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A New Determination of the Free-Space Velocity of Electromagnetic Waves

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A new determination of the free-space velocity of electromagnetic waves

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[Plate 1]

The paper describes the determination of the free-space phase velocity of electromagnetic waves *in vacuo* by means of a millimetre-wave interferometer capable of high precision. As already provisionally announced (Froome 1958) the result is:

$$c_0 = 299\,792{\cdot}50 \pm 0{\cdot}10 \text{ km/s},$$

where the variation represents the standard deviation of a single determination in statistical combination with estimated systematic errors.

1. Introduction

An advanced form of the 'four horn' millimetre-wave interferometer previously described (Froome 1954) has been used to measure the phase velocity of electromagnetic waves with the highest accuracy possible by this method. The operating frequency of the instrument was 72·006 Gc/s, corresponding to a wavelength of approximately 4 mm.

As before, the basis of the determination consisted of the simultaneous measurement of the free-space wavelength, and frequency, generated by a microwave source. The interferometer was used for the wavelength measurement and was of the 'two-beam' type. It employed a movable carriage supporting a pair of receiving apertures at opposite extremities, symmetrically disposed between an identical pair of transmitting horns energized with microwave power conveyed by waveguide from the common source. The axes of the four horns were carefully alined and the motion of the carriage was also arranged parallel to this direction. The two received signals were mixed together to produce interference, the resultant output undergoing a minimum for every half-wave displacement of the carriage. The separation between each transmitter and the corresponding receiving horn was of the order of 10 m and as the effective aperture dimensions were squares of about 6 cm side, the receiving horns were well within the Fraunhofer diffraction field of the transmitters. The received wavefronts were thus approximately spherical, and of radius equal to the appropriate transmitting-receiving horn separation. It is the change of these radii with the position of the movable receiving carriage that gives rise to the diffraction correction, the evaluation of which was the primary function of the 24 Gc/s prototype equipment.

The principal innovation with the 72 Gc/s interferometer was the use of a cavity resonator refractometer for the direct determination of the refractive index of the air in the neighbourhood of the interferometer. This replaced the less accurate method of using separate observations of atmospheric pressure, temperature and humidity with refractive index formulae such as those derived by Essen & Froome (1951) or Essen (1953).

During the course of the new determination a small error was discovered in the results obtained with the prototype apparatus. This error was due to the assignment of too high a value to the 1 m length-standard used in it.

Consequently the published values of c_0 obtained with the prototype are too high by 0·2 km/s. All the conclusions arrived at concerning the operation of the prototype instrument are, of course, unaffected by the change.

The movable carriage of the new interferometer was displaced through 970 half-waves (roughly 2 m, twice the prototype displacement and six times the order of interference) by means of end-standards known in terms of a light-wave standard of length. The value of microwave wavelength so obtained, when multiplied by the air refractive index and the microwave frequency gave a vacuum phase velocity which had still to be corrected for the effect of diffraction before the true free-space value could be derived. This was done by making seven velocity measurements, each for a different transmitting-receiving horn separation.

By virtue of (*a*) the greater carriage displacement, (*b*) the shorter wavelength, and (*c*) the improved location, all major error-producing influences arising from the method of microwave transmission, propagation, and reception were reduced relative to their effect on the prototype machine by more than one order of magnitude. It is interesting to note that the greatest single uncertainty in the whole measurement arises from the use of the length-standards.

2. Description of apparatus

Figure 1 shows in diagrammatic form all the essential details of the interferometer and associated equipment. The source of microwaves was the first harmonic of a Pound stabilized Q-band klystron oscillator operating at 36·003 Gc/s. The greater part of the output (about 10 mW) from this oscillator was fed by means of a waveguide switch into one of two silicon crystal distorter units tuned for maximum harmonic output at 72·006 Gc/s. One harmonic generator was used to supply the interferometer itself, the other for operating the refractometer by means of which the refractive index of the air in the neighbourhood of the equipment could be measured. This refractometer has been described elsewhere (Froome 1955). A smaller portion of the klystron power was used for driving the Pound frequency stabilizer and a minute fraction for frequency measurement.

The measurement of microwave frequency was accomplished by comparing the klystron output against a high harmonic of a 5 Mc/s quartz crystal standard. The 5 Mc/s was multiplied in stages of two to five times up to 600 Mc/s and then fed into a silicon crystal harmonic generator mounted in waveguide, so that the harmonic at exactly 36 Gc/s could be mixed with a small fraction of the klystron output. The beat frequency between the two was detected by means of a calibrated communications receiver. When the refractometer was being used the klystron frequency had to be variable (instead of constant at 36·003 Gc/s) between 36·002 and 36·014 Gc/s and was set to the nearest 100 kc/s calibration point on the communications receiver corresponding to approximate resonance of the refractometer cavity, the setting to perfect resonance then being made by means of a small tuning plunger attached to the cavity, in the manner already described (Froome 1955; Essen 1953; Essen & Froome 1951). The accuracy of frequency determination was always at least as good as 1 part in 10^8 and was thus the most precise measurement associated with the whole velocity determination.

When the interferometer was being used to make a wavelength measurement the 72·006 Gc/s output from the appropriate harmonic generator was guided to a hybrid junction ('magic-T'), which served as beam divider, and eventually through two long arms of cylindrical waveguide to the transmitting horns. The matching-stub to the left of the beam divider, together with the constant phase

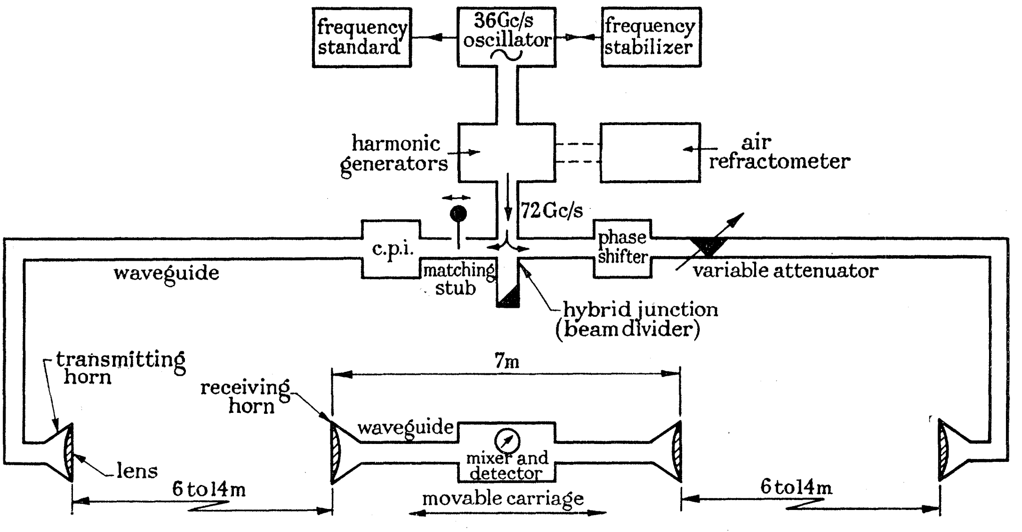

FIGURE 1. Diagram of 72 Gc/s interferometer.

waveguide interferometer (c.p.i.), constituted a device for altering the amplitude of the energy transmitted down this arm without producing a phase displacement (Froome 1954). The phase-shifter to the right, together with the variable attenuator, was required in order to adjust the position and balance of the first interference minimum on the movable receiving carriage.

After passing through the attenuators, the energy in each arm was transmitted via a transducer (for the purpose of exciting the low-loss H_{01} circular mode) to cylindrical waveguide (12·5 mm bore) for transmission over the relatively long journey to the transmitting horns. Before the energy entered each horn another transducer served to convert to the H_{01} mode of propagation in the standard-size rectangular waveguide leading to the horn throat.

The movable part of the interferometer consisted of a pair of receiving horns mounted on a large carriage situated centrally between the transmitting horns. Both transmitting and receiving horns were identical in every respect and each was fitted with a polystyrene lens at its mouth to correct the radiated waveform (or equiphase receiving surface) to planar. The horn apertures were rectangles of approximately 8×6 cm, the larger dimension being in the plane of the magnetic field component, thereby making the radiation pattern as symmetrical as possible.

The central platform of the receiving carriage held three wheels arranged to run on a cast-iron bed of 3 m length taken from a precision measuring machine. Beneath the surface of the bed ran a counterweight equal in mass to the carriage, but arranged to move in the opposite direction so as to maintain a constant moment on the concrete block supporting the bed and on the floor of the room beneath the block (figure 2).

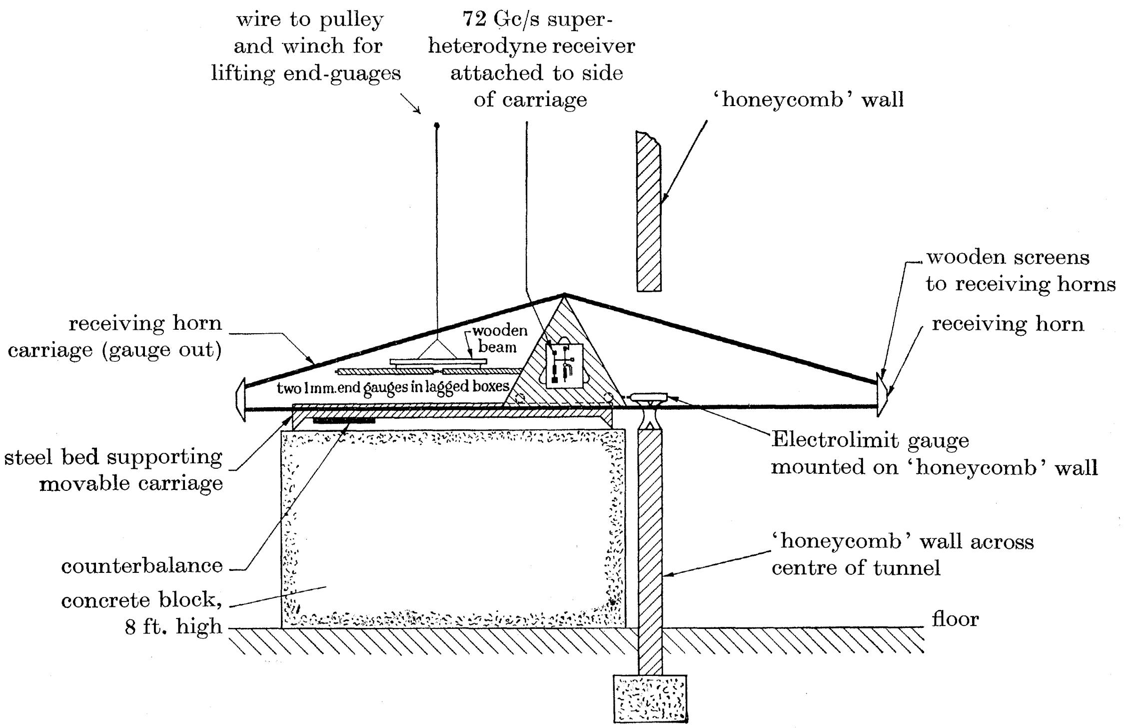

FIGURE 2. Method of mounting the receiving horn carriage of the 72 Gc/s microwave interferometer in the duplex wind tunnel room.

The whole apparatus was installed in a very large wind-tunnel room. Across the approximate centre of the room is a 'honeycomb' anti-turbulence wall and this was used to form a convenient datum point for measurements of displacement of the receiving carriage. Part of the carriage actually passed through a hole cut in this wall. Figure 3, plate 1, is a photograph of the receiving carriage showing the 'honeycomb' wall.

A highly sensitive electrical measuring head (T.T. & H. Electrolimit, divided in units of 10μin. ($0{\cdot}25\mu$) was mounted upon the 'honeycomb' wall in such a manner as to be entirely independent of the steel bed (and concrete block) supporting the carriage, except when a small ball mounted on the centre of the carriage (in line with the receiving horn axes) was in contact with the sensing anvil of the Electrolimit head. Provision was made for an independent slow-motion adjustment of the carriage.

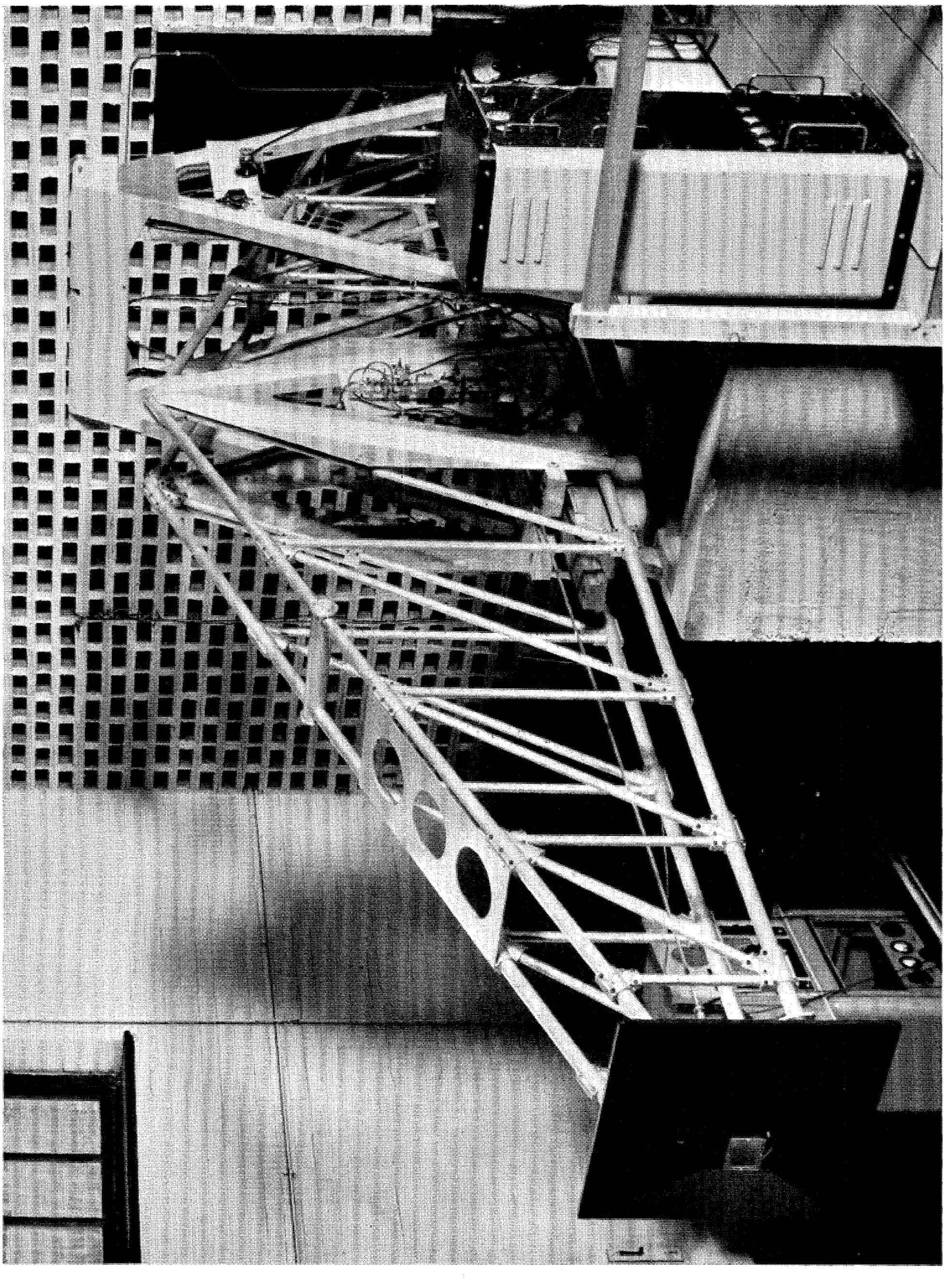

FIGURE 3. Photograph of the movable receiving carriage. The 'honeycomb' wall built across the wind-tunnel room can also be seen.

The energy picked up by the receiving horns was transmitted along oversize rectangular waveguide to a hybrid junction mixer mounted on the central platform of the carriage. The 'mixed', or interference signal, was detected by means of a superheterodyne receiver involving a second stabilized Q-band klystron and crystal harmonic generator as 'local oscillator'. This was affixed to the side of the movable carriage (see figures 2, 3). The intermediate frequency, chosen to be 60 Mc/s, was first amplified by a wide-band 'pre-amplifier' and then by a v.h.f. communications receiver with a final bandwidth of 5 kc/s. The output from the detector stage of this receiver was indicated by a micro-ammeter placed close to the receiving carriage slow-motion adjustment and the precision of setting on a minimum was about $0{\cdot}5\mu$. When the interferometer was not in use, the output from the second Q-band klystron could be conveyed by means of a waveguide switch to the detector associated with the refractometer.

The receiving carriage was constructed from aluminium tubing and it was found that the positions of the receiving horns did not shift relative to the mixer by more than ± 5 μin. as the carriage was traversed along the supporting bed. By use of the H_{01} mode of propagation in the oversize waveguide between receiving horns and mixer (so that the guide phase velocity approaches the free-space value) the effect of carriage distortion was further reduced because, providing the waveguide follows the carriage distortion, the position of an interference minimum would be largely independent of the positions of the receiving horns on the carriage.

The geometrical axes of the transmitting horns, the receiving horns, the ball on the carriage, and the anvil of the Electrolimit gauge-head could be alined accurately by means of an alinement telescope. The mounting of the receiving horns was such that they could be rotated by 180° about their (horizontal) geometrical axes; such rotation was part of the experimental procedure for eliminating errors arising from any difference between the electrical and geometrical axes if the carriage should happen to tilt a little as it is displaced. Corresponding errors in the transmitting horns are of the same form as the leading term in the diffraction correction (see equation (1), §6) and were eliminated by the evaluation of that correction.

The 2 m length-standard used for measuring the carriage displacement through 970 minima (i.e. 970 half-waves) consisted of two 1 m end-bars set up in line, with slip gauges wrung to each face and separated in the centre by a 6 mm steel ball. When the end-standard was 'inserted' into position (i.e. the interferometer was set on the 971st minimum) wheels attached at the Airy points of each gauge automatically and accurately located it on rails fixed to the 3 m bed so that the standard was free to move longitudinally as the setting on the 971st minimum was made. The diameter of the 6 mm ball was measured by optical interferometry under the load (200 g) imposed by the Electrolimit head. All slip gauges used were likewise measured by optical interferometry.

The determination of the length of the two 1 m end-bars needs some elaboration. This was accomplished by means of the apparatus of Sears & Barrell (1932), originally designed for the determination of the metre and the yard in terms of the wavelength of the red line of the cadmium spectrum. With it one can measure a special 1 m end-bar of X-section in terms of any suitable light-wave standard. This X-bar can then be compared against other 1 m end-standards either by interferometry or by mechanical comparison. For the reasons given in §8 the X-bar was measured in terms of the cadmium red radiation as emitted by the internationally specified form of Michelson lamp.

When used on the microwave interferometer, the two 1 m end-bars were placed inside well-lagged wooden boxes and fitted with a thermostatic control operated by a sensitive contact thermometer controlling at 20 °C. The actual temperature of the gauges was obtained by means of ten thermocouples related to a mercury-in-glass thermometer calibrated to $\pm 0{\cdot}002$ °C. The whole length-bar assembly could be lowered into position by means of a winch and pulley arrangement.

3. The constant phase waveguide interferometer (c.p.i.)

Although a full description of this device has already been published (Froome 1954), for the purpose of clarity a further summary of its mode of operation is desirable. The basic theory is very simple: if two equal vectors, initially parallel (equal waves in phase), are rotated in opposite directions through the same angle, the direction of their resultant is unchanged while its amplitude is reduced.

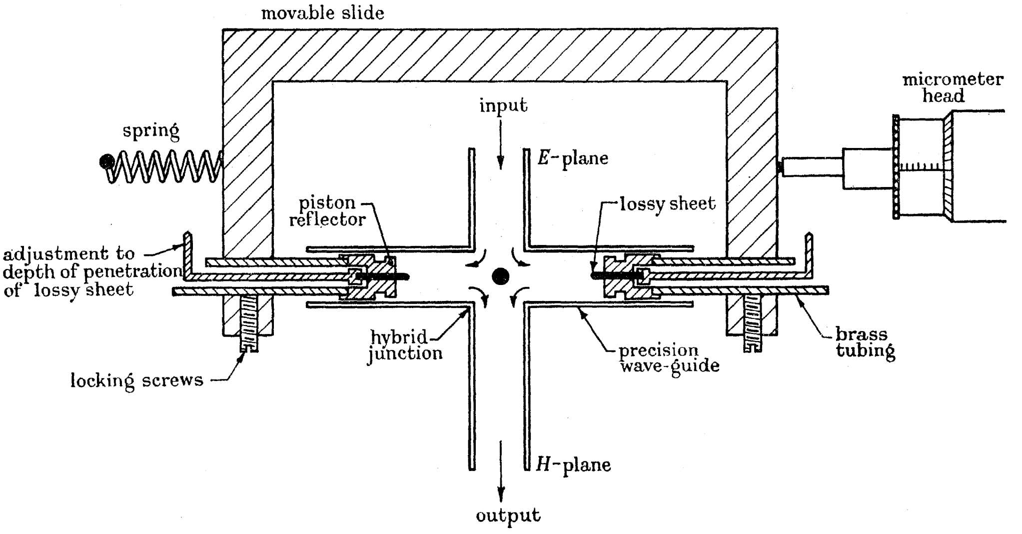

Figure 4. Auxiliary constant phase interferometer (c.p.i.)

Figure 4 is a diagram of the c.p.i. It consists of a waveguide interferometer utilizing a hybrid junction for the dual purpose of beam division and recombination. Energy from the beam divider of the four-horn interferometer enters the E-plane arm of the c.p.i. and at the centre of the junction divides into two equal parts and enters the arms containing the mechanically linked movable-piston reflectors. The waves reflected from these pistons interfere at the centre of the junction and normally some energy is transmitted through the H-plane output arm and the remainder is returned through the input arm. In figure 1 the matching stub between c.p.i. and the main interferometer beam divider prevents further reflexion of this returned energy from the main beam divider. Thus, it is apparent that when the c.p.i. pistons are so adjusted as to give the maximum transmitted output, a rotation of the c.p.i. micrometer in either direction will cause a reduction of output energy. When the movable carriage of the principal interferometer was in the 'gauges inserted' position (i.e. set on the 971st minimum) and the amplitude of the waves picked up by the receiving horns was balanced by a clockwise rotation of the c.p.i., the c.p.i. micrometer was said to be at 'position A'. Anti-clockwise rotation to produce amplitude balance led to 'position B'. If the c.p.i. micrometer was moved from one position of maximum transmission to the next the phase of its output was changed by π so that two new positions (A^1 and B^1) of the micrometer were now possible, for equal output, when the device was used for less than maximum transmission.

4. The inherent errors of the microwave system

All errors were of the same form, but greatly reduced in magnitude, as those detailed in the description of the prototype interferometer. Apart from the effect of diffraction, which reduces with increasing transmitting-receiving horn separation and can be precisely eliminated (§6), errors arise from: (1) multiple reflexion of microwave energy between transmitting and receiving horns; (2) transmitted energy scattered into the receiving horns from some fixed external surface such as the floor; (3) errors inherent in the measurement of the refractive index of the atmosphere, and thus in the reduction of wavelength measurements made in air to the vacuum condition; (4) errors due to imperfections in the c.p.i.; (5) the errors, already discussed, due to possible differences between the geometrical and microwave axes of the four horns.

Error (1) depends only upon the separation between the transmitting and receiving horns and reverses for a $\frac{1}{4}\lambda$ displacement of the receiving carriage. Thus, this error was eliminated by making at least two wavelength measurements, the carriage for the second being displaced by $\frac{1}{4}\lambda$ from its initial setting.

The disturbance due to the second effect, always less than 1 part in 10^6, was eliminated by making a number of wavelength measurements for slightly differing transmitting-receiving horn separations, the transmitting horns being set in steps a total of a few wavelengths either side of the mean position selected for a given diffraction effect. Thus, the phase of the direct transmission to the receiving horns was varied relative to scattered radiation without in any way upsetting the evaluation of diffraction.

The estimated accuracy of the refractive index measurement was $\pm 1{\cdot}1$ parts in 10^7. Great care had to be exercised in order to ensure equilibrium between the water vapour in the air sample passing through the refractometer and water atoms occluded by the cavity walls. It was necessary to circulate the air *for at least* 2 h before making a refractive index measurement (i.e. before evacuating the cavity resonator and noting the change of resonant frequency); thus only one determination could be made at the middle of the period occupied by each wavelength measurement. Actually, the interferometer could only be used on cloudy dry days, or days of constant drizzle, when the water vapour content of the air was very stable.

When used in the manner described in the next section no error arising from the c.p.i. could be found, even when deliberately maladjusted. Remembering the success of the c.p.i. used at 24 Gc/s, this result is not surprising, for the precision of manufacture of the 72 Gc/s c.p.i. was two orders of magnitude better. The waveguide portions of the 72 Gc/s device were constructed from optically flat stainless-steel pieces and it was estimated that the internal dimensions were uniform to within ± 1 μin. over the distance of the piston movements.

5. Experimental procedure

In order to make wavelength measurements free from all disturbing influences, the following procedure was adopted.

(1) The observer, after having performed a preliminary run to adjust (by means of the slip gauges) the 2 m combination end-standard to be closely equal to 970 half-waves, set the movable receiving carriage in contact with the sensing anvil of the Electrolimit gauge mounted on the 'honeycomb' wall.

(2) The c.p.i. having been set at a position of maximum transmission (the mean of the A and B positions), the variable attenuator and phase-shifter (see figure 1) were adjusted to give zero-detector current at the position of the first minimum.

(3) The temperature of the end-bar assembly was taken.

(4) The observer made four settings on the first minimum, reading the Electrolimit gauge each time.

(5) The receiving carriage was run to the limit of its movement and the end-bar assembly lowered into position between the Electrolimit head and the fiduciary ball on the carriage.

(6) The c.p.i. was turned to position A and the observer noted the Electrolimit gauge reading for four settings on the 971st minimum.

(7) Operation (6) was repeated for the position B of the c.p.i.; the mean of the Electrolimit readings for the A and B positions being taken as the true position of the 971st minimum.

(8) The length-standard was removed and operation (4) repeated to correct for drift, after which the temperature of the length-bar was again taken.

(9) The c.p.i. micrometer was moved to the next position of full transmission (defined as the mean of the A^1 and B^1 positions) and the Electrolimit head moved in its clamp by $\frac{1}{4}\lambda$ so that the position of the 1st minimum was again within the range of the Electrolimit gauge.

(10) Operations (3) to (8) were repeated using the c.p.i. in the A^1 and B^1 positions instead of the A and B positions.

By taking the mean of the two wavelength measurements so far made, the effects of multiple reflexions between transmitting and receiving horns were eliminated, together with all the c.p.i. errors except one. The operations were continued:

(11) The refractive index of the air was determined by shutting off the slow air circulation through the refractometer cavity and measuring the change of frequency upon evacuation.

(12) A quarter-wave spacer was inserted in the waveguide of the output arm of the c.p.i. The two receiving horns were rotated through 180° about their (horizontal) geometrical axes, thereby reversing the effect of any difference between microwave and geometrical axis.

(13) Operations (1) to (10) were repeated. The mean of all the four wavelength measurements thus made was then free from all errors arising from the microwave system, with the exception of that due to diffraction and scatter from fixed objects.

(14) The effect of scatter was removed by the method described in the previous section.

(15) To determine and eliminate the effect of diffraction, fully corrected (operations (1) to (14)) wavelength measurements were made for seven different transmitting-receiving horn separations. The actual experiments took place between January and July 1956.

6. The diffraction correction

Since the full analysis of the diffraction correction has already been described (Froome 1954), only the results of it will be given here. As before, experiments were so planned that the transmitted wavefronts, by the time they reached the receiving horns, were nearly true spheres. If z is the distance separating one transmitting horn aperture from its corresponding receiver the effect of diffraction is to cause an addition retardation of the received wave (over and above z) by D_z where

$$D_z = \frac{A_1}{z} + \frac{A_2}{z^2} + \frac{A_3}{z^3} + \frac{A_4}{z^4} + \dots. \tag{1}$$

The constants A_1, A_2, etc., are a function of horn-aperture field distributions and are independent of z. For plane wavefronts at the mouth of the transmitting horns and plane receiving surfaces the even-order terms A_2, A_4, etc., vanish and the series converges rapidly. When phase errors are present at the horn apertures the convergence is somewhat slower and the experiments must be planned so that orders higher than $1/z^3$ are negligible. The first term of the series corresponds to the truly spherical received wavefront condition and dominated all the other terms for the experiments with the four-horn interferometer. The method of evaluating the diffraction correction is such that the greater the ratio of the A_1/z term to all the other terms in the series, the less need be known about the actual aperture field distributions.

The interference equation may be written

$$\tfrac{1}{2}N\lambda = \Delta_z - (D_{z_1} - D_{z_2}), \tag{2}$$

where N is the number of half-waves (970) in the carriage displacement, Δz. ($\Delta z = z_2 - z_1$) z_1 and z_2 are the transmitter-receiver separations for each end of the carriage displacement. (The carriage was disposed symmetrically between the transmitting horns.) λ is the true free-space wavelength in air.

In order to minimize the effect of uncertainty in the knowledge of horn aperture distributions, (2) is best written

$$\tfrac{1}{2}N\lambda = \Delta_z - K(D_{z_1} - D_{z_2}), \tag{3}$$

where K is a constant (ideally unity) determined by a least squares solution of the experimental observations. In terms of phase velocity *in vacuo*, equation (3) becomes

$$c_0 = c - 2Knf(D_{z_1} - D_{z_2})/N; \tag{4}$$

c_0 is the true free-space phase velocity and c the apparent (measured) velocity $(2nf\Delta z/N)$. n is the measured refractive index of the air and f is the measured microwave frequency.

The results were evaluated from a least squares solution of equation (4) giving both c_0 and K. If the assumptions concerning horn aperture distributions are roughly correct, then K comes out nearly equal to unity and the 'best' value of c_0 is obtained. For the ideal 'spherical wave' case where only the first term in equation (1) is significant, equation (4) gives the same result for c_0 regardless of the nature of the assumed aperture distributions and regardless of the value of K. The experiments with the 72 Gc/s interferometer were planned so as to make the derived value of c_0 very insensitive to the exact nature of the horn aperture distributions (i.e. very insensitive to K).

For ideal rectangular-section horns propagating the H_{01} mode and free from aperture phase errors the theoretical form of Dz is

$$\begin{aligned} D_z &= [0{\cdot}18943a^2 + \tfrac{1}{3}b^2 + 0{\cdot}0253\lambda^2]/z \\ &\quad - [0{\cdot}104a^4 + 0{\cdot}289b^4 + 0{\cdot}063a^2b^2 + (0{\cdot}1795a^6 + 0{\cdot}6127b^6)/\lambda^2]/z^3 \\ &\quad + 0(1/z^5) + \dots, \end{aligned} \tag{5}$$

where the horn aperture dimensions are $2a$ in the H-plane and $2b$ in the E-plane.

If smoothly varying aperture phase errors (zero on horn axis) of magnitude Pa^2, Qb^2 (cm) at the horn edges are introduced, D_z becomes

$$D_z = (1/k)\,[\lambda\beta_1/z - \lambda^2\beta_2/z^2 - \lambda^3\beta_3/z^3 + O(1/z^4) + \dots], \tag{6}$$

$$\begin{aligned} \text{where } k &= 2\pi/\lambda, \\ \beta_1 &= 4(X_1 + U_1), \\ \beta_2 &= (2/\pi)(Y_1 + V_1) + 4(Y_2 + V_2 + 2X_1Y_1 + 2U_1V_1), \\ \beta_3 &= (2/\pi)[(3X_2 + 5X_1^2 - 5Y_1^2) + (3U_2 + 5U_1^2 - 5V_1^2) + 4(X_1U_1 - Y_1V_1)] \\ &\quad + 8[\tfrac{1}{3}X_3 + 3X_1X_2 - 3Y_1Y_2 + 10X_1Y_1^2 - 10X_1^3/3 \\ &\quad + \tfrac{1}{3}U_3 + 3U_1U_2 - 3V_1V_2 + 10U_1V_1^2 - 10U_1^3/3], \end{aligned}$$

$$\begin{aligned} \text{with } X_1 &= a_1^2(0{\cdot}2976 - 0{\cdot}0227p^2 - 0{\cdot}0004p^4), \\ X_2 &= a_1^4(0{\cdot}1942 - 0{\cdot}0308p^2 - 0{\cdot}0002p^4), \\ X_3 &= a_1^6(0{\cdot}1662 - 0{\cdot}0380p^2 - 0{\cdot}0001p^4), \\ Y_1 &= a_1^2(0{\cdot}1057p - 0{\cdot}0020p^3 - 0{\cdot}0002p^5), \\ Y_2 &= a_1^4(0{\cdot}1084p - 0{\cdot}0043p^3 - 0{\cdot}0002p^5), \\ U_1 &= b_1^2(0{\cdot}5236 - 0{\cdot}0328q^2 - 0{\cdot}0033q^4 - 0{\cdot}0003q^6), \\ U_2 &= b_1^4(0{\cdot}4935 - 0{\cdot}0618q^2 - 0{\cdot}0052q^4 - 0{\cdot}0003q^6), \\ U_3 &= b_1^6(0{\cdot}5537 - 0{\cdot}0957q^2 - 0{\cdot}0073q^4 - 0{\cdot}0004q^6), \\ V_1 &= b_1^2(0{\cdot}2193q + 0{\cdot}0069q^3), \\ V_2 &= b_1^4(0{\cdot}2953q + 0{\cdot}0059q^3 - 0{\cdot}0003q^5 - 0{\cdot}0001q^7), \end{aligned}$$

and
$$p = 2kPa^2/\pi, \qquad q = 2kQb^2/\pi,$$
$$a_1 = a/\lambda, \qquad b_1 = b\lambda.$$

(The series converge for p, q, $\leqslant 1$, i.e. for phase errors of not more than $\frac{1}{2}\pi$ at the horn aperture edges. A spherical form of phase error is given by $P = Q$.)

7. Results

Table 1 gives the measured phase velocities for the seven different positions of the transmitting horns.

Table 2 shows the corresponding values of c_0 after removal of the diffraction effect.

Table 1. Measured velocities (reduced to vacuum values)

separation of transmitting and receiving horns with movable carriage in centre of displacement (cm)	measured phase velocity (km/s)
629·5	299796·020
751·5	4·981
875·0	4·283
999·0	3·802
1120·5	3·583
1247·5	3·482
1367·5	3·259

Table 2. Final values of c_0

horn separation (cm)	c_0 (km/s)
629·5	299792·513
751·5	2·529
875·0	2·476
999·0	2·414
1120·5	2·478
1247·5	2·588
1367·5	2·512

mean $c_0 = 299\,792{\cdot}501 \pm 0{\cdot}059$ [s.d.]

It is seen that the standard deviation of a single determination of velocity at any particular value of the horn separation is just under 2 parts in 10^7. It should be noted that the variation of the c_0 results in table 2 includes, in addition to the uncertainty of setting on a minimum, the random uncertainties of the air refractometer and the temperatures of the length-bar and the air.

As with the other microwave interferometers (Froome 1952, 1954) it was found that the actual diffraction correction was less than the purely theoretical value. That is, insertion of the geometrical horn apertures ($a = 4$ cm, $b = 3$ cm) in equation (5) gave a value of K less than unity when the results were evaluated using equation (4), although the value of c_0 obtained was the same as that finally accepted. The values of c_0 in table 2 have been derived by putting $a = 3{\cdot}6$ cm, $b = 2{\cdot}6$ cm in equation (5) thereby obtaining from equation (4) a value of K closely equal to unity. A small allowance has also been made for the effect of possible aperture phase errors. This was done by substituting in equation (4) D_z values obtained from equation (6) on the assumption of bad phase errors near the rim of the horn apertures. On the basis of this investigation it was decided to reduce the c_0 values obtained for supposed perfect plane waveforms (i.e. equation (5)) by 0·02 km/s with an uncertainty of $\pm 0{\cdot}01$ km/s. Table 2 incorporates this additional correction.

Table 3 lists the estimates of the possible systematic errors present in the determination.

TABLE 3. SYSTEMATIC ERRORS

source of error	magnitude expressed in km/s
length measurement (± 2 in 10^7)	$\pm 0{\cdot}060$
refractive index of air ($\pm 1{\cdot}1$ in 10^7)	$\pm 0{\cdot}033$
length-bar temperature ($\pm 0{\cdot}006$ °C)	$\pm 0{\cdot}020$
air temperature ($\pm 0{\cdot}03$ °C)	$\pm 0{\cdot}010$
c.p.i. residual	$\pm 0{\cdot}015$
diffraction residual	$\pm 0{\cdot}010$
frequency (± 1 in 10^8)	$\pm 0{\cdot}003$

The standard deviation of a single determination (table 2) is increased to 0·10 km/s when modified by statistical combination with the estimated systematic errors (table 3), so the result is

$$c_0 = \mathbf{299\,792{\cdot}50 \pm 0{\cdot}10}\,\text{km/s}.$$

8. DISCUSSION

When the same value is assigned to the part of the length-bar combination used for the experiment described herein and with the earlier equipment, the value for c_0 obtained with the prototype becomes $299\,792{\cdot}7_5 \pm 0{\cdot}3$ km/s. In view of this agreement there can be no doubt that the methods used with the prototype for eliminating the disturbing effects of diffraction, scatter, etc., must have been very satisfactory—in particular the accuracy of assessment of diffraction must have been excellent, for corrections of up to 100 km/s were applied at the smallest transmitting-receiving horn separations. Since the 72 Gc/s interferometer has been used in an identical manner, but with a figure of improvement for those effects of more than one order of magnitude, the estimated accuracy of this interferometer should also be correct.

The estimated accuracy of the determination of the length-bar combination refers to interferometric measurement in terms of the red line of the cadmium spectrum as emitted by the standard Michelson lamp running at the specified conditions. The most recent information placed before the Consultative Committee for the Definition of the Metre, in September 1957, shows that this source is reproducible to 1 part in 10^7, or slightly better, and this is considered to be the accuracy of determination of the length of the 1 m X-section end-bar. The additional 1 part in 10^7 given in table 3 is due to further uncertainties arising from the comparisons of the two 1 m cylindrical section end-gauges against the X-bar, and in the measurement of the slip gauges and the 6 mm steel ball which were also a part of the final form of the length standard.

In 1960 it is hoped to replace the International Prototype Metre by a light-wave standard, the Consultative Committee having already agreed to recommend the adoption of the orange-red line of krypton-86 ($\lambda_{\text{vac.}} = 0{\cdot}605\,780\,21\,\mu$) emitted by a specially cooled lamp. The actual value of this wavelength has been obtained from the mean of a number of comparisons by various standards laboratories against the red line of cadmium ($\lambda_{\text{vac.}} = 0{\cdot}644\,024\,91\,\mu$) as emitted by the same form of lamp used for the length measurements connected with the new velocity determination. In any case,whatever wavelength standard is finally adopted *the wavelength of the cadmium line will not be altered.*

The new value is in good agreement with other recent velocity measurements of high precision, the most recent and readily available discussion of these being that of Mulligan & McDonald (1957) and Bergstrand (1957). By means of his Geodimeter, Bergstrand has made the most precise determinations of the velocity of light using geodetic bases of the order of 10 km in length. In this latest paper he summarizes all the Geodimeter results and gives two values for c_0, namely, $299\,792{\cdot}75 \pm 0{\cdot}34$ km/s and $299\,792{\cdot}85 \pm 0{\cdot}16$ km/s. In each case the value quoted incorporates the weighted mean of results from a number of bases, the second set being obtained with an improved form of apparatus. For this latter set he states the estimated accuracy of a determination from a single base-line (itself the weighted mean of a number of velocity measurements) to be about $\pm 0{\cdot}4$ km/s of which half lies in the uncertainty of length measurement. It is also in excellent agreement with the other N.P.L. determinations using microwaves: Essen's (1950) cavity resonator gave $c_0 = 299\,792{\cdot}5 \pm 1$ km/s; Froome's microwave interferometers gave $299\,792{\cdot}6 \pm 0{\cdot}7$ (1952) and $299\,792{\cdot}7_5 \pm 0{\cdot}3$ (1954, corrected 1958). The microwave determinations of the velocity of electromagnetic waves up to 1956 have been reviewed by Froome (1956). Of interest also is the recent (1957) decision of the XIIth General Assembly of the International Scientific Radio Union (U.R.S.I.) to recommend the adoption of $c_0 = 299\,792{\cdot}5 \pm 0{\cdot}4$ km/s. This value was also accepted by the International Union for Geodesy and Geophysics (I.U.G.G.).

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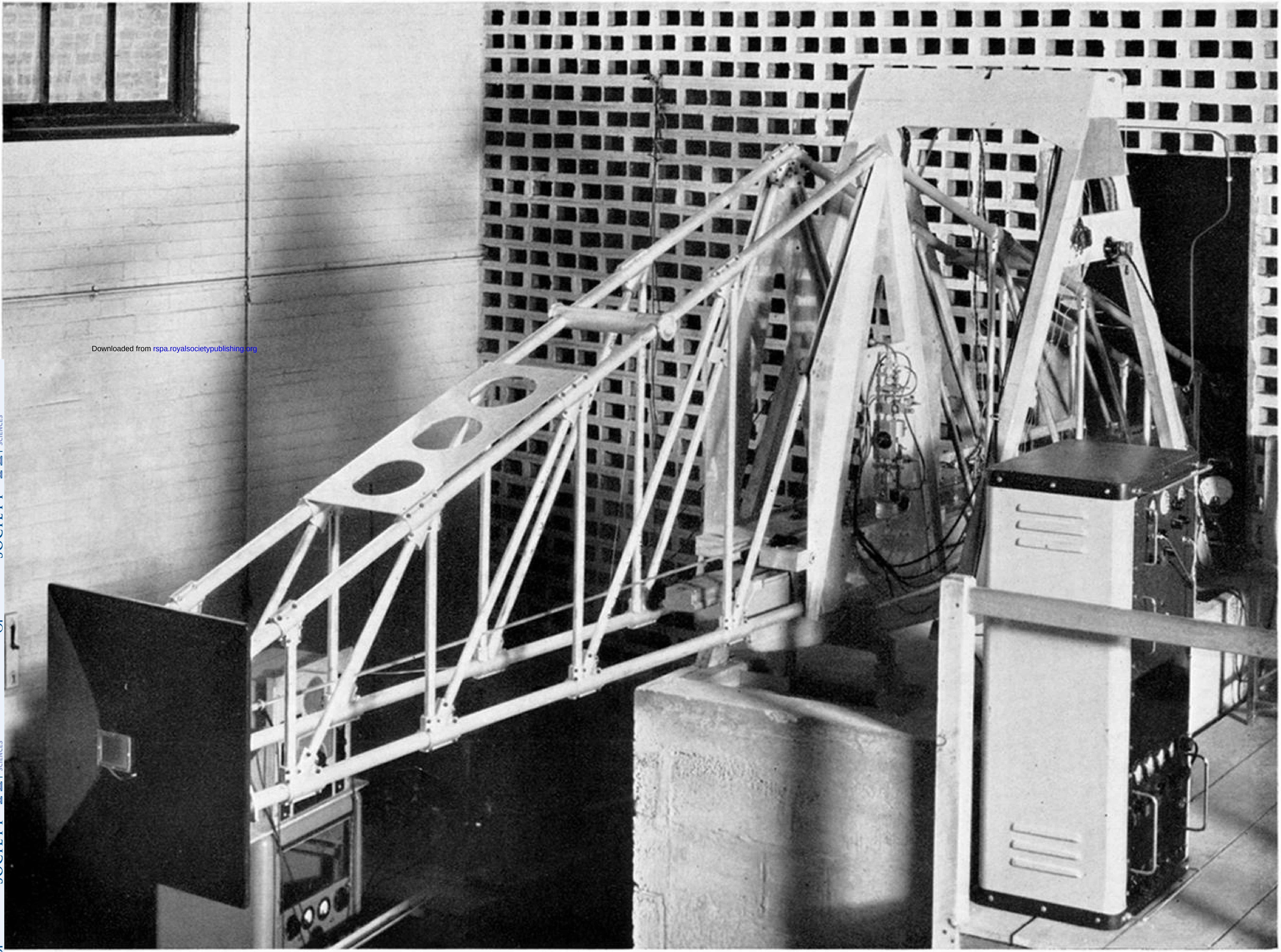

IGURE 3. Photograph of the movable receiving carriage. The 'honeycomb' wall built across the wind-tunnel room can also be seen.

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