Off-Design Datasheet

Off-Design Analysis Methods
- Classical Method
- Simplified Method

⚠ Ref. Values → design values
⚠ std. values → Static Sea Level values $T_{std} = 288.16\ K$, $P_{std} = 101325\ Pa$
⚠ rel. values → Off-design value / design value

⚠ Don't use Polytropic eff. in off-design analysis.

* Corrected Parameters :-

* $\delta_i = \dfrac{P_{t\,i}}{P_{std.}}$ → Total pressure
* $\theta_i = \dfrac{T_{t\,i}}{T_{std.}}$ → Total temp.
* $N_{C_i} = \dfrac{N}{\sqrt{\theta_i}}$ → Rotational speed
* $\dot{m}_{C_i} = \dfrac{\dot{m}_i \sqrt{\theta_i}}{\delta_i}$ → Mass flow rate
* $F_c = \dfrac{F}{\delta_0}$ → Thrust
* $S_c = \dfrac{S}{\sqrt{\theta_0}}$ → S.F.C
* $\dot{m}_{f_c} = \dfrac{\dot{m}_f}{\delta_2 \sqrt{\theta_2}}$ → Fuel flow rate

  2 → station number

Section 8.1.3 in the ref.
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Classical Methods:

Equations:

$$\star \quad \frac{\dot m_4 N}{P_{t4}} = \frac{\dot m_2 \sqrt{T_{t2}}}{P_{t2}} \; \frac{P_{t2}}{P_{t3}} \; \frac{N}{\sqrt{T_{t2}}} \; \frac{P_{t3}}{P_{t4}} \; \frac{\dot m_4}{\dot m_2} \qquad \boxed{1}$$

steady Comp. turb. state Continuity

$$\star \quad \frac{\Delta T_{t4-5}}{T_{t4}} = \frac{\Delta T_{t2-3}}{T_{t2}} \; \frac{c_{pc}}{\eta_m c_{pt}} \; \frac{\dot m_2}{\dot m_4} \; \frac{T_{t2}}{T_{t4}} \qquad \boxed{2}$$

Power balance

$$\star \quad \frac{N}{\sqrt{T_{t4}}} = \frac{N}{\sqrt{T_{t2}}} \; \frac{\sqrt{T_{t2}}}{\sqrt{T_{t4}}} \qquad \boxed{3}$$

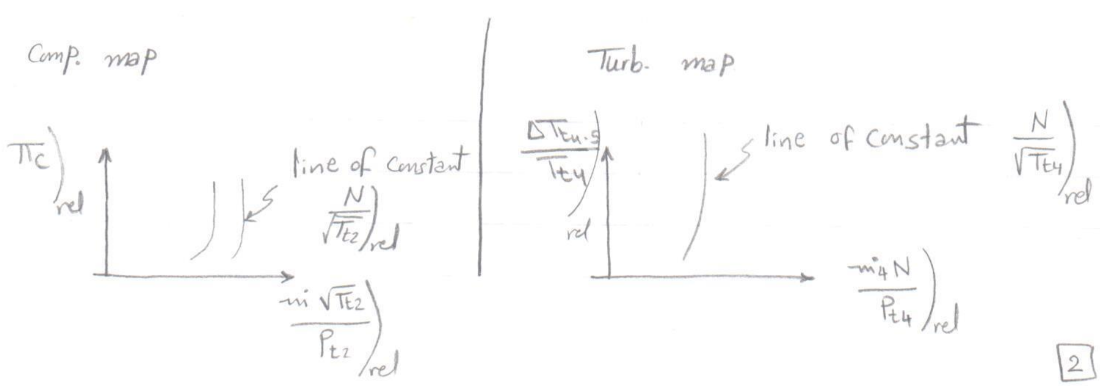
rotational speed Compatibility

In Relative Form:

$$\boxed{1} \quad \left.\frac{\dot m_4 N}{P_{t4}}\right)_{rel} = \left.\frac{\dot m_2 \sqrt{T_{t2}}}{P_{t2}}\right)_{rel} \; \left.\frac{1}{\pi_c}\right)_{rel} \; \left.\frac{N}{\sqrt{T_{t2}}}\right)_{rel}$$

$$\boxed{2} \quad \left.\frac{\Delta T_{t4-5}}{T_{t4}}\right)_{rel} = (\tau_c - 1)_{rel} \; \left.\frac{T_{t2}}{T_{t4}}\right)_{rel} \; \left.\frac{1}{(1+f)(1-\underbrace{\epsilon}_{\text{bleed}})}\right)_{rel}$$

$$\boxed{3} \quad \left.\frac{N}{\sqrt{T_{t4}}}\right)_{rel} = \left.\frac{N}{\sqrt{T_{t2}}}\right)_{rel} \sqrt{\left.\frac{T_{t2}}{T_{t4}}\right)_{rel}}^{\,\star}$$



Procedure: For given $\left.\frac{T_{t4}}{T_{t2}}\right)_{rel}$ or $\left.\frac{\dot{m}_f \sqrt{T_{t2}}}{P_{t2}}\right)_{rel}$

1) Assume an operating point on Comp. map $\longrightarrow$ get $\left(\frac{\dot{m}_2\sqrt{T_{t2}}}{P_{t2}}, \pi_c, \eta_c, \frac{N}{\sqrt{T_{t2}}}\right)_{rel}$

2) Use eq$\underline{n}$. $\boxed{1}$ get $\left.\frac{\dot{m}_4 N}{P_{t4}}\right)_{rel}$

3) use eq$\underline{n}$. $\boxed{2}$ & the value of $\left.\frac{T_{t2}}{T_{t4}}\right)_{rel} \longrightarrow$ get $\left.\frac{\Delta T_{t4-5}}{T_{t4}}\right)_{rel}$

4) Use $\frac{\dot{m}_4 N}{P_{t4}}$ & $\frac{\Delta T_{t4-5}}{T_{t4}}$ $\longrightarrow$ locate the Point on the Turb. map

5) Check rotational speed using eq$\underline{n}$. $\boxed{3}$ and iterate till Convergence.

★ **Fuel Flow Parameter:-** $\left(\frac{\dot{m}_f\sqrt{T_{t2}}}{P_{t2}}\right) \longrightarrow$ How to can use it to get $\left.\frac{T_{t4}}{T_{t2}}\right)_{rel}$!?

★ $\dot{m}_f = \dot{m}_0 \star f \longrightarrow \therefore \dot{m}_f = \dot{m}_0 \frac{C_{p_t} T_{t4} - C_{p_c} T_{t3}}{\eta_b h_{PR} - C_{p_t} T_{t4}}$

$$\longrightarrow \frac{\dot{m}_f}{P_{t2}\sqrt{T_{t2}}} = \frac{\dot{m}_0\sqrt{T_{t2}}}{P_{t2}} \star \frac{C_{p_t}\frac{T_{t4}}{T_{t2}} - C_{p_c}\tau_c}{\eta_b h_{PR} - C_{p_t} T_{t4}}$$

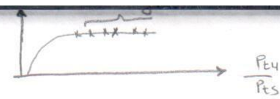
$$\therefore \left.\frac{\dot{m}_f}{P_{t2}\sqrt{T_{t2}}}\right)_{rel} = \left.\frac{\dot{m}\sqrt{T_{t2}}}{P_{t2}}\right)_{rel} \star \frac{\left.\frac{T_{t4}}{T_{t2}}\right)_{off} - \left.\frac{C_{p_c}}{C_{p_t}}\tau_c\right)_{off}}{\left.\frac{T_{t4}}{T_{t2}}\right)_{ref} - \left.\frac{C_{p_c}}{C_{p_t}}\tau_c\right)_{ref}} \star \underbrace{\frac{\eta_b h_{PR} - C_{p_t} T_{t4})_{ref}}{\eta_b h_{PR} - C_{p_t} T_{t4})_{off}}}_{\simeq 1}$$

$\therefore$ we can get $\left.\frac{T_{t4}}{T_{t2}}\right)_{rel} = f\left(\frac{\dot{m}_f}{P_{t2}\sqrt{T_{t2}}}\right)_{rel}$

$$\boxed{\left.\frac{T_{t4}}{T_{t2}}\right)_{rel} = \left.\frac{T_{t2}}{T_{t4}}\right)_{ref} \star \left[\frac{\left.\frac{\dot{m}_f}{P_{t2}\sqrt{T_{t2}}}\right)_{rel} \star \left(\left.\frac{T_{t4}}{T_{t2}}\right)_{ref} - \left.\frac{C_{p_c}}{C_{p_t}}\tau_c\right)_{ref}\right)}{\left.\frac{\dot{m}\sqrt{T_{t2}}}{P_{t2}}\right)_{rel}} + \left.\frac{C_{p_c}}{C_{p_t}} \star \tau_c\right)_{rel} \star \tau_c\right)_{ref}\right]}$$

Then solve the same procedure to get the Requirements. #

Case of chocked Turb. NGV :-



in this case $\longrightarrow$ $\left.\frac{\dot{m}\sqrt{T_{t4}}}{A_4 P_{t4}}\right)_{rel}$ = Constant

Sub. into $\boxed{1}$ $\longrightarrow$ $\therefore$ $\boxed{\left.\Pi_c\right)_{rel} = \left.\frac{\dot{m}\sqrt{T_{t2}}}{P_{t2}}\right)_{rel} * \left.\sqrt{\frac{T_{t4}}{T_{t2}}}\right)_{rel}}$

Gas Generator - Nozzle Matching :-

$$\left.\frac{\dot{m}_8\sqrt{T_{t8}}}{A_8 P_{t8}}\right)_{GG} = \frac{\dot{m}_2\sqrt{T_{t2}}}{A_2 P_{t2}}\left[\frac{P_{t2}}{P_{t5}}\sqrt{\frac{T_{t5}}{T_{t2}}}\ \frac{\dot{m}_5}{\dot{m}_2}\right]\left[\frac{\dot{m}_8}{\dot{m}_5}\ \frac{P_{t7}}{P_{t8}}\ \frac{P_{t5}}{P_{t7}}\right]\frac{A_2}{A_8} \qquad \boxed{1}$$

$$\frac{\dot{m}_8\sqrt{T_{t8}}}{A_8 P_{t8}} = f\left(\frac{P_{t8}}{P_o}\right) \rightarrow \text{from nozzle map } \boxed{2}$$

For matching $\left.MFP\right)_{\boxed{1}} = \left.MFP\right)_{\boxed{2}}$

* 1) Variable geom. $A_8 \neq ct.$

 $\hookrightarrow$ Select $\frac{N}{\sqrt{T_{t2}}}$, $\frac{T_{t4}}{T_{t2}}$ independently to get $\frac{P_{t5}}{P_{t2}}$, $\frac{T_{t5}}{T_{t2}}$

 Then change A_8 till $\left.MFP\right)_{8\,GG} = \left.MFP\right)_{8\,nozzle}$

* 2) Fixed geom. $A_8 = ct.$

 $\hookrightarrow$ iterate on GG operating point (values of $\frac{N}{\sqrt{T_{t2}}}$, $\frac{T_{t4}}{T_{t2}}$)

 till $\left.MFP\right)_{8\,GG} = \left.MFP\right)_{8\,nozzle}$

Simplified Methods:

Assumptions:

* $(1+f)$ = Constant but $f \neq$ Constant
* $\eta_c, \eta_f, \eta_b, \eta_{th}, \eta_{tl}, \eta_{mh}, \eta_{ml}$ = Constant
* π_b, π_d, π_n = Constant
* The flow is chocked at the high pressure turbine entrance nozzle

Equations

* $$\left.\frac{\dot m_2 \sqrt{T_{t2}}}{P_{t2}}\right)_{rel} = \left.\frac{\dot m_4 \sqrt{T_{t4}}}{P_{t4}}\right)_{rel} \frac{\pi_c)_{rel}}{\sqrt{\left.\frac{T_{t4}}{T_{t2}}\right)_{rel}}} \qquad \boxed{1}$$

* $$\tau_c - 1)_{rel} = (1-\tau_t)_{rel} \left(\frac{T_{t4}}{T_{t2}}\right)_{rel} \qquad \boxed{2}$$

* $$MFP)_4 = f(\pi_t) \qquad \boxed{3}$$

* $$\frac{\sqrt{\tau_t})_{rel}}{\pi_t)_{rel}} (MFP_4)_{rel} = \left(\frac{A_8}{A_4}\right)_{rel} MFP_8)_{rel} \qquad \boxed{4}$$

* $$MFP_8 = f(M_8, \gamma, R) \qquad \boxed{5}$$

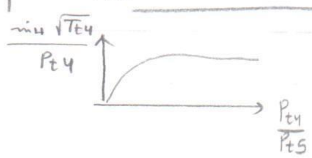
$$\rightarrow \quad M_8 = \sqrt{\frac{2}{\gamma_t - 1}\left[\left(\frac{P_{t8}}{P_8}\right)^{\frac{\gamma_t - 1}{\gamma_t}} - 1\right]}$$

Duct

$$\rightarrow \quad \frac{P_{t8}}{P_0} = \begin{cases} \pi_r \pi_d \pi_c \pi_b \pi_t \pi_D \pi_N & (CN) \\ \pi_r \pi_d \pi_c \pi_b \pi_t \pi_D \sqrt{\pi_N} & (CDN) \end{cases}$$

* For chocked nozzle and fixed A_8:

$$\boxed{\frac{\sqrt{\tau_t})_{rel}}{\pi_t)_{rel}} MFP_4)_{rel} = 1} \quad \boxed{*} \longrightarrow \quad \pi_t = \left(1 - \frac{1-\tau_t}{\eta_t}\right)^{\frac{\gamma_t}{\gamma_t - 1}} \quad \therefore \pi_t = f(\tau_t) \quad (1)$$



From this $\quad MFP_4)_{rel} = f(\tau_t)$ or $f(\pi_t) \quad (2)$

From ① & ② $\longrightarrow$ the solution of Continuity eqn. $\boxed{\star}$ necessitates that the relative values of $\left.\tau_t\right)_{rel}$, $\left.\pi_t\right)_{rel}$, $\left.MFP_4\right)_{rel}$ be 1 (turbine operates at the reference (design) point)

$$\therefore \quad \left.\pi_t\right)_{rel} = 1 \quad , \quad \left.\tau_t\right)_{rel} = 1 \quad , \quad \left.MFP_4\right)_{rel} = 1$$

In this case $\rightarrow$

$$\boxed{\left.\tau_c - 1\right)_{rel} = \left.\frac{T_{t4}}{T_{t2}}\right)_{rel} = \left.\left(\frac{N}{\sqrt{T_{t2}}}\right)^2\right)_{rel}} \quad \boxed{2}$$

$$\tau_c = \pi_c^{\frac{\gamma_c - 1}{\gamma_c}} \quad , \quad \boxed{\left.\frac{\dot{m}_2\sqrt{T_{t2}}}{P_{t2}}\right)_{rel} = \frac{\left.\pi_c\right)_{rel}}{\sqrt{\left.\frac{T_{t4}}{T_{t2}}\right)_{rel}}}} \quad \boxed{1}$$

* Matching Procedure for fixed A_8 : (Given T_{t4} & Flight Conditions)

① Assume nozzle is chocked
② calculate $\left.\tau_c\right)_{rel}$ from eqn $\boxed{2}$
③ check nozzle chocking by $\frac{P_{t8}}{P_0}$
$\rightarrow$ if nozzle is chocked $\longrightarrow$ the assumption is correct
$\rightarrow$ ④ calculate $\dot{m}_2$ from eqn. $\boxed{1}$

$\longrightarrow$ if the assumption is not correct $\longrightarrow$ Use un-chocked nozzle Procedure

① Assume a value of τ_t & through η_t get π_t
② Get τ_c from $\boxed{2}$
③ $\frac{P_{t8}}{P_0} = \frac{P_{t8}}{P_8}$ (un-chocked nozzle condition) $\longrightarrow$ get M_8 then MFP_8

$$MFP_8 = \sqrt{\frac{\gamma}{R}}\, M_8 \left(1 + \frac{\gamma_t - 1}{2} M_8^2\right)^{\frac{-(\gamma_t+1)}{2(\gamma_t-1)}}$$

④ Get τ_t from eqn $\boxed{4}$
⑤ Check with τ_t assumed $\longrightarrow$ iterate for convergence.
⑥ Get $\left.\frac{\dot{m}_2\sqrt{T_{t2}}}{P_{t2}}\right)_{rel}$ & $\dot{m}_2$ from eqn $\boxed{1}$

Variable chocked nozzle : $A_8 \neq$ Constant

The same procedure but $\boxed{\left.\frac{\sqrt{\tau_t}}{\pi_t}\right)_{rel} = A_8)_{rel.}}$ not $= 1$

* Given Rotational speed :

* Procedure : (For C ocked nozzle)

$$\left.\tau_c - 1\right)_{rel} = \left(\left(\frac{N}{\sqrt{T_{t_2}}}\right)^2\right)_{rel} \quad \boxed{2}$$

the same procedue

* Nozzle area ratio : slide $\boxed{32}$

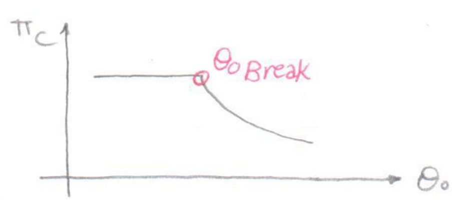
* Engine Limits :

$$\theta_0 = \frac{T_{t_0}}{T_{std}} = \frac{T_0}{T_{std}} \tau_r \quad \text{where } \boxed{T_{t0} = T_{t2}}$$

$$\therefore \theta_0 = \frac{T_0}{T_{std}} * \left(1 + \frac{\gamma_c - 1}{2} M_0^2\right)$$

* At Constant M_0 $\longrightarrow$ $h \uparrow$ $T_0 \downarrow$ $\theta_0 \downarrow$
* At Constant h $\longrightarrow$ $M \uparrow$ $\theta_0 \uparrow$

From eq$^{\underline{n}}$



$$\begin{cases} \theta_0 < \theta_{0\,Break} \longrightarrow \pi_c = \left.\pi_c\right)_{max}, & T_{t4} < \left.T_{t4}\right)_{max} \\ \theta_0 > \theta_{0\,Break} \longrightarrow \pi_c < \left.\pi_c\right)_{max}, & T_{t4} = \left.T_{t4}\right)_{max} \end{cases}$$