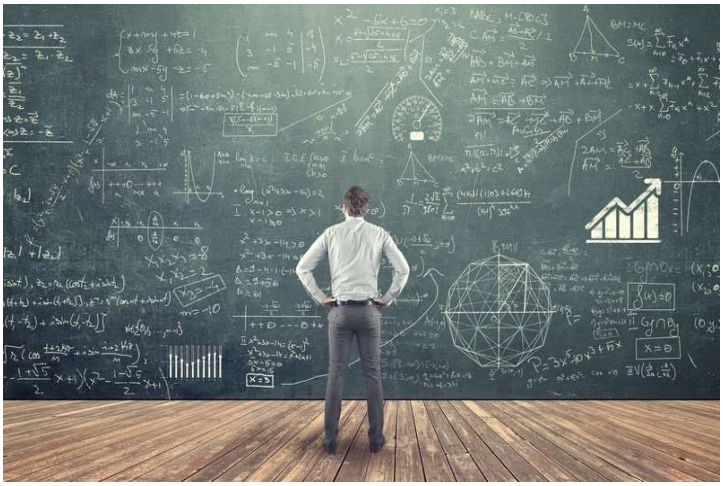

Lottery

The Business of Lottery and Probability

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INTRODUCTION:

Probability science is a very important and essential science that we nearly use in our everyday lives without realizing it, starting from your personal expectations for the result of a certain match and ending with the application of the mathematical probability laws in your field of study or work. Thus, if we investigate any certain field, we will find out that in one way or another a part of it is based on the science of probability, or let us say, is an application for probability. For example, in the medical field, clinical estimate of probability affects the physician's belief and diagnosis whether the patient has a disease or not. As a result, this belief/diagnosis determines the action that should be taken; to rule out, to treat or to do more tests.

Also, probability is an essential science when it comes to industry. The expected quality of products and the productivity of machines or factories for non-defective products are mainly based on the science of probability. As for the field of communication, to deliver signals and bits through communication channels, you must be aware that some of them are lost or fail during delivery. To make up for such a loss, engineers must be aware of the expected percentage of signals that usually fail. This will not happen unless they use techniques of probability.

Finally, when it comes to the business field, probability is a major pillar. It is essential for predicting future sales or a company's profits. It is essential for every branch and every type of business, even the lottery business! Wait, is the lottery considered as a business?

THE LOTTERY BUSINESS AND ECONOMY:

The answer is: Yes! Lotteries and prize drawings are big businesses throughout the world. They attract significant annual investments from individuals who dream of scooping up a great and potentially life-changing cash prize. Cheap ticket prices and the lure of large prizes make lottery a popular gambling platform; players do not have much to lose, but plenty to gain. In fact, it has been reported that up to 70% of UK adults, play the lottery regularly. As a result, those inexpensive lottery tickets add up to serious funds. Thus, it is a big business opportunity.

Profits generated by national lotteries are understandably huge. However, it is important to understand that in most countries, lotteries are primarily meant to give back to the community. If you trace the history of lotteries all the way back to ancient Rome, you will find out that they were used in Renaissance Europe to raise money for churches and other government projects. As for today, a large ratio of the lotteries' proceeds goes to public sectors, including education, park services, and funds for veterans and seniors. This happens through the enforced taxes on such a business**. That is why the lottery business contributes to national economies.**

In the UK, according to the law, large lotteries must donate a certain percentage of their revenue to good causes. According to the Gambling Commission of UK, in 2015-16, lotteries raised £1.9 billion in donations.

Moving on to the US, in 2014, Americans spent $70.15 billion on lottery tickets alone. That is more than the total spending on music, books, sports teams, movies, and video games combined, according to CNN Money. Thanks to the enforced lottery taxes laws in the US, the government gains huge funds from such a business. There is a law in the US states that if you win over $600 in the lottery, you will owe federal income taxes on that money.

THE LOTTERY AND ITS TYPES:

Now, we can all agree that the lottery is not just a business. It is a huge effective one, whether to national economies or even to the players themselves. However, some of us might still wonder ***what the lottery itself is.***

According to Oxford's dictionary, lottery is a means of raising money by selling numbered tickets and giving prizes to the holders of numbers drawn at random. In other words, it is a popular form of gambling, encouraging people to pay a small sum of money to be in with a chance of winning a big jackpot. However, it is a low-odds game of chance as winners are selected randomly. Lottery can be played in many ways with many rules. The following are some of the most famous lottery types:

1- The Lotto:

The main game is still the lotto, present in most countries and boasting the biggest payouts. The concept is simple, all you have to do is select 6 numbers between 0 and 47 (or other ranges), mark them on your card and enter. On the day of the draw, 6 numbers will be drawn from that range. Then, you can check your numbers to see if your chosen numbers match the winning numbers.

The numbers are usually printed onto colorful balls and placed in a special machine. Once in motion, the machine will mix the balls up and drop 6 of the numbers into another section of the machine. These will be the winning numbers.

Matching 6 numbers may be the dream but matching 5 or less numbers can sometimes also pay off. Winning with 5 matching numbers can sometimes mean you walk away with up to 1 million dollars in some games. Your odds will always be greater if you enter more than once.

2- The Powerball:

The odds of winning in Powerball are greater than walking away with the grand jackpot in a traditional lotto. However, each play is usually $2 (more than a Lotto ticket). The jackpot is won by matching 5 white balls and a 6th red ball in any order. The 5 balls are drawn from a drum of 59 balls with a separate drum of red balls where 1 ball will be selected. Your odds are better in the Powerball than the lotto. However, this may change as the winnings get higher and more people buy their tickets. This means that you will have to split your winnings with more people.

3- Raffles:

Raffles are usually run alongside the main Lotto, often seen as a bonus game. The raffle is played differently to a traditional lottery as you cannot choose your own numbers and have a pre-selected number assigned to you. Specific codes are printed on your ticket & if that code matches the one selected on the draw day, you win.

4- Scratch-Off Instant Games:

Instant games are your quick and easy online "scratch cards." These popular games are all over the internet and can have both small and large payouts, depending on how much you are willing to spend. Players can "scratch" their choice of card and win instant prizes if they are successful. The odds are sometimes as good as 1:3, a definite bonus for those wanting a fun yet effective game.

We have gained much information about the lottery business, its economic importance, its nature, and its types. Still, we have not mentioned what part of it is an application for the science of probability, or even is related to it.

PROFIT EXPECTATION & PROBABILITY:

Businesses are shaped by expectations. If any facet of business does not meet expectations, the business suffers, especially when it comes to the financial facet. Thus, an appropriate updated financial expectation is placed every now and then in any business, to determine the expected profits and the productivity of such a business.

Since after all lottery is a business, funds and profits are the greatest matter in such a field, whether to the entities or governments that control it or to the players themselves. Thus, profits expectation is a normal act in such an industry. However, profit expectation is affected by several aspects and factors, such as the expected amount for pay-offs, the expected number of participants, the probability of winning, the rules of each type of lottery, …etc. All those factors are determined through the different mathematical laws and techniques of probability. As a result, profit expectation in the lottery business is mainly based on the science of probability. **Thus, profit expectation is a probability application.**

Each type of lottery has its own rules and its own effect on the resulted profit. Therefore, we will only focus on the most famous type of them, which is "The Lotto".

To study the application of probability on profit expectation in Lotto, we will concentrate on 3 main aspects:

1- The probability of winning. (A Counting Techniques application).
2- The expected pay-off. (A Random Variable application).
3- The risk of such a type of investment for players. (A Random Variable application).

A) THE PROBABILITY OF WINNING:

It is a great matter to both the player and the business owner to be aware of the possibility of a certain number to win. For the player, it helps him to approximately determine his chances of winning. If it is a big ratio, then he would not hesitate and purchase a ticket immediately, but if it is not, he might reconsider it as possibly it would be wasted money. Even if that player calculates that probability after purchasing the ticket, it would be like an indication for him if he is likely to win or not. Moreover, it would either enhance him to purchase other different-numbered tickets or to lose hope and not build high expectations on winning.

As for the business owners, high probability would encourage players to purchase more tickets, and assuming that few of them – or maybe none – will not pick the right numbers, the profit for the company or the government controlling the game would be greater. In case of a low probability of winning, this encourages them to elevate the prize to encourage more people to purchase their tickets, knowing that it would be difficult for most of them to achieve it. So, how is that probability determined? ***Based on the techniques of counting,*** **let us study our probability on a game with the following rules, before generalizing the concept:**

*Consider a lottery in which a person ***picks 6 numbers*** from ***1 to 50*** without repetition. A player wins the 1st prize if the 6 selected numbers match the selected winning numbers by the entity (their order is not considered). So, when picking 6 numbers there are 7 possible scenarios: to guess 0,1,2,3,4,5 or 6 numbers correctly. Based on counting techniques, we can calculate number of combinations for every scenario.

Assuming that you pick 0 correct numbers, this means that you have picked neither of the 6 winning numbers AND 6 out of the losing numbers, which are: 50 – 6 = 44. So, to determine the number of possible combinations in this case (where order does not matter), we use the technique of mathematical combination as follow:

$$^{44}C_6 \times {}^{6}C_0$$

Similarly, if you pick only 1 correct number, this means that you have picked 1 number out of the 6 winning numbers AND 5 numbers out of the remaining 44 losing numbers, so the number of combination will be:

$$^{44}C_5 \times {}^{6}C_1$$

So, if we generalize a formula for calculating the number of combinations for our example, it will be as follow:

$$^{44}C_{6-n} \times {}^{6}C_n$$

Where n is the number of correct number a player has picked up.

As a result, we can construct the following table:

n	No. of Possible Combinations
0	7059052
1	6516048
2	2036265
3	264880
4	14190
5	264
6	1
Total no	15890700

← Winning Combination!

So, to calculate the probability of every possible pick, we use the following relation:

$$\frac{\text{No of possible combination for n correct numbers}}{\text{Total no of possible combinations}}$$

Thus, we can construct the following table:

n	P(n)
0	0.4442254
1	0.4100542
2	0.1281419
3	0.0166689
4	0.000893
5	$1.661349091 \times 10^{-5}$
6	$6.292988981 \times 10^{-8}$
Sum	1

← Winning Probability!

GENERALIZING THE IDEA:

If a player picks 6 different numbers from ***k to m.*** There are 7 possible scenarios: 0,1,2,3,4,5 or 6 numbers are correct (n numbers are correct). So, the number of possible combinations is:

$$^{i-6}C_{6-n} \times {}^{6}C_{n}$$

Where $\mathbf{i = m + 1 - k}$

Therefore, the probability of such a combination is calculated as follow:

$$\frac{^{i-6}C_{6-n} \times {}^{6}C_{n}}{\sum_{n=0} {}^{i-6}C_{6-n} \times {}^{6}C_{n}}$$

So, if you want to know the probability of winning: substitute n with 6:

$$\frac{{}^{i-6}C_0 \times {}^{6}C_6}{\sum_{n=0}^{} {}^{i-6}C_{6-n} \times {}^{6}C_n}$$

B) THE EXPECTED PAY-OFF:

Another important factor that must be put into consideration when expecting profit or when considering purchasing a lottery ticket is the average amount of money expected to be paid-off. There is a variation in prizes where not only the ones who pick 6 numbers win, but also those who pick 3 or more winning numbers win as well, perhaps a smaller prize, still they have a share of the cake. So, how do we calculate the expected pay-off? *It is mainly based on the concept of* ***random variables and the expected means.*** We can continue studying this through the previously mentioned game with the same rules:

*Assume that a player is paid if his numbers ***matched 3 or more*** numbers from the winning ones, with varying prizes that are determined as follow:

No of matching numbers	Value of pay-off
0	0
1	0
2	0
3	3
4	89
5	1268
6	4000000

Let random variable X = number of matches in a ticket.
h(X) = the pay-off for the event {X=x}, if x numbers were matched.

So, to determine the expected payoff (h(x)), we calculate the expected mean for the function h(x), as follow:

$E(h(X)) = \Sigma\, P(X=x).h(x)$ ⟶ (1)

P(X=x) can be calculated using the formula used in "the probability of winning" part. Thus, the probability density function will be as follow:

x	P(X=x)
0	0.4442254
1	0.4100542
2	0.1281419
3	0.0166689
4	0.000893
5	$1.661349091 \times 10^{-5}$
6	$6.292988981 \times 10^{-8}$
Sum	1

By substituting in (1):

$E(h(X)) = 0.4442254 \times 0 + 0.4100542 \times 0 + 0.1281419 \times 0 + 0.0166689 \times 3 + 0.000893 \times 89 + 1.661349091 \times 10^{-5} \times 1268 + 6.292988981 \times 10^{-8} \times 4000000$

$E(h(X)) = 0.40226 \approx 0.4$

This means that *on average every player would regain 0.4$.*

GENERALIZING THE IDEA:

Knowing how to calculate the probability for every possible number of matches -as discussed in detail in the previous section-, you can use the following formula to determine the average payoff for each player:

$$E(h(X)) = \Sigma\, P(X=x).h(x)$$

Where: X = number of matches in a ticket, & h(X) = the pay-off for the event {X=x} (which varies from one game/event to another).

C) THE RISK OF INVESTMENT (FOR PLAYERS):

Based on the previous part, the expected paid-off value is usually very small. Actually, it is very rarely that a player could get back anything from that one-dollar investment. Thus, it is a risky investment. However, people believe numbers rather than theories and based logic. So, how can "numbers" show that risk? That is so simple. Calculate ***the standard deviation.***

Continuing studying the same example, assume that the player pays ***1$ per ticket*** and the expected pay-off value is ***0.4$*** (as calculated previously). To know the risk, calculate the deviation from the average pay-off:
Let random variable X = number of matches in a ticket.
h(X) = the pay-off for the event {X=x}, if x numbers were matched.
E(h(X)) = 0.4 and pay-off table & PDF are as shown:

x	P(X=x)
0	0.4442254
1	0.4100542
2	0.1281419
3	0.0166689
4	0.000893
5	$1.661349091 \times 10^{-5}$
6	$6.292988981 \times 10^{-8}$

x	h(x)
0	0
1	0
2	0
3	3
4	89
5	1268
6	4000000

1st: Variance: $V(h(x)) = E((h(x))^2) - (E(h(x)))^2 = (3^2 \times 0.0167 + 89^2 \times 0.00089 + 1268^2 \times 1.66\times10^{-5} + 4000000^2 \times 6.293 \times 10^{-8}) - (0.4)^2 = 1006912.012\ \$^2$

2nd: Standard Deviation: $\sigma = \sqrt{V(h(x))} = \sqrt{1006912.012} = 1003.45\$$

The more the value deviates from the expected pay-off, the riskier the investment is. So, here in our examples, it is kind of risky to invest much money is such tickets.

GENERALIZING THE IDEA:

Generally speaking, the standard deviation determines the risk of investments in the field of business; the greater its value deviates, the riskier the investment is. As for the lottery business, knowing the average pay-off, a player can determine the risk of investing his money by purchasing tickets through the following formula:

$$\sigma = \sqrt{V(h(x))} = \sqrt{E((h(x))^2) - (E(h(x))^2}$$

Where h(x) is the pay-off (determined by the entity controlling the game).

IN REAL LIFE (MATLAB CODE):

Expecting profits & advising players whether to purchase tickets in a certain lottery or not has become a business itself nowadays. Many companies offer players to determine for them if playing a certain lottery would be profitable for them or not. They'd tell them the probability they 'd win, the expected payoff they'd get, how risky is it, and other things based on probability & the rules of the game. Actually, there are free online websites & apps that offer such services. So, we will show you a simple approach of how those websites operate through the following MATLAB code:

```
%1st: determining probability of winning first prize (for players)
start=input('Enter the start of range you are allowed to pick numbers from ');
ending=input('Enter the end of that range ');
x=ending-start-5;
sum=0.0;
for n = 0 : 6
    sum = sum + (factorial(x) / (factorial(x - 6 + n)*factorial(6 - n))) *
(factorial(6)/(factorial(6 - n)*factorial(n)));
end
probwin = 1/sum;
disp(['probability of a ticket to be the winning one = ', num2str(probwin)])
if probwin > 1/593775
    disp('Go for it and purchase one or more tickets!')
else
    disp('Reconsider it or purchase ONLY one ticket!')
end


%2nd: determining the expected pay-off for every player and risk of investment:
mean = 0;
mean2 = 0;
for n = 0 : 6
    prompt = ['Enter pay off value for ',num2str(n),' matching numbers! '];
    pay = input(prompt);
    prob = ((factorial(x) / (factorial(x - 6 + n)*factorial(6 - n))) *
(factorial(6)/(factorial(6 - n)*factorial(n)))/sum);
    mean = mean + prob*pay;
    mean2 = mean2 + power(pay,2) * prob;
end
disp(['The average pay-off you will get = ',num2str(mean),'$'])
var = mean2 - power(mean , 2);
standdev = power(var , 0.5);
disp(['The risk of investment: The payoff deviates by ',num2str(standdev)])
if standdev > 100*mean
    disp('It is a relatively high risk')
else
    disp('Risk is relatively not very high!')
end
```

That MATLAB program is mainly aimed at helping players make decisions on purchasing lottery tickets (just as those companies and websites do):

1st: it prompts the user for the start and the end of the range from which he would pick his numbers.

2nd: using the rule of combination, a loop is used to compute $\sum {}^{i-6}C_{6-n} \times {}^{6}C_{n}$

3rd: it computes the probability of choosing six winning numbers (at n = 6).

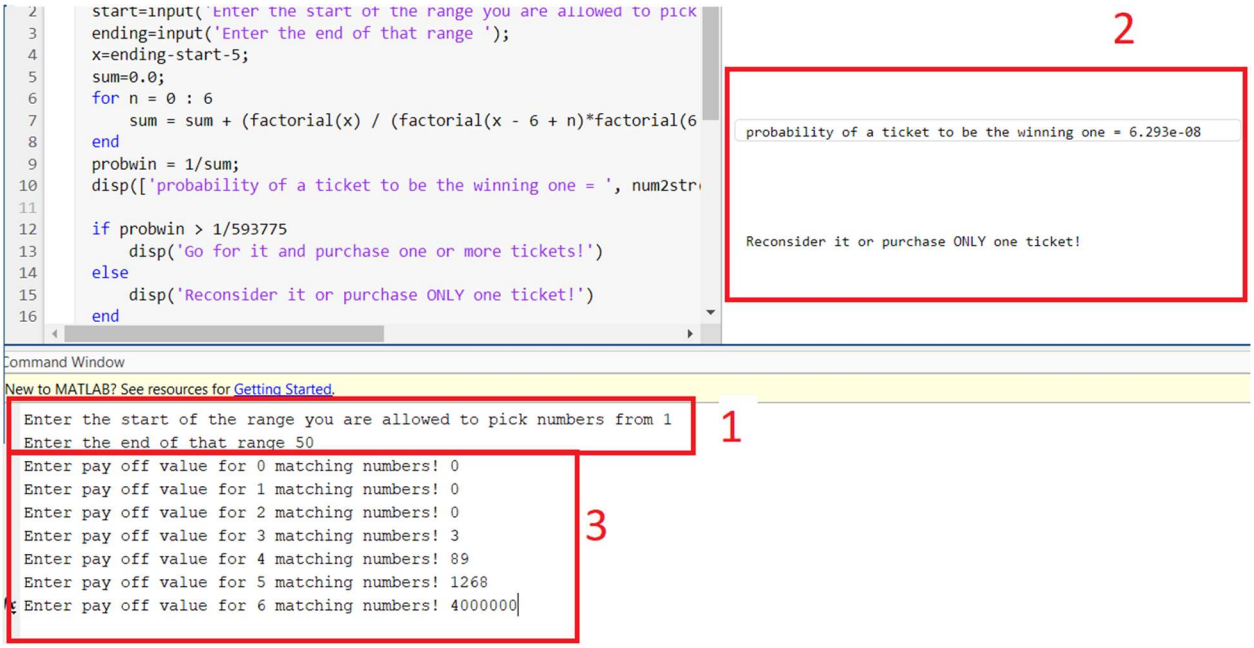
4th: It's most common in "lotto's that ranges would roughly vary between (1-10 & 1-50), where 1-10 range has greater probability of winning than 1-50. So, the average range would be: 1-30. The program uses the probability of winning in this range as an indication to whether the previously computed probability is a large one or not. If it is greater than that of the average range, it encourages the user to buy 1 or more tickets. Else, it advices him to reconsider it.

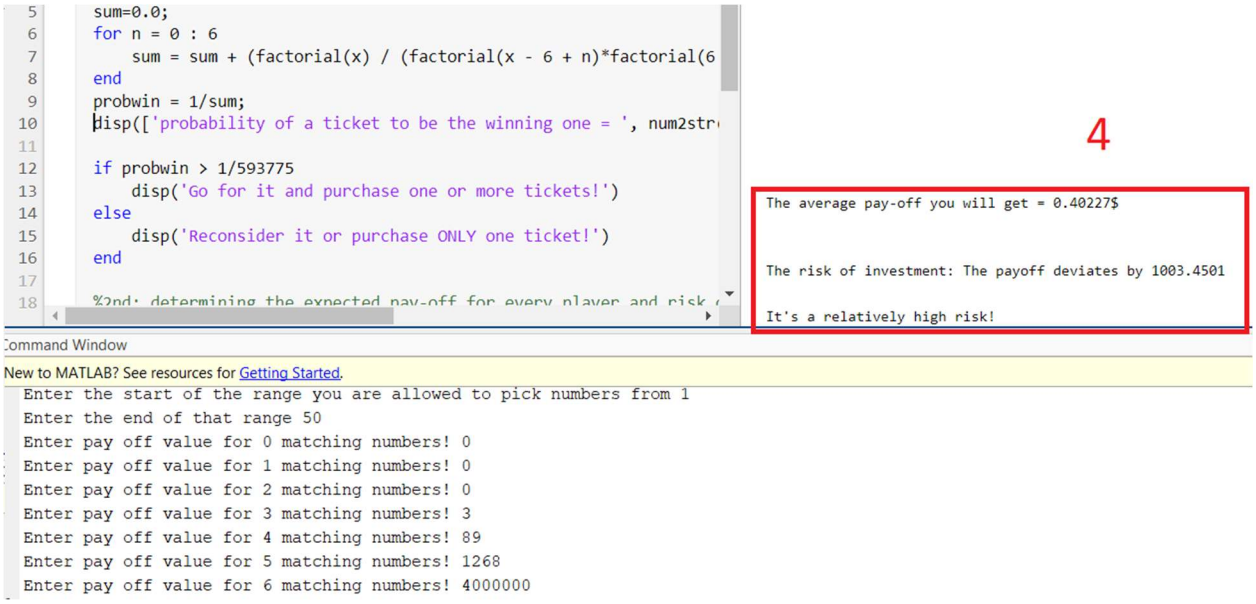
5th: it prompts the user for the pay-off values (which is given from the company controlling the game)

6th: as a result, it shows the user the average pay-off he would get using the expected mean formula. Thus, it helps expect the gain from such an investment.

7th: Also, it determines the risk of investing in those tickets using the standard deviation formula. Whether the deviation is high or low is an issue that is determined through studies by experts that varies from one data type to another. So, for our application, we have assumed that an investment is too risky if the payoff deviates the average value by more than 100 times.

The following are screenshots for a sample run:





```
sum=0.0;
for n = 0 : 6
    sum = sum + (factorial(x) / (factorial(x - 6 + n)*factorial(6
end
probwin = 1/sum;
disp(['probability of a ticket to be the winning one = ', num2str
if probwin > 1/593775
    disp('Go for it and purchase one or more tickets!')
else
    disp('Reconsider it or purchase ONLY one ticket!')
end
%2nd: determining the expected pay-off for every player and risk
```

4

```
The average pay-off you will get = 0.40227$

The risk of investment: The payoff deviates by 1003.4501

It's a relatively high risk!
```

Command Window

New to MATLAB? See resources for Getting Started.

```
Enter the start of the range you are allowed to pick numbers from 1
Enter the end of that range 50
Enter pay off value for 0 matching numbers! 0
Enter pay off value for 1 matching numbers! 0
Enter pay off value for 2 matching numbers! 0
Enter pay off value for 3 matching numbers! 3
Enter pay off value for 4 matching numbers! 89
Enter pay off value for 5 matching numbers! 1268
Enter pay off value for 6 matching numbers! 4000000
```

CONCLUSION:

This research is not written in support of gambling and none of us supports it. However, it was just an analysis aimed at showing that lottery nowadays is more than just a luck game. It is not just about purchasing tickets and earning prizes. No. It has internationally become a huge effective business with branching smaller business (such as: expecting wins and pay-offs websites and companies) that greatly depends on the science of probability. So, do not play the lottery! Just know about its business.

REFERENCES:

1. *The Profitability of the Lottery Business* from *"How to Start a Lottery Business" article* from *Startups blog.*
   Link: https://startups.co.uk/guides/how-to-start-a-lottery-business/

2. *The Effect of Lottery Business on Economy* from "The Economics of Lottery" article by *Amelia Josephon* from *Smartasset's website.*
   Link: https://smartasset.com/taxes/the-economics-of-the-lottery

3. *The Effect of Lotter Business on Economy* from *"The Good and the Bad of National Lotteries" article* by *Lewis Hemphries* from *investopedia's website.*
   Link: https://www.investopedia.com/financial-edge/0512/the-good-and-bad-of-national-lotteries.aspx

4. *The Definition of Lottery* from *"Lottery" article* by *Daniel Liberto* from *investopedia's website.*
   Link: https://www.investopedia.com/terms/l/lottery.asp

5. *Types of Lottery* from *Lottocop's website.*
   Link: http://www.lottocop.com/lottery-tips/the-different-types-of-lotteries-to-play/

6. *Probability vs Possibility in Lottery* from *Data Data Data's blog by Kyla Scanlon.*
   Link: https://kylascanlon.com/2020/04/26/probability-vs-possibility-lottery-analysis-in-quarantine/

7. *Lottery: A life application for Probability* from *OPRE6301, a study document from the University of Texas, Dallas.*

8. *Risk in Investments & Standard Deviation* from *"How is Standard Deviation used to Determine Risk?" article* by *Brian Beers* from *Investopedia's website.*
   Link: https://www.investopedia.com/ask/answers/021915/how-standard-deviation-used-determine-risk.asp#:~:text=One%20of%20the%20most%20common,an%20investment%20will%20be%20risky

9. An Example for a Website that Calculates Probability of Winning in Detail → webmath.com
   Link: http://www.webmath.com/lottery.html