F104-A Aircraft Modes

SPC 409: Flight Dynamics and Control Course Project

Dr. Mohannad Draz

Mariam Wagdy
201801585

Mohamed Ismail
201800595

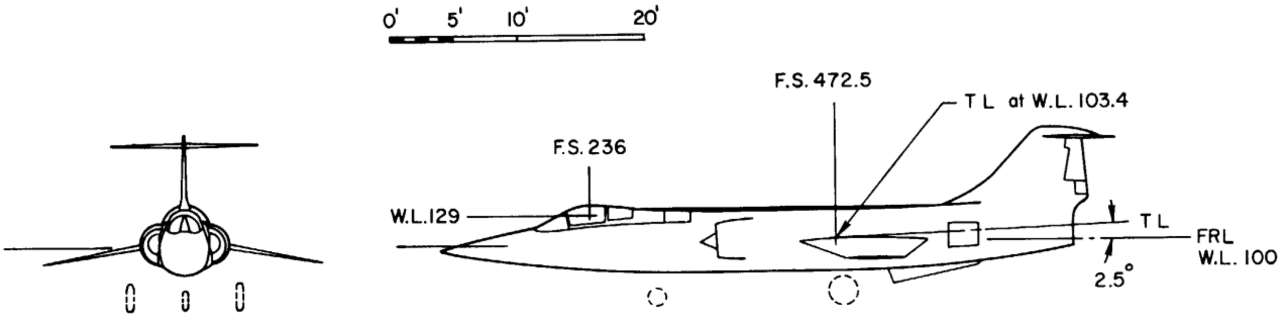
Nader Fawzy
201800322

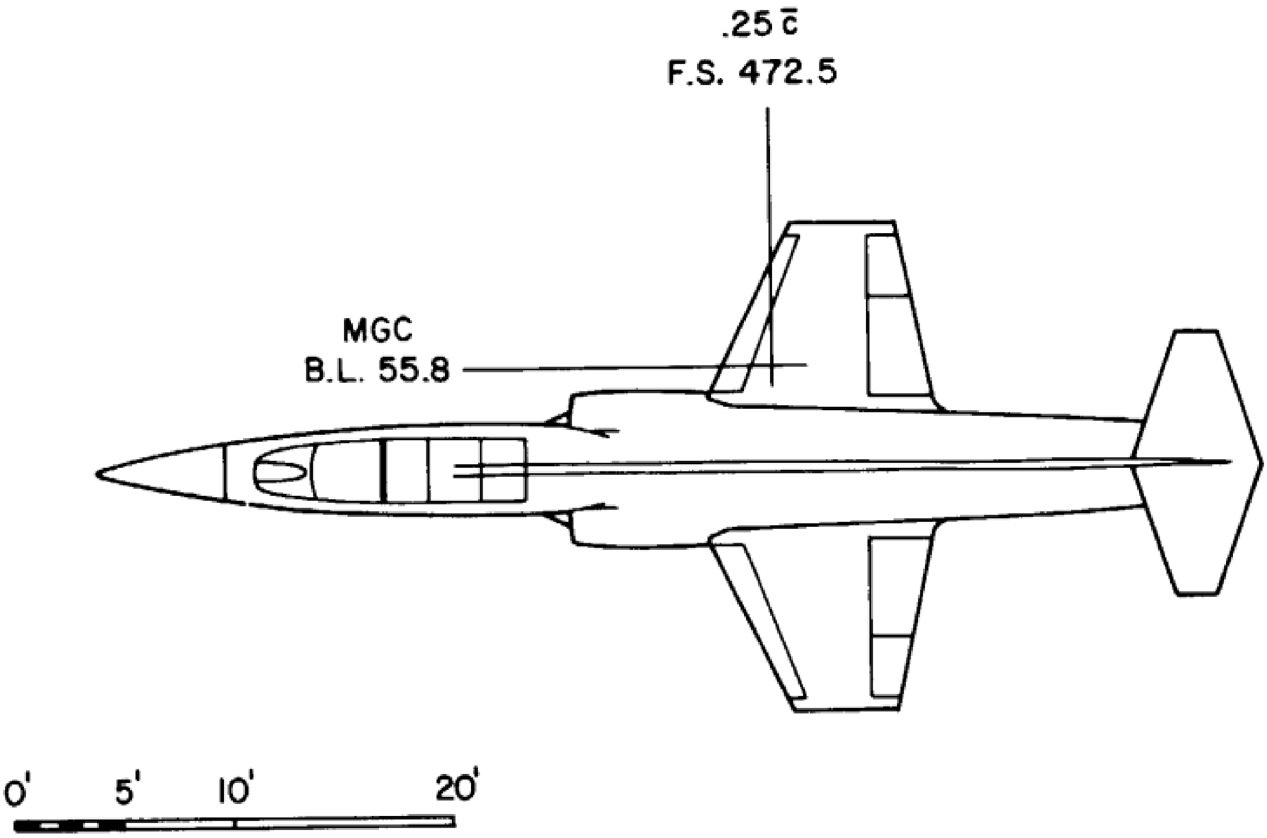
Contents

Aircraft Parameters

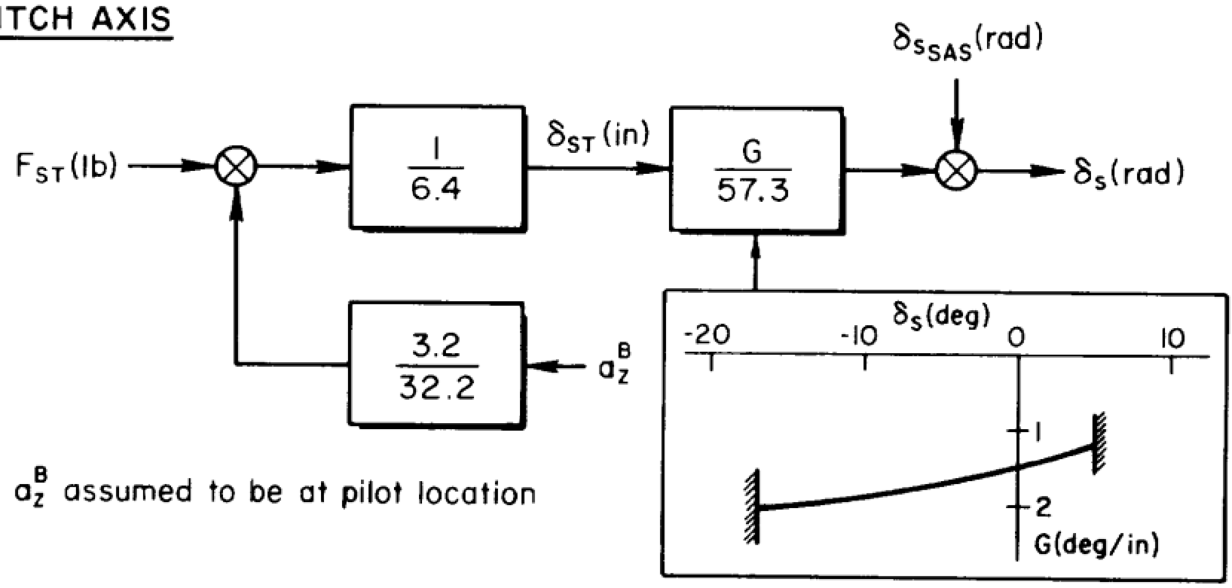
The F-IO4A is a single place_ lightweight_ supersonic air superiority fighter powered by a single turbojet engine with afterburner. The wing has a full span leading edge flap. Trailing edge flaps have a blowing-type boundary layer control system. Control is provided by conventional ailerons and rudder and an all-movable stabilizer. Pitch_ roll_ and yaw dampers are incorporated_ however their effect is not shown here. Pitch and roll controls are fully irreversible while the yaw control is a cable-actuated rudder without boost. A bobweight is used in the longitudinal feel system. Its position is assumed to be at the pilot's location. The primary source of data was LR 10794. Drag information was obtained from LR-12873. The nominal configuration used here is the combat loading for the F-]O4A based on actual weight and balance data. The PA configuration is a typical loading at flight manual approach speeds.

For nominal Configuration:

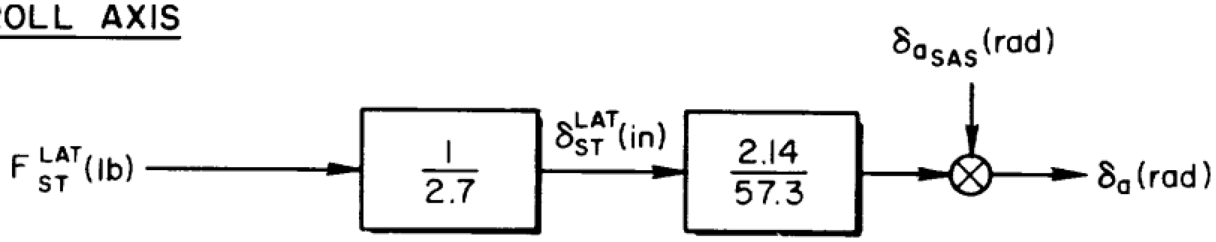




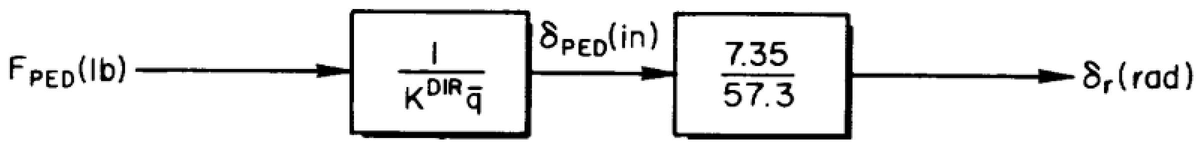
PITCH AXIS



ROLL AXIS



YAW AXIS



$\delta_s = [-17,5]$
$c_L = 0.735$
$c_D = 0.263$
$C_{L_\alpha} = 3.44[1/rad]$
$c_{D_\alpha} = 0.45[1/rad]$
$c_{m_\alpha} = -0.64[1/rad]$
$c_{m_{\dot\alpha}} = -1.6[1/rad]$
$c_{m_q} = -5.8[1/rad]$
$c_{L_{\delta_s}} = 0.68[1/rad]$
$c_{m_{\delta_s}} = -1.46[1/rad]$

$c_{y_\beta} = -1.17[1/rad]$
$c_{n_\beta} = 0.5[1/rad]$

$c_{\mathcal{L}_\beta} = -0.175[1/rad]$
$c_{\mathcal{L}_p} = -0.285[1/rad]$
$c_{n_p} = -0.14[1/rad]$
$c_{\mathcal{L}_r} = 0.265[1/rad]$
$c_{n_r} = -0.75[1/rad]$
$c_{n_{\delta_a}} = 0.0042[1/rad]$
$c_{\mathcal{L}_{\delta_a}} = 0.039[1/rad]$
$c_{y_{\delta_r}} = 0.208[1/rad]$
$c_{\mathcal{L}_{\delta_r}} = 0.045[1/rad]$
$c_{n_{\delta_r}} = 0.16[1/rad]$
$c_{y_{\delta_d}} = 0.0325[1/rad]$
$c_{n_{\delta_d}} = -0.025[1/rad]$
$c_{\mathcal{L}_{\delta_d}} = -0.0044[1/rad]$

Working Conditions:

F/C #	1
H(FT)	SL
M(-)	.257
VTO(FPS)	287.
VTO(KTAS)	170.
VTO(KCAS)	170.
W(LBS)	14126.
C.G.(MGC)	.164
IX (SLUG-FT SQ)	3582.
IY (SLUG-FT SQ)	55802.
IZ (SLUG-FT SQ)	56669.
IXZ(SLUG-FT SQ)	2658.
EPSILON(DEG)	-2.86
Q(PSF)	97.8
QC(PSF)	99.5
ALPHA(DEG)	2.30
GAMMA(DEG)	0.
LXP(FT)	19.0
LZP(FT)	-2.40
ITH(DEG)	-2.50
XI(DEG)	-2.50
LTH(FT)	0.

$h = 0\ (sea\ level)$
$M = 0.257$
$v_{To} = 287[ft/s]$
$\alpha_o = 2.3^o$
$\delta_s = -7.1^o$
$Weight\ (W) = 14126\ [lb]$
$c.g\ at\ 0.164\ \bar{c}$
$I_x = 3582\ [slug\ ft^2]$;
$I_y = 55802\ [slug\ ft^2]$;
$I_z = 56669\ [slug\ ft^2]$
$I_{xz} = 2658\ [slug\ ft^2]$
$\epsilon = -2.86^o$
$S = 196.1\ [ft^2]$
$b = 21.95\ [ft]$
$\bar{c} = 9.55\ [ft]$
$Q = 97.8\ [lb^2 ft]$
$\gamma = 0^o$

XU *	-.0737
ZU *	-.204
MU *	.000294
XW	.0631
ZW	-.570
MW	-.00732
ZWD	0.
ZQ	0.
MWD	-.000304
MQ	-.317
XD S	1.19
ZD S	-29.7
MD S	-4.79
XD TH	.00228
ZD TH	.994E-4
MD TH	0.

Рисунок 1 Longitudinal-Directional Dimensional Derivatives

$X_u = -0.737$
$Z_u = -0.204$
$M_u = 0.000294$

$X_w = 0.0631$
$Z_w = -0.57$
$M_w = -0.00732$

$Z_{\dot{w}} = 0$
$Z_q = 0$
$M_{\dot{w}} = -0.000304$
$M_q = -0.317$

$X_{\delta_s} = 1.19$
$Z_{\delta_s} = -29.7$
$M_{\delta_s} = -4.79$

$X_{\delta_T} = 0.00228$
$Z_{\delta_T} = 0.0000994$
$M_{\delta_T} = 0$

DENOMINATOR	
Z(DET)1	.238
W(DET)1	.152
Z(DET)2	.324
W(DET)2	1.51
NUMERATORS	
N(U /DS)	
A(U)	1.19
1/T(U)1	43.8
Z(U)1	.740
W(U)1	1.25
N(W /DS)	
A(W)	-29.7
1/T(W)1	46.6
1/T(W)2	(.256)
1/T(W)3	(.150)
N(THE/DS)	
A(THE)	-4.79
1/T(THE)1	.104
1/T(THE)2	.496
N(HD /DS)	
A(HD)	29.7
1/T(HD)1	.0504
1/T(HD)2	-4.69
1/T(HD)3	5.12
N(AZP/DS)	
A(AZP)	61.2
1/T(AZP)1	-.00775
1/T(AZP)2	.0575
Z(AZP)1	.0887
W(AZP)1	3.41

Рисунок 4 Stabilizer Transfer Function Factors

DENOMINATOR	
Z(DET)1	.239
W(DET)1	.152
Z(DET)2	.324
W(DET)2	1.51
NUMERATORS	
N(U /DTH)	
A(U)	.00228
1/T(U)1	.000361
Z(U)1	.323
W(U)1	1.51
N(W /DTH)	
A(W)	.994E-4
1/T(W)1	.00157
Z(W)1	(-.118)
W(W)1	(-4.09)
N(THE/DTH)	
A(THE)	-.242E-8
1/T(THE)1	24.0
1/T(THE)2	-64.1
N(HD /DTH)	
A(HD)	-.795E-5
1/T(HD)1	-64.1
Z(HD)1	.139
W(HD)1	1.45
N(AZP/DTH)	
A(AZP)	.994E-4
1/T(AZP)1	-.00451
1/T(AZP)2	-4.79
Z(AZP)1	.194
W(AZP)1	1.50

Рисунок 3 Thrust Transfer Functions

YV	-.178
YB	-51.1
LB'	-20.9
NB'	2.68
LP'	-1.38
NP'	-.0993
LR'	1.16
NR'	-.157

Рисунок 2 Lateral-Directional Dimensional Derivatives

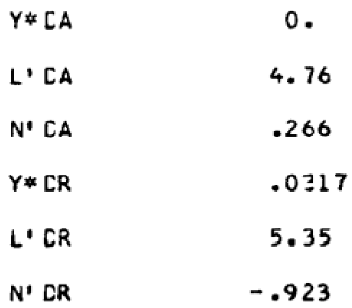
$Y_v = -0.178$
$Y_\beta = -51.1$

$\mathcal{L}'_\beta = -20.9$
$N'_\beta = 2.68$

$\mathcal{L}'_p = -1.38$
$N'_p = -0.0993$

$\mathcal{L}'_r = 1.16$
$N'_r = -0.157$

$Y_{\delta a} = 0$
$\mathcal{L}'_{\delta_a} = 4.76$



Y*CA	0.
L'CA	4.76
N'CA	.266
Y*CR	.0317
L'CR	5.35
N'CR	-.923

Рисунок 5 Lateral-Directional Dimensional Derivatives

$N'_{\delta_a} = 0.266$

$Y_{\delta_r} = 0.0317$
$\mathcal{L}'_{\delta_r} = 5.35$
$N'_{\delta r} = -0.923$

Longitudinal Mode

Ignoring derivatives to $\dot{w}$,

$$v_o = p_o = q_o = r_o = \psi_o = \varphi_o = w_o$$

$$\dot{x_{4*1}} = A_{4*4} x_{4*1} + B_{4*2} u_{2*1}$$

$$\begin{pmatrix} \dot{u} \\ \dot{w} \\ \dot{q} \\ \dot{\theta} \end{pmatrix} = A \begin{pmatrix} u \\ w \\ q \\ \theta \end{pmatrix} + B \begin{pmatrix} \Delta\delta_s \\ \Delta\delta_T \end{pmatrix}$$

$$A = \begin{pmatrix} X_u & X_w & X_q & -g \\ Z_u & Z_w & Z_q + u_o & 0 \\ M_u & M_w & M_q & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} -0.737 & 0.0631 & & -32.2 \\ -0.204 & -0.57 & 287 & 0 \\ 0.000294 & -0.00732 & -0.317 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$B = \begin{pmatrix} X_{\delta_s} & X_{\delta_T} \\ Z_{\delta_s} & Z_{\delta_T} \\ M_{\delta_s} & M_{\delta_T} \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 1.19 & 0.00228 \\ -29.7 & 0.0000994 \\ -4.79 & 0 \\ 0 & 0 \end{pmatrix}$$

LONG PERIOD APPROXIMATION

$$\begin{pmatrix} \dot{w} \\ \dot{q} \end{pmatrix} = \begin{pmatrix} Z_w & Z_q + u_o \\ M_w & M_q \end{pmatrix} \begin{pmatrix} w \\ q \end{pmatrix} + \begin{pmatrix} Z_{\delta_s} & Z_{\delta_T} \\ M_{\delta_s} & M_{\delta_T} \end{pmatrix} \begin{pmatrix} \Delta\delta_s \\ \Delta\delta_T \end{pmatrix}$$

$$\begin{pmatrix} \dot{w} \\ \dot{q} \end{pmatrix} = \begin{pmatrix} -0.57 & 287 \\ -0.00732 & -0.317 \end{pmatrix} \begin{pmatrix} w \\ q \end{pmatrix} + \begin{pmatrix} -29.7 & 0.00000994 \\ -4.79 & 0 \end{pmatrix} \begin{pmatrix} \Delta\delta_s \\ \Delta\delta_T \end{pmatrix}$$

SHORT PERIOD APPROXIMATION

$$\begin{pmatrix} \dot{u} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} X_u & -g \\ -\dfrac{Z_u}{u_o} & 0 \end{pmatrix} \begin{pmatrix} u \\ \theta \end{pmatrix} + \begin{pmatrix} X_{\delta_s} & X_{\delta_T} \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \Delta\delta_s \\ \Delta\delta_T \end{pmatrix}$$

$$\begin{pmatrix} \dot{u} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} -0.737 & -32.2 \\ \dfrac{0.204}{287} & 0 \end{pmatrix} \begin{pmatrix} u \\ \theta \end{pmatrix} + \begin{pmatrix} 1.19 & 0.00228 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \Delta\delta_s \\ \Delta\delta_T \end{pmatrix}$$

Nondimensionalize the eigen vector:

$$\begin{pmatrix} \dot{u} \\ \dot{w} \\ \dot{q} \\ \dot{\theta} \end{pmatrix} \rightarrow \begin{pmatrix} (\dot{u}/u_o)/\dot{\theta} \\ \alpha/\dot{\theta} \\ (\dot{q}\bar{c}/2u_o)/\dot{\theta} \\ \dot{\theta}/\dot{\theta} \end{pmatrix}$$

	Pole	Damping	Frequency (rad/s)	Time Constant (s)
Full System	$-5.20e-01 \ \pm 1.22i$	3.92e-01	1.33	1.92
	-5.69e-01	1	5.69e-01	1.76
	4.26e-02	-1	4.26e-02	-2.35e+01

	Pole	Damping	Frequency (rad/s)	Time Constant(s)
Long Period	-7.81e-01	1 (error=0)	7.81e-01(error=37.26%)	1.28
	4.40e-02	-1 (error =0)	4.40e-02 (error=3.29%)	-2.28e+01
Short Period	-4.14e-01 ± 1.19i	0.33 (error=15%)	1.26 (error=5.26%)	2.41

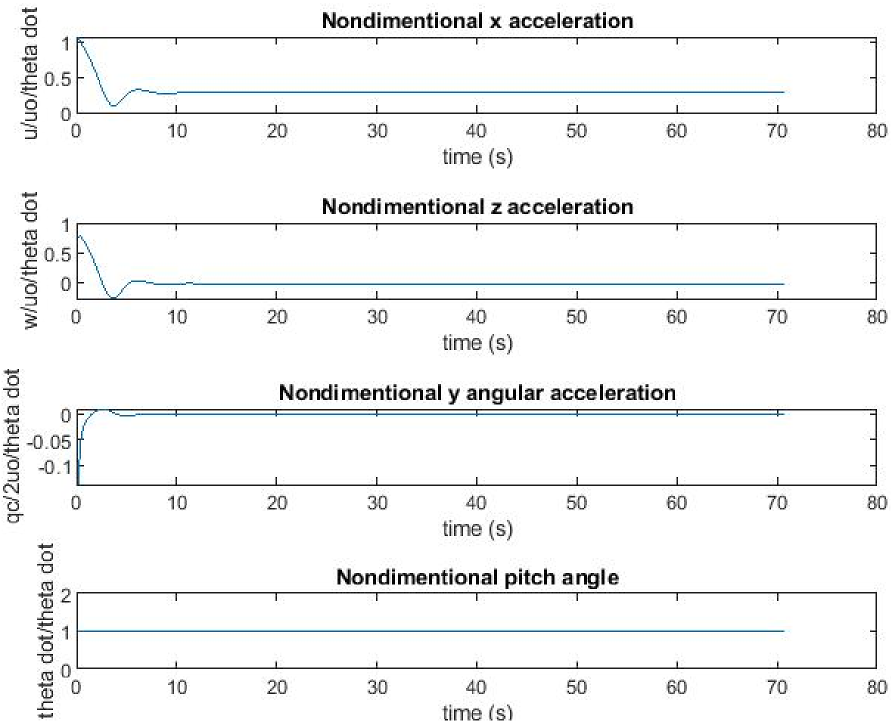
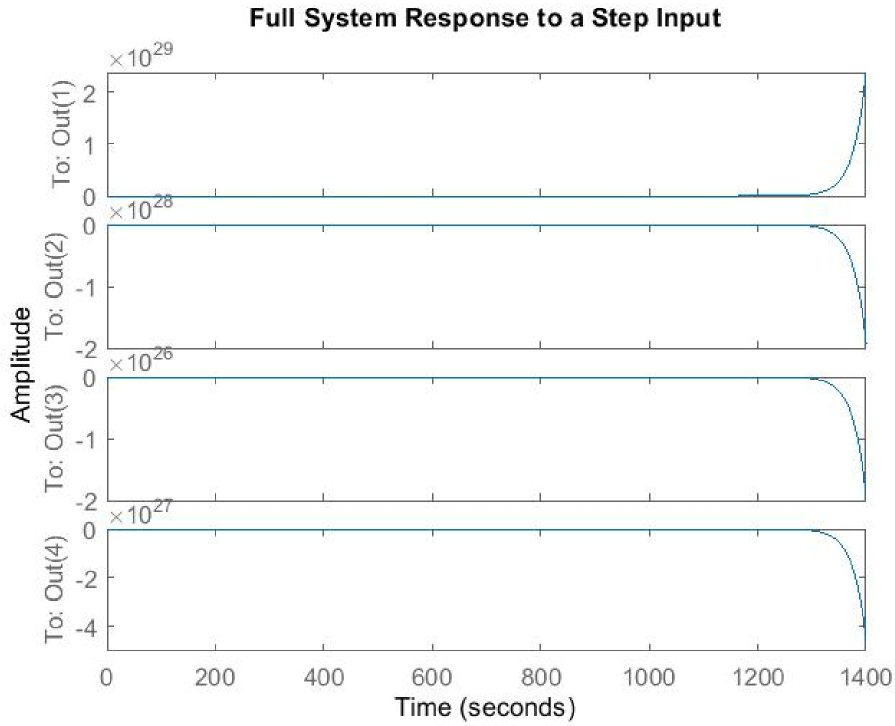


Рисунок 6 Impulse Input



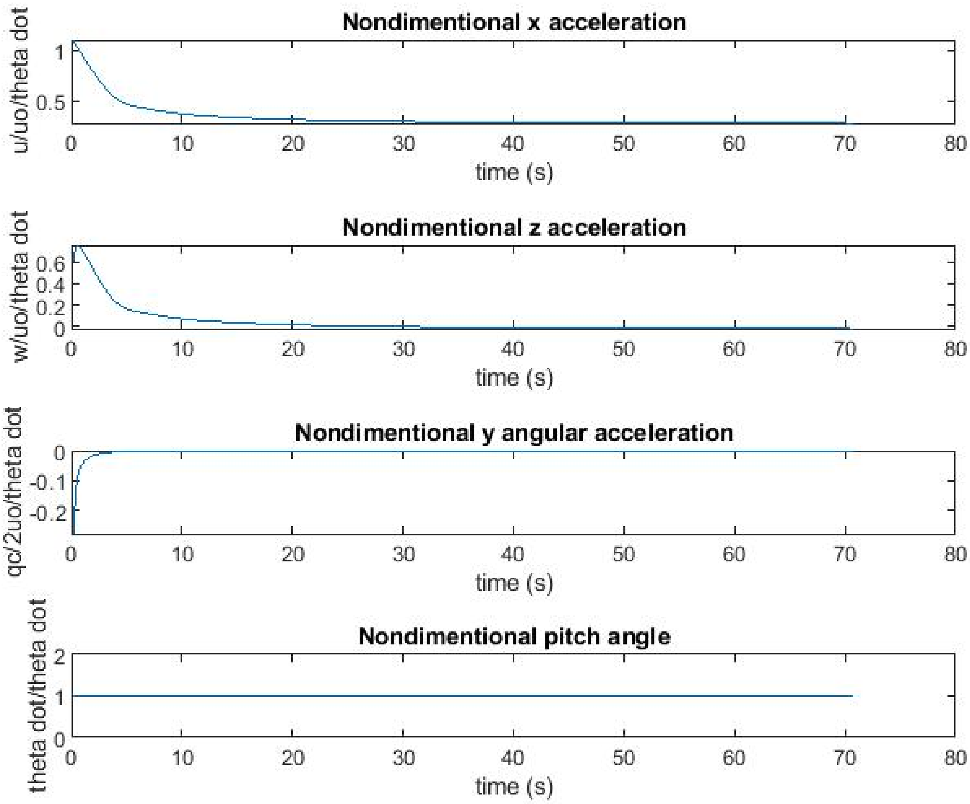
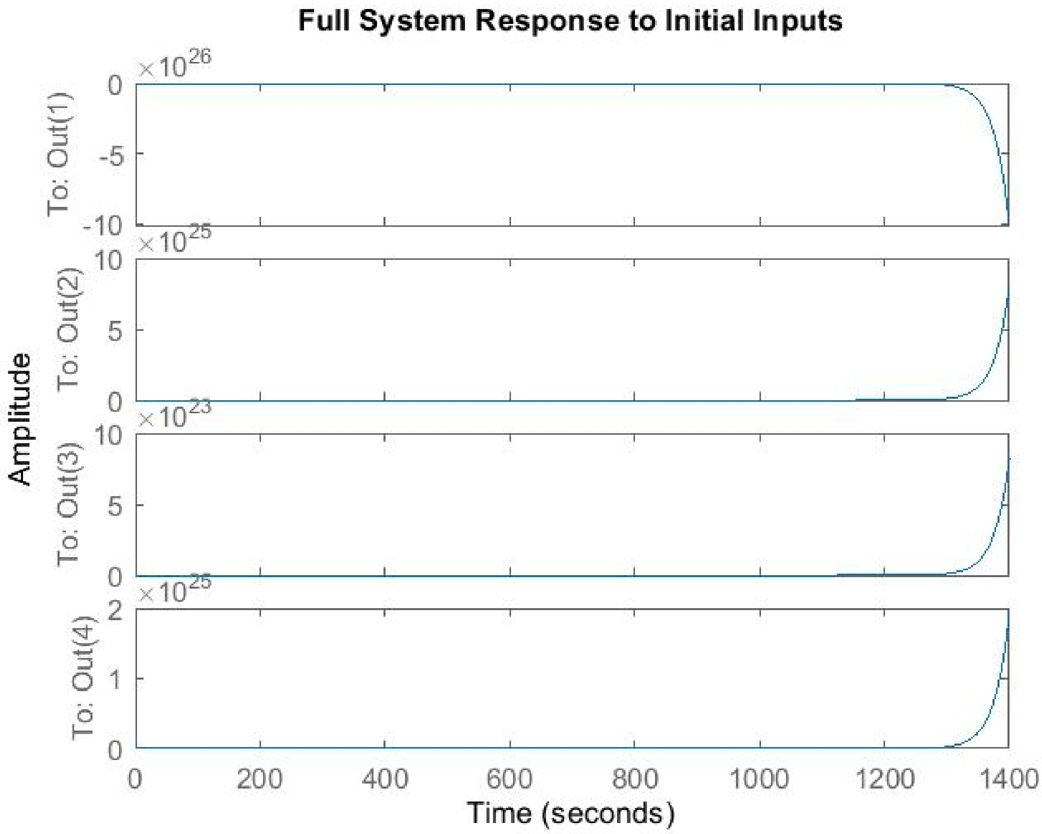
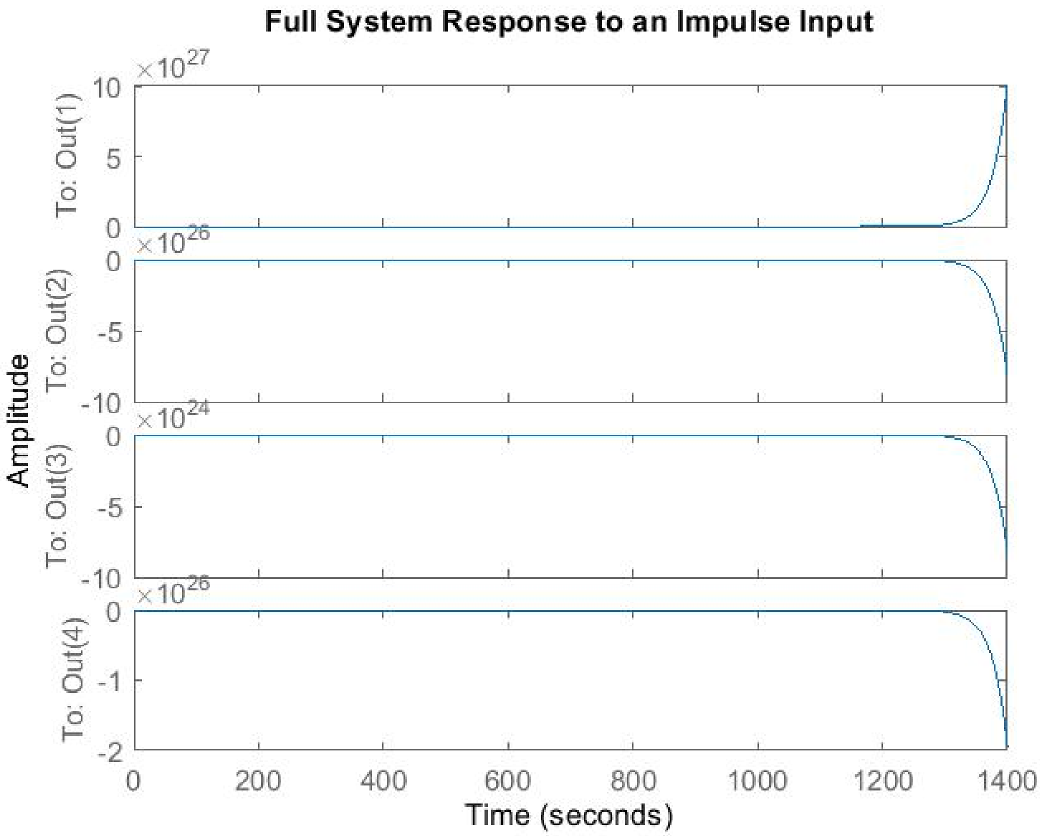
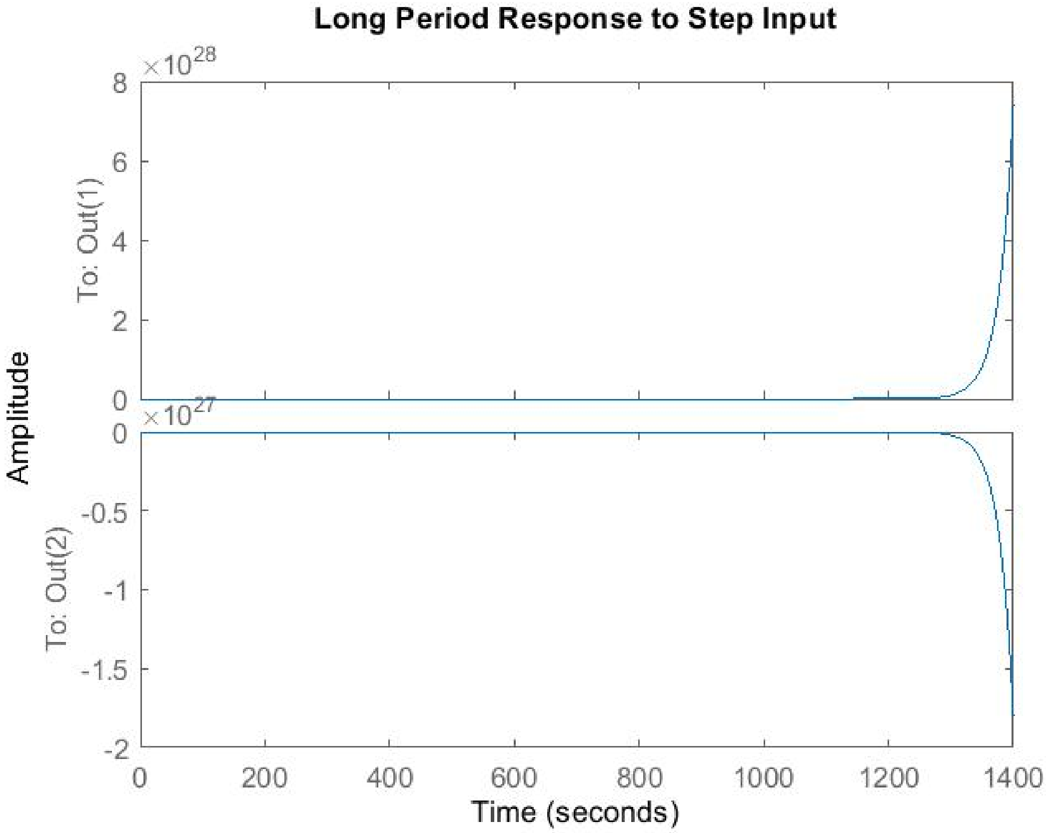


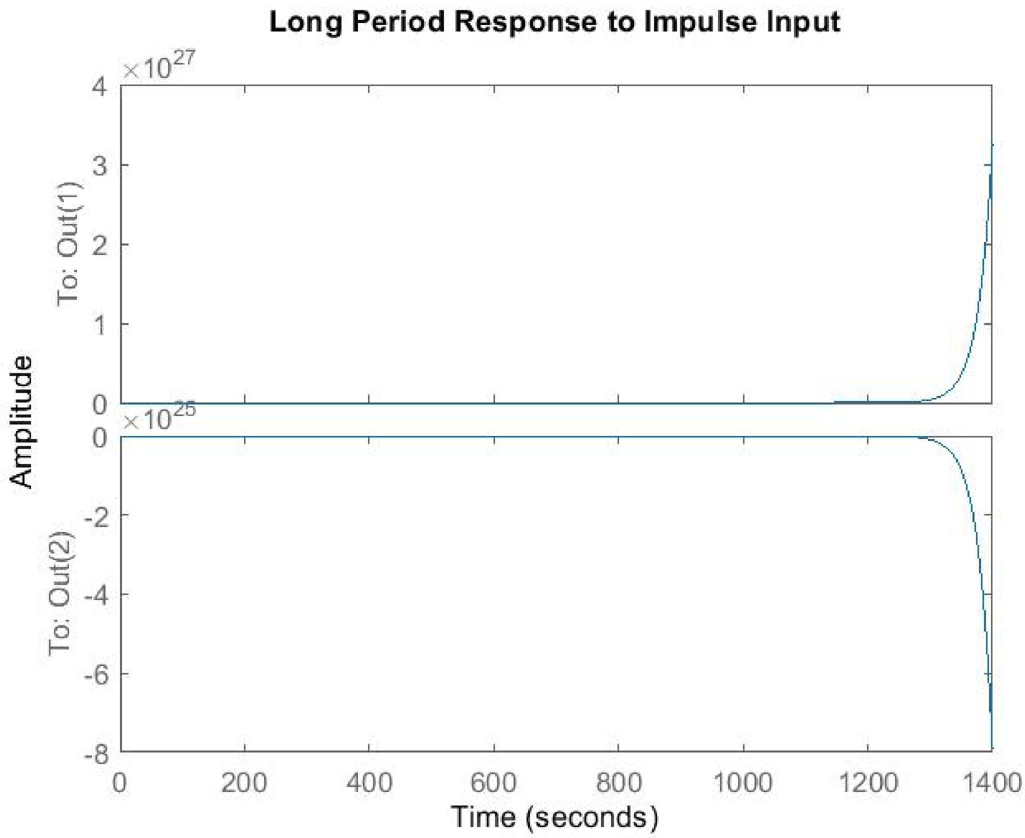
Рисунок 7 Step Input

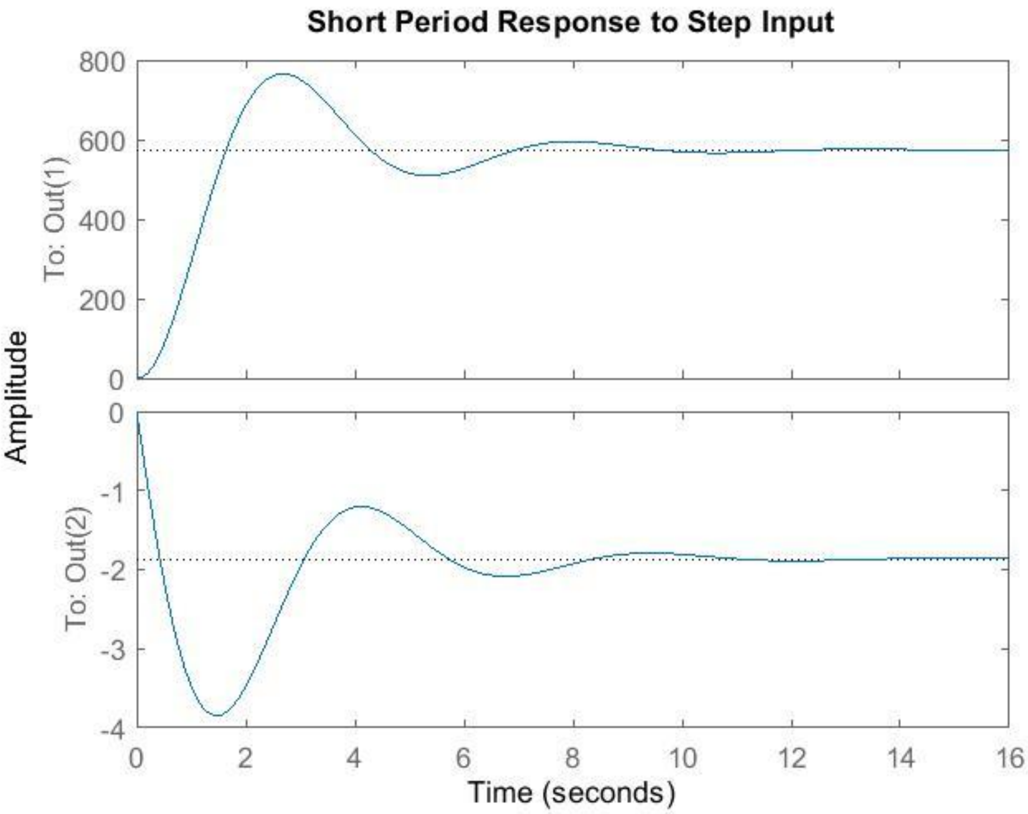

Full System Response to Initial Inputs
Amplitude
To: Out(1)
To: Out(2)
To: Out(3)
To: Out(4)
Time (seconds)

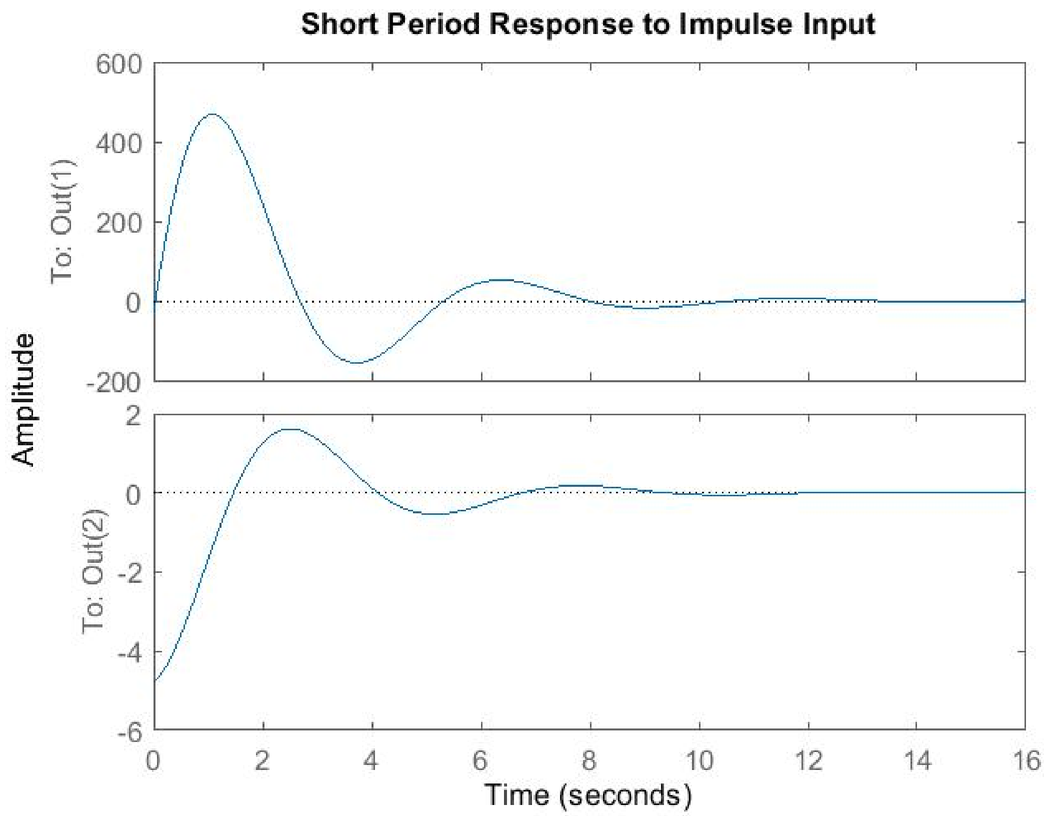

Full System Response to an Impulse Input
Amplitude
To: Out(1)
To: Out(2)
To: Out(3)
To: Out(4)
Time (seconds)

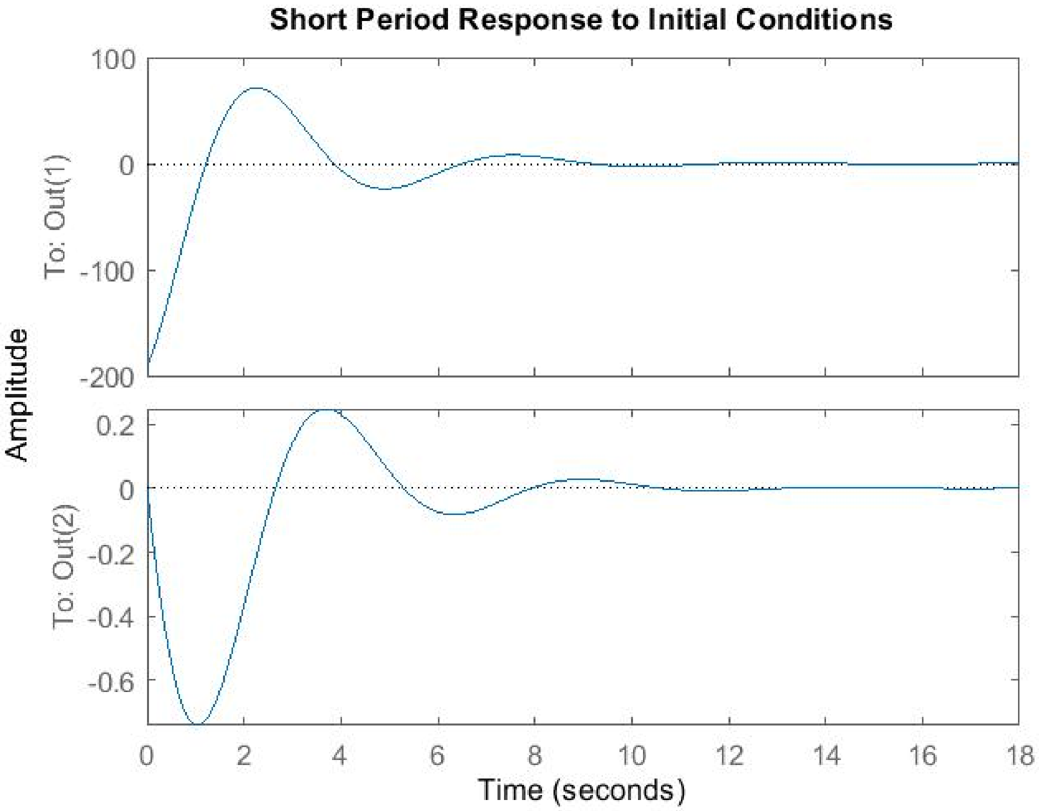

Long Period Response to Step Input
×10^28
Amplitude
To: Out(1)
To: Out(2)
×10^27
Time (seconds)


Long Period Response to Initial Conditions


Long Period Response to Impulse Input
×10^27
Amplitude
To: Out(1)
To: Out(2)
×10^25
Time (seconds)


Short Period Response to Step Input
Amplitude
To: Out(1)
To: Out(2)
800
600
400
200
0
0
-1
-2
-3
-4
0
2
4
6
8
10
12
14
16
Time (seconds)


Short Period Response to Impulse Input
Amplitude
To: Out(1)
To: Out(2)
600
400
200
0
-200
2
0
-2
-4
-6
0
2
4
6
8
10
12
14
16
Time (seconds)


Short Period Response to Initial Conditions
Amplitude
To: Out(1)
To: Out(2)
Time (seconds)
100
0
-100
-200
0.2
0
-0.2
-0.4
-0.6
0
2
4
6
8
10
12
14
16
18

Lateral Mode

$$\begin{pmatrix} \frac{Y_\beta}{u_o} & \frac{Y_p}{u_o} & \frac{Y_r}{u_o}-1 & \frac{g\cos\theta_o}{u_o} \\ \mathcal{L}'_\beta & \mathcal{L}'_p & \mathcal{L}'_r & 0 \\ N'_\beta & N'_p & N'_r & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} -0.178 & 0 & -287 & 0 & 0 \\ -20.9 & -1.38 & 1.16 & 0 & 0 \\ 2.68 & -0.0993 & -0.157 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} Y_{\delta_a} & Y_{\delta_r}/u_o \\ \mathcal{L}'_{\delta_a} & \mathcal{L}'_{\delta_r} \\ N'_{\delta_a} & N'_{\delta r} \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0.0317/287 \\ 4.76 & 5.35 \\ 0.266 & -0.923 \\ 0 & 0 \end{pmatrix}$$

Nondimensionalize the eigen vector:

$$\begin{pmatrix} \dot\beta \\ \dot p \\ \dot r \\ \dot\varphi \end{pmatrix} \rightarrow \begin{pmatrix} \dot\beta/\dot\varphi \\ (\dot p b/2u_o)/\dot\varphi \\ (\dot r b/2u_o)/\dot\varphi \\ \dot\varphi/\dot\varphi \end{pmatrix}$$

ROLLING MODE APPROXIMATION:

This motion can be approximated by the single degree of freedom rolling motion.

$$\dot p = \mathcal{L}'_p\, p + \mathcal{L}'_{\delta_a}\delta_a$$

$$A_{roll} = (\mathcal{L}'_p)$$
$$B_{roll} = (\mathcal{L}'_{\delta_a})$$
$$C_{roll} = (1)$$
$$D_{roll} = (0)$$
$$\lambda_{r_{approx}} = \mathcal{L}'_p = -1.38$$

The magnitude of the roll damping $\mathcal{L}'_p$, is dependent on the size of the wing and tail surfaces.

$$TF_{Rolling} = \frac{p(s)}{\delta_a(s)} = \frac{\mathcal{L}'_{\delta_a}}{s-\mathcal{L}'_p} = \frac{4.76}{s+1.38}$$

DUTCH ROLL APPROXIMATION:

$$\begin{pmatrix} \dot\beta \\ \dot r \end{pmatrix} = \begin{pmatrix} \frac{Y_\beta}{u_o} & \frac{Y_r}{u_o}-1 \\ N_\beta & N_r \end{pmatrix} \begin{pmatrix} \beta \\ r \end{pmatrix} + \begin{pmatrix} Y_{\delta_a} & Y_{\delta_r}/u_o \\ N'_{\delta_a} & N'_{\delta_r} \end{pmatrix} \begin{pmatrix} \delta_a \\ \delta_r \end{pmatrix}$$

SPIRAL MODE APPROXIMATION:

Neglecting the side force equation and $\Delta\varphi$,

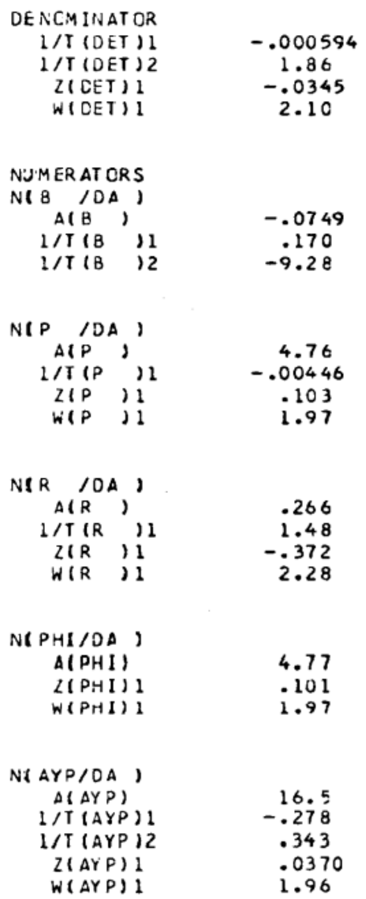
$$\mathcal{L}'_\beta \Delta\beta + \mathcal{L}'_r \Delta r = 0$$
$$\Delta\dot{r} = N_\beta\, \Delta\dot{\beta} + N_r \Delta r$$
$$\Delta\dot{r} + (\mathcal{L}'_r N_\beta - \mathcal{L}'_\beta N_r)\, \Delta r / \mathcal{L}'_\beta \ = 0$$
$$A_{spiral} = \left(\frac{(\mathcal{L}'_r N_\beta - \mathcal{L}'_\beta N_r)}{\mathcal{L}'_\beta}\right)$$
$$B_{spiral} = (0)$$
$$C_{spiral} = (1)$$
$$D_{spiral} = (0)$$
$$\lambda_{spiral} = \frac{(-\mathcal{L}'_r N_\beta + \mathcal{L}'_\beta N_r)}{\mathcal{L}'_\beta} = -8.254 * 10^{-3}$$

	Pole	Damping	Frequency (rad/s)	Time Constant (s)
Full System	0	-1	0	Inf
	39.2	-1	3.92e+01	-2.55e-02
	-40.3	1	4.03e+01	2.48e-02
	-5.65e-03	1	5.65e-03	1.77e+02
	-6.22e-01	1	6.22e-01	1.61

	Pole	Damping	Frequency (rad/s)	Time Constant(s)
Spiral M	8.25e-03	-1	8.25e-03	-1.21e+02
Dutch Mode	$-1.67 \pm 1.64i$	1.02e-01	1.65e+00	5.97e+00
Roll Mode	-1.38	1	1.38	0.725

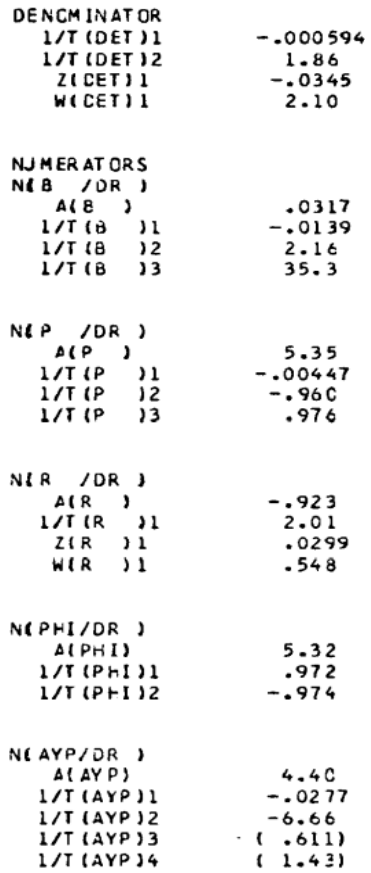
	Spiral Mode	Dutch Mode	Roll Mode
Rise Time	0	__	1.5920
Settling Time	0	__	2.8348
Settling Min	0	__	3.1199

Settling Max	0	__	3.4492
Overshoot	Inf	__	0
Undershoot	0	__	0
Peak	0	__	3.4492
Peak Time	0	__	7.6419



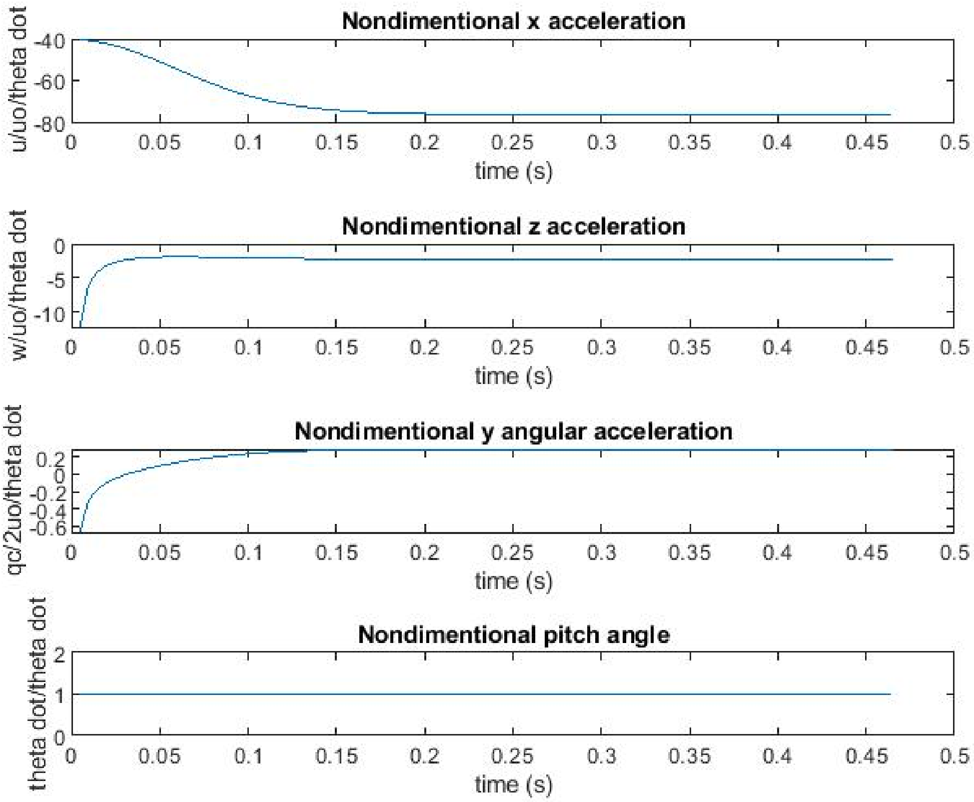
```
DENOMINATOR
   1/T(DET)1        -.000594
   1/T(DET)2         1.86
   Z(DET)1          -.0345
   W(DET)1           2.10

NUMERATORS
N(B   /DA )
   A(B   )          -.0749
   1/T(B   )1        .170
   1/T(B   )2       -9.28

N(P   /DA )
   A(P   )           4.76
   1/T(P   )1       -.00446
   Z(P   )1          .103
   W(P   )1          1.97

N(R   /DA )
   A(R   )           .266
   1/T(R   )1        1.48
   Z(R   )1         -.372
   W(R   )1          2.28

N(PHI/DA )
   A(PHI)            4.77
   Z(PHI)1           .101
   W(PHI)1           1.97

N(AYP/DA )
   A(AYP)            16.5
   1/T(AYP)1        -.278
   1/T(AYP)2         .343
   Z(AYP)1           .0370
   W(AYP)1           1.96
```

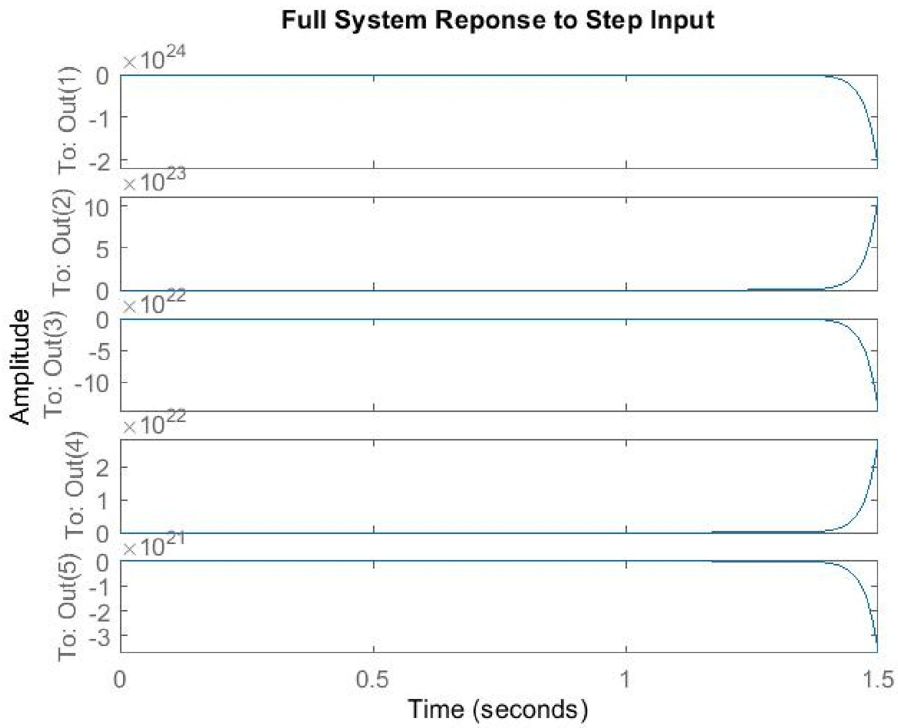
Рисунок 9 Aileron Transfer Function Factors

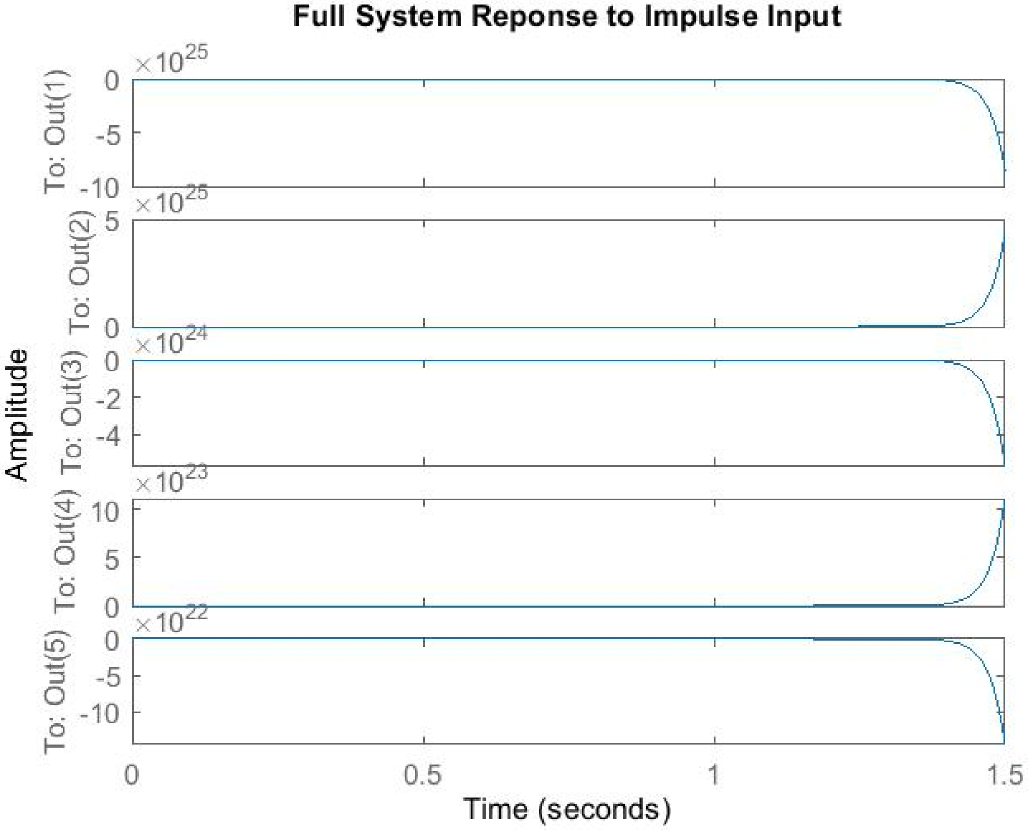


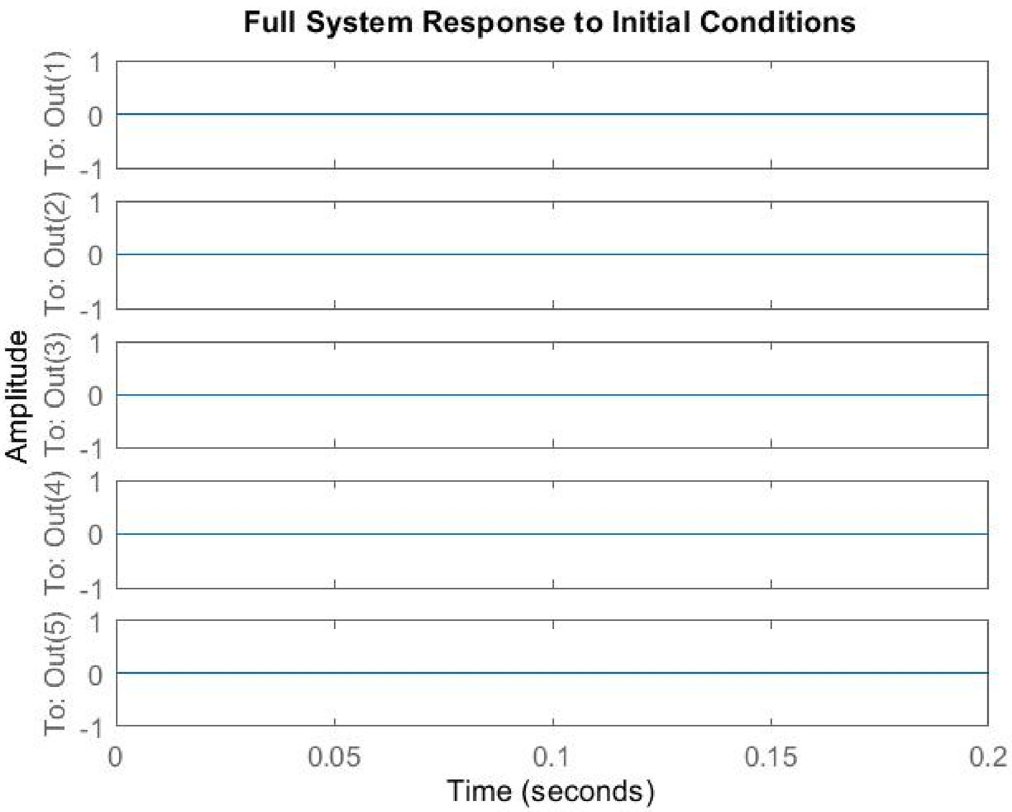
```
DENOMINATOR
   1/T(DET)1        -.000594
   1/T(DET)2         1.86
   Z(DET)1          -.0345
   W(DET)1           2.10

NUMERATORS
N(B   /DR )
   A(B   )           .0317
   1/T(B   )1       -.0139
   1/T(B   )2        2.16
   1/T(B   )3        35.3

N(P   /DR )
   A(P   )           5.35
   1/T(P   )1       -.00447
   1/T(P   )2       -.960
   1/T(P   )3        .976

N(R   /DR )
   A(R   )          -.923
   1/T(R   )1        2.01
   Z(R   )1          .0299
   W(R   )1          .548

N(PHI/DR )
   A(PHI)            5.32
   1/T(PHI)1         .972
   1/T(PHI)2        -.974

N(AYP/DR )
   A(AYP)            4.40
   1/T(AYP)1        -.0277
   1/T(AYP)2        -6.66
   1/T(AYP)3       ( .611)
   1/T(AYP)4       ( 1.43)
```

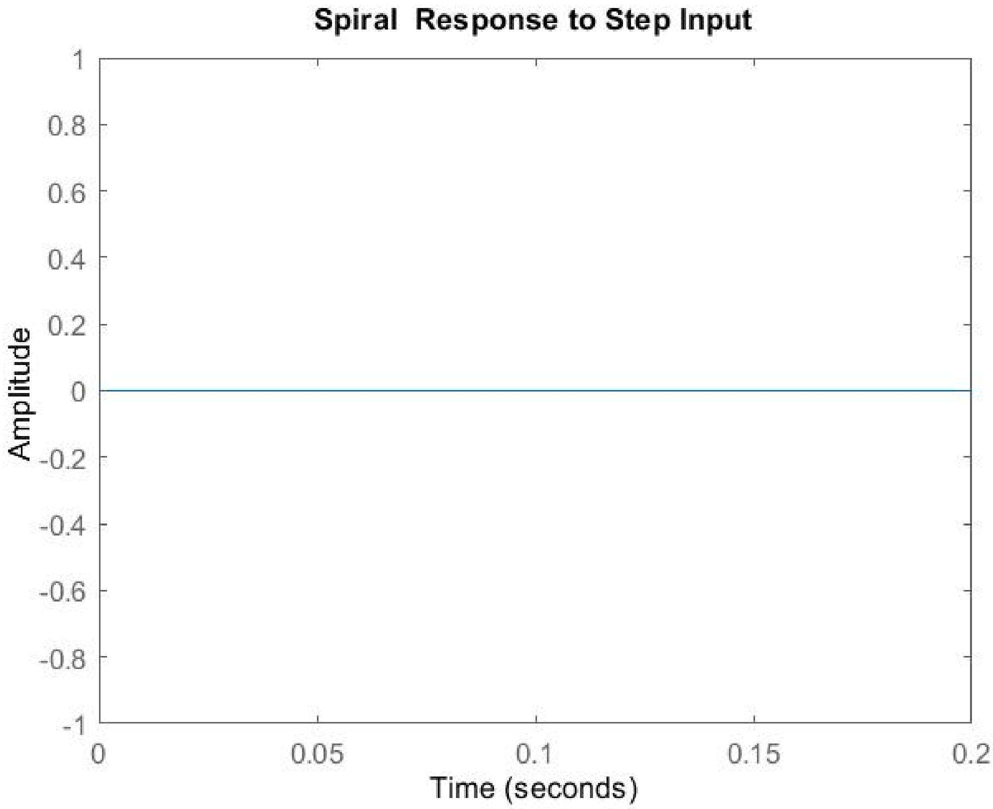
Рисунок 8 Rudder Transfer Function Factors

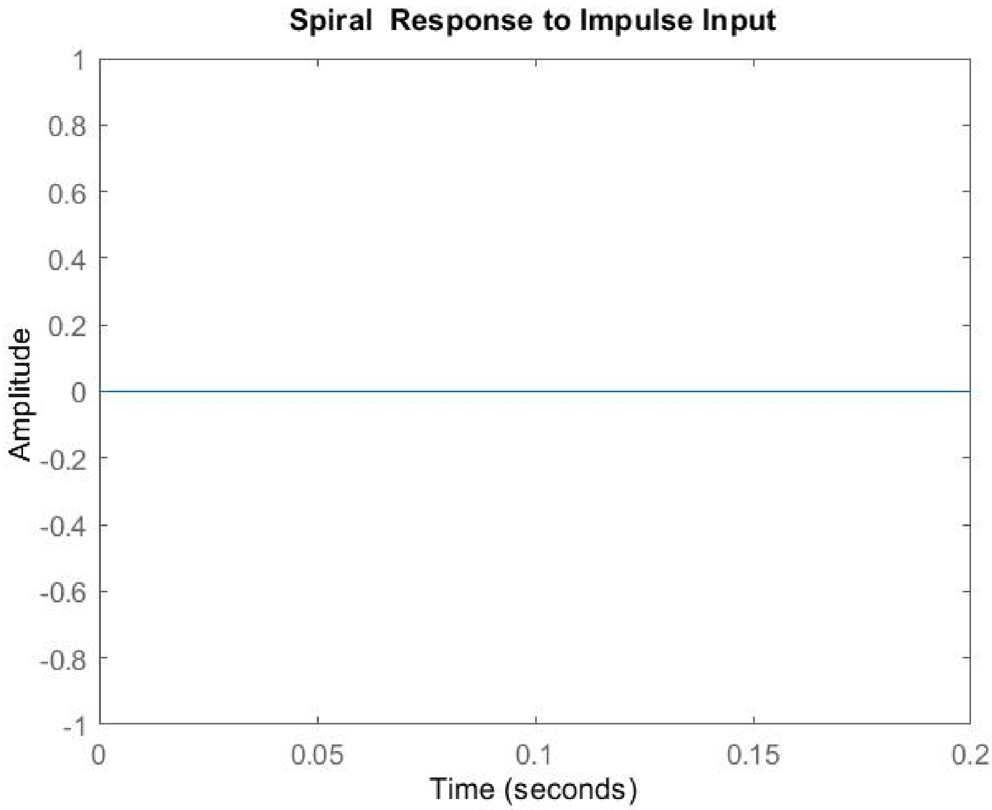

Nondimentional x acceleration
u/uo/theta dot
time (s)
Nondimentional z acceleration
w/uo/theta dot
time (s)
Nondimentional y angular acceleration
qc/2uo/theta dot
time (s)
Nondimentional pitch angle
theta dot/theta dot
time (s)

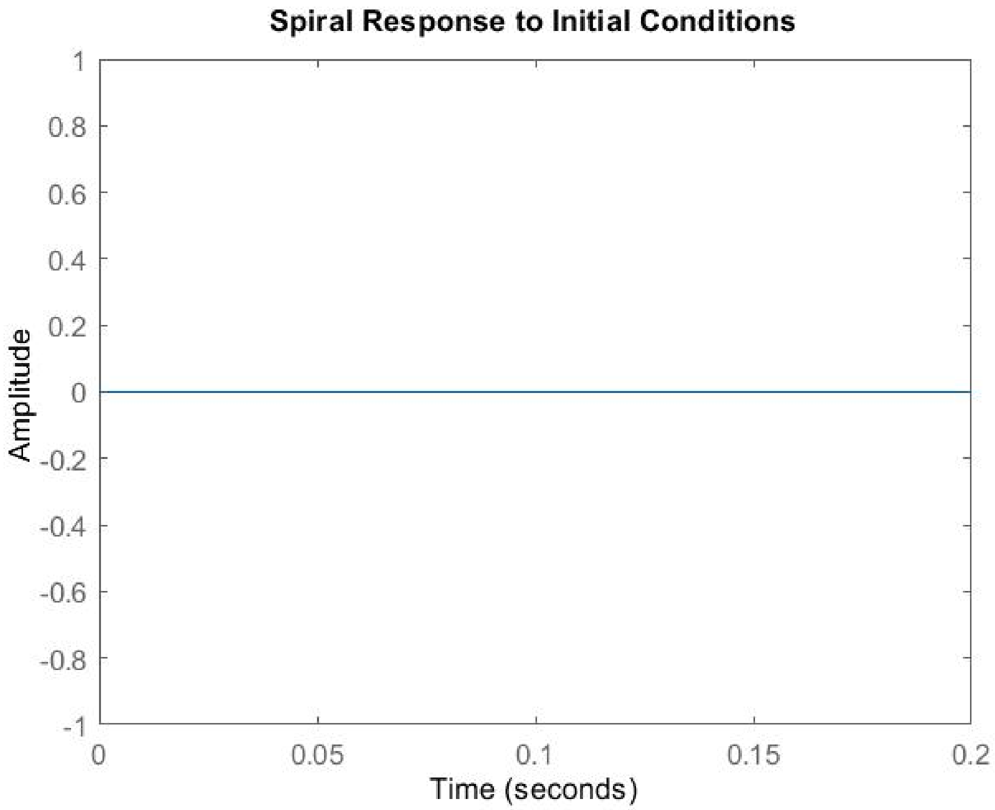

Full System Reponse to Step Input
Amplitude
To: Out(1)
To: Out(2)
To: Out(3)
To: Out(4)
To: Out(5)
Time (seconds)

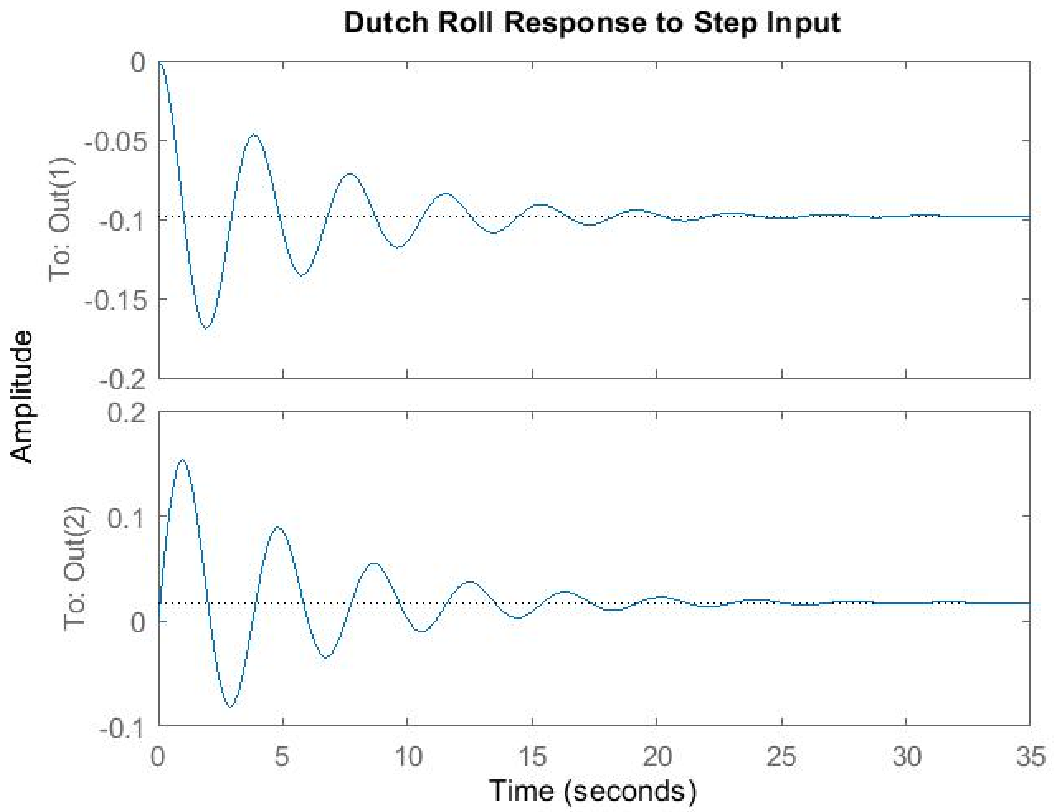

Full System Reponse to Impulse Input
Amplitude
To: Out(1)
To: Out(2)
To: Out(3)
To: Out(4)
To: Out(5)
×10^25
×10^25
×10^24
×10^23
×10^22
0
-5
-10
5
0
0
-2
-4
10
5
0
0
-5
-10
0
0.5
1
1.5
Time (seconds)

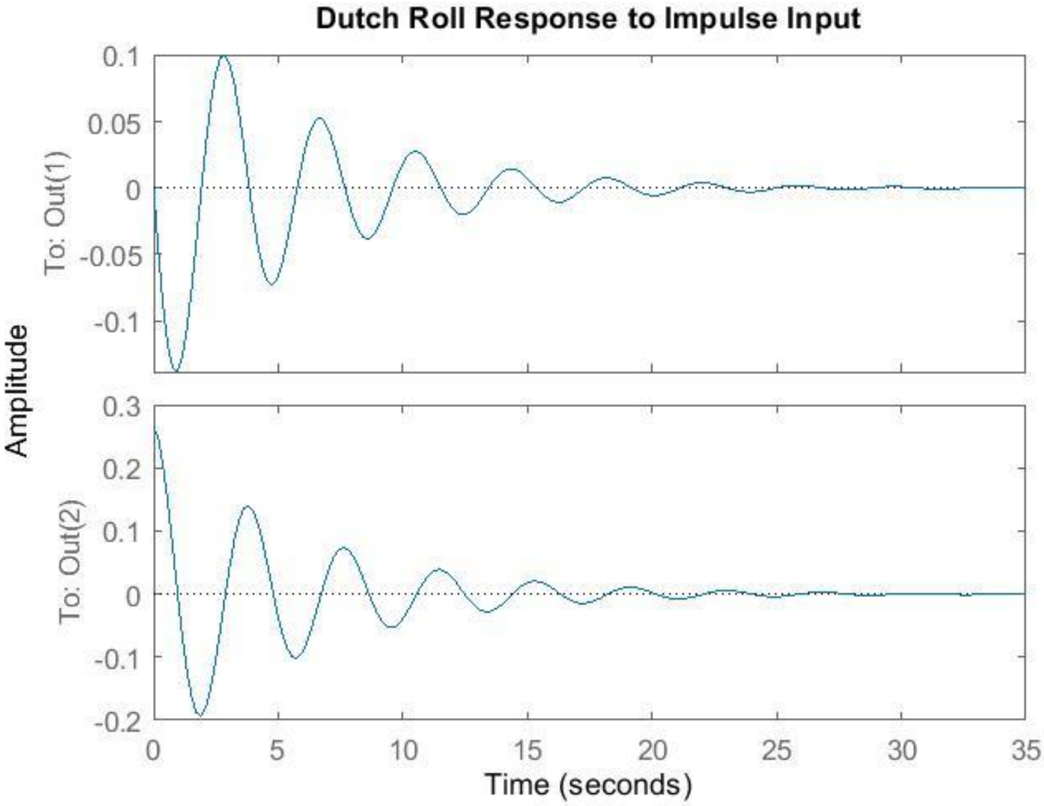

Full System Response to Initial Conditions
Amplitude
To: Out(1)
To: Out(2)
To: Out(3)
To: Out(4)
To: Out(5)
1
0
-1
0
0.05
0.1
0.15
0.2
Time (seconds)

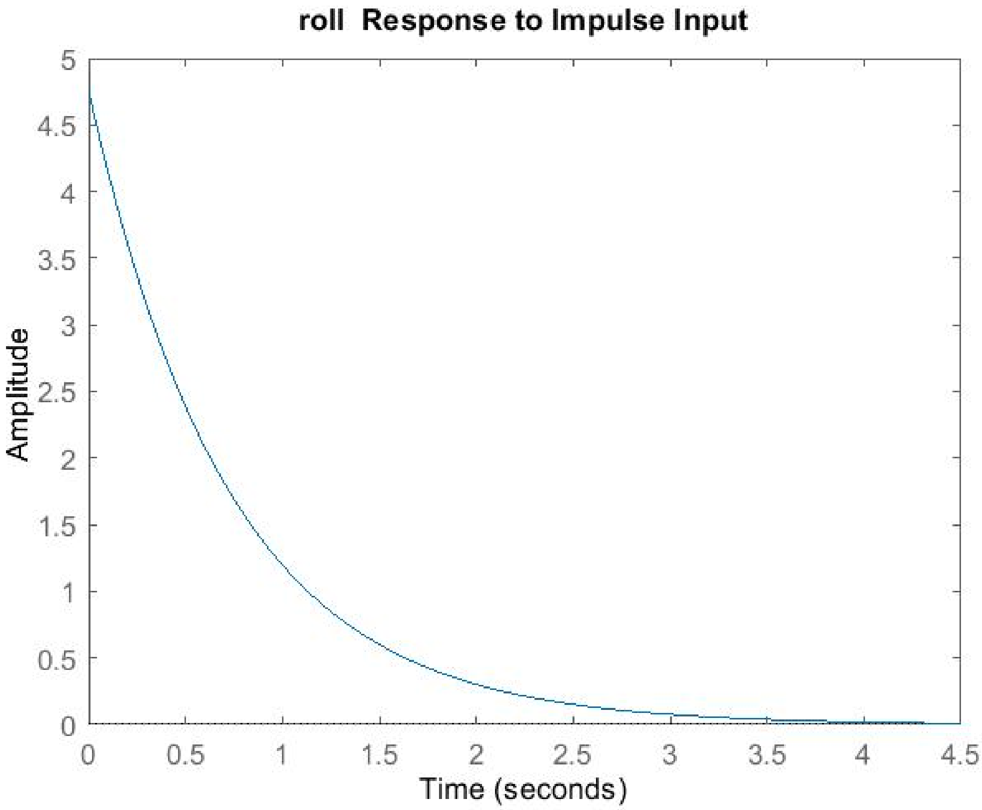

Spiral Response to Step Input
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
Amplitude
0
0.05
0.1
0.15
0.2
Time (seconds)

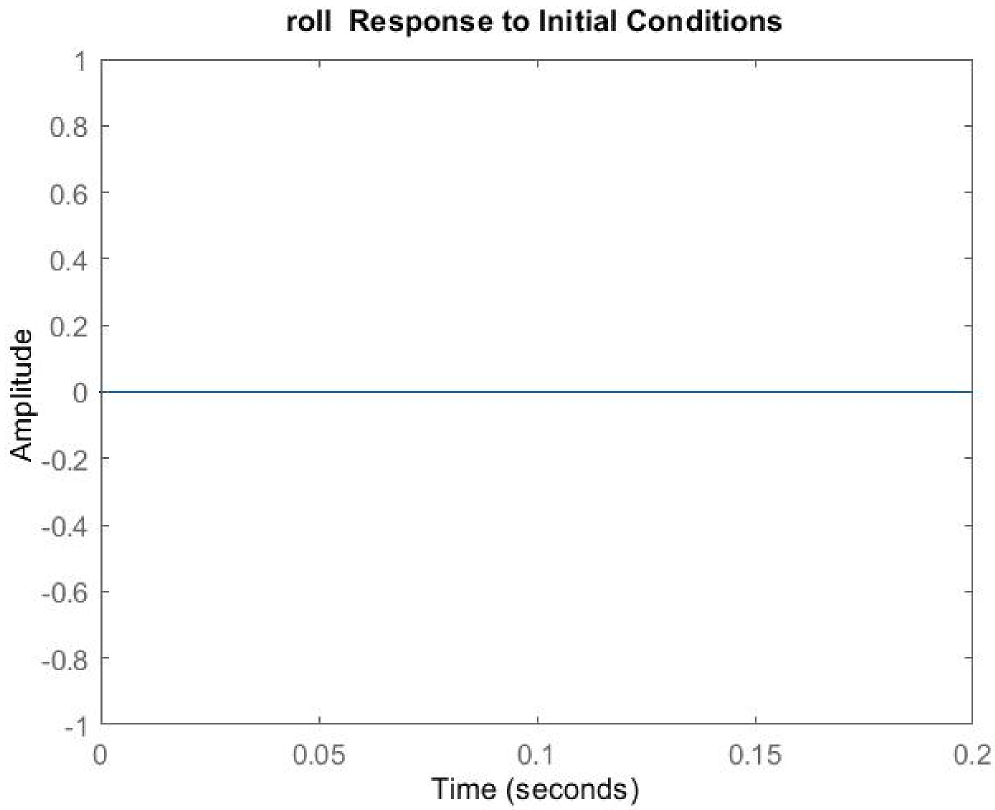

Spiral Response to Impulse Input
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
Amplitude
0
0.05
0.1
0.15
0.2
Time (seconds)

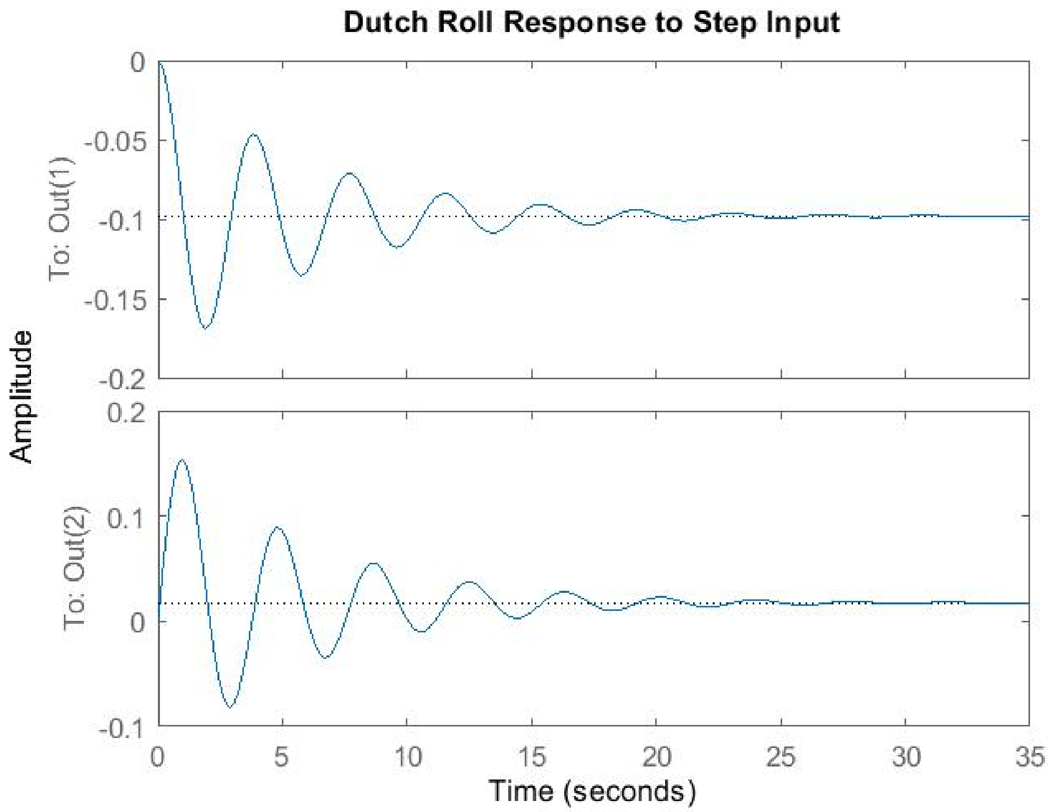

Spiral Response to Initial Conditions
Amplitude
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
0.05
0.1
0.15
0.2
Time (seconds)

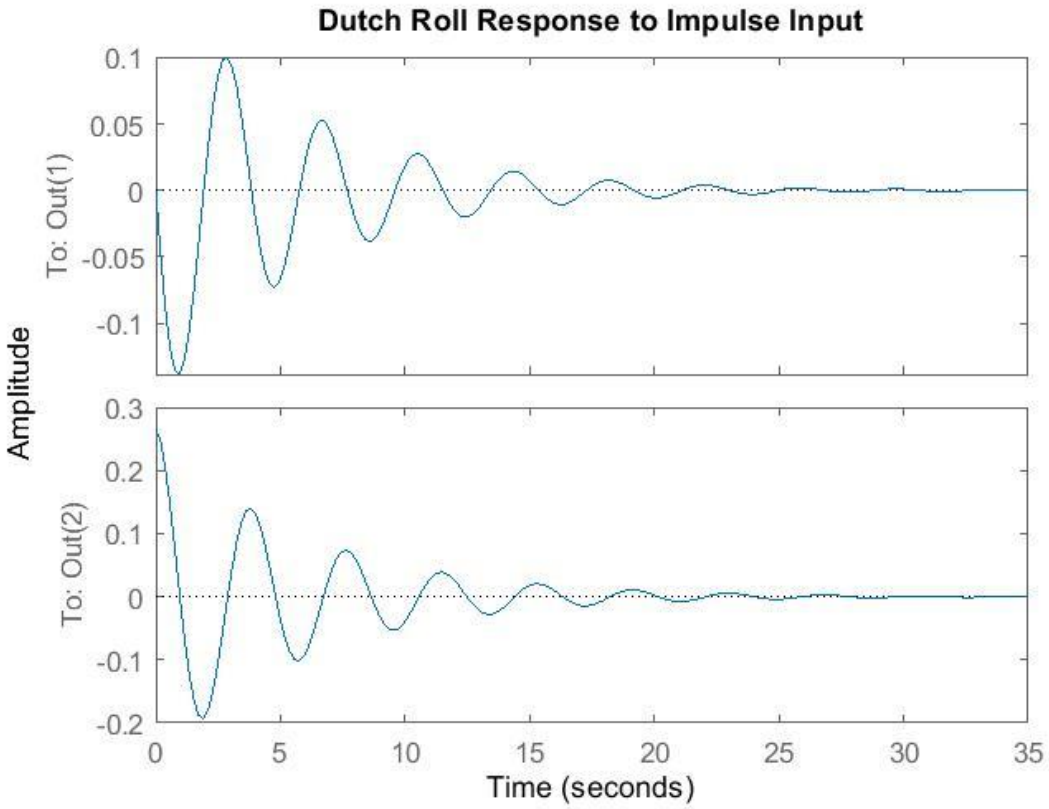

Dutch Roll Response to Step Input
Amplitude
To: Out(1)
0
-0.05
-0.1
-0.15
-0.2
To: Out(2)
0.2
0.1
0
-0.1
0
5
10
15
20
25
30
35
Time (seconds)


Dutch Roll Response to Impulse Input
Amplitude
To: Out(1)
To: Out(2)
0.1
0.05
0
-0.05
-0.1
0.3
0.2
0.1
0
-0.1
-0.2
0
5
10
15
20
25
30
35
Time (seconds)


roll Response to Impulse Input
Amplitude
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
Time (seconds)


roll Response to Initial Conditions
Amplitude
Time (seconds)
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
0.05
0.1
0.15
0.2


Dutch Roll Response to Step Input
Amplitude
To: Out(1)
To: Out(2)
0
-0.05
-0.1
-0.15
-0.2
0.2
0.1
0
-0.1
0
5
10
15
20
25
30
35
Time (seconds)


Dutch Roll Response to Impulse Input
Amplitude
To: Out(1)
To: Out(2)
0.1
0.05
0
-0.05
-0.1
0.3
0.2
0.1
0
-0.1
-0.2
0
5
10
15
20
25
30
35
Time (seconds)

Autopilot Controller Design

In this part of project, we are working on designing two autopilot for two different moving

The Longitudinal Autopilot

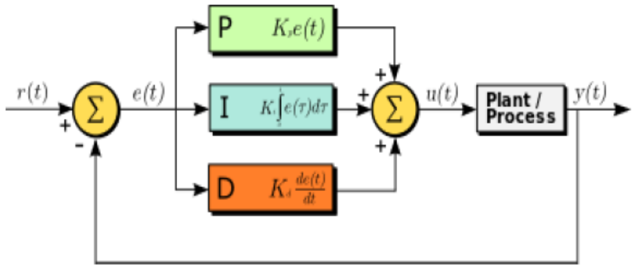
The role of the longitudinal autopilot is to control the motion in the longitudinal plane by controlling the elevator and thrust to achieve the desired command of altitude or Climb angle.

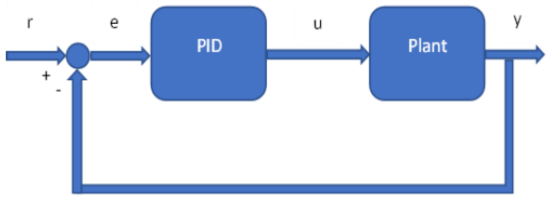
The Lateral Autopilot

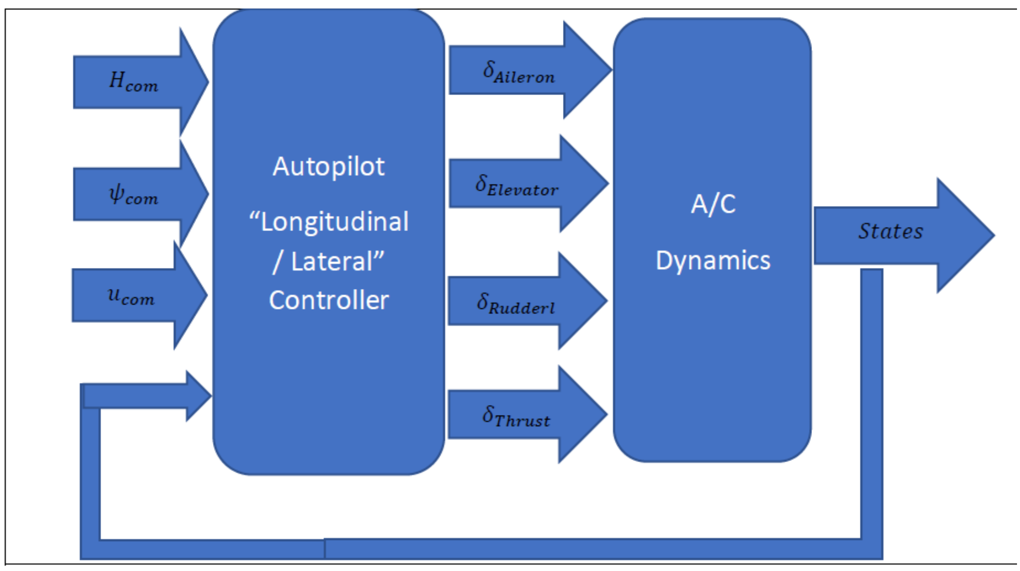
The rule of the lateral autopilot is to control the motion in the lateral-directional plane by controlling the rudder and aileron to achieve a coordinated turn.

We will use the linearized state-space model of the aircraft dynamics to represent the motion of the airplane, which is a multi-input multi-output system, and to design our controllers we will use our previous studies about the linear time invariant single input single output system to design our controllers. This method is called "Successive loop closure". Which is a PID controller for control loop mechanism employing feedback that is widely used in industrial control systems and a variety of other applications requiring continuously modulated control. A PID controller continuously calculates an error value e(t) as the difference between a desired set point (SP) and a measured process variable and applies a correction based on proportional, integral, and derivative terms (denoted P, I, and D respectively), hence the name.

$$u(t) = K_p + K_D e(t) + K_I \int e(t)\,dt$$







LATERAL CONTROLLER

In Lateral Controller we have 3 motions Turn & Coordinated Turn & Coordinated Level turn

Turn Motion:

Turning is a maneuver in which the airplane's heading angle is changed (airplane turns) but experiencing lateral acceleration, i.e. ($a_z \neq 0$).

In terms of Aerodynamics, turning is a maneuver in which the airplane's heading angle is changed (airplane turns) but with "slipping or skidding", i.e., the side slip angle ($\beta \neq 0$).

Coordinated turn Motion:

In terms of mechanics, the coordinated turn is a maneuver in which the airplane's heading angle is changed (airplane turns) without experiencing any lateral acceleration, i.e., (az=0). We may call it "zero-lateral acceleration turn"

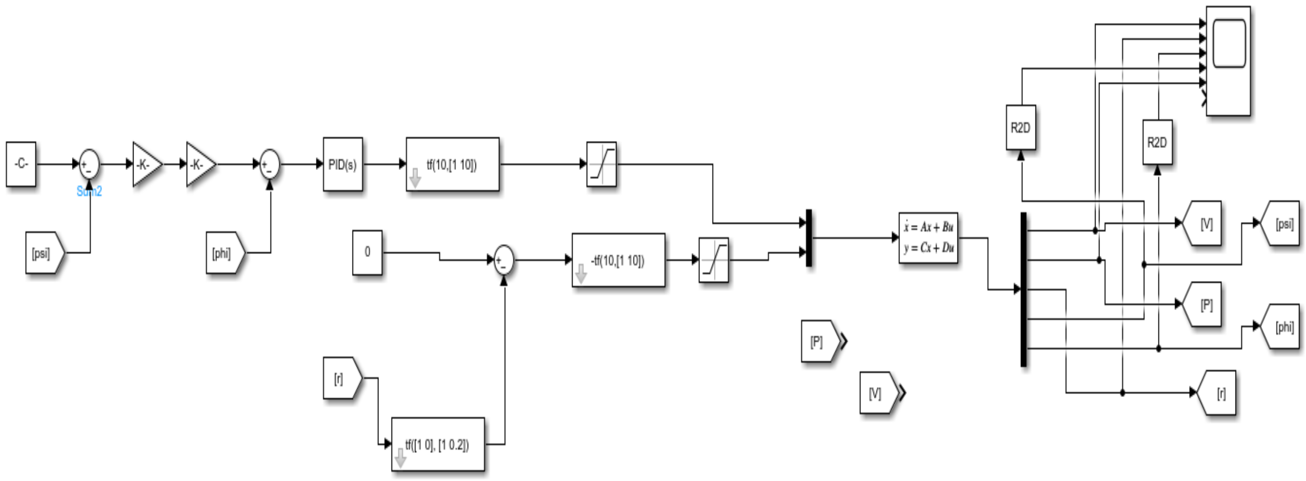
In terms of Aerodynamics, the coordinated turn is a maneuver in which the airplane's heading angle is changed (airplane turns) without "slipping or skidding", i.e., the side slip angle (β = 0).

Coordinated Level Turn Motion:

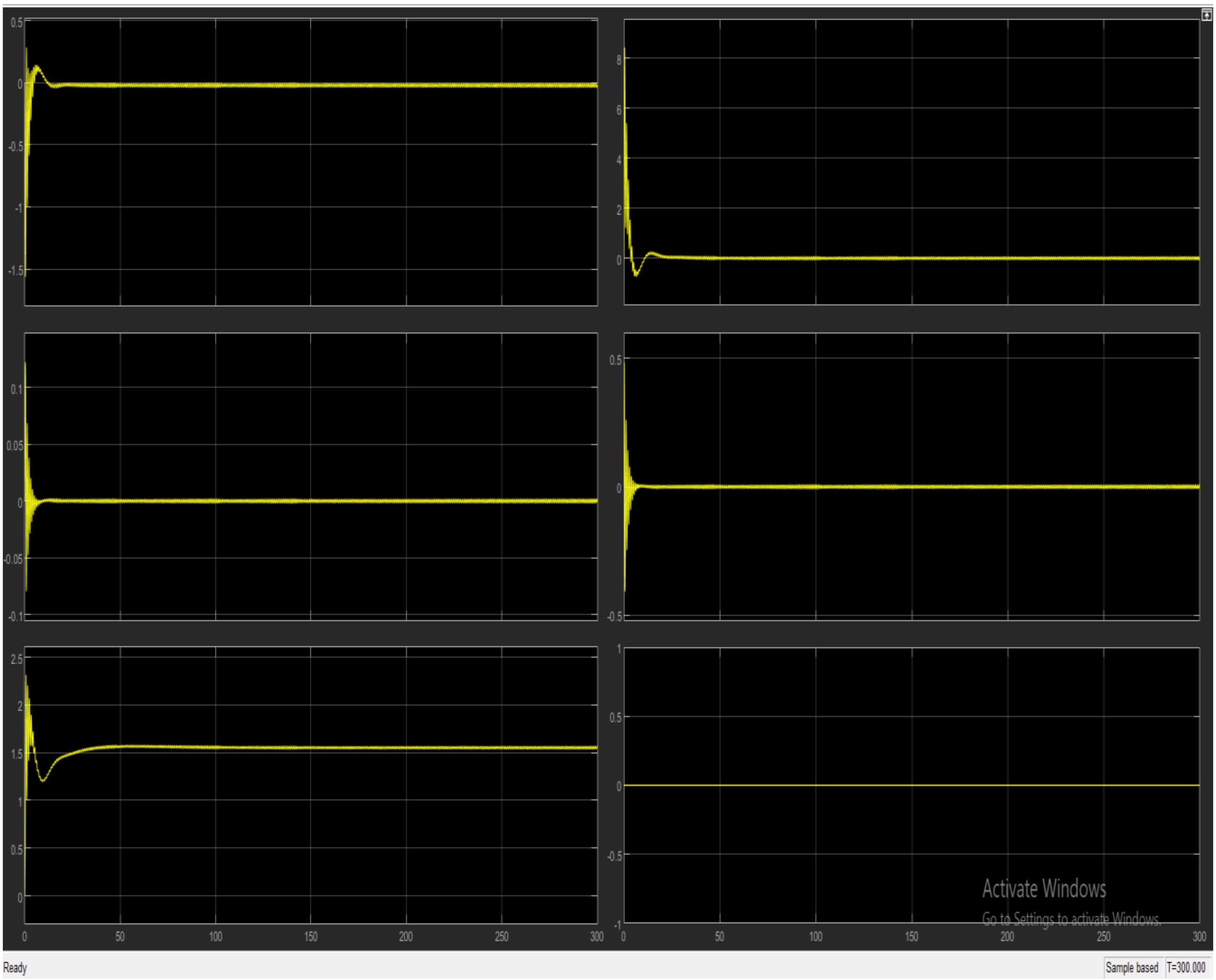
In terms of mechanics, the coordinated level turn is a maneuver in which the airplane's heading angle is changed (airplane turns) without experiencing any lateral acceleration, i.e., (az=0). Without losing altitude.

In terms of Aerodynamics, the coordinated level turn is a maneuver in which the airplane's heading angle is changed. We will view that with and without controller.

With controller (PID)



After trying PID is more suitable than any other controller



P= 0.363250528215049

I = 0.0138253028436041

D = 2.12060034133301

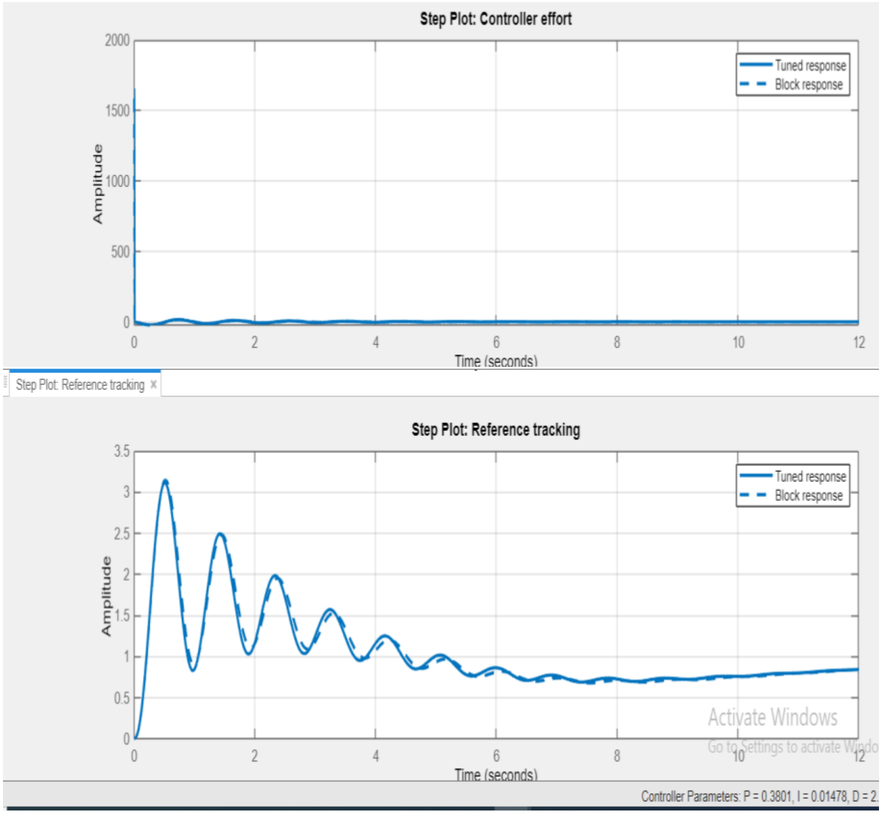
But the aircraft is not stable as the poles has positive one pole, so I will do some poles placement to make the aircraft stable. The A & B& C &D were

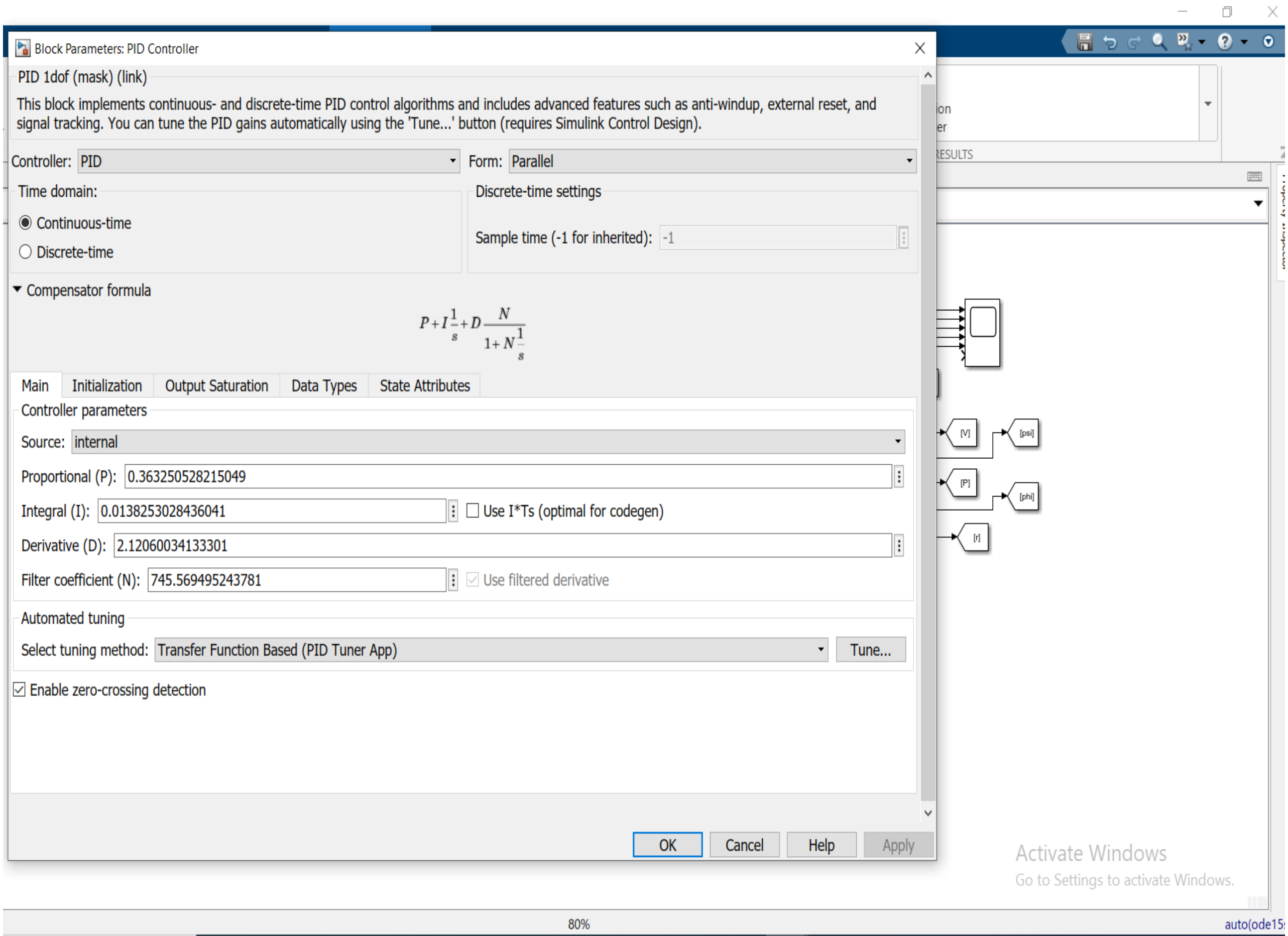
```
A_latr_full =

   -0.1780  -51.1000  191.2212  -32.1740         0
   -0.0048   -1.3800    1.1600         0         0
    0.0003    0.0993   -0.1570         0         0
         0    1.0000         0         0         0
         0         0    1.0000         0         0


B_latr_full =

         0    0.0317
    4.7600    5.3500
    0.2660   -0.9230
         0         0
         0         0


C_latr_full =

     1     0     0     0     0
     0     1     0     0     0
     0     0     1     0     0
     0     0     0     1     0
     0     0     0     0     1


D_latr_full =

     0     0
     0     0
     0     0
     0     0
     0     0

    Pole          Damping        Frequency       Time Constant
                               (rad/TimeUnit)      (TimeUnit)

  0.00e+00      -1.00e+00       0.00e+00              Inf
 -1.62e+00       1.00e+00       1.62e+00         6.17e-01
  2.95e-01      -1.00e+00       2.95e-01        -3.39e+00
 -7.58e-02       1.00e+00       7.58e-02         1.32e+01
 -3.14e-01       1.00e+00       3.14e-01         3.18e+00
```

Now, doing poles placement with changing the sign of negative pole to be positive by getting K and then

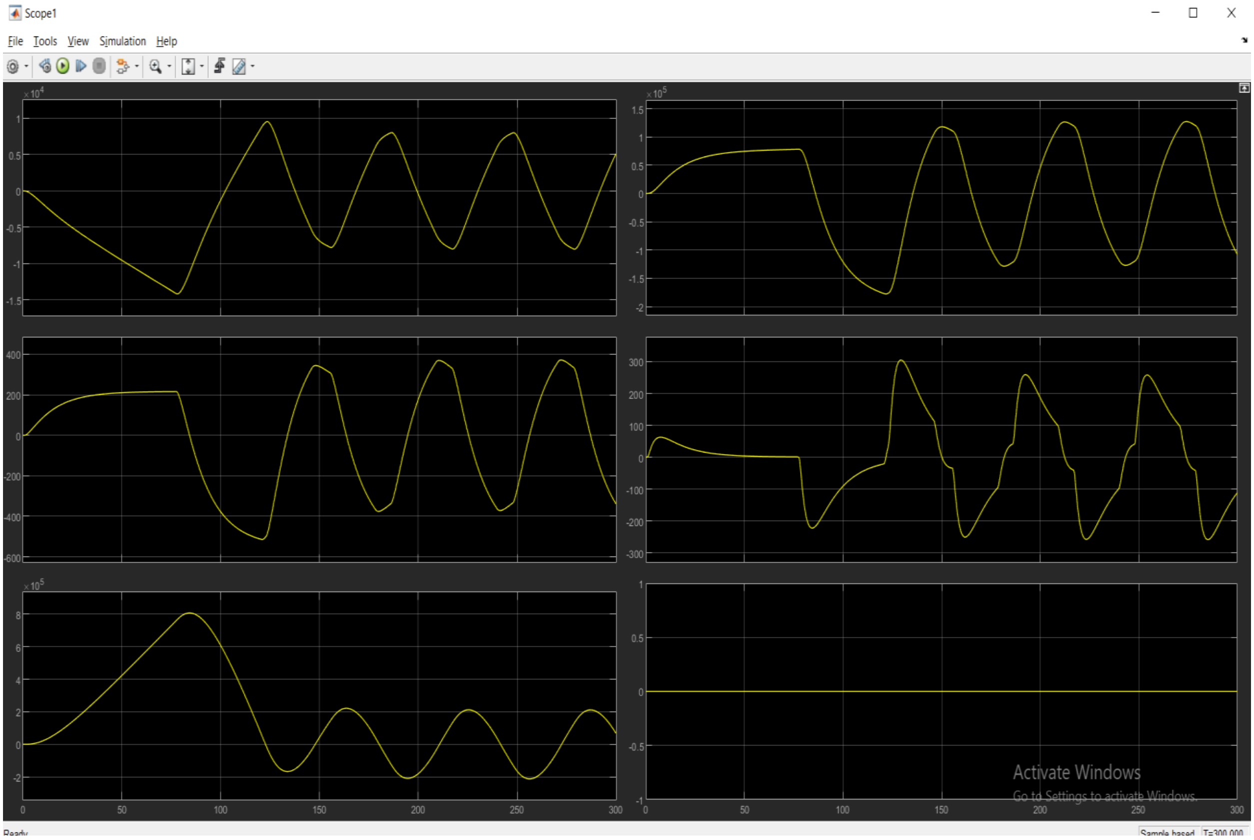
$$A_{stable} = A - B * K$$

```
p =

         0   -0.0758   -0.2950   -0.3140   -1.6200


K =

    0.0177   -7.4939   29.2806   -4.7615    0.0339
   -0.0084    3.1481  -12.6004    2.0818   -0.0148


A_latr_full1 =

   -0.1777  -51.1998  191.6206  -32.2400    0.0005
   -0.0444   17.4487  -70.8039   11.5273   -0.0824
   -0.0121    4.9983  -19.5758    3.1880   -0.0226
         0    1.0000         0         0         0
         0         0    1.0000         0         0


   Pole        Damping       Frequency      Time Constant
                           (rad/seconds)     (seconds)

 -5.18e-13     1.00e+00       5.18e-13        1.93e+12
 -7.58e-02     1.00e+00       7.58e-02        1.32e+01
 -2.95e-01     1.00e+00       2.95e-01        3.39e+00
 -3.14e-01     1.00e+00       3.14e-01        3.18e+00
 -1.62e+00     1.00e+00       1.62e+00        6.17e-01
```


Step Plot: Controller effort
Tuned response
Block response
Amplitude
Time (seconds)
Step Plot: Reference tracking
Step Plot: Reference tracking
Tuned response
Block response
Amplitude
Time (seconds)
Activate Windows
Go to Settings to activate Windo
Controller Parameters: P = 0.3801, I = 0.01478, D = 2.


Block Parameters: PID Controller
PID 1dof (mask) (link)
This block implements continuous- and discrete-time PID control algorithms and includes advanced features such as anti-windup, external reset, and signal tracking. You can tune the PID gains automatically using the 'Tune...' button (requires Simulink Control Design).
Controller: PID
Form: Parallel
Time domain:
Continuous-time
Discrete-time
Discrete-time settings
Sample time (-1 for inherited): -1
Compensator formula
Main
Initialization
Output Saturation
Data Types
State Attributes
Controller parameters
Source: internal
Proportional (P): 0.363250528215049
Integral (I): 0.0138253028436041
Use I*Ts (optimal for codegen)
Derivative (D): 2.12060034133301
Filter coefficient (N): 745.569495243781
Use filtered derivative
Automated tuning
Select tuning method: Transfer Function Based (PID Tuner App)
Tune...
Enable zero-crossing detection
OK
Cancel
Help
Apply
Activate Windows
Go to Settings to activate Windows.
80%

Without controller



As you can see without controller, it's not stable and there are many fluctuations.

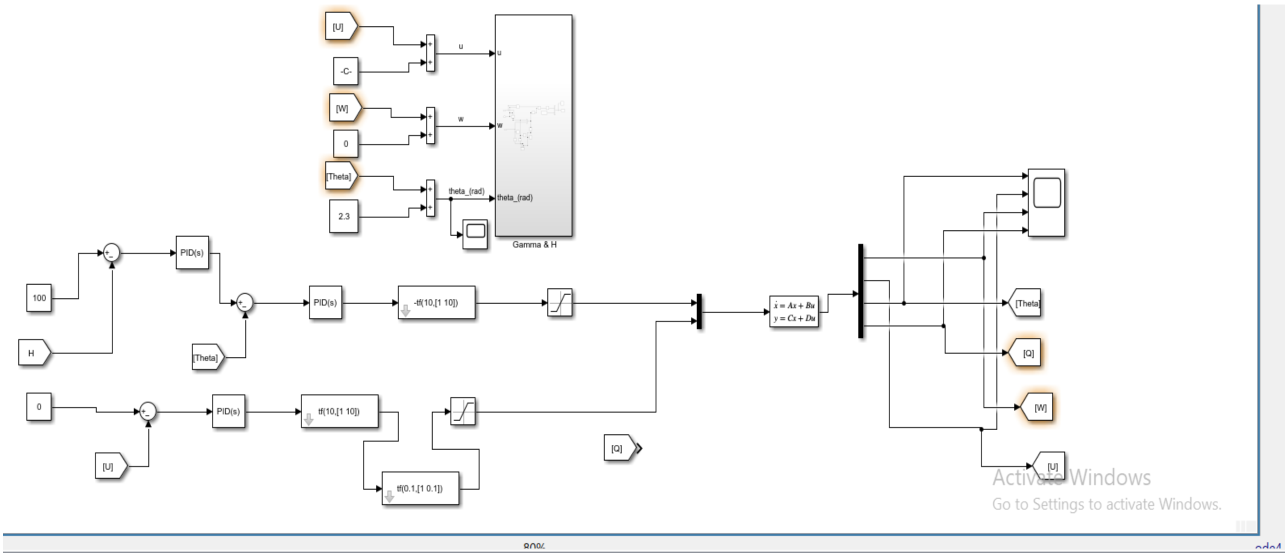
LONGITUDINAL CONTROLLER

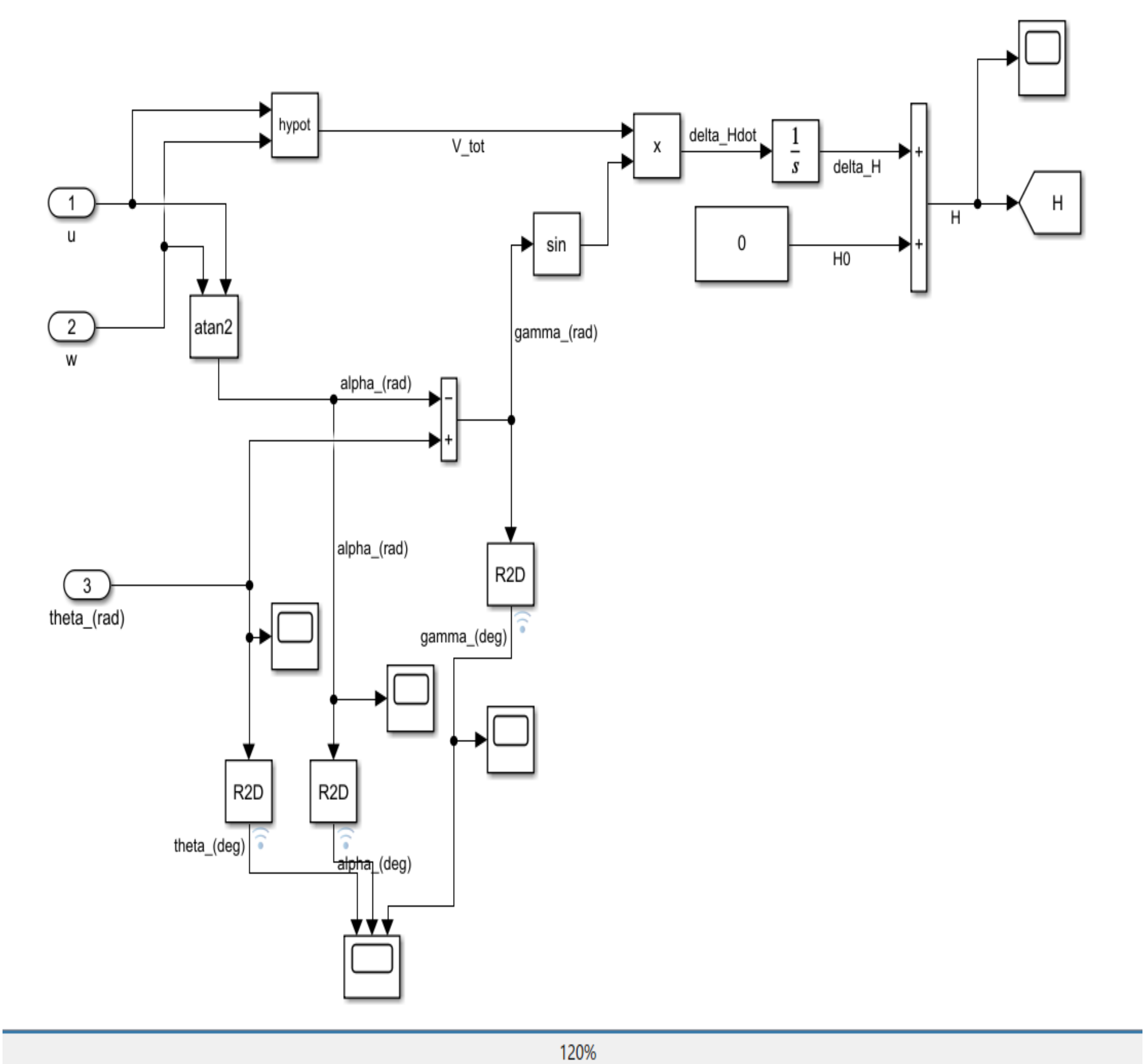
The longitudinal autopilot in cruise conditions essentially controls two of the four longitudinal states, and also guarantees that the other states are within limits. Typically, it controls the altitude h and the longitudinal speed u, while it makes sure that the pitch and the pitching velocity are within limits.

The longitudinal controls are the elevator and throttle (thrust) deflections. These two controls are supposed to achieve the required values of h and u, and there are two options for doing so:

The first option is to achieve the required altitude by controlling the pitch by feeding back the altitude signal and then position the elevator deflection by feeding back the pitch signal. To achieve the required aircraft velocity, the velocity signal is fed back to position the throttle lever.

The second option is to control the aircraft altitude directly by δT , and use pitch feedback to position δe and control the aircraft velocity by this elevator deflection.


[U]
-C-
[W]
0
[Theta]
2.3
u
w
theta_(rad)
Gamma & H
100
H
PID(s)
PID(s)
-tf(10,[1 10])
[Theta]
0
[U]
PID(s)
tf(10,[1 10])
tf(0.1,[1 0.1])
[Q]
[Theta]
[Q]
[W]
[U]


hypot
V_tot
x
delta_Hdot
1/s
delta_H
0
H0
H
H
1
u
2
w
atan2
sin
gamma_(rad)
alpha_(rad)
alpha_(rad)
3
theta_(rad)
R2D
gamma_(deg)
R2D
R2D
theta_(deg)
alpha_(deg)
120%

With controller (PID)

After trying PID is more suitable than any other controller

P = 0.390554703212597

I = 0.00167275571490297

D = 12.1787166967996

But the aircraft is not stable as the poles has positive one pole, so I will do some poles placement to make the aircraft stable.

The A & B& C &D were

```
C_long_full
     1     0     0     0
     0     1     0     0
     0     0     1     0
     0     0     0     1
```

```
A_long_full
   -0.7370    0.0631 -214.0174  -32.1740
   -0.2040   -0.5700 -191.2212         0
    0.0004    0.0075   -0.2588         0
         0         0    1.0000         0
```

```
B_long_full
    1.1900    0.0023
  -29.7000    0.0001
   -4.7810   -0.0000
         0         0
```

```
damp(A_long_full)

         Pole              Damping       Frequency      Time Constant
                                       (rad/seconds)     (seconds)

  4.26e-02               -1.00e+00       4.26e-02        -2.35e+01
 -5.69e-01                1.00e+00       5.69e-01         1.76e+00
 -5.20e-01 + 1.22e+00i    3.92e-01       1.33e+00         1.92e+00
 -5.20e-01 - 1.22e+00i    3.92e-01       1.33e+00         1.92e+00
```

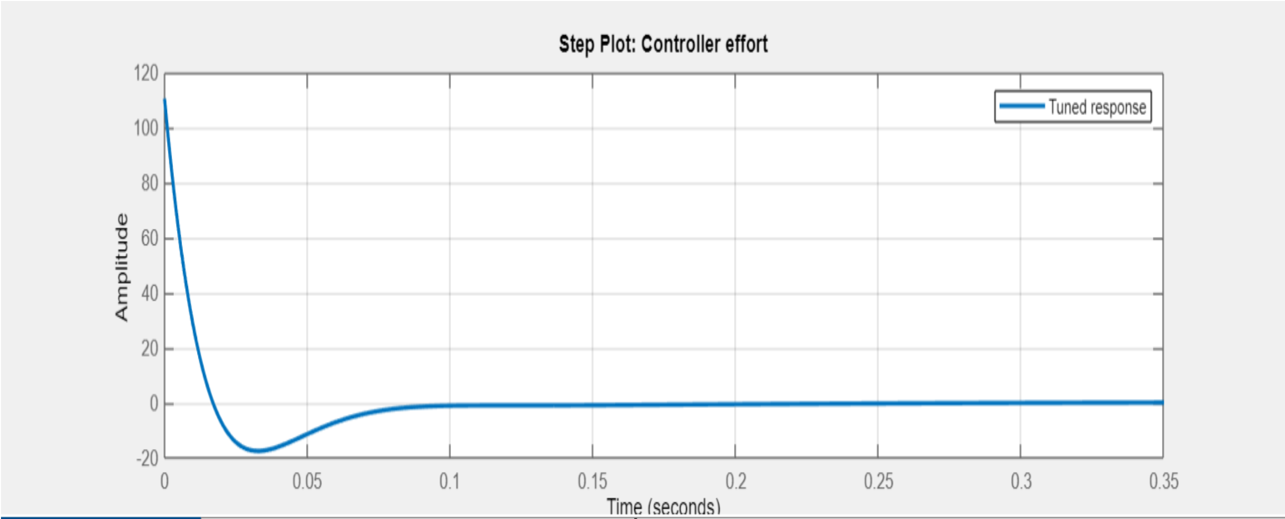
Now, doing poles placement with changing the sign of negative pole to be positive by getting K and then

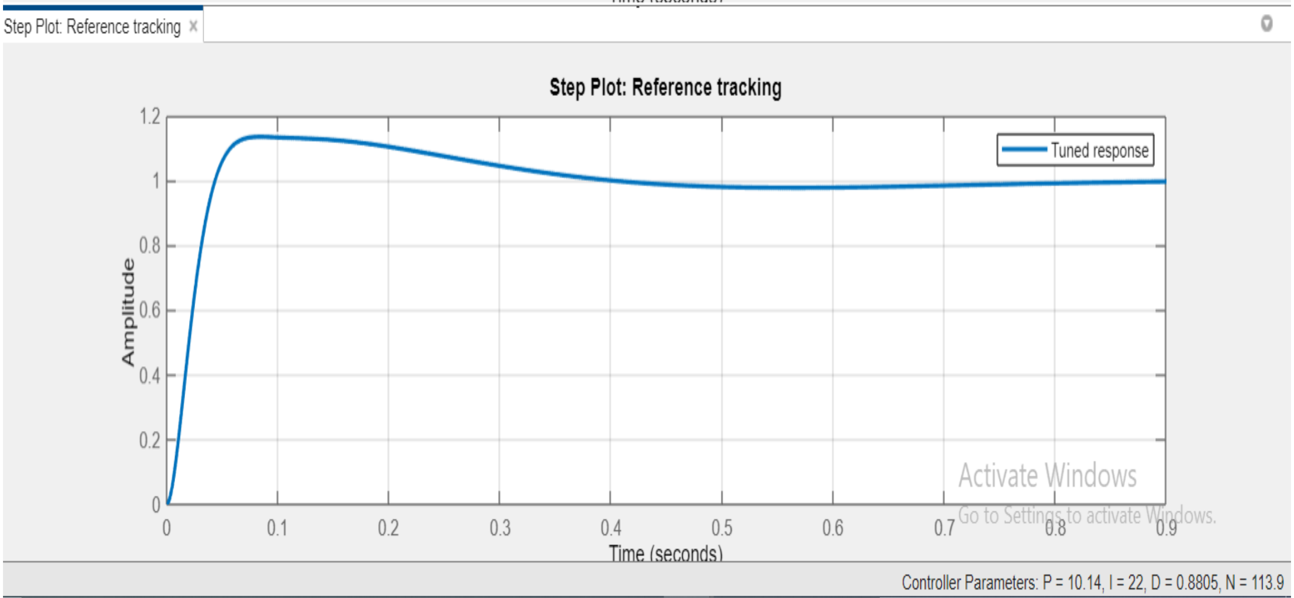
$$A_{stable} = A - B * K$$

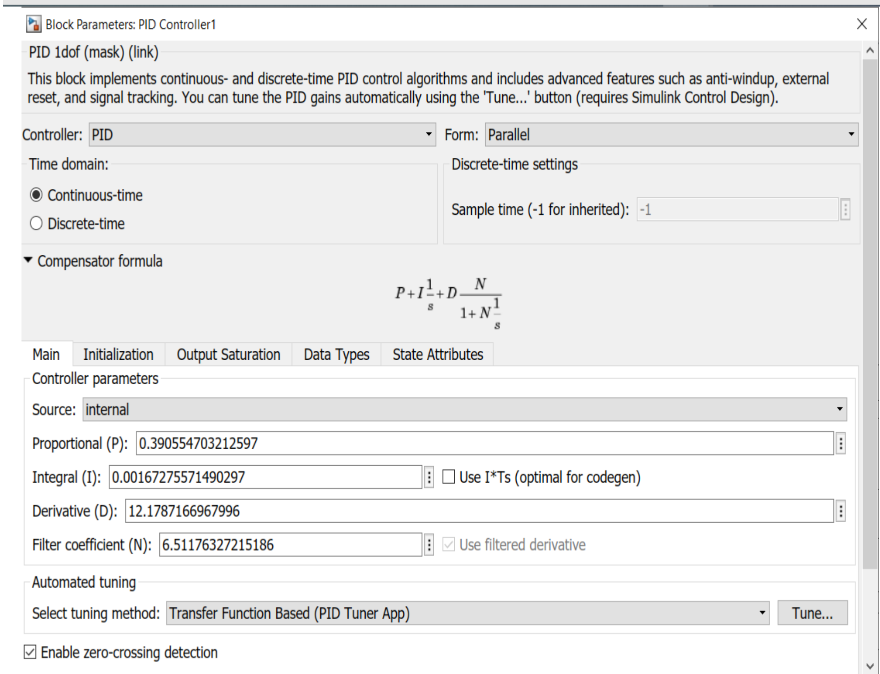
```
Klong =

   1.0e+04 *

    0.0000   -0.0000   -0.0000   -0.0000
   -0.0072    0.0015   -8.4952   -1.4436


A_long_full1 =

   -0.5730    0.0281  -20.2671    0.7736
   -0.1729   -0.5748 -184.2831    0.5833
    0.0042    0.0070   -0.5038   -0.1375
         0         0    1.0000         0
```

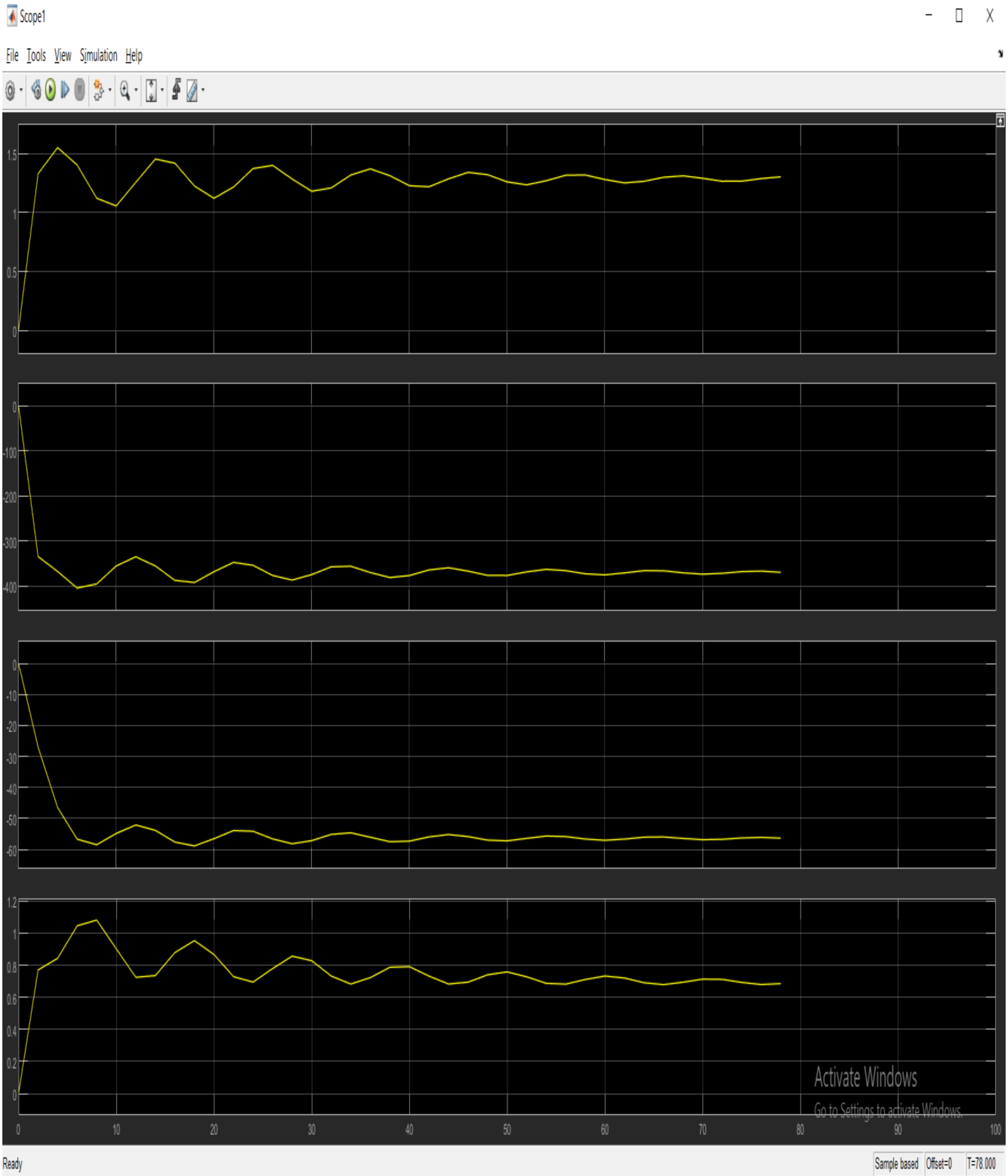
The new poles →

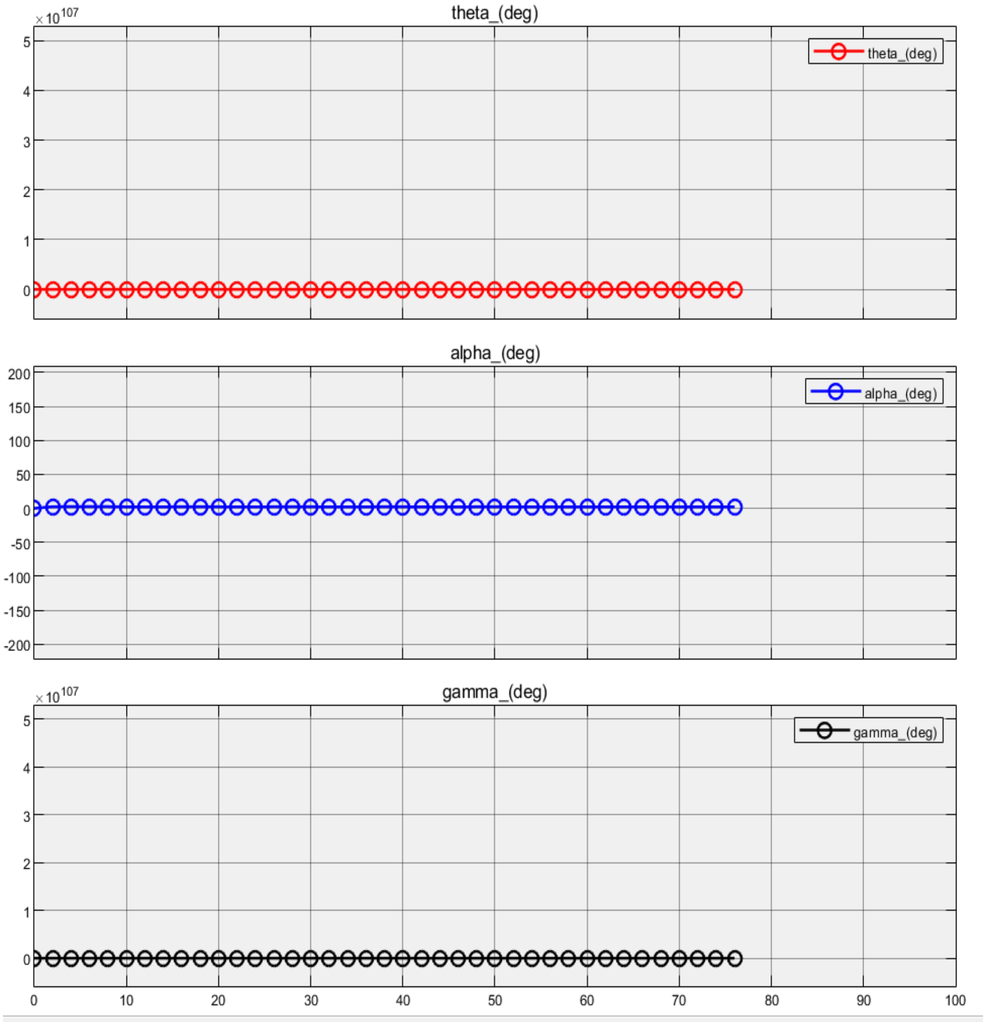
```
plong =

  -0.0426 + 0.0000i  -0.5690 + 0.0000i  -0.5200 + 1.2200i  -0.5200 - 1.2200i
```


Step Plot: Controller effort
Tuned response
Amplitude
120
100
80
60
40
20
0
-20
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
Time (seconds)

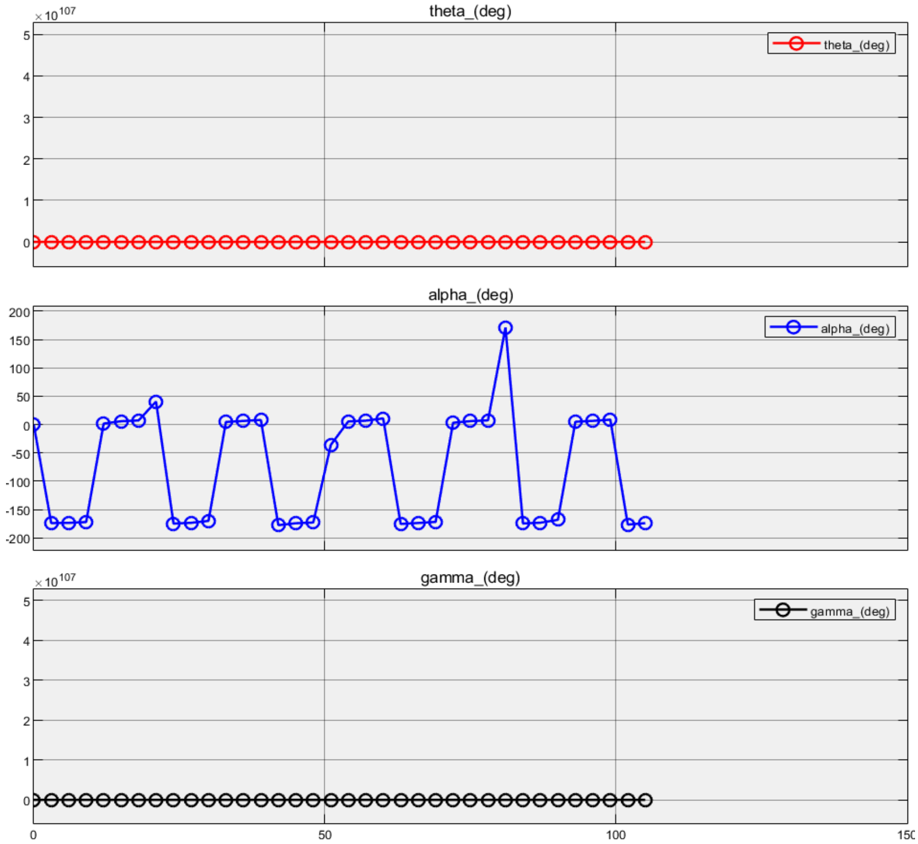

Step Plot: Reference tracking ×
Step Plot: Reference tracking
Tuned response
Amplitude
1.2
1
0.8
0.6
0.4
0.2
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
Time (seconds)
Activate Windows
Go to Settings to activate Windows.
Controller Parameters: P = 10.14, I = 22, D = 0.8805, N = 113.9

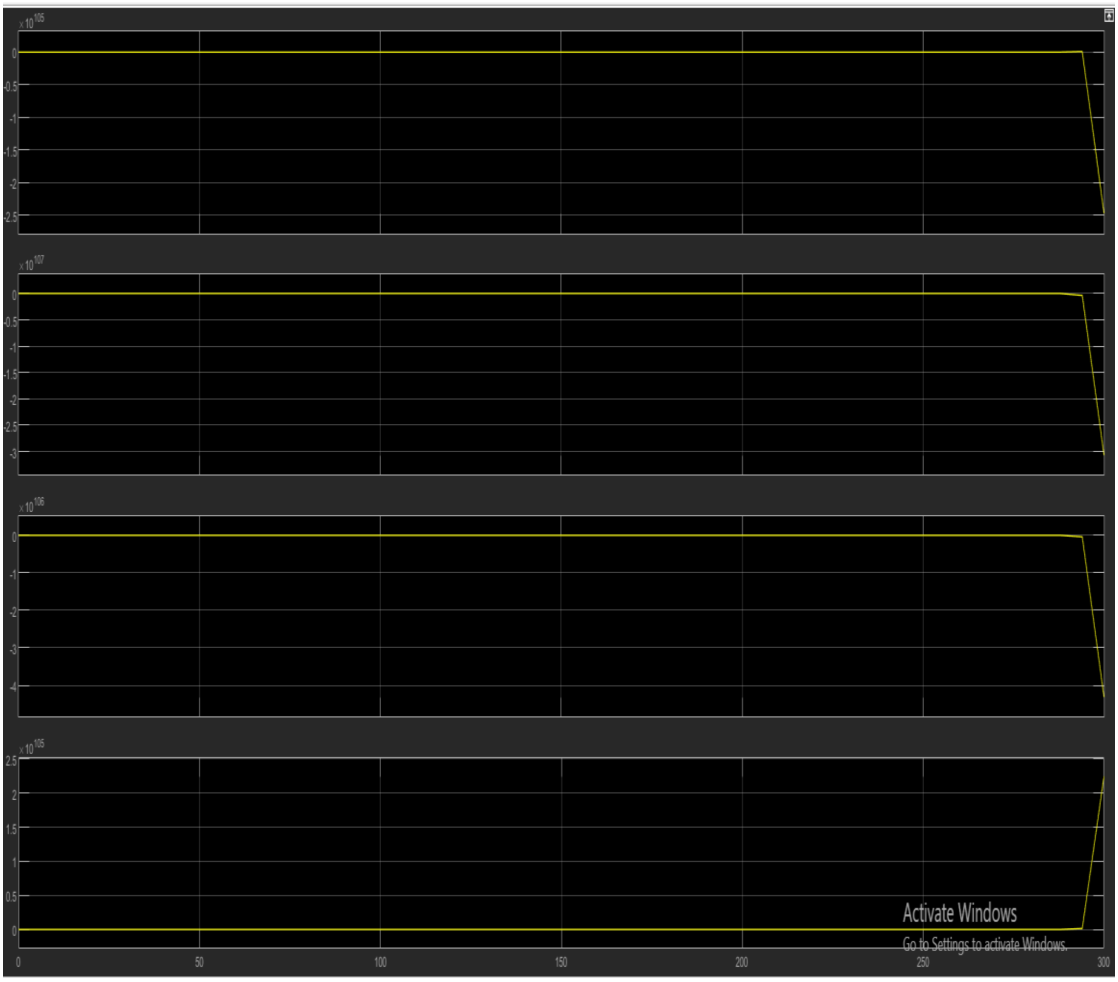

Block Parameters: PID Controller1
PID 1dof (mask) (link)
This block implements continuous- and discrete-time PID control algorithms and includes advanced features such as anti-windup, external reset, and signal tracking. You can tune the PID gains automatically using the 'Tune...' button (requires Simulink Control Design).
Controller: PID
Form: Parallel
Time domain:
Continuous-time
Discrete-time
Discrete-time settings
Sample time (-1 for inherited): -1
Compensator formula
$P+I\frac{1}{s}+D\frac{N}{1+N\frac{1}{s}}$
Main
Initialization
Output Saturation
Data Types
State Attributes
Controller parameters
Source: internal
Proportional (P): 0.390554703212597
Integral (I): 0.00167275571490297
Use I*Ts (optimal for codegen)
Derivative (D): 12.1787166967996
Filter coefficient (N): 6.51176327215186
Use filtered derivative
Automated tuning
Select tuning method: Transfer Function Based (PID Tuner App)
Tune...
Enable zero-crossing detection


Scope1
File Tools View Simulation Help
Activate Windows
Go to Settings to activate Windows.
Ready
Sample based
Offset=0
T=78.000


theta_(deg)
×10^107
theta_(deg)
alpha_(deg)
alpha_(deg)
gamma_(deg)
×10^107
gamma_(deg)

Without Controller





As you can see without controller, it’s not stable and there are many fluctuations and it diverges.