.0002765	−0.0025	2.273	22 010.482	299 792.382
10.25–10.29	19.1	9.5	749.1	4.2	0.11	1.0002764	−0.0024	2.276	22 010.487	299 792.305
10.34–10.38	19.0	9.9	749.1	4.6	0.11	1.0002764	−0.0023	2 272	22 010.482	299 792.374
10.45–10.48	19.2	9.7	749.1	4.3	0.11	1.0002762	−0.0023	2.270	22 010.485	299 792.342
10.53–10.57	19.3	9.9	749.1	4.4	0.13	1.0002761	−0.0028	2.269	22 010.485	299 792.330
11.02–11.06	19.5	9.8	749.1	4.3	0.18	1.0002759	−0.0036	2.272	22 010.492	299 792.246
Arith. mean	18.90	9.63	749.12	4.39	0.123	1.00027656	−0.00258	2.2777	22 010.4847	299 792.3393
Stand. dev.	0.44	0.17	0.11	0.12	0.020	0.00000044	0.00038	0.0121	0.0072	0.0975
M. suc. dif.	0.14	0.11	0.06	0.13	0.012	0.00000014	0.00023	0.0092	0.0082	0.1119
							RMS error =		0.0022	0.0294
18.06.1970										
14.08–14.11	21.2	11.0	748.4	4.8	0.09	1.0002741	−0.0020	2.224	22 010.485	299 792.334
14.16–14.19	21.4	11.0	748.3	4.7	0.10	1.0002739	−0.0020	2.215	22 010.480	299 792.404
14.25–14.27	21.4	11.1	748.3	4.8	0.10	1.0002739	−0.0021	2.229	22 010.496	299 792.190
14.32–14.36	21.1	11.0	748.3	4.9	0.09	1.0002742	−0.0020	2.227	22 010.488	299 792.299
14.48–14.51	21.2	11.0	748.0	4.8	0.08	1.0002739	−0.0018	2.219	22 010.485	299 792.338
14.55–14.59	21.1	11.0	748.0	4.8	0.08	1.0002741	−0.0017	2.225	22 010.488	299 792.290
15.14–15.18	21.3	10.9	747.9	4.7	0.08	1.0002738	−0.0018	2.231	22 010.499	299 792.148
15.23–15.25	21.3	11.0	747.9	4.7	0.09	1.0002739	−0.0020	2.221	22 010.488	299 792.300
15.31–15.34	21.2	11.1	747.9	4.9	0.10	1.0002739	−0.0022	2.228	22 010.494	299 792.208
15.39–15.43	21.1	10.8	747.8	4.6	0.10	1.0002740	−0.0021	2.218	22 010.482	299 792.372
15.50–15.55	21.2	10.8	747.7	4.5	0.10	1.0002739	−0.0021	2.205	22 010.472	299 792.517
15.59–16.02	21.3	10.6	747.7	4.3	0.08	1.0002738	−0.0018	2.242	22 010.509	299 792.008
Arith. mean	21.24	10.95	748.02	4.72	0.092	1.00027394	−0.00197	2.2236	22 010.4888	299 792.2839
Stand. dev.	0.10	0.15	0.25	0.16	0.008	0.00000011	0.00016	0.0092	0.0097	0.1321
M. suc. dif.	0.11	0.09	0.08	0.11	0.006	0.00000011	0.00011	0.0099	0.0102	0.1391
							RMS error =		0.0028	0.0381
21.06.1970										
19.53–19.56	16.0	7.9	748.1	3.9	0.17	1.0002789	−0.0035	1.133	17 709.158	299 792.266
20.03–20.05	15.9	7.8	747.8	4.0	0.16	1.0002790	−0.0032	1.135	17 709.159	299 792.244
20.10–20.14	15.7	7.6	747.9	3.9	0.16	1.0002792	−0.0033	1.132	17 709.153	299 792.359
20.19–20.22	15.5	7.6	747.9	3.9	0.16	1.0002793	−0.0033	1.146	17 709.164	299 792.167
20.50–20.53	14.7	7.2	748.0	3.9	0.17	1.0002802	−0.0035	1.148	17 709.151	299 792.393
20.57–21.00	14.5	7.1	748.1	3.9	0.17	1.0002804	−0.0035	1.150	17 709.149	299 792.420
21.04–21.07	14.3	7.0	748.1	3.9	0.18	1.0002806	−0.0037	1.137	17 709.131	299 792.721
21.10–21.15	14.0	6.9	748.2	4.0	0.19	1.0002810	−0.0039	1.131	17 709.120	299 792.918
21.18–21.21	13.7	6.8	748.2	4.0	0.22	1.0002812	−0.0044	1.161	17 709.145	299 792.484
21.26–21.31	13.5	6.7	748.2	4.0	0.23	1.0002814	−0.0045	1.163	17 709.143	299 792.526
21.34–21.38	13.3	6.6	748.2	4.0	0.23	1.0002816	−0.0046	1.163	17 709.139	299 792.596
21.41–21.45	13.2	6.3	748.2	3.8	0.26	1.0002818	−0.0050	1.166	17 709.139	299 792.592
Arith. mean	14.53	7.15	748.06	3.95	0.192	1.00028039	−0.00386	1.1471	17 709.1459	299 792.4737
Stand. dev.	1.04	0.50	0.14	0.08	0.035	0.00000106	0.00062	0.0135	0.0126	0.2138
M. suc. dif.	0.23	0.13	0.07	0.07	0.009	0.00000023	0.00016	0.0078	0.0082	0.1394
							RMS error =		0.0036	0.0617
22.06.1970										
7.15– 7.20	12.5	7.7	749.8	5.5	0.12	1.0002830	−0.0026	1.195	17 709.149	299 792.414
7.33– 7.36	12.7	7.8	749.9	5.5	0.12	1.0002828	−0.0024	1.209	17 709.167	299 792.121
7.50– 7.52	13.0	7.8	749.9	5.4	0.11	1.0002825	−0.0024	1.172	17 709.136	299 792.648

Table 1 (*cont.*)

Time (Reflector)	Temperature Dry	Wet	Press		Ref. colt.	Refractive beam Index	Curv.	Rest length	Distance	Velocity
7.55– 8.00	13.4	8.2	749.9	5.6	0.11	1.0002821	−0.0023	1.173	17 709.142	299 792.534
8.06– 8.12	13.5	7.4	749.9	4.7	0.11	1.0002821	−0.0022	1.163	17 709.133	299 792.687
Arith. mean	13.02	7.77	749.91	5.31	0.113	1.00028249	−0.00238	1.1823	17 709.1454	299 792.4808
Stand. dev.	0.41	0.31	0.04	0.37	0.007	0.00000039	0.00013	0.0189	0.0135	0.2278
M. suc. dif.	0.18	0.34	0.04	0.35	0.003	0.00000017	0.00007	0.0143	0.0132	0.2238
							RMS error		0.0060	0.1019

23.06.1970

Excentr. = 0.0022,
Stations weight for temp. and press 0.0, 0.175, 0.347, 0.325, 0.153

17.58–18.02	20.8	11.0	749.7	5.0	0.13	1.0002749	−0.0026	2.247	17 709.146	299 792.471
18.06–18.10	20.8	11.1	749.7	5.1	0.13	1.0002750	−0.0028	2.246	17 709.144	299 792.502
18.15–18.18	20.7	11.1	749.7	5.1	0.13	1.0002751	−0.0027	2.248	17 709.145	299 792.494
18.43–18.46	20.4	11.0	749.7	5.2	0.13	1.0002753	−0.0027	2.245	17 709.137	299 792.616
19.21–19.24	19.9	10.6	749.7	5.0	0.12	1.0002758	−0.0025	2.259	17 709.143	299 792.523
19.31–19.34	19.7	10.8	749.8	5.3	0.12	1.0002760	−0.0024	2.262	17 709.142	299 792.534
19.40–19.43	19.6	10.4	749.8	4.9	0.13	1.0002761	−0.0028	2.276	17 709.153	299 792.347
19.49–19.52	19.3	10.3	749.7	4.9	0.13	1.0002763	−0.0027	2.288	17 709.163	299 792.183
19.57–19.59	19.2	10.2	749.7	4.8	0.13	1.0002764	−0.0027	2.273	17 709.146	299 792.479
20.09–20.13	18.9	10.1	749.8	4.9	0.14	1.0002767	−0.0028	2.285	17 709.152	299 792.363
20.21–20.24	18.7	9.6	749.9	4.5	0.14	1.0002770	−0.0030	2.290	17 709.152	299 792.375
20.36–20.38	18.3	9.8	749.9	4.9	0.17	1.0002774	−0.0035	2.300	17 709.155	299 792.326
20.45–20.47	18.1	9.6	750.0	4.8	0.17	1.0002776	−0.0034	2.303	17 709.153	299 792.346
21.00–21.04	17.3	9.4	750.0	5.0	0.19	1.0002784	−0.0038	2.311	17 709.148	299 792.439
21.09–21.13	17.0	9.3	750.0	5.0	0.19	1.0002787	−0.0038	2.326	17 709.157	299 792.283
21.18–21.21	16.6	9.4	750.1	5.3	0.19	1.0002790	−0.0038	2.320	17 709.145	299 792.492
21.25–21.29	16.3	9.1	750.0	5.1	0.20	1.0002793	−0.0039	2.331	17 709.151	299 792.392
21.32–21.35	16.0	8.9	750.1	5.0	0.20	1.0002796	−0.0040	2.327	17 709.142	299 792.535
21.39–21.43	15.7	8.9	750.1	5.2	0.20	1.0002799	−0.0041	2.341	17 709.150	299 792.400
Arith. mean	18.60	10.03	749.86	4.99	0.154	1.00027709	−0.00316	2.2883	17 709.1486	299 792.4264
Stand. dev.	1.68	0.76	0.16	0.19	0.032	0.00000165	0.00060	0.0319	0.0062	0.1053
M. suc. dif.	0.23	0.16	0.04	0.16	0.008	0.00000023	0.00015	0.0069	0.0055	0.0937
							RMS error =		0.0014	0.0242

24.06.1970

8.05– 8.10	17.8	10.2	751.4	5.5	0.13	1.0002784	−0.0027	2.317	17 709.155	299 792.318
8.15– 8.21	18.0	9.8	751.4	5.1	0.13	1.0002782	−0.0027	2.327	17 709.168	299 792.105
8.25– 8.30	18.0	9.8	751.4	5.1	0.13	1.0002782	−0.0027	2.302	17 709.142	299 792.538
8.35– 8.40	18.1	9.9	751.5	5.0	0.15	1.0002781	−0.0032	2.308	17 709.150	299 792.404
8.45– 8.49	18.3	9.8	751.5	4.8	0.12	1.0002780	−0.0026	2.289	17 709.133	299 792.687
8.54– 9.00	18.4	9.8	751.5	4.8	0.14	1.0002779	−0.0028	2.304	17 709.150	299 792.396
9.05– 9.09	18.7	9.9	751.5	4.7	0.14	1.0002776	−0.0029	2.287	17 709.139	299 792.594
9.14– 9.19	18.9	10.1	751.5	4.8	0.12	1.0002774	−0.0024	2.286	17 709.142	299 792.537
9.25– 9.30	19.1	10.1	751.5	4.9	0.13	1.0002773	−0.0027	2.293	17 709.151	299 792.395
9.35– 9.39	19.2	10.1	751.5	4.7	0.13	1.0002771	−0.0026	2.280	17 709.140	299 792.567
9.45– 9.48	19.6	10.4	751.5	4.9	0.13	1.0002767	−0.0027	2.276	17 709.143	299 792.519
9.54– 9.58	19.6	10.5	751.4	5.0	0.10	1.0002767	−0.0021	2.265	17 709.133	299 792.688
Arith. mean	18.64	10.03	751.47	4.95	0.129	1.00027764	−0.00268	2.2943	17 709.1455	299 792.4789
Stand. dev.	0.64	0.25	0.04	0.21	0.013	0.00000060	0.00026	0.0178	0.0098	0.1655
M. suc. dif.	0.14	0.11	0.02	0.12	0.012	0.00000013	0.00024	0.0096	0.0092	0.1563
							RMS error		0.0028	0.0478

24.06.1970

19.16–19.20	20.4	11.4	750.4	5.6	0.13	1.0002756	−0.0027	2.263	17 709.151	299 792.392
19.25–19.31	20.3	11.3	750.4	5.6	0.14	1.0002757	−0.0029	2.259	17 709.145	299 792.492
19.42–19.46	20.2	11.3	750.4	5.6	0.14	1.0002757	−0.0029	2.247	17 709.131	299 792.717
19.50–19.58	20.0	11.3	750.4	5.6	0.14	1.0002759	−0.0029	2.273	17 709.155	299 792.325

Table 1 (*cont.*)

Time	Temperature				Ref.	Refractive beam		Rest		
(Reflector)	Dry	Wet	Press		colt.	Index	Curv.	length	Distance	Velocity
20.03–20.08	19.8	11.2	750.4	5.6	0.15	1.0002761	−0.0031	2.273	17 709.151	299 792.382
20.15–20.19	19.6	11.1	750.5	5.7	0.16	1.0002763	−0.0033	2.281	17 709.155	299 792.318
20.24–20.30	19.5	11.0	750.5	5.6	0.16	1.0002764	−0.0034	2.293	17 709.165	299 792.143
20.48–20.54	18.8	10.8	750.6	5.7	0.19	1.0002771	−0.0038	2.297	17 709.156	299 792.298
21.00–21.03	18.7	10.7	750.6	5.7	0.20	1.0002772	−0.0040	2.292	17 709.149	299 792.418
Arith. mean	19.71	11.12	750.47	5.66	0.156	1.00027622	−0.00321	2.2752	17 709.1509	299 792.3874
Stand. dev.	0.61	0.23	0.07	0.05	0.024	0.00000060	0.00045	0.0169	0.0093	0.1570
M. suc. dif.	0.21	0.08	0.02	0.03	0.008	0.00000020	0.00015	0.0083	0.0080	0.1349
							RMS error =		0.0031	0.0523
25.06.1970										
15.06–15.09	23.1	12.6	748.9	5.8	0.09	1.0002725	−0.0018	2.202	17 709.145	299 792.494
15.12–15.14	23.3	12.7	748.9	5.7	0.08	1.0002723	−0.0017	2.218	17 709.164	299 792.165
15.16–15.19	23.3	12.8	748.9	5.9	0.09	1.0002723	−0.0019	2.194	17 709.141	299 792.553
15.23–15.27	23.4	12.9	748.8	6.0	0.10	1.0002722	−0.0020	2.218	17 709.167	299 792.123
15.35–15.39	23.2	12.7	748.7	5.9	0.08	1.0002723	−0.0018	2.198	17 709.144	299 792.506
16.10–16.13	23.1	12.8	748.5	6.0	0.11	1.0002724	−0.0023	2.228	17 709.173	299 792.015
16.15–16.17	23.3	13.0	748.5	6.2	0.10	1.0002722	−0.0022	2.218	17 709.167	299 792.118
16.19–16.20	23.2	13.0	748.5	6.2	0.10	1.0002722	−0.0021	2.200	17 709.148	299 792.443
16.23–16.25	23.1	12.8	748.5	6.0	0.09	1.0002723	−0.0020	2.202	17 709.149	299 792.422
16.28–16.31	23.0	12.8	748.4	6.0	0.09	1.0002724	−0.0019	2.208	17 709.153	299 792.358
16.35–16.39	23.1	12.8	748.5	6.0	0.10	1.0002723	−0.0022	2.206	17 709.152	299 792.377
17.49–17.53	23.0	12.9	748.2	6.2	0.12	1.0002723	−0.0025	2.220	17 709.165	299 792.145
18.02–18.06	22.9	13.1	748.1	6.4	0.12	1.0002724	−0.0025	2.229	17 709.173	299 792.013
18.31–18.35	22.3	12.9	748.2	6.6	0.12	1.0002729	−0.0025	2.241	17 709.175	299 791.973
18.40–18.44	22.1	12.9	748.2	6.6	0.12	1.0002731	−0.0026	2.219	17 709.151	299 792.390
18.55–18.59	21.6	12.9	748.2	6.8	0.13	1.0002736	−0.0026	2.229	17 709.152	299 792.361
19.14–19.18	21.4	13.1	748.2	7.2	0.13	1.0002738	−0.0028	2.240	17 709.160	299 792.241
19.26–19.30	20.9	12.2	748.2	6.4	0.14	1.0002742	−0.0029	2.246	17 709.158	299 792.276
19.35–19.39	21.1	12.4	748.2	6.4	0.14	1.0002741	−0.0029	2.247	17 709.161	299 792.216
19.46–19.50	21.0	12.6	748.2	6.8	0.13	1.0002742	−0.0028	2.258	17 709.170	299 792.057
19.56–20.00	20.9	13.3	748.2	7.7	0.14	1.0002742	−0.0029	2.252	17 709.164	299 792.167
20.06–20.10	20.8	12.7	748.2	7.0	0.14	1.0002743	−0.0030	2.248	17 709.157	299 792.280
20.14–20.18	20.6	12.6	748.2	7.0	0.15	1.0002745	−0.0030	2.261	17 709.167	299 792.108
20.26–20.30	20.4	12.6	748.2	7.1	0.15	1.0002747	−0.0031	2.251	17 709.154	299 792.330
20.51–20.55	19.8	12.5	748.2	7.2	0.15	1.0002752	−0.0030	2.272	17 709.165	299 792.147
Arith. mean	22.14	12.78	748.41	6.44	0.116	1.00027316	−0.00244	2.2282	17 709.1590	299 792.2512
Stand. dev.	1.17	0.24	0.26	0.54	0.023	0.00000100	0.00045	0.0223	0.0090	0.1669
M. suc. dif.	0.18	0.20	0.05	0.23	0.006	0.00000017	0.00012	0.0104	0.0100	0.1698
							RMS error =		0.0020	0.0334
26.06.1970										
16.43–16.45	16.3	13.1	745.4	9.8	0.13	1.0002774	−0.0026	2.295	17 709.150	299 792.402
16.47–16.48	16.4	13.1	745.5	9.8	0.13	1.0002773	−0.0027	2.314	17 709.170	299 792.057
16.51–16.53	16.6	13.1	745.5	9.6	0.13	1.0002771	−0.0027	2.298	17 709.158	299 792.265
16.57–17.01	16.5	12.8	745.5	9.3	0.13	1.0002772	−0.0027	2.302	17 709.161	299 792.212
17.09–17.13	16.3	12.4	745.4	8.9	0.13	1.0002774	−0.0027	2.297	17 709.153	299 792.360
17.29–17.32	16.7	12.0	745.4	8.2	0.14	1.0002771	−0.0028	2.304	17 709.165	299 792.147
17.36–17.40	16.7	11.9	745.3	8.1	0.13	1.0002770	−0.0027	0.291	17 709.153	299 792.358
17.45–17.49	16.7	11.7	745.4	7.8	0.12	1.0002771	−0.0026	2.288	17 709.150	299 792.410
17.58–18.00	16.9	11.8	745.3	7.9	0.12	1.0002768	−0.0025	2.284	17 709.150	299 792.406
18.04–18.08	16.8	11.6	745.3	7.7	0.12	1.0002769	−0.0025	2.283	17 709.147	299 792.452
18.14–18.18	16.8	11.6	745.3	7.6	0.13	1.0002769	−0.0026	2.291	17 709.155	299 792.319
18.34–18.38	16.9	11.9	745.2	8.0	0.14	1.0002768	−0.0029	2.271	17 709.137	299 792.627
18.44–18.48	17.0	11.7	745.2	7.7	0.13	1.0002768	−0.0028	2.267	17 709.133	299 792.685
18.59–19.03	16.9	11.7	745.3	7.7	0.13	1.0002769	−0.0027	2.292	17 709.157	299 792.286
19.14–19.18	16.7	11.5	745.3	7.6	0.13	1.0002770	−0.0027	2.281	17 709.143	299 792.526

Table 1 (*cont.*)

Time (Reflector)	Temperature Dry	Wet	Press		Ref. colt.	Refractive beam Index	Curv.	Rest length	Distance	Velocity
19.29–19.33	16.7	11.6	745.3	7.7	0.13	1.0002770	−0.0028	2.306	17 709.168	299 792.099
19.44–19.48	16.6	11.6	745.3	7.8	0.13	1.0002771	−0.0028	2.291	17 709.151	299 792.380
19.54–19.58	16.5	11.5	745.3	7.8	0.13	1.0002773	−0.0028	2.300	17 709.158	299 792.270
Arith. mean	16.68	12.04	745.34	8.26	0.129	1.00027707	−0.00269	2.2919	17 709.1533	299 792.3479
Stand. dev.	0.21	0.60	0.08	0.79	0.005	0.00000020	0.00010	0.0119	0.0098	0.1659
M. suc. dif.	0.10	0.15	0.03	0.18	0.003	0.00000010	0.0006	0.0095	0.0095	0.1615
							RMS error =		0.0023	0.0391

Reflectors

The prism system in Pihnari included 24 prisms and each individual prism has been given its own correction for eccentricity. In Naurisjoki only three prisms were used. The eccentricities were also eliminated here.

Frequencies

The geodimeter was operated on a single measuring frequency and switching between different oscillators was avoided in order to keep the frequency constant. The oscillator was checked against the cesium standard at FOA before and after the observations. During the field observations we used the frequency of the Finnish Geodetic Institute.

Day	Frequency (Hz)
12.6.1970	$f_1 = 299\ 69992^a$
24.6.1970	$299\ 69987^b$
25.6.1970	$299\ 69987^b$
26.6.1970	$299\ 69986^b$
30.6.1970	$299\ 69985^a$

[a] Cesium standard
[b] Finnish standard in field

During the second week we only found a drift of 2 periods which corresponds to 1:15 000 000. We estimate the corresponding contribution to the standard error of the velocity of light.

$s_f = \pm 20$ m/sec

The following frequencies were used for the computations

Day	Frequency (Hz)
16.6.1970	299 69992
17.6.1970	299 69992
18.6.1970	299 69992
21.6.1970	299 69992
22.6.1970	299 69992
23.6.1970	299 69987
24.6.1970	299 69987
25.6.1970	299 69987
26.6.1970	299 69987

Meteorological observations

Dry temperatures, wet temperatures and pressures have been observed in all five observation towers. Vertical gradients have been determined in all stations at least for some days. The following instruments have been used:

Temperature: Assman, Friedrich (electrical with platinum electrodes)
Pressure: Baromec, Feuss

In the normal geodetic application of electro-optical distance measuring we can only expect to have meteorological data for the end points of the actual line, and a prediction of the intermediate data will be rather difficult. In our present determination of the velocity of light we are fortunate enough to have the very important high observation towers in the line of sight. Still we have to find a procedure that gives the best prediction of the integrated value of the meteorological parameters. In our solution we used a weakly stationary stochastic process for the prediction of the intermediate data from the available observations. Then we could compute the "mean values" from a numerical or analytical integration. For this study the distance between the observation towers had to replace the time as the basic parameter in our stochastic process. The covariances between the stations were first computed as a function of the distances and then the final solution was obtained under the condition of covariance stationarity. We refrain from giving any definite estimates of the prediction errors in this part of the study. However, we have found sets of observations where the root mean square difference between the temperature observations at two different towers was as low as $\pm 0\overset{\circ}{.}2$ for a longer period. The integrated mean for the whole base line should have a systematic error not higher than $\pm 0\overset{\circ}{.}1$. The accidental error can hardly exceed 1°. The accidental error should be eliminated (when using a large number of observations during several days). We estimate the various meteorological contributions to the standard error of the velocity of light:

s_t	$= \pm 30$ m/sec	temperature
s_h	$= \pm\ 3$ m/sec	humidity
s_p	$= \pm 10$ m/sec	pressure

The last day we obtained the following mean temperatures (Celsius)

Niinisalo	$16\overset{\circ}{.}78$	(Assman)
Luojos	$16\overset{\circ}{.}76$	(Friedrich)
Pihnari	$16\overset{\circ}{.}77$	(Assman)

The consistency was remarkable.

The measurements were made during rather stable weather conditions. There was a wind of 4–7 m/sec and no clouds during the first week. The second week was only cloudy the last day. This day we found an extreme homogeneity during the time 17.36–18.38. We conclude that the most favourable conditions were found a cloudy day with moderate wind in the "equilibrium" between day and afternoon. The refraction coefficient was 0.12–0.14 and the refractive index varied between 1.0002768 and 1.0002774. For this cloudy day we had a least squares standard deviation of only ± 0.00000020 for three hours of observations. The standard deviation of the refractive index estimated from the mean square successive differences was as small as ± 0.00000010. For the clear days the corresponding value was normally around ± 0.00000015. This means that the quality of the meteorological observations increased considerably during the cloudy period.

The accidental errors of the meteorological determinations should be well presented by the mean square successive differences. The following results were obtained for the last days.

	Day	Dry temp.	Wet temp.	Press (mm)	
	23.6.70	$\pm 0\overset{\circ}{.}23$	$\pm 0\overset{\circ}{.}16$	± 0.04	Clear
Morning	24.6.70	0.14	0.11	0.02	Clear
Afternoon	24.6.70	0.21	0.08	0.02	Clear
	25.6.70	0.18	0.20	0.05	Clear
	26.6.70	0.10	0.15	0.03	Cloudy

Thus we find the best values for "dry temperature" the cloudy day. For the pressure no significant differences are found.

If we study the final determination of velocities for different days then we find that the mean square successive difference (s_H) is almost unaffected of the weather conditions.

Refractive index

The refractive index varied from the maximum value 1.000 2830 at 07.15 22.6.1970 to the minimum value 1.000 2722 at 16.15 25.6.1970. If we use the simple model of one stochastic variable for the refractive index then we find very large differences of the standard deviation from the least squares estimate (s_0) and the mean square successive differences (s_H). For the observations 25.6 we have for example

$$s_0 = \pm 0.00000100 \quad n = 25$$

$$s_H = \pm 0.00000017$$

Thus we have the ratio

$$\frac{s_H^2}{s_0^2} = 0.029$$

The correspond ng value in a theoretical distribution according to Hart will be 0.709 for a risk level of 5 %. This means that this simple stochastic model is not under control and it will be natural to use a stochastic process for the presentation.

The stochastic processes

All the observed parameters dry temperature, wet temperature, pressure and vertical angles belong to stochastic processes. The time parameter is continuous but the outcomes are given as discrete sets. This type of stochastic process is normally called a "continuous parameter chain". These stochastic processes are normally weakly stationary when using the natural time as a parameter.

Measurements have only been made at the five observation towers and it is requested to make a prediction of meteorological data for all points along the line of sight. However, for our practical computation we only need the integrated value for the complete base line.

Refraction coefficient

The refraction coefficient was defined in the classical way from the relation between the radius of the earth at the actual point and the radius of the line of sight. Measurements were made with theodolites (Wild T2) at the ends of the base line. The refraction coefficient was rather constant during the day observations and varied mostly between 0.10 and 0.15. Observations from 04.52–07.45 in the morning 19.6.1970 indicated extreme variation of the refraction coefficient from 0.69 to 0.13. The systematic effect of this variation was clearly indicated by the ratio between the standard deviation from the least squares estimate and the mean square successive differences.

In order to get a reliable elimination of the errors from the photomultiplier, we used the Niinisalo-Naurisjoki basis as a reference against the Niinisalo-Pihnari basis for the last week of observations. During the first week ground reflectors at a distance of approximately 200 m from Niinisalo were used for the same purpose.

1. Mean square successive difference test

We are going to make a hypothesis testing of all observation sets. In this study we use weights proportional to the actual distances for all meteorological computations. The hypothesis testing is based on the variances estimated with the method of least squares and the mean square successive differences.

Null hypothesis:

$$\sigma_0^2 = \sigma_H^2$$

where

σ_0^2 = variance within a set

σ_H^2 = variance from mean square successive differences

Alternative hypothesis:

$$\sigma_0^2 > \sigma_H^2$$

The observations will be tested with the distribution of Hart.

Numerical results (standard deviations for a single observation):

Set	s_0 (m/sec)	s_H (m/sec)	$s_{\bar{x}}$ (m/sec)	n	H	H^*
16.6.1970 p.m., a.m.	±219	±191	± 55	16	0.761	0.614
17.6.1970 a.m.	128	111	34	14	0.752	0.591
17.6.1970 p.m.	120	82	35	12	0.467	0.564
18.6.1970 a.m.	98	112	29	11	1.306	0.548
18.6.1970 p.m.	132	139	38	12	1.109	0.564
21.6.1970 p.m.	214	139	62	12	0.422	0.564
22.6.1970 a.m.	288	224	102	5	0.965	0.410
23.6.1970 p.m.	105	94	24	19	0.801	0.642
24.6.1970 a.m.	166	156	48	12	0.883	0.564
24.6.1970 p.m.	157	135	52	9	0.739	0.512
25.6.1970 p.m.	167	170	33	25	1.036	0.677
26.6.1970 p.m.	166	162	39	18	0.952	0.633

Table 2. *Example of the computation of covariance matrices*

26.06.1970

Covariance: temperature dry

0.0674	0.0294	0.0288	0.0075	0.0171
0.0294	0.0376	0.0259	0.0210	0.0294
0.0288	0.0259	0.0882	0.0086	0.0082
0.0075	0.0210	0.0086	0.0896	0.1057
0.0171	0.0294	0.0082	0.1057	0.1835

Adjusted covariance: temperature dry

0.0920	0.0530	0.0203	0.0121	0.0247
0.0530	0.0920	0.0415	0.0161	0.0130
0.0203	0.0415	0.0920	0.0429	0.0181
0.0121	0.0161	0.0429	0.0920	0.0464
0.0247	0.0130	0.0181	0.0464	0.0920

Covariance: temperature wet

0.0508	0.0462	0.0738	0.0840	0.0840
0.0462	0.0877	0.1758	0.1380	0.1380
0.0738	0.1758	0.6558	0.4219	0.4219
0.0840	0.1380	0.4219	0.3909	0.3909
0.0840	0.1380	0.4219	0.3909	0.3909

Adjusted covariance: temperature wet

0.3100	0.2723	0.2109	0.1416	0.0697
0.2723	0.3100	0.2564	0.1946	0.1296
0.2109	0.2564	0.3100	0.2586	0.2031
0.1416	0.1946	0.2586	0.3100	0.2636
0.0697	0.1296	0.2031	0.2636	0.3100

Covariance: pressure

0.0038	0.0044	0.0034	0.0037	0.0036
0.0044	0.0111	0.0074	0.0068	0.0064
0.0034	0.0074	0.0075	0.0046	0.0042
0.0037	0.0068	0.0046	0.0059	0.0056
0.0036	0.0064	0.0042	0.0056	0.0058

Adjusted covariance: pressure

0.0067	0.0058	0.0050	0.0045	0.0044
0.0058	0.0067	0.0056	0.0048	0.0045
0.0050	0.0056	0.0067	0.0056	0.0049
0.0045	0.0048	0.0056	0.0067	0.0057
0.0044	0.0045	0.0049	0.0057	0.0067

Polynomial coefficients:

C(0) = 9.2035867219D-02 C(1) = −1.0093625611D-05 C(2) = 3.1780217084D-10

Inverted covariance matrix: temperature dry

17.3597	−10.0738	1.0336	0.9687	−3.9249
−10.0738	19.6188	−7.1530	0.7409	0.9691
1.0336	−7.1530	17.0919	−7.4038	1.1040
0.9687	0.7409	−7.4038	18.0997	−8.0263
−3.9249	0.9691	1.1040	−8.0263	15.6064

Inverted covariance matrix: temperature wet

15.8663	−14.6993	−0.3924	−0.3678	3.1482
−14.6993	24.7893	−10.3263	0.0540	−0.3400
−0.3924	−10.3263	21.2312	−10.7201	−0.3873
−0.3678	0.0540	−10.7201	22.6839	−12.2075
3.1482	−0.3400	−0.3873	−12.2075	13.2953

Inverted covariance matrix: pressure

655.4494	−522.3157	7.7702	7.2821	−92.3403
−522.3157	914.6934	−379.5902	2.9775	7.0694
7.7702	−379.5902	783.8118	−394.0337	8.0528
7.2821	2.9775	−394.0337	837.2123	−429.0612
−92.3403	7.0694	8.0528	−429.0612	564.1563

The following notations are introduced

s_0^2 = estimated variance within sets (least squares)

s_H^2 = estimated variance from mean square successive difference

$s_{\bar{x}}$ = standard deviation of the mean of the set (least squares)

H = s_H^2/s_0^2

H^* = theoretical value of H for a risk level of 5 %

n = number of observations of the set

We reject the null hypothesis at the risk level of 5 % for the days 17.6 p.m. and 21.6.

There seems to be a small systematic effect that is not fully compensated when data in only five points are used for the meteorological analysis. We note that the Hart ratio is excellent for the cloudy day (26.6). For this last day we have computed the arithmetic mean using two rather different weight combinations for the meteorological observations 10-1-1-1-10 and 1-10-10-10-1.

The difference between the two solutions was only 0.9 mm. This means that the solution is very well under control. It is obvious that our original weight relation for the analysis gives some systematic effects. In our final solution we have to choose "optimum weights" in order to obtain series that can be accepted by the Hart test.

2. Analysis of variances

We use the Fisher distribution for an analysis of the variances from all different sets.

Null hypothesis:

$$\sigma_i^2 = \sigma_m^2$$

where

$$\sigma_i^2 = \text{variance within sets}$$
$$\sigma_m^2 = \text{variance among sets}$$

Alternative hypothesis:

$$\sigma_i^2 < \sigma_m^2$$

First we make a preceeding test according to Barlett in order to verify that pooling is permitted. The pooled variance from the twelve sets gives the variance within sets

$$s_i^2 = \frac{15 \times 219^2 + 13 \times 128^2 + 11 \times 120^2 + 10 \times 98^2 + 11 \times 132^2 + 11 \times 214^2 + 4 \times 228^2 + 18 \times 105^2 + 11 \times 166^2 + 8 \times 157^2 + 24 \times 167^2 + 17 \times 166^2}{153}$$

Degrees of freedom = 153

$s_i^2 = 25\ 665\ (\text{m/sec})^2$

$s_i = \pm 160$ m/sec

The variance among sets (s_m^2) gave the result

$s_m^2 = 90\ 256\ (\text{m/sec})^2$

$s_m = \pm 300$ m/sec

We formed the variance ratio

$$F\frac{s_m^2}{s_i^2} = 3.52$$

The corresponding theoretical value for a Fisher distribution with 11 and 153 degrees of freedom and a risk level 5 % is finally = 1.85. Thus we have to reject our primary hypothesis of equal variances. We conclude that this stochastic process has an instantaneous noise (s_i) that can be estimated to

$s_i = \pm 160$ m/sec

For this stochastic process the standard deviation among sets (s_m) is furthermore

$s_m = \pm 300$ m/sec

All these estimates refer to a "single electro-optical observation". The standard deviation of the mean of all observations is finally

$s_{\bar{x}} = \pm 23$ m/sec (standard deviation among sets)

$s_{\bar{x}} = \pm 13$ m/sec (least squares standard deviation)

Summary of results

Day	Velocity (m/sec)	Number of observations
16.6.1970 a.m./p.m.	$c = 299\ 792\ 315 \pm 55$	16
17.6.70 a.m.	438 ± 34	14
17.6.70 p.m.	322 ± 35	12
18.6.70 a.m.	339 ± 29	11
18.6.70 p.m.	284 ± 38	12
21.6.1970 p.m.	474 ± 62	12
22.6.1970 a.m.	481 ± 102	5
23.6.1970 p.m.	426 ± 24	19
24.6.1970 a.m.	479 ± 48	12
24.6.1970 p.m.	387 ± 52	9
25.6.1970 p.m.	251 ± 33	25
26.6.1970 p.m.	348 ± 39	18
	$c = 299\ 792\ 364 \pm 23$	165

Final error estimate

We summarize the contributions from the various error sources using the propagation law of independent errors and obtain the final standard error (s)

$$s = \sqrt{s_f^2 + s_t^2 + s_h^2 + s_p^2 + s_r^2 + s_c^2 + s_g^2 + s_{\bar{x}}^2}$$

where $s_f = \pm 20$ m/sec (frequency), $s_t = \pm 30$ m/sec (temperature), $s_h = \pm\ 3$ m/sec (humidity), $s_p = \pm 10$ m/sec (pressure), $s_c = \pm 1$ m/sec (colour) $s_r = \pm 30$ m/sec (refractive formula), $s_g = \pm 30$ m/sec (geodetic contribution), $s_{\bar{x}} = \pm 23$ m/sec (stochastic standard deviation among sets).

Thus we have

$s \approx \pm 60$ m/sec

This error estimate includes the stochastical part as well as estimated uncompensated residual errors. We anticipate that observations a.m. and p.m. should have equal means and we balance the final mean in order to correct for the remaining systematic error.

For all observations we use the final estimation of the mean value of the velocity of light

299 792 375 $\pm$ 60 m/sec

It is of interest to note that the present international value 299 792.5 $\pm$ 0.5 has mainly been obtained from studies with cavity resonators and radio interferometry. If we take the last electro-optical determinations of the velocity of light we find

Schöldström	1955	299 792.4 $\pm$ 0.2
Edge	1956	299 792.4 $\pm$ 0.11 and 299 792.2 $\pm$ 0.13
Karolus	1967	299 792.44 $\pm$ 0.2
Bjerhammar	1971	299 792.375 $\pm$ 0.06

Our new determination was fulfilled with laser technology using a very long base of utmost accuracy. The electro-optical observations were carried out during two whole weeks. Together with the determinations of light made after 1954 we have a convincing proof that the international velocity of light[1] gives a systematic error. Therefore we recommend the following value of the velocity of light.

299 792.4 $\pm$ 0.1 km/sec (vacuum)

This study has here been fulfilled using the Wiener-Hopf approach for the determination of weights.

[1] In 1957, the International Union of Geodesy and Geophysics adopted the value 299 792.5 km/sec for velocity *in vacuo*.

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ОПРЕДЕЛЕНИЕ СКОРОСТИ СВЕТА С ПОМОЩЬЮ ПРИНЦИПА ДВОЙНОГО СУПЕРГЕТЕРОДИНИРОВАНИЯ

Большинство новых электрооптических способов определения скорости света основывается на принципе модуляции Бергштранда. В 1952–54 гг. автор разработал «принцип двойного супергетеродинирования» для электрооптических измерений расстояний. Этот принцип модуляции теперь используется в приборах AGA и автор использовал Геодиметр-8 для нового определения скорости света. Данное исследование было проведено в тесной кооперации с Финским геодезическим Институтом, который предоставил автору свою опорную линию Ниинисало–Пихнари (22 219 848,3 $\pm$1,78 *мм*). Конечным результатом двухнедельных наблюдений является значение $c = 299\ 792\ 375 \pm 60$ *м*/сек. Окончательный статистический анализ проводился с помощью техники фильтрации Винера–Хопфа. Проведена также проверка гипотез по Фишеру и Харту.