LINEAR AND NONLINEAR PROGRAMMING: MATH 404

Interior Point Methods for Linear Programming

University of Science and Technology at Zewail City

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I. Table of Contents

II. Introduction

The attracted files:

1. Initial: function, which takes (A, b, c) and returns x_0, s_0, λ_0.
2. Main: tests different algorithms
3. CenPa: central Path function, which takes (A, b, c, alpha, sigma, decision) and returns p^*, d^*, and x^*, respectively.
   - A, b, and c stand for the problem matrices and vectors.
   - Alpha is step size, and is ignored if adaptive sizing option is used, but must be entered.
   - Sigma is centering parameter. It is used in adaptive sizing in the first iteration only.
   - Decision: whether sizing is fixed (0) or adaptive (1 or otherwise).
4. cenpa_previous_working_version: central path with adaptive sizing on alpha only using another method.
5. Mehrotra function: which takes (A, b, c, sigma) and returns p^*, d^*, and x^*, respectively.

Stopping condition was constant for each method: $\mu<0.0001$

III. Example 1:

$$Min \quad z = -2x_1 - x_2$$

$$\begin{aligned} S.t. \quad & 3x_1 + 4x_2 + x_3 \qquad = 6 \\ & 6x_1 + \ \ x_2 + \qquad x_4 = 3 \\ & \ x_1, x_2, x_3, x_4 \geq 0 \end{aligned}$$

$$\boldsymbol{X} = [\,x_1\; x_2\; x_3\; x_4]$$
$$\boldsymbol{C} = [-2 - 1\; 0\; 0]$$
$$\boldsymbol{A} = \begin{bmatrix} 3 & 4 & 1 & 0 \\ 6 & 1 & 0 & 1 \end{bmatrix}$$
$$\boldsymbol{b} = \begin{bmatrix} 6 \\ 3 \end{bmatrix}$$

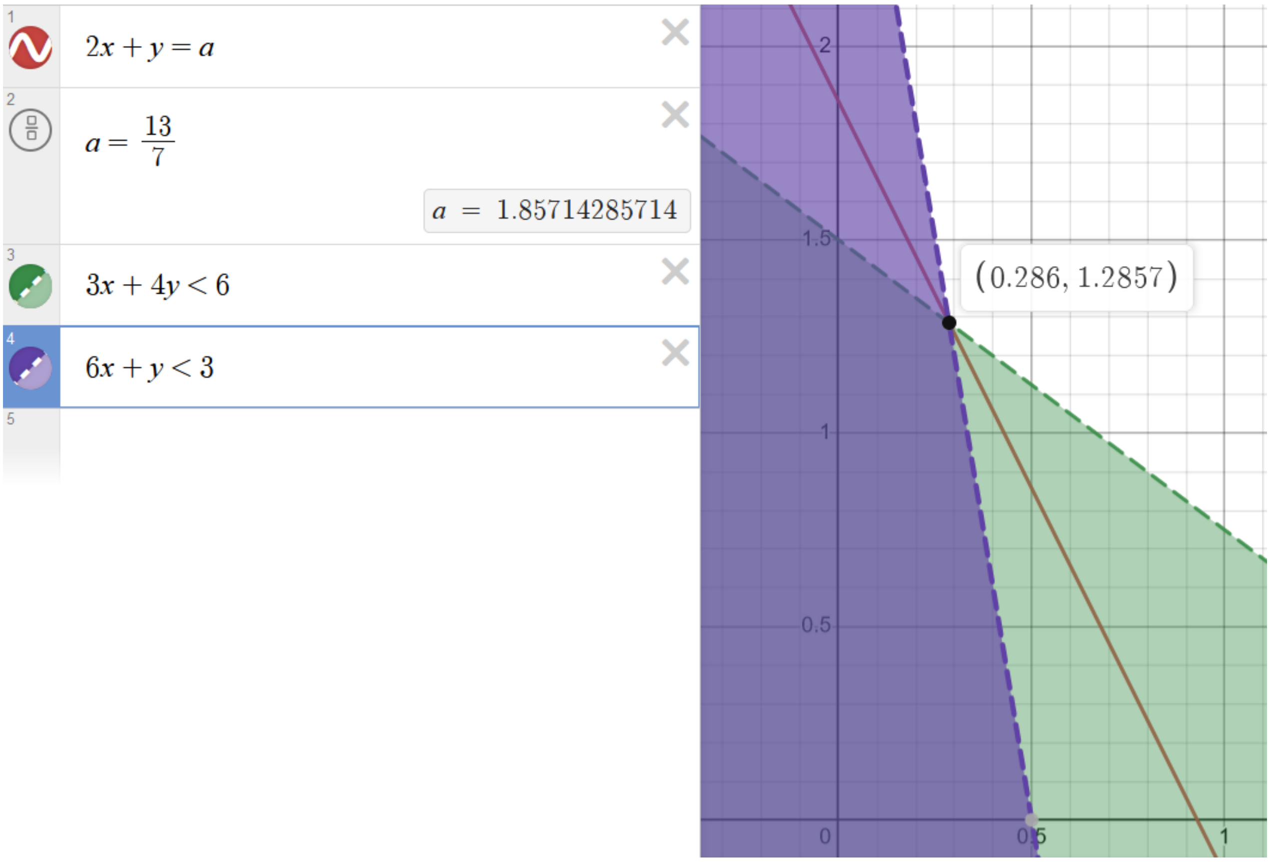


Figure 1 A graphical representation of the feasible region, highlighting the vertices and the optimum point with respect to the maximum objective function contour. (Desmos | Graphing Calculator, n.d.)

Central Path

Fixed alpha and sigma

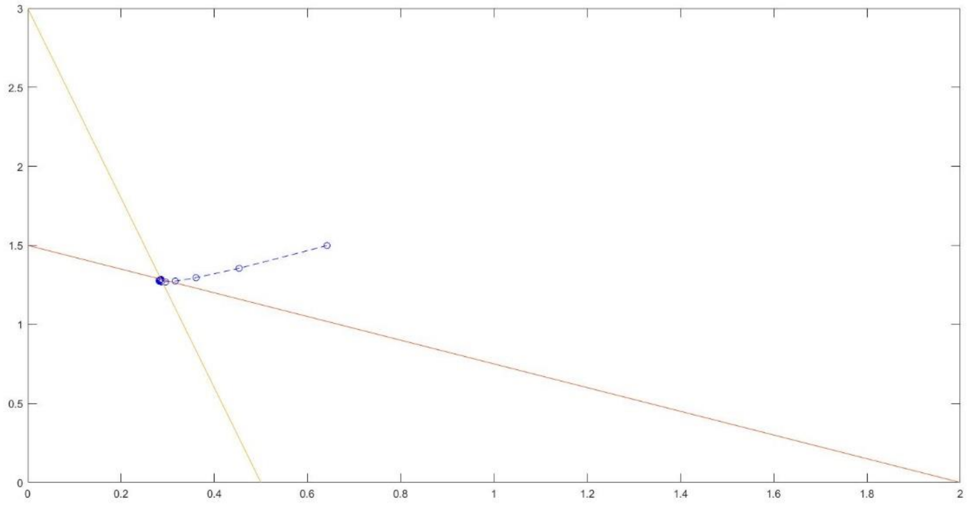


Figure 2 Feasible Region and path taken σ=0.5, α=0.5

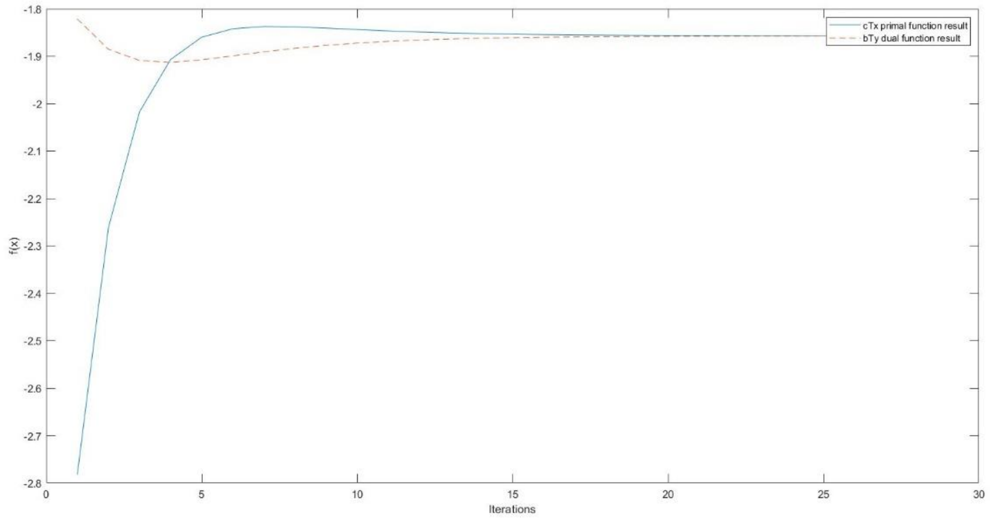


Figure 3 Duality Test σ=0.5, α=0.5

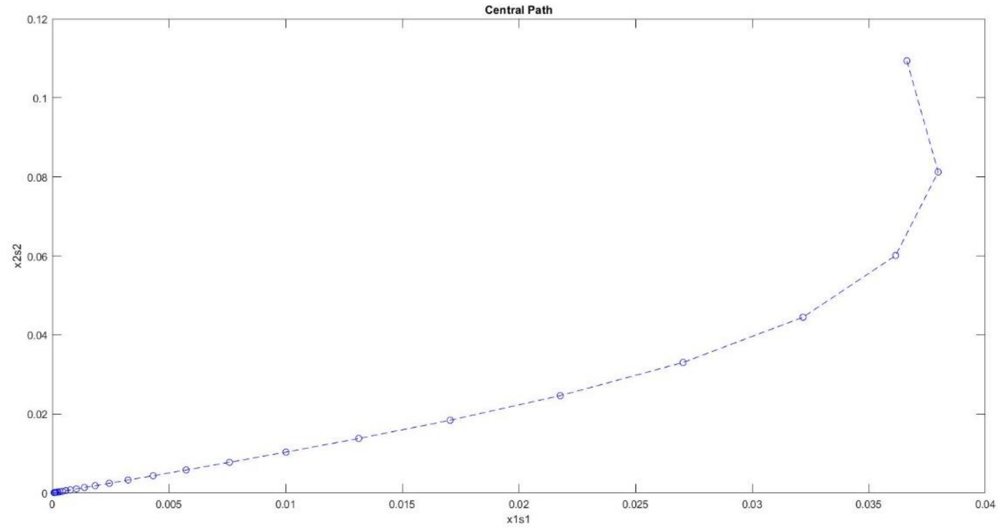


Figure 4 Central path σ=0.5, α=0.5

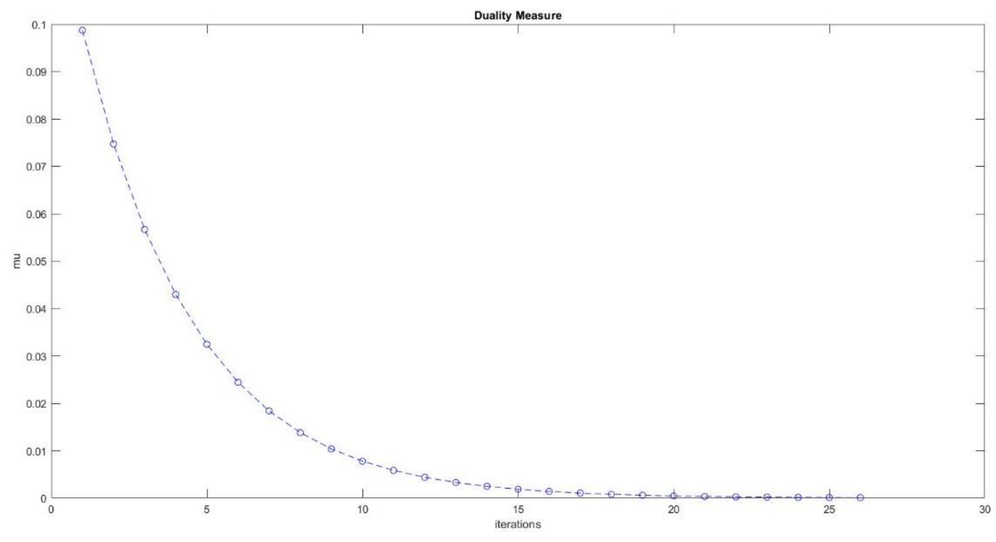


Figure 5 duality measure mu function of iteration σ=0.5, α=0.5

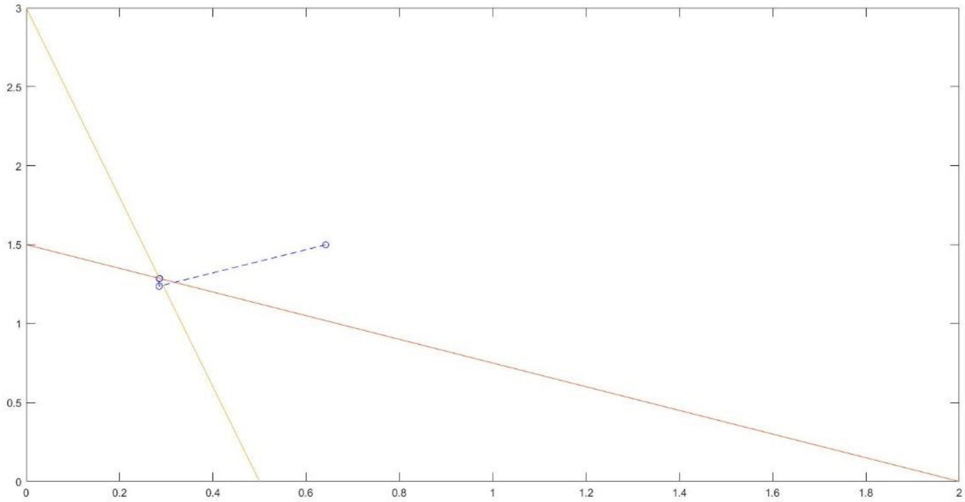


Figure 6 Feasible Region and path taken σ=0, α=0.5

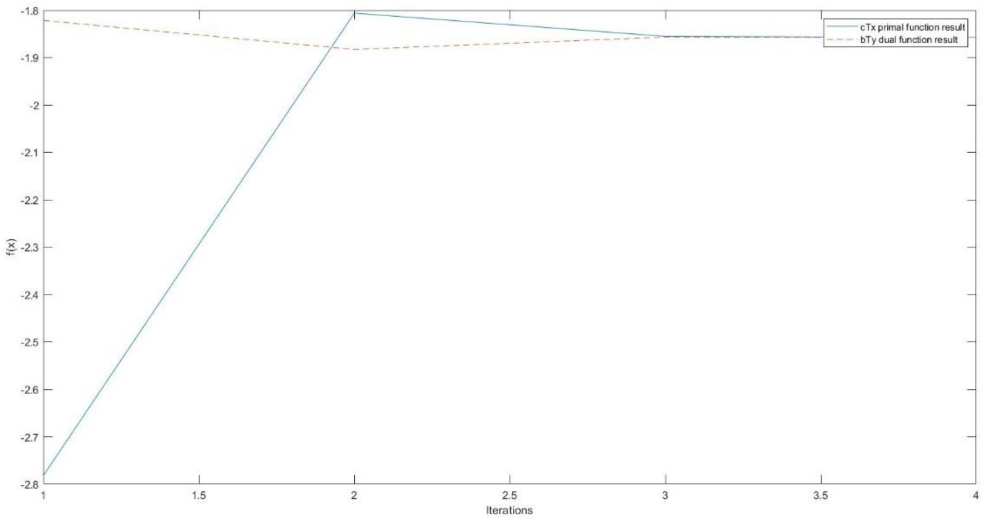


Figure 7 Duality Test σ=0, α=0.5

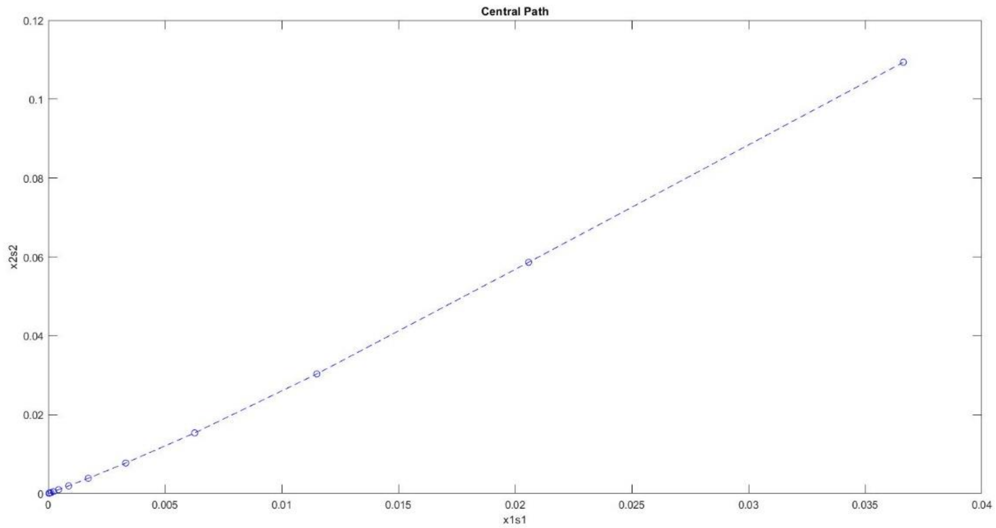


Figure 8 Central path σ=0, α=0.5

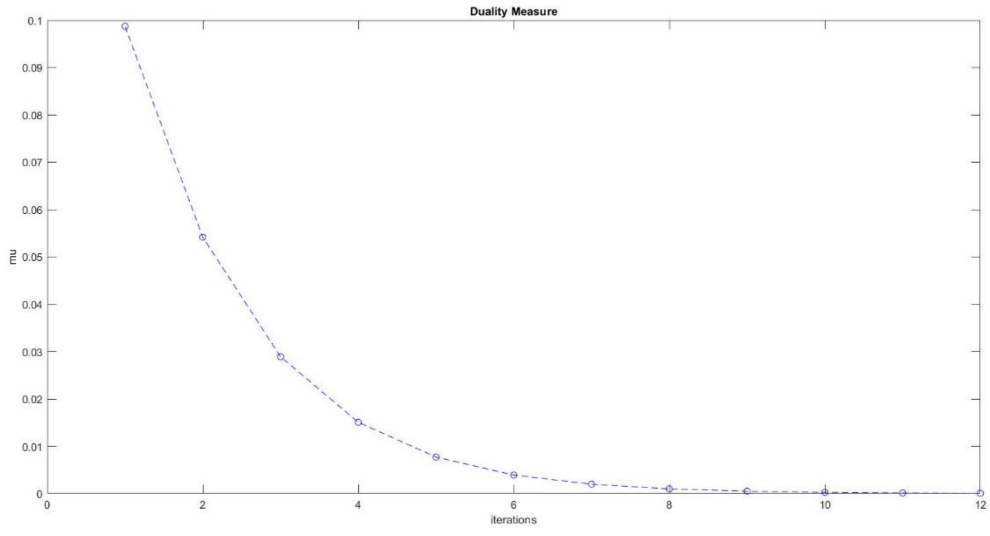


Figure 9 duality measure mu function of iteration σ=0, α=0.5

Adaptive alpha

A method of choosing maximum alpha was used by starting from maximum value (α=1) and checking if any value of $x_{k+1}^T s_{k+1} < 0$. If this condition is false, the algorithm continues. If it is true, the alpha is decreased by subtracting s small value e of it, and recalculating x and rechecking the condition.

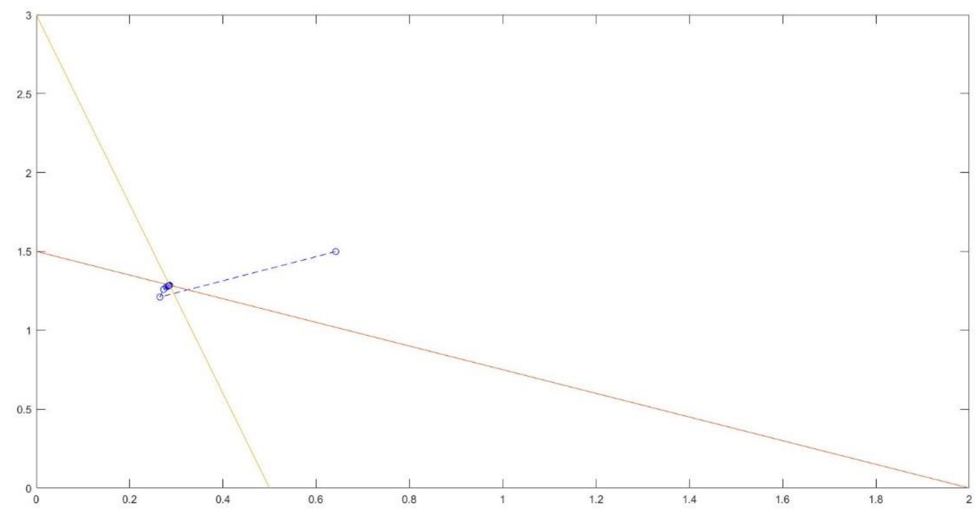


Figure 10 Feasible Region and path taken σ=0.5

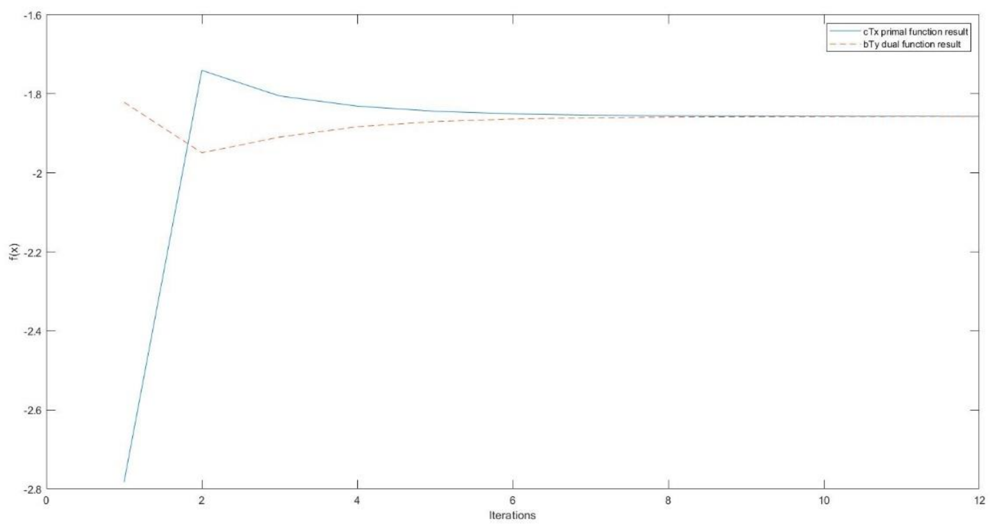


Figure 11 Duality Test σ=0.5

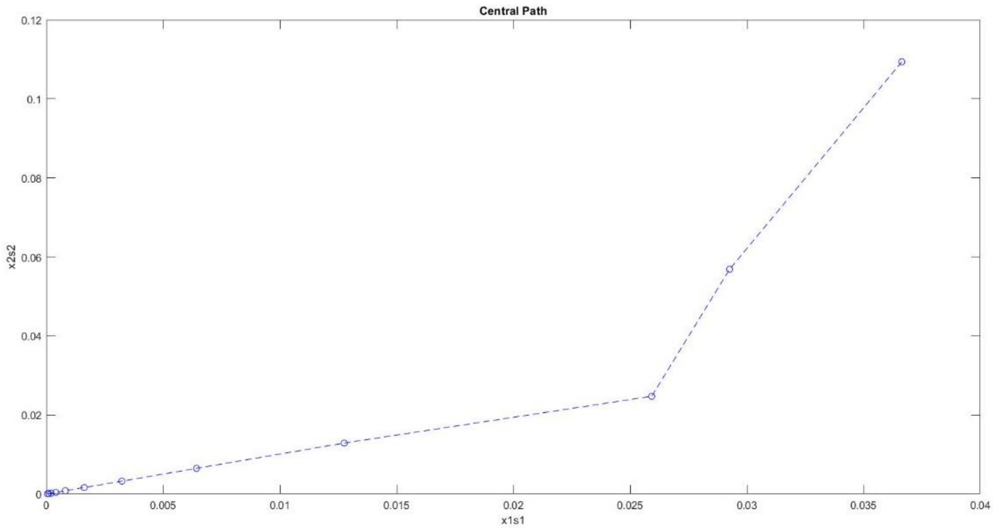


Figure 12 Central path σ=0.5

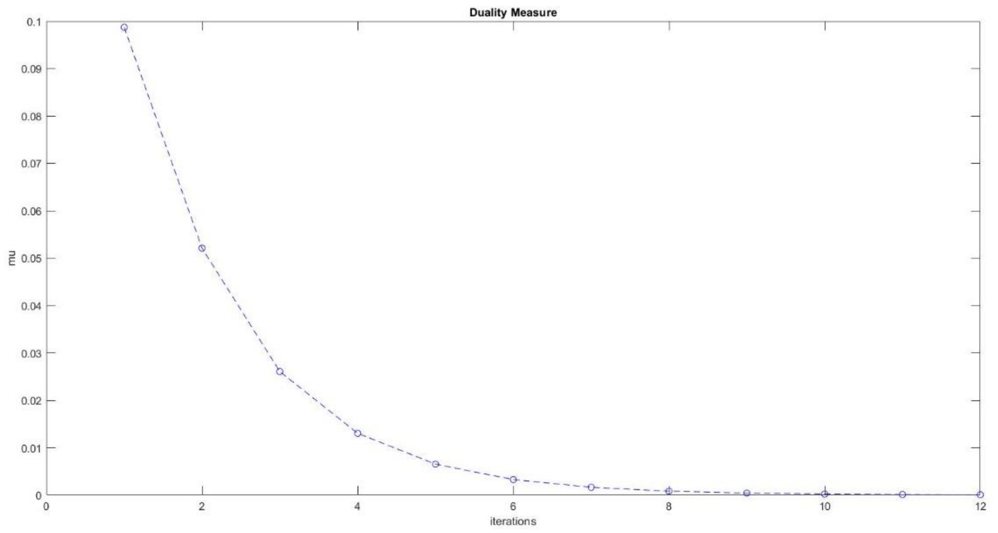


Figure 13 duality measure mu function of iteration σ=0.5

Adaptive alpha and sigma

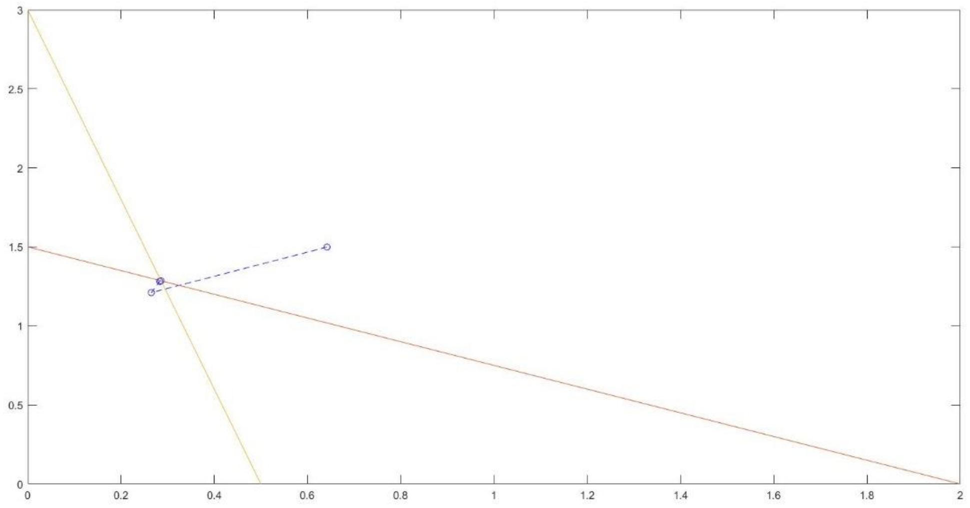


Figure 14 Feasible Region and path taken σ0=0.5

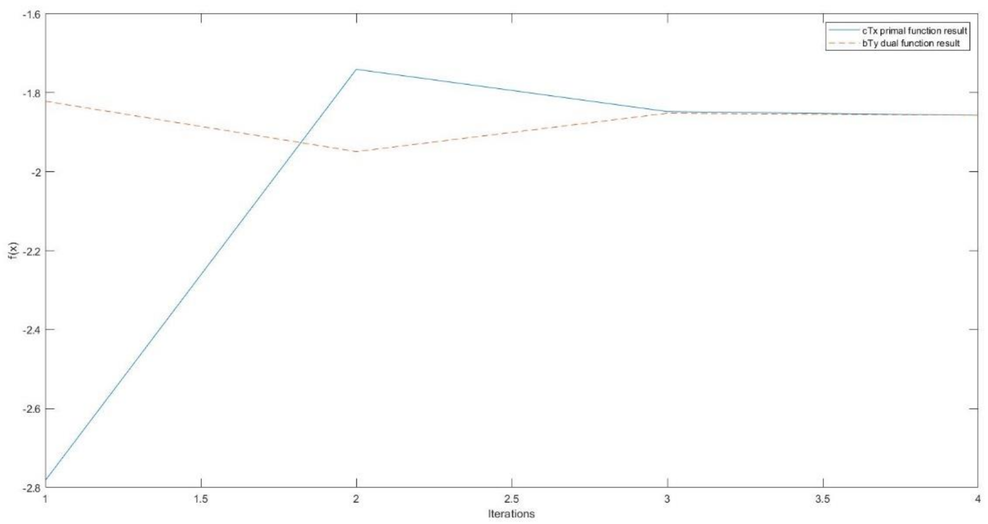


Figure 15 Duality Test σ0=0.5

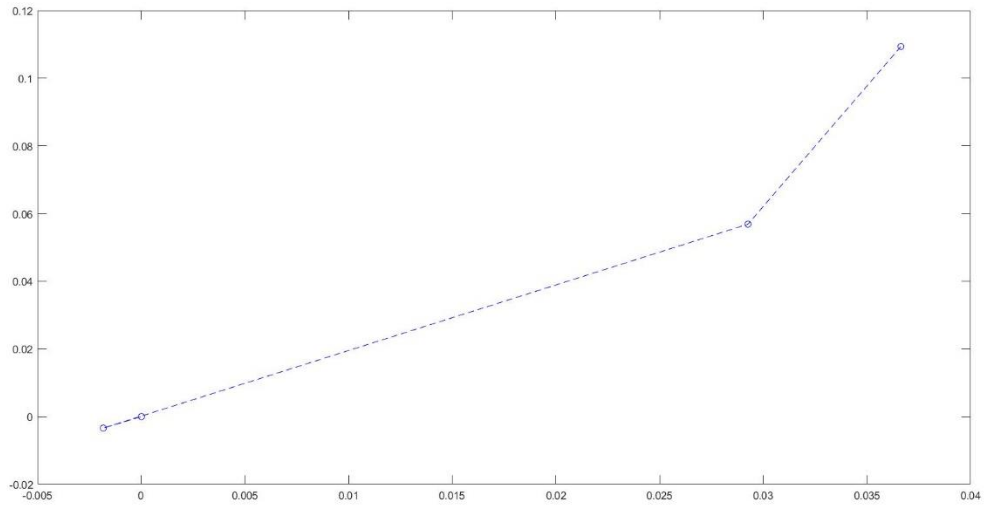


Figure 16 Central path σ0=0.5

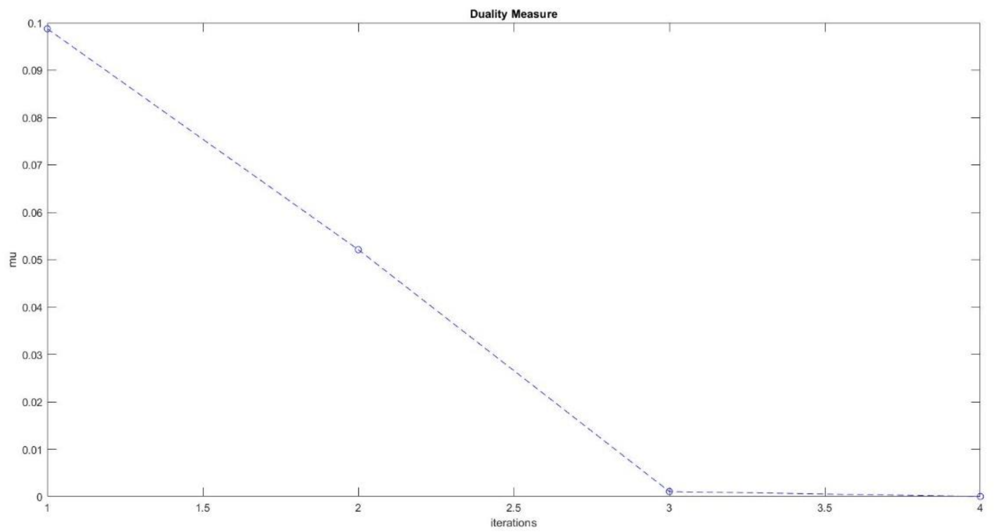


Figure 17 duality measure mu function of iteration σ0=0.5

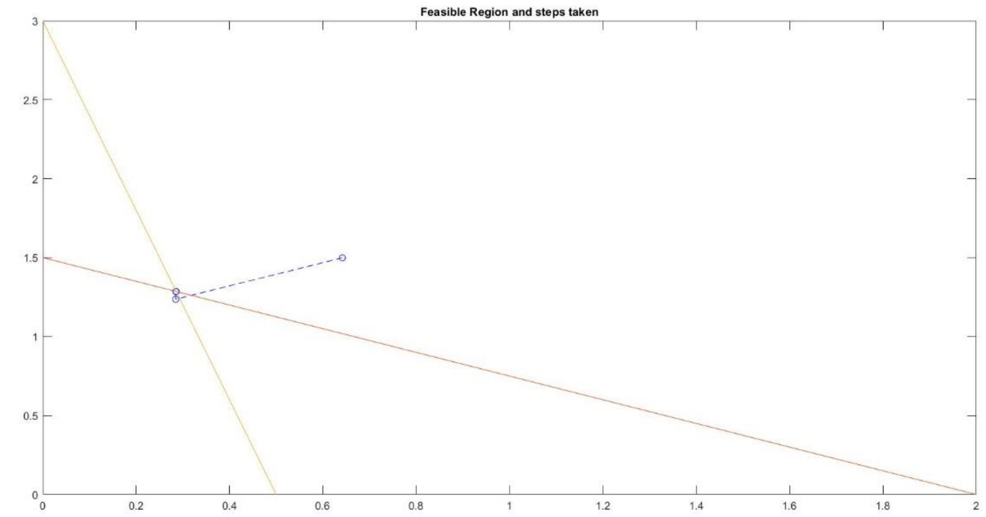


Figure 18 Feasible Region and path taken σ0=0

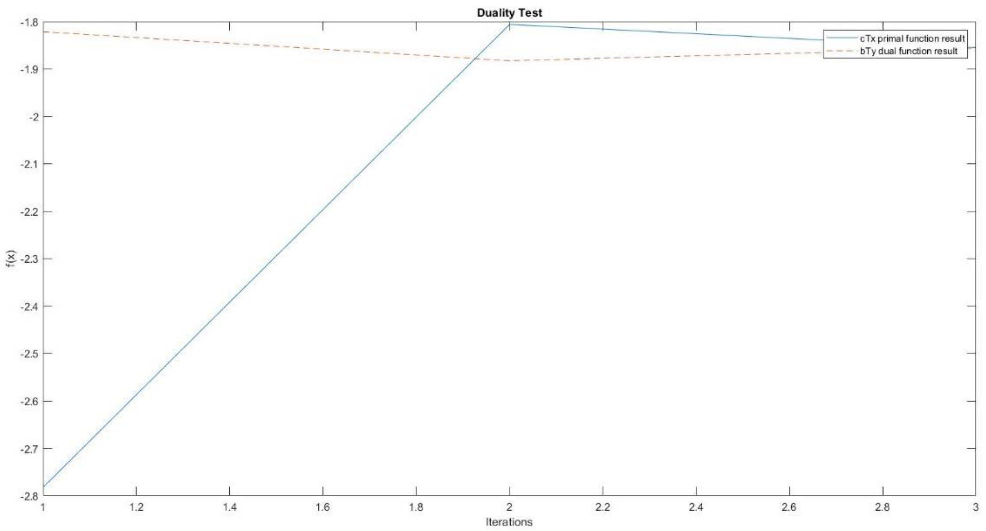


Figure 19 Duality Test σ0=0

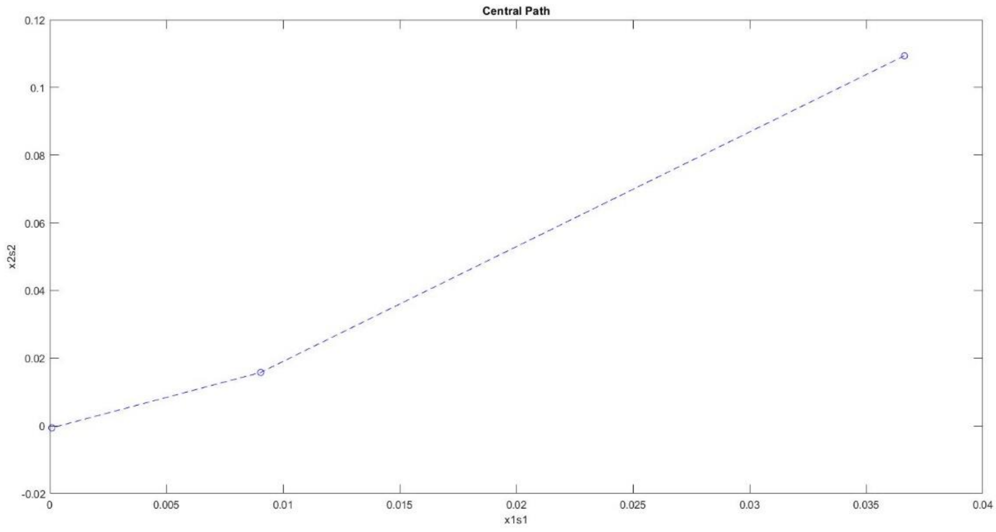


Figure 20 Central path σ0=0

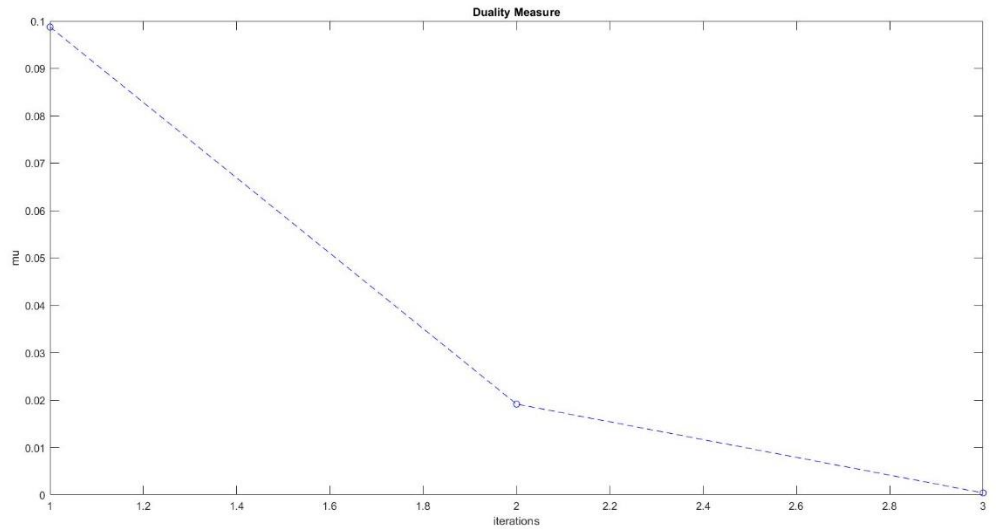


Figure 21 duality measure mu function of iteration σ=0

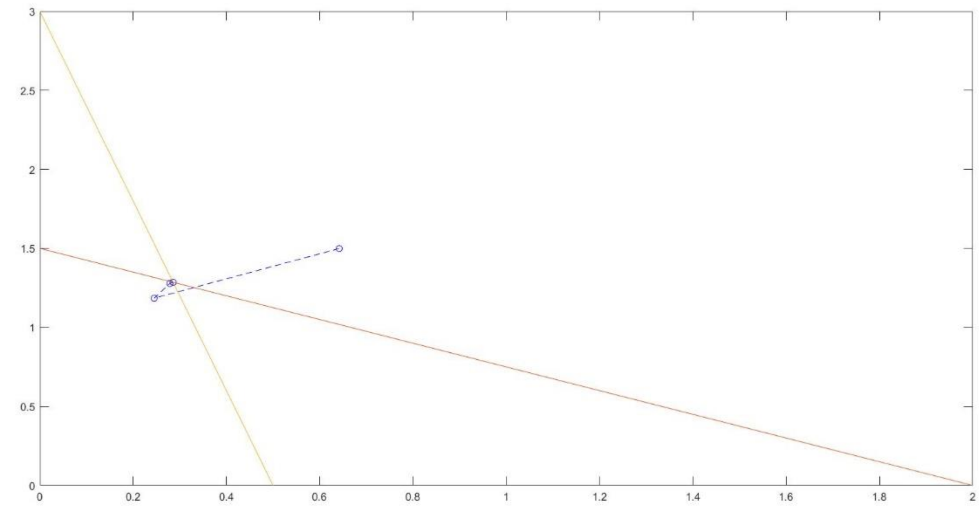


Figure 22 Feasible Region and path taken σ0=1

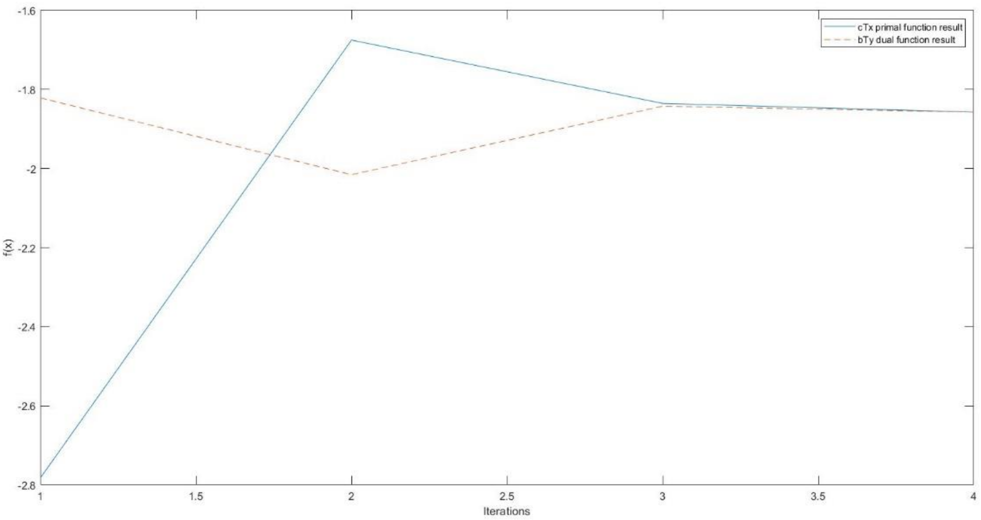


Figure 23 Duality Test σ0=1

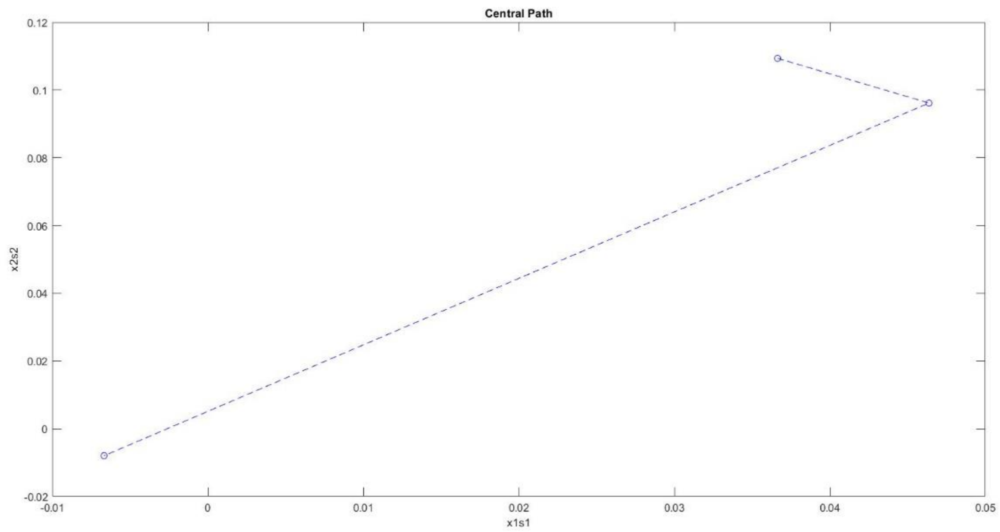


Figure 24 Central path σ0=1

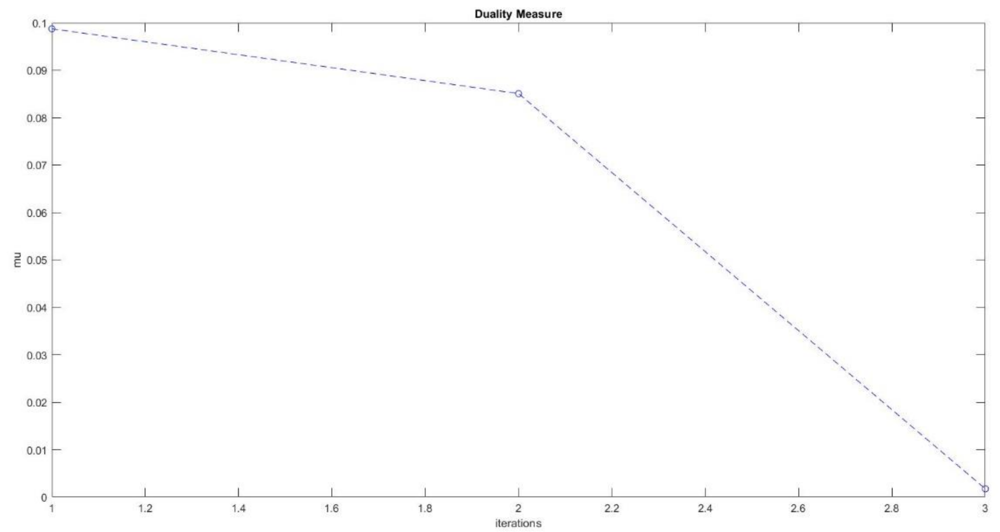


Figure 25 duality measure mu function of iteration σ=1

Mehrotra

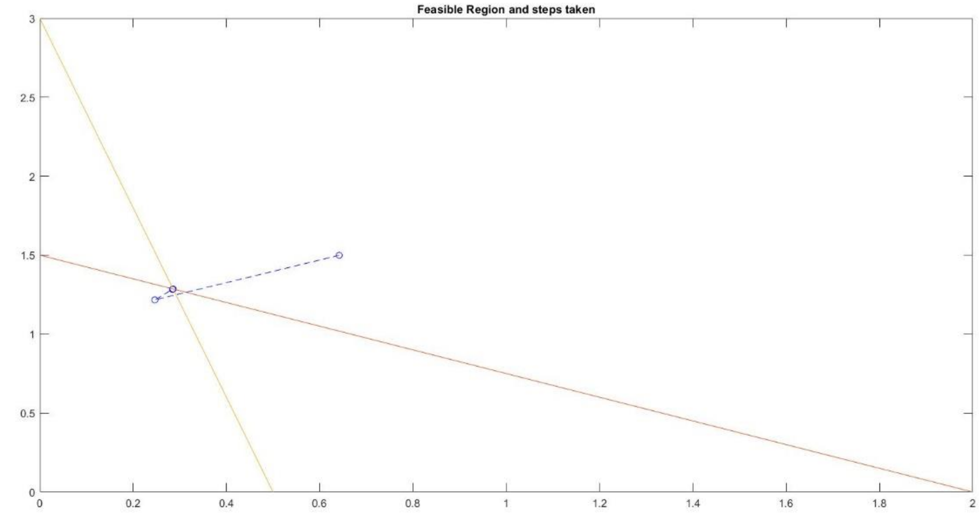


Figure 26 Feasible Region and path taken

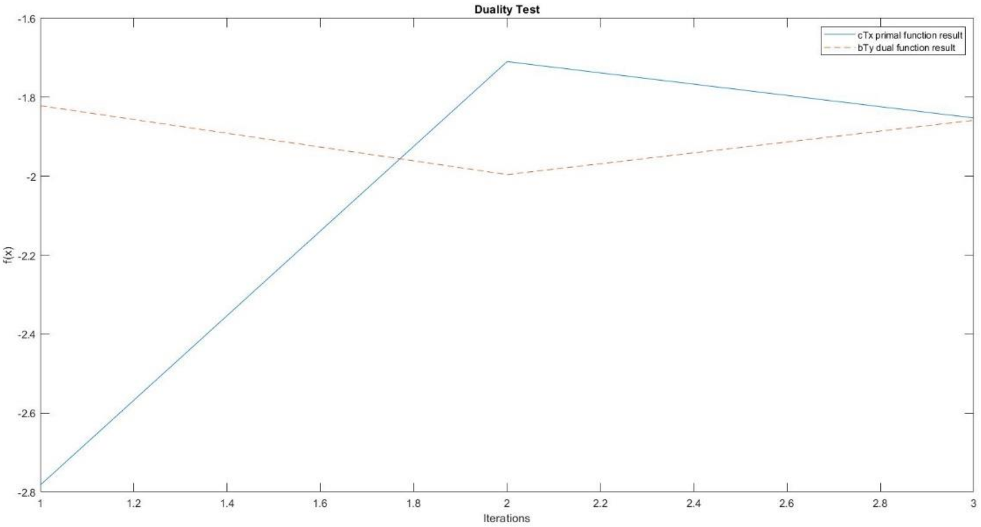


Figure 27 Duality Test

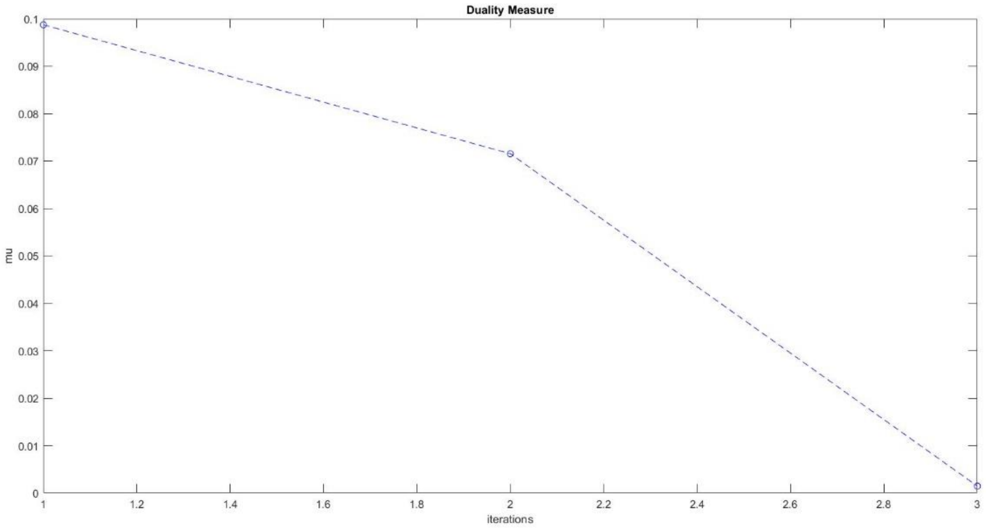


Figure 28 duality measure μ function of iteration

IV. Example 2 Mehrotra:

$$\begin{aligned}
&Max \quad z = x_1 - 8x_2 \\
&s.t. \quad -3x_1 - 2x_2 \leq -6 \\
&\qquad x_1 - x_2 \leq 6 \\
&\qquad 9x_1 + 7x_2 \leq 108 \\
&\qquad 3x_1 + 7x_2 \leq 70 \\
&\qquad -2x_1 + 5\,x_2 \leq 35 \\
&\qquad x_1, x_2 \geq 0
\end{aligned}$$

$$\boldsymbol{C} = [-1 \quad 8 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]$$

$$\boldsymbol{A} = \begin{bmatrix} -3 & -2 & 1 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 1 & 0 & 0 & 0 \\ 9 & 7 & 0 & 0 & 1 & 0 & 0 \\ 3 & 7 & 0 & 0 & 0 & 1 & 0 \\ -2 & 5 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \qquad \boldsymbol{b} = \begin{bmatrix} -6 \\ 6 \\ 108 \\ 70 \\ 35 \end{bmatrix}$$

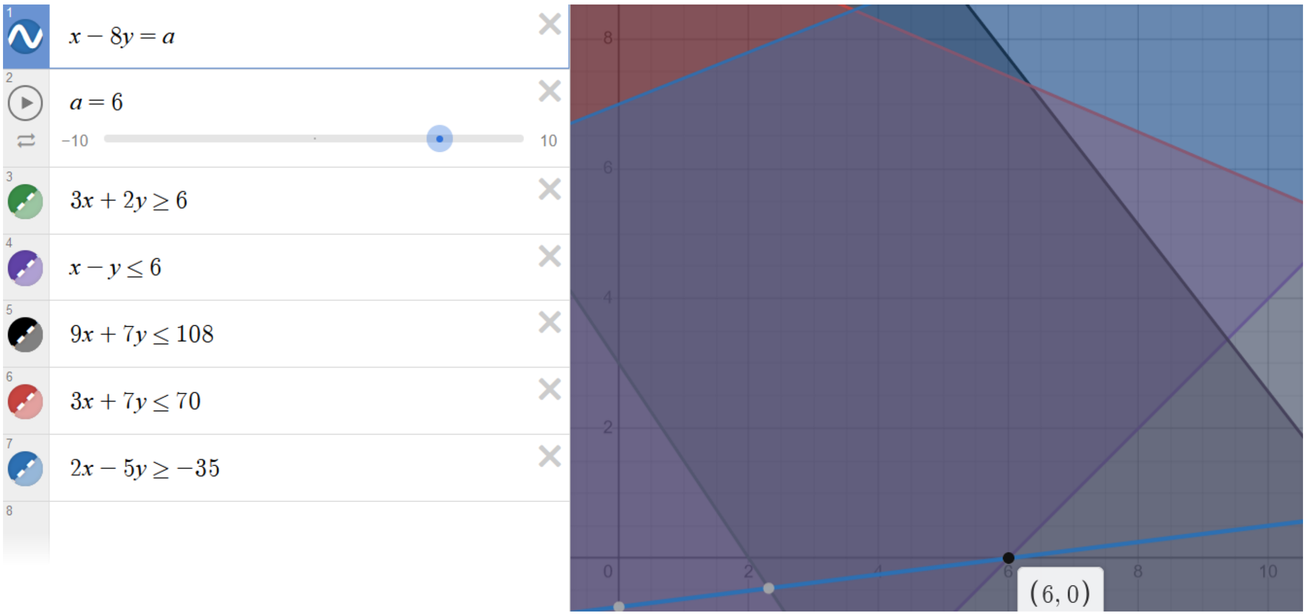


Figure 29 A graphical representation of the feasible region, highlighting the vertices and the optimum point with respect to the maximum objective function contour (Desmos | Graphing Calculator, n.d.)

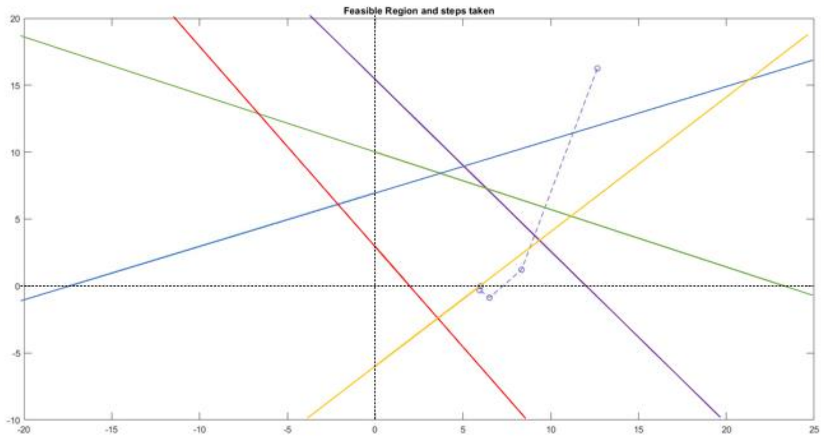


Figure 30 Feasible Region and path taken, manually enhanced

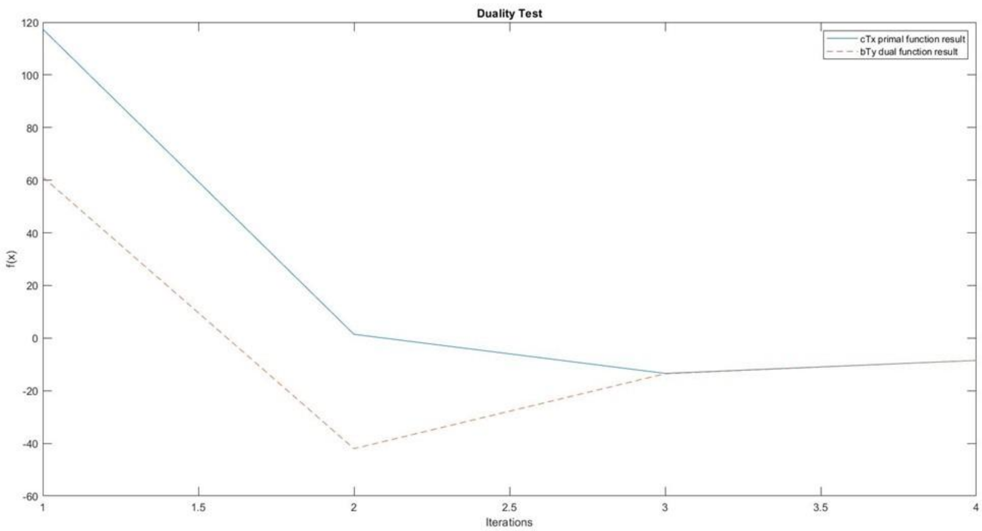


Figure 31 Duality Test

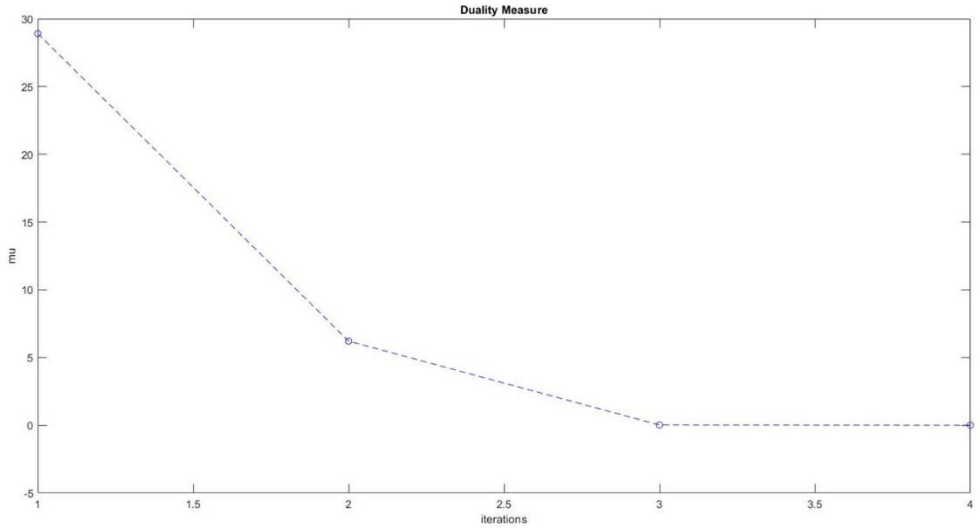


Figure 32 duality measure mu function of iteration

V. Example 3 Mehrotra:

$$Max \quad z = 50x_1 + 100x_2$$

$$\begin{aligned} s.t. \quad 2x_1 + \ \ x_2 &\leq 1250 \\ 2x_1 + 5x_2 &\leq 1000 \\ 2x_1 + 3x_2 &\leq 900 \\ x_2 &\leq 152 \end{aligned}$$

$$x_1, x_2 \geq 0$$

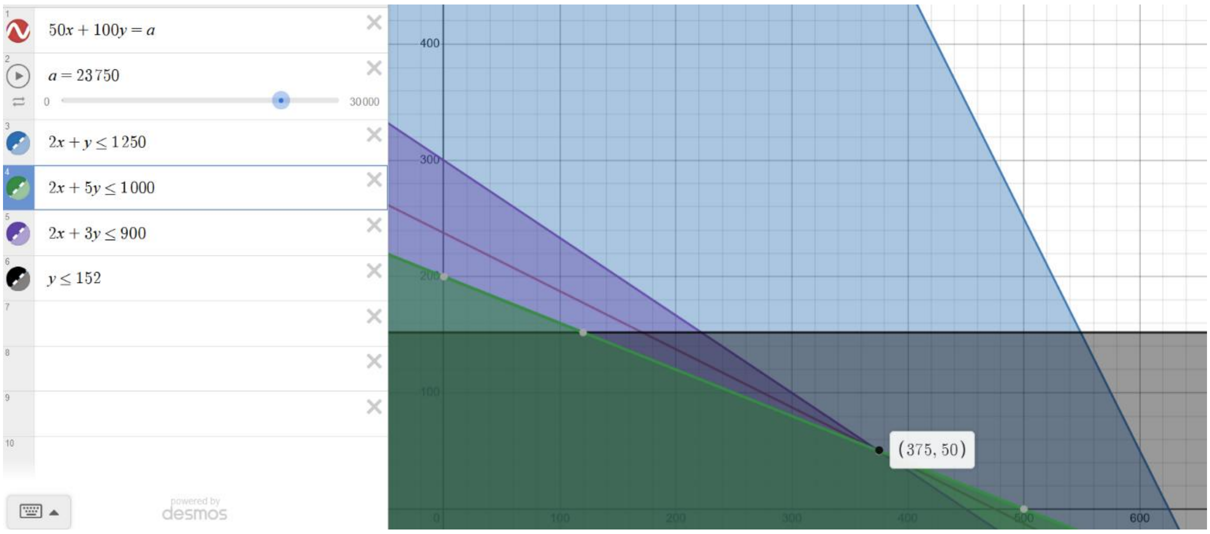


Figure 33 A graphical representation of the feasible region, highlighting the vertices and the optimum point with respect to the maximum objective function contour (Desmos | Graphing Calculator, n.d.)

$$\boldsymbol{C} = [-50 \quad -100 \quad 0 \quad 0 \quad 0 \quad 0]$$

$$\boldsymbol{A} = \begin{bmatrix} 2 & 1 & 1 & 0 & 0 & 0 \\ 2 & 5 & 0 & 1 & 0 & 0 \\ 2 & 3 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}, \qquad \boldsymbol{b} = \begin{bmatrix} 1250 \\ 1000 \\ 900 \\ 152 \end{bmatrix}$$

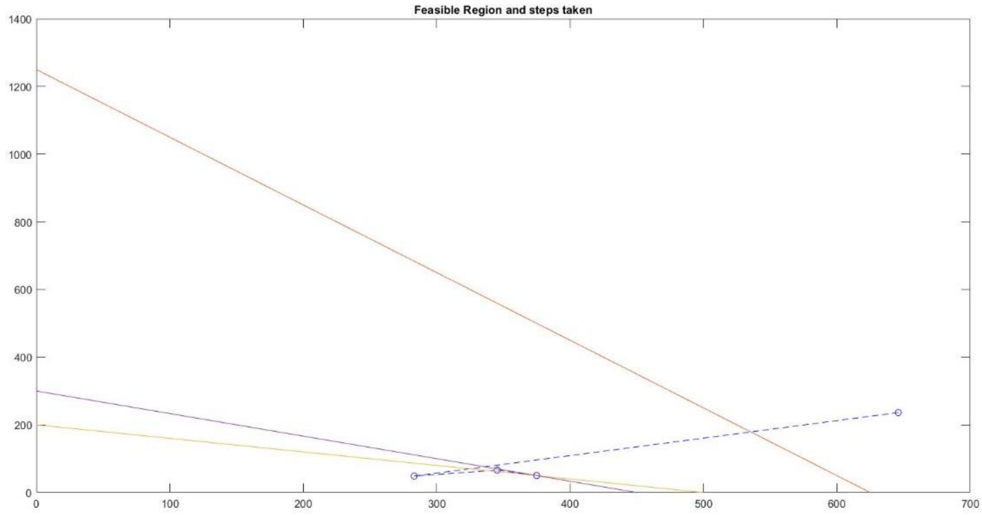


Figure 34 Feasible Region and path taken

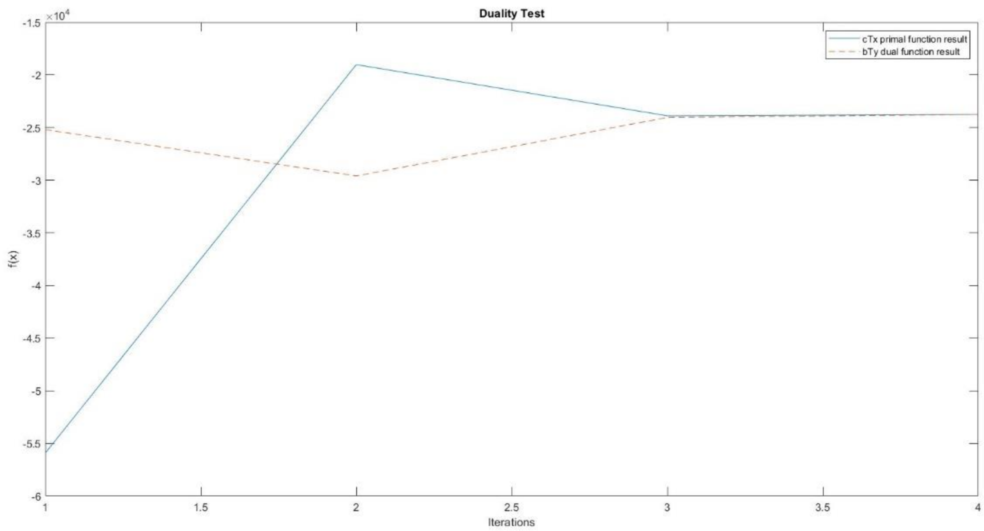


Figure 35 Duality Test

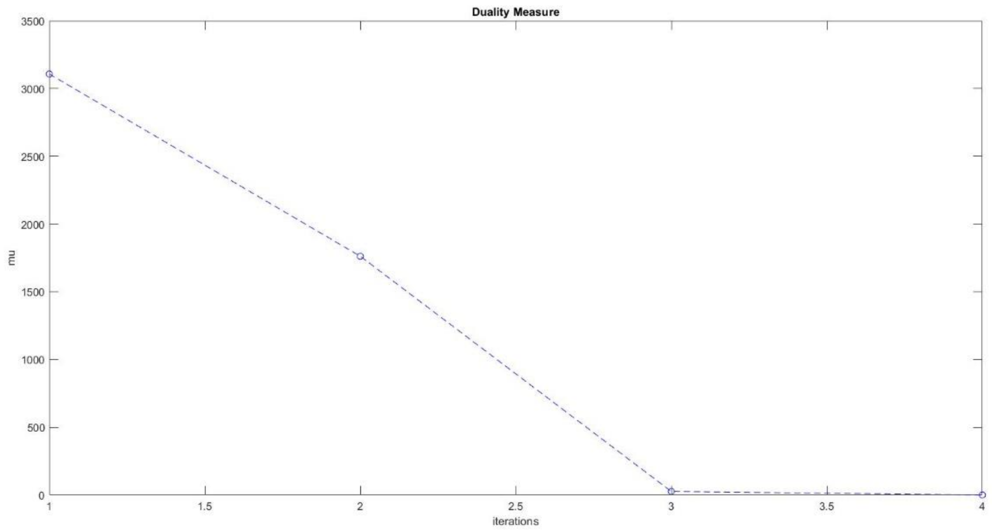
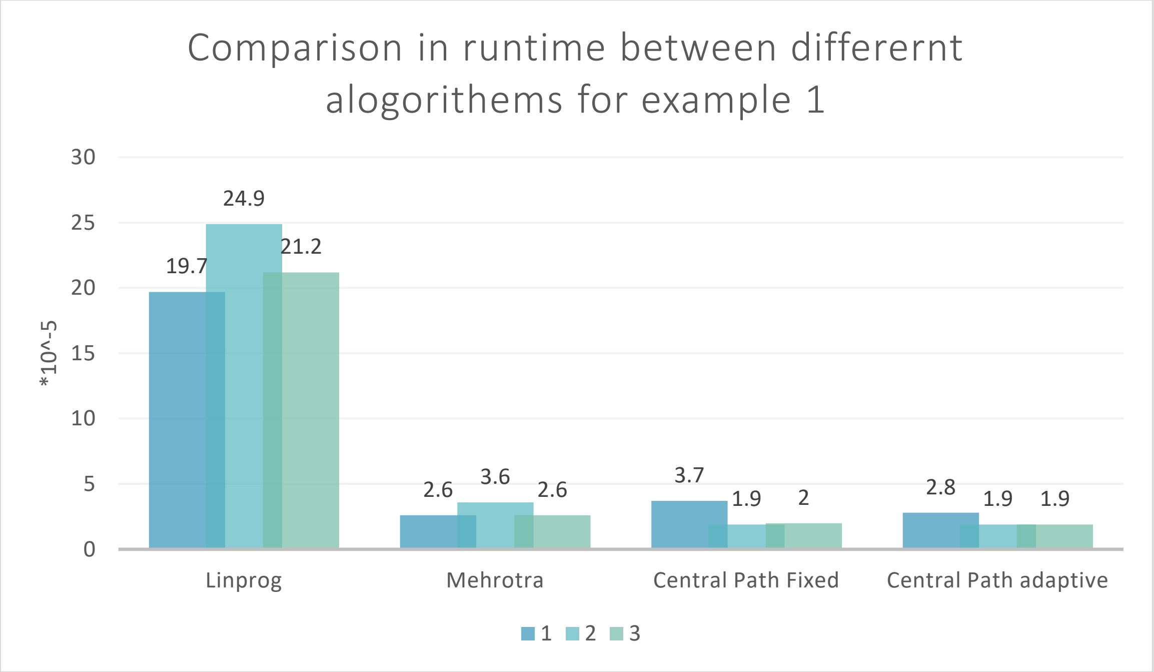


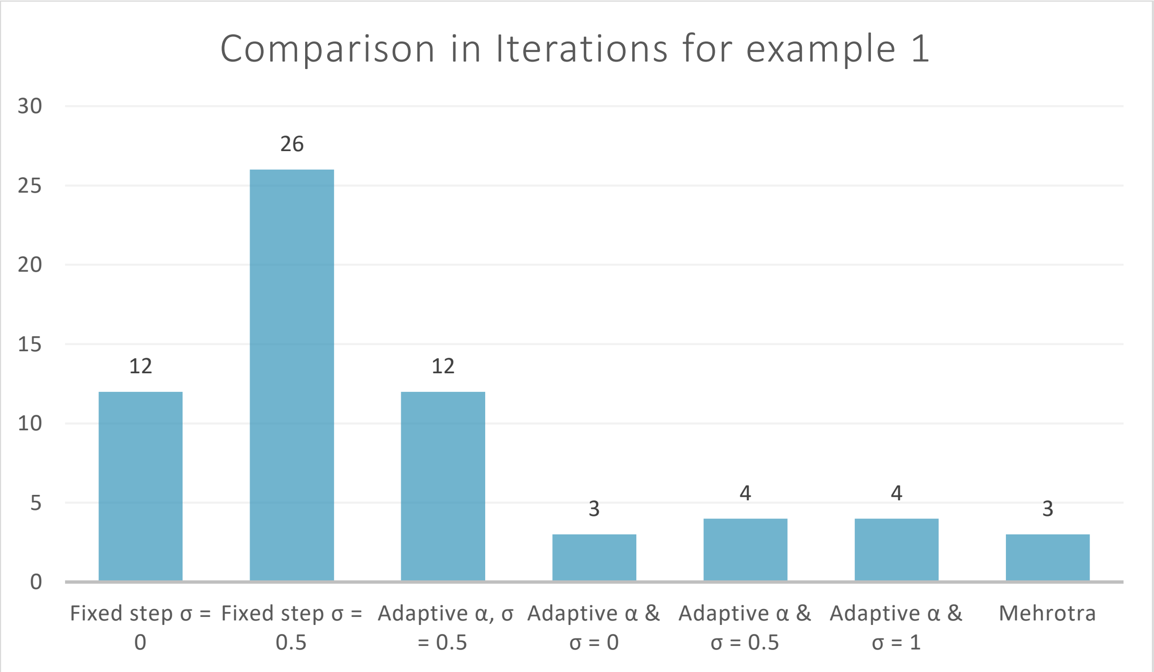
Figure 36 duality measure mu function of iteration

VI. Comparison

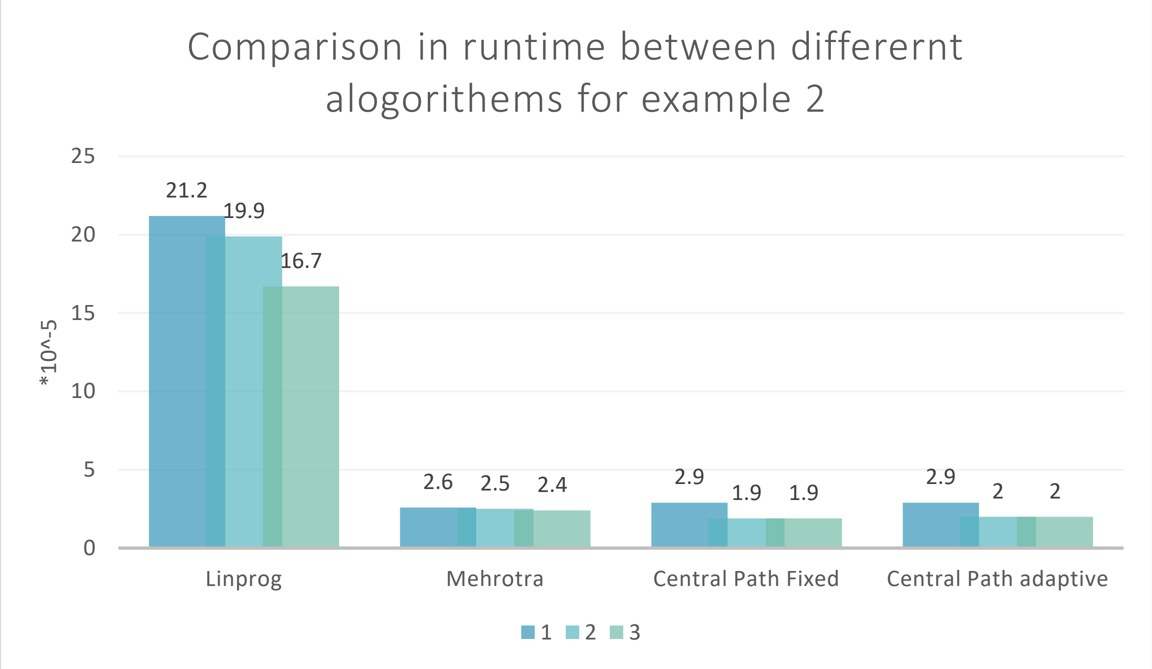
Example 1:



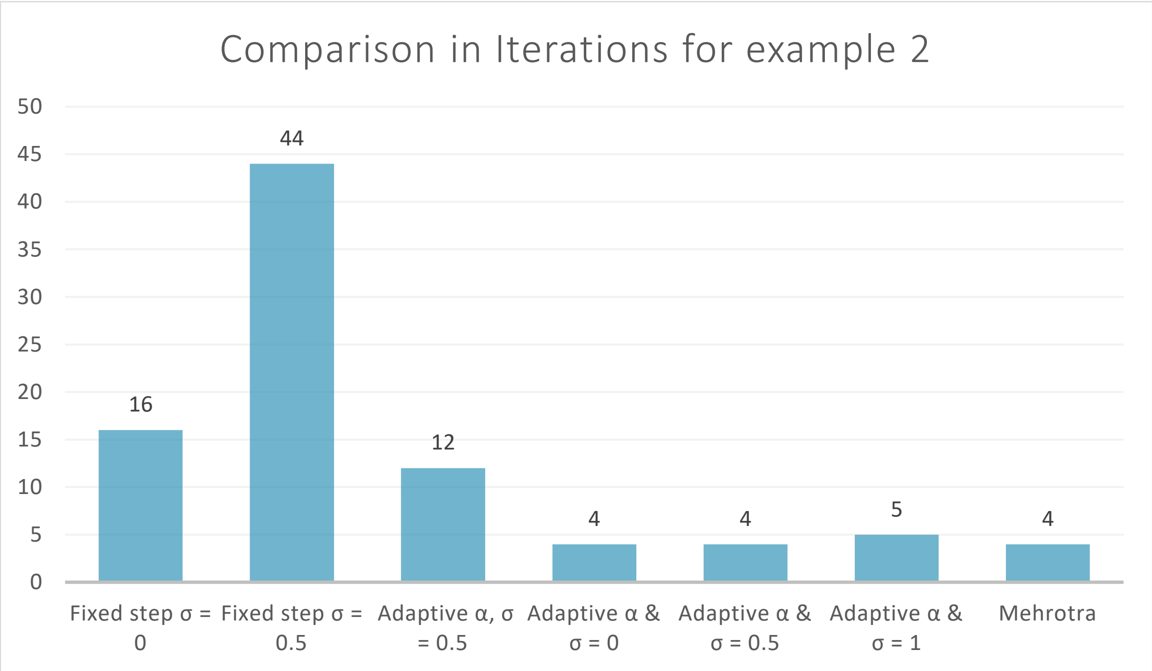
Three samples were taken for each algorithm for better comparison.



Example 2:

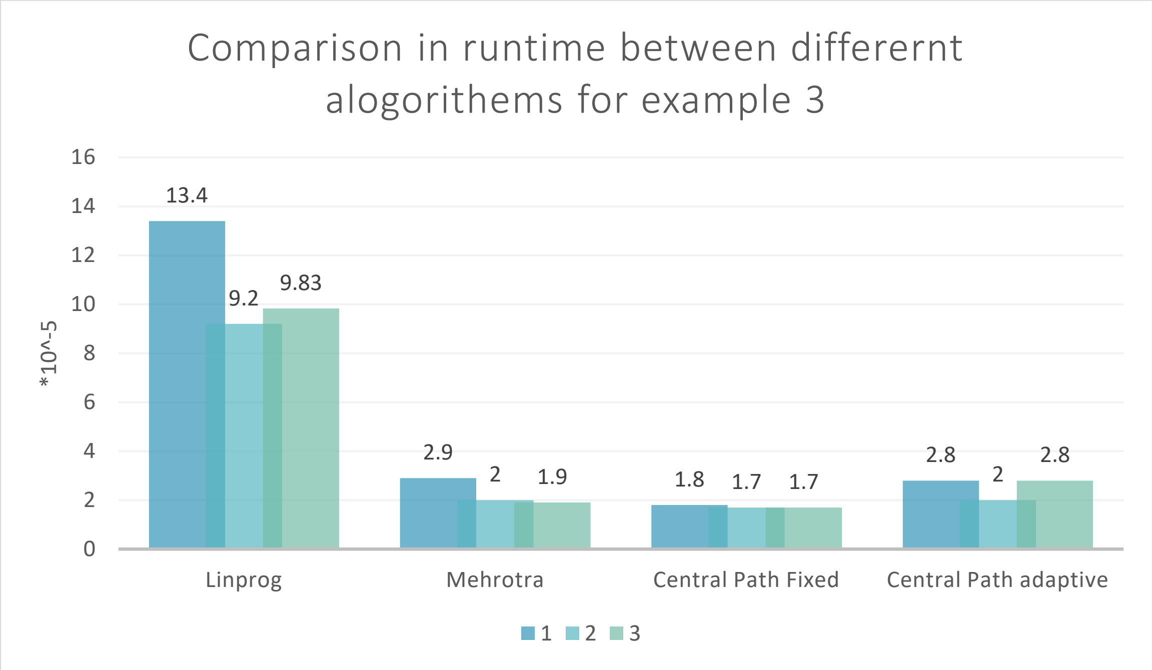


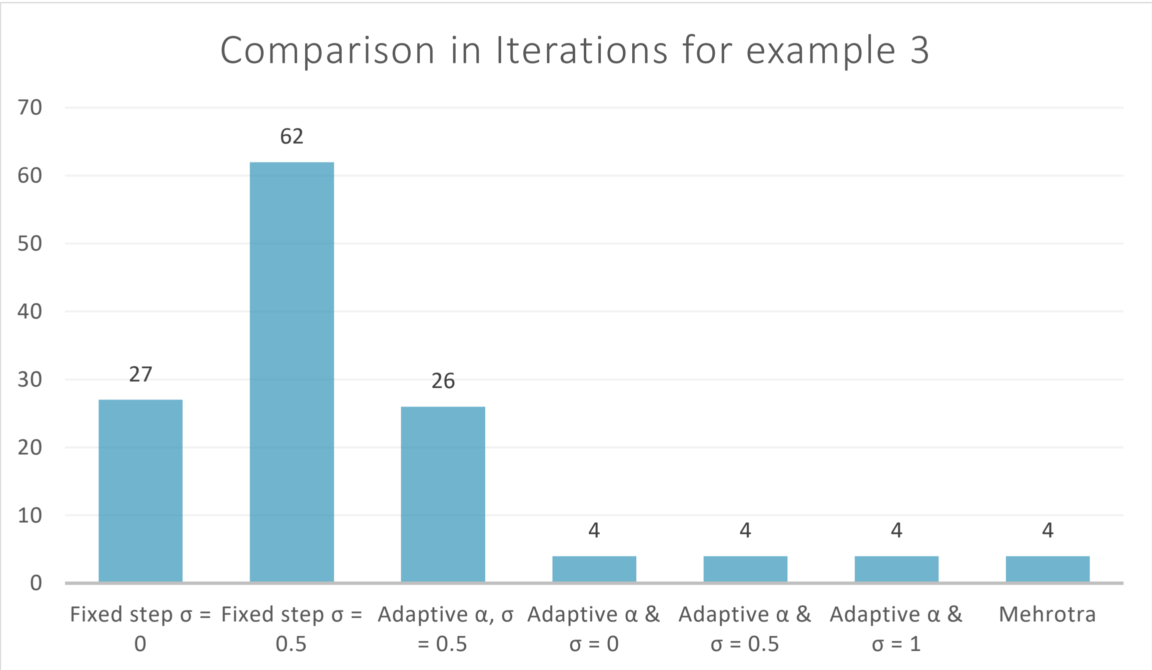
Comparison in runtime between differernt alogorithems for example 2
25
20
15
10
5
0
*10^-5
21.2
19.9
16.7
2.6
2.5
2.4
2.9
1.9
1.9
2.9
2
2
Linprog
Mehrotra
Central Path Fixed
Central Path adaptive
1
2
3



Comparison in Iterations for example 2
50
45
40
35
30
25
20
15
10
5
0
16
44
12
4
4
5
4
Fixed step σ = 0
Fixed step σ = 0.5
Adaptive α, σ = 0.5
Adaptive α & σ = 0
Adaptive α & σ = 0.5
Adaptive α & σ = 1
Mehrotra

Example 3:


Comparison in runtime between differernt alogorithems for example 3
*10^-5
16
14
12
10
8
6
4
2
0
13.4
9.2
9.83
2.9
2
1.9
1.8
1.7
1.7
2.8
2
2.8
Linprog
Mehrotra
Central Path Fixed
Central Path adaptive
1
2
3


Comparison in Iterations for example 3
70
60
50
40
30
20
10
0
27
62
26
4
4
4
4
Fixed step σ = 0
Fixed step σ = 0.5
Adaptive α, σ = 0.5
Adaptive α & σ = 0
Adaptive α & σ = 0.5
Adaptive α & σ = 1
Mehrotra

VII. References

[1] Matrix decomposition for solving linear systems (no date) *Matrix decomposition for solving linear systems - MATLAB*. Available at: https://www.mathworks.com/help/matlab/ref/decomposition.html (Accessed: January 8, 2023).

[2] Solve System of Linear Equations (no date) *Solve System of Linear Equations - MATLAB & Simulink*. Available at: https://www.mathworks.com/help/symbolic/solve-a-system-of-linear-equations.html (Accessed: January 8, 2023).

[3] Solve systems of linear equations (no date) *Solve systems of linear equations Ax = B for x - MATLAB mldivide *. Available at: https://www.mathworks.com/help/matlab/ref/mldivide.html (Accessed: January 8, 2023).