Time Series

- Is a collection of observations of well-defined data items obtained through repeated measurements over time.

- An ordered sequence of values of a variable at equally spaced time intervals.

- For example, measuring the value of retail sales each month of the year would comprise a time series. This is because sales revenue is well defined, and consistently measured at equally spaced intervals. Data collected irregularly or only once are not time series.

- Time Series Analysis

➢ Analyzing time series data in order to extract meaningful statistics and other characteristics of the data.

- Time Series Forecasting

➢ Estimating many future aspects of a business or other operation based on the current time series.

Goals of Times Series

There are two main goals

1) Identifying the nature of the phenomenon represented by the sequence of observations.

2) Forecasting (predicting future values of the time series variable).

Important of Time Series

- A very popular tool for Business Forecasting.
- Basis for understanding past behavior.
- Can forecast future activities/planning for future operations.
- Evaluate current accomplishments of performance.
- Facilitates comparison Importance of time series.

- Stock & Watson (2007) say that the assumption that the future will be like the past is an important one in time series regression. If the future is like the past, then the historical relationships can be used to forecast the future. But if the future differs fundamentally from the past, then the historical relationships might not be reliable guides to the future. Therefore, in the context of time series regression, the idea that historical relationships can be generalized to the future is formalized by the concept of **stationarity**.

- Why are stationary time series so important? According to Gujarati (2003, 2011), there are at least two reasons. *First*, if a time series is nonstationary, we can study its behavior only for the time period under consideration. Each set of time series data will therefore be for a particular episode. As a result, it is not possible to generalize it to other time periods. Therefore, for the purpose of forecasting or policy analysis, such (nonstationary) time series may be of little practical value. *Second*, if we have two or more nonstationary time series, regression analysis involving such time series may lead to the phenomenon of **spurious** or **nonsense regression** (Gujarati, 2011; Asteriou, 2007).

- He was trying to explain changes in the price level across time. He built a model in which he was using the money supply as the explanatory variable. he found that there was some sort of correlation between these variables and when he run the regression he found and R^2 of 0,995
- Then he did a regression between prices and a variable C_t which had even better correlation with prices when he looked at the graph of P_t against C_t ,and found an R^2 of 0,998. It looked like he had found a better model.

But…. C_t was the humidity in the UK.

And why did he get these results?

He was regressing one nonstationary variable on another nonstationary variable.

This is called a **spurious regression.**

- Granger and Newbold came up with a rule which is a way of diagnosing when your model has spurious regression going on.
- The idea is that if you have an $R^2 >$ DW statistic then you might have a spurious relationship.
- If we have a low value of DW then this means that you have large runs of positive residuals and then we have large runs of negative residuals. (AR(1) serial correlation).

Stationarity

- ❖ Time series is said to be stationary if its **mean** and **variance** are **constant over time**

and

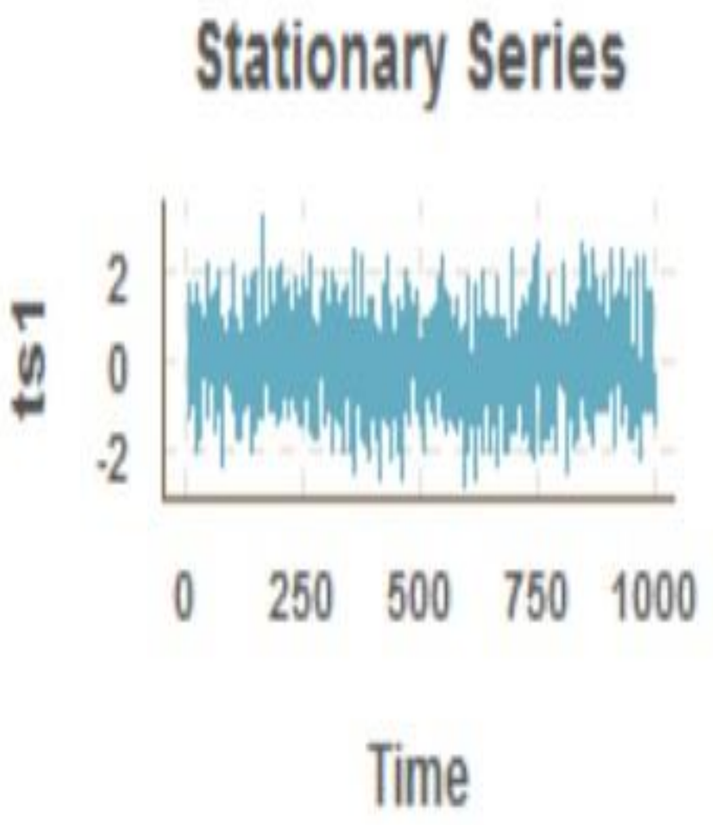
- ❖ The value of the covariance between the two periods depends only on the distance or gap or lag between the two time periods and not the actual time at which the covariance is computed

Stationarity

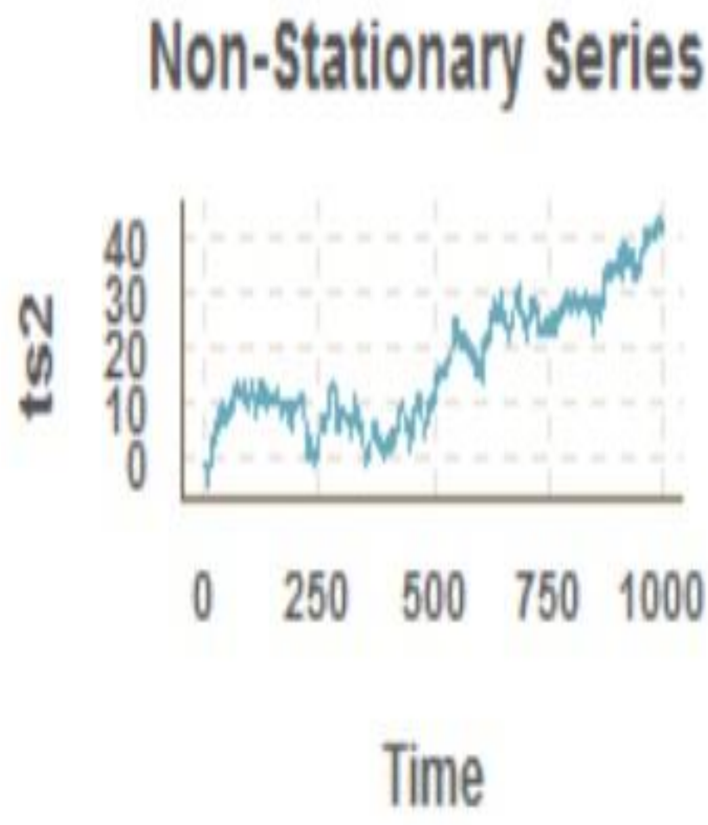
1) $$E\{Xt\} = \mu$$

2) $$Var\{Xt\} = \sigma^2$$

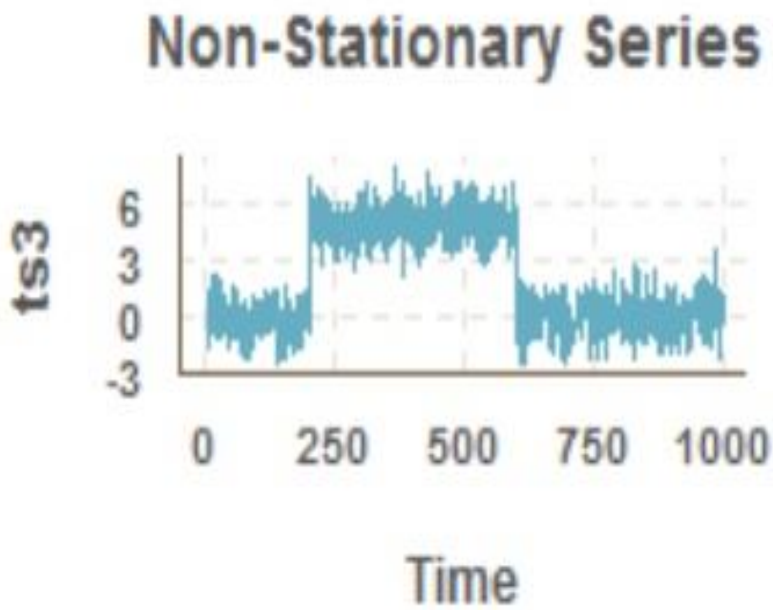
3) $$Cov\{Xt, Xt + h\} = f(h) \quad \neq g(t)$$



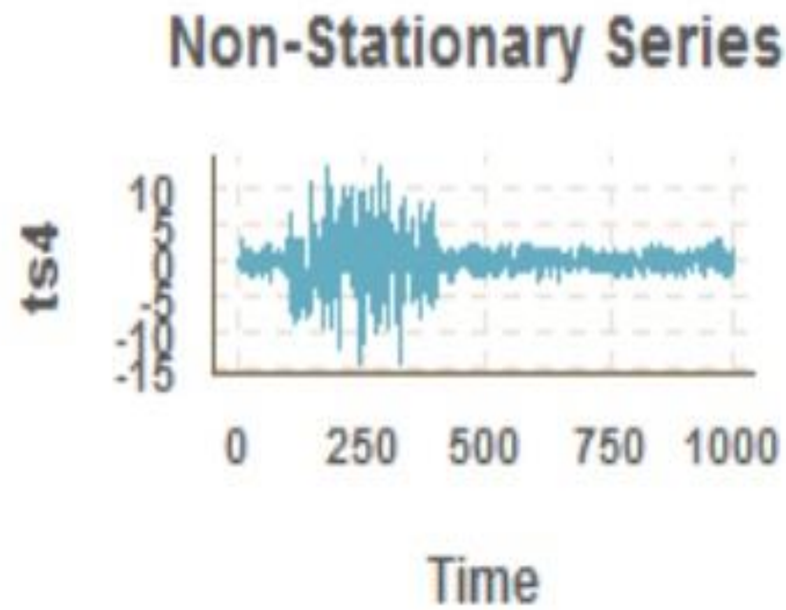
stationary – E{Xt}=μ, Var{Xt}=σ^2



Non-stationary in mean



Non-stationary - mean



Non-stationary – covariance

Why stationarity is so important?

- First, if a time series is nonstationary, we can study its behavior only for the time period under consideration. Each set of time series data will therefore be for a particular episode. As a result, it is not possible to generalize it to other time periods. Therefore, for the purpose of forecasting or policy analysis, such (nonstationary) time series may be of little practical value.
- Second, if we have two or more nonstationary time series, regression analysis involving such time series may lead to the phenomenon of **spurious or nonsense regression**

Testing for non-Stationarity -Dickey-Fuller test

$$X_{t}= \alpha + \rho X_{t-1} + \varepsilon_t$$

α=0, α≠0

$$X_t - X_{t-1} = \alpha + (\rho - 1)X_{t-1} + \varepsilon_t$$

$$\Delta X_t = \alpha + \delta X_{t-1} + \varepsilon_t$$

Now ΔXt is stationary so we can estimate δ

Ho: ρ=1

H1: ρ<1

t statistic < DFcritical

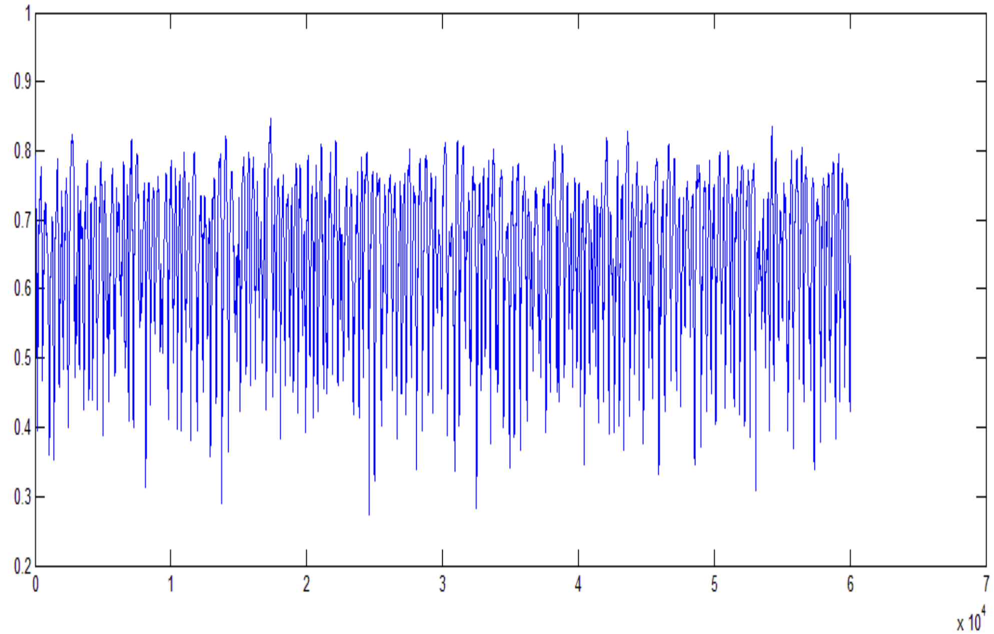
→reject the null hypothesis

White noise

- $E\{\varepsilon_t\} = 0$ (fluctuations around zero)
- $Var\{\varepsilon_t\} = \sigma^2$ (homoskedasticity)
- $Cov\{\varepsilon_{t,}\ \varepsilon_{t+h}\} = o \quad h \neq 0$ (no serial correlation)

$$\varepsilon_t \sim IID(0,\sigma^2)$$

I - Independent
I - Identically
D - Distrubeted



Random Walks

A random walk is an AR(1) model where ρ=1, meaning the series is not quit dependent.

Random Walks

A random walk is an AR(1) model where ρ=1, meaning the series is not quit dependent.

Random walk with no drift

The **random walk without drift** is defined as follow. Suppose *ut* is a white noise error term with mean 0 and variance 2. The Y_t is said to be a random walk if:

$$Y_t = Y_{t-1} + u_t$$

The basic idea of a random walk is that the value of the series tomorrow Y_{t+1} is its value today (Y_t) plus an unpredicted change , u_{t+1}

$$Y_1 = Y_0 + u_1$$
$$Y_2 = Y_1 + u_2 = Y_0 + u_1 + u_2$$
$$Y_3 = Y_2 + u_3 = Y_0 + u_1 + u_2 + u_3$$
$$Y_4 = Y_3 + u_4 = Y_0 + u_1 + \ldots + u_4$$
$$Y_t = Y_{t-1} + u_t = Y_0 + u_1 + \ldots + u$$

$u_t \sim (0, \sigma 2)$

In general, if the process started at some time 0 with a value Y_o, we have $Y_t = Y_o + \sum u_t$

- $E\{Y_t\}=E\ \{\ Y_0+\sum u_t\} = 0$
- $Var\{Y_t\}=E\ \{Y_0\ +\sum u_t + Y_0\ \} = t\sigma^2$

We can see that while the mean is constant, the variance is non-stationary function of time, and so, the series is non-stationary. Non stationary due to variance. It could as well be non stationary due to mean.

We can see now that re-writting our equation we have : $Y_t = Y_{t-1} + u_t \rightarrow$

$$Y_t - Y_{t-1} = u_t \rightarrow$$

$$\Delta Y_t = u_t$$

which is stationary, as u_t is stationary

Random walk with drift

$Y_t = \delta + Y_{t-1} + u_t$

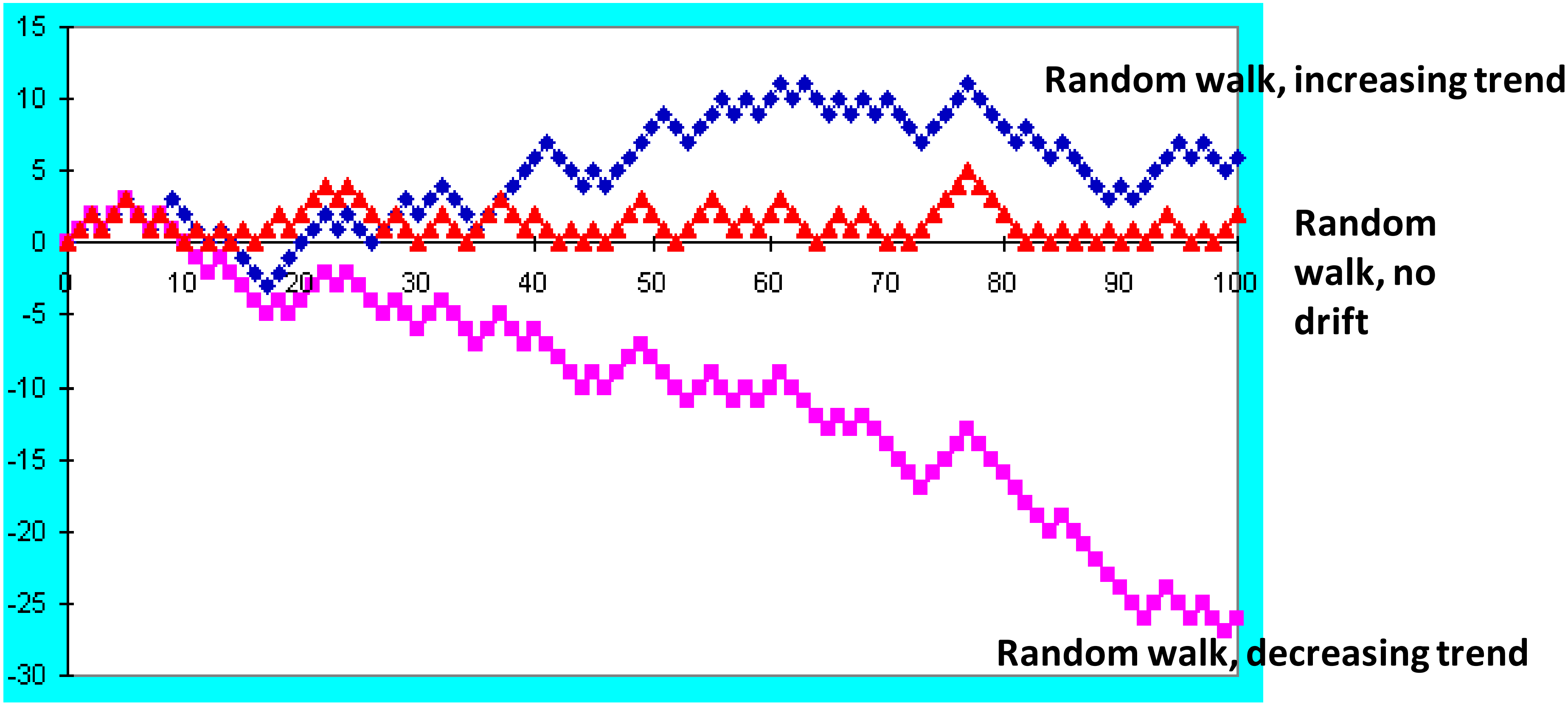
where is known as the drift parameter. The name drift comes from the fact that if we write the preceding equation as:

$Y_t - Y_{t-1} = \Delta Y_t = \delta + u_t$

it shows that Y_t drifts upward or downward, depending on being positive or negative. We can easily show that, the random walk with drift violates both conditions of stationarity:

- $E\{Y_t\} = \alpha t$
- $Var\{Y_t\} = t\sigma^2$

In other words, both mean and variance of Yt depends on *t, its distribution depends on t, that is, it is nonstationary.*


15
10
5
0
-5
-10
-15
-20
-25
-30
0
10
20
30
40
50
60
70
80
90
100
Random walk, increasing trend
Random walk, no drift
Random walk, decreasing trend

Testing for non-Stationarity -Dickey-Fuller test

$$X_t = \alpha + \rho X_{t-1} + \varepsilon_t$$

α=0, α≠0

$$X_t - X_{t-1} = \alpha + (\rho - 1) X_{t-1} + \varepsilon_t$$

$$\Delta X_t = \alpha + \delta X_{t-1} + \varepsilon_t$$

Now ΔX_t is stationary so we can estimate δ

Ho: ρ=1

H1: ρ<1

t statistic<DFcritical

→reject the null hypothesis

If $\rho=1$ then we have a random walk without drifts, which as we saw previous, is not stationary.

If ρ is in fact 1, we face what is known as the unit root problem, that is, a situation of nonstationarity. The name unit root is due to the fact that $\rho=1$

Augmented Dickey Fuller tests

- $\Delta X_t = \alpha + \delta X_{t-1} + \varepsilon_t$, AR(1)
- $H_0 : \delta = 0$
- $H_1 : \delta < 0$
- But how do we go ahead and test for a unit root in a presence of higher order precesses?
- $\Delta X_t = \alpha + \delta X_{t-1} + \beta \Delta X_{t-1} + \varepsilon_t$ AR(2)
- $\Delta X_t = \alpha + \delta X_{t-1} + \Sigma \beta_i \Delta X_{t-i} + \varepsilon_t$, where i=1...h AR(i)
- $H_0 : \delta = 0$, non-stationary t test for adding or not
- $H_1 : \delta < 0$, stationary

Integration

A stationary variable is called integrated of order 0.

Variable Y_t of order d if it can be transformed into a stationary variable after differencing d times.

In simple words, the number of the times a variable needs to be differenciated, is the order of the integration

Levels vs Differences Regression

$$Y_{t=}\alpha + \beta X_t + \varepsilon_t$$

$$\Delta Y_{t=}\alpha + \gamma \Delta X_t + u_t$$

We can see that if we have a long run relationship in levels, that implies that there will be a stable relationship in first-differences.

$$Y_t - Y_{t-1} = \alpha + \beta X_t + \varepsilon_t - \alpha - \beta X_{t-1} - \varepsilon_{t-1}$$

$$\Delta Y_{t=}\alpha + \beta \Delta X_t + \Delta \varepsilon_t$$

- But the converse is not necesserilly true. The reason behind this is due to the error term.if there is some consistent error, if ΔYt changes by ΔXt and Ut, it doesn’t take long for the effect of these random errors to build up and actually, to force Yt and Xt to move in different ways and with not a long run constant relationship between them

Integration and possible problems

- Regression I(1) vs I(1) – spurious regression
- Regression I(0) vs I(1) – mistakes in statistical inference (test statistics for variable significance is not t-distributed)

Solutions

- Transformation of the series (differencing, logarithm)
- Respecification of the model
- ***<u>Error correction and cointegration</u>***

- Solution one, (differencing I(1) data), makes OLS adequate tool

BUT

Long term relationship cannot be found because they are lost from the data.

Cointegration

- Dependencies between non-stationary variables
- Are called COINTEGRATING relationships

Cointegration

Dependencies between non-stationary variables

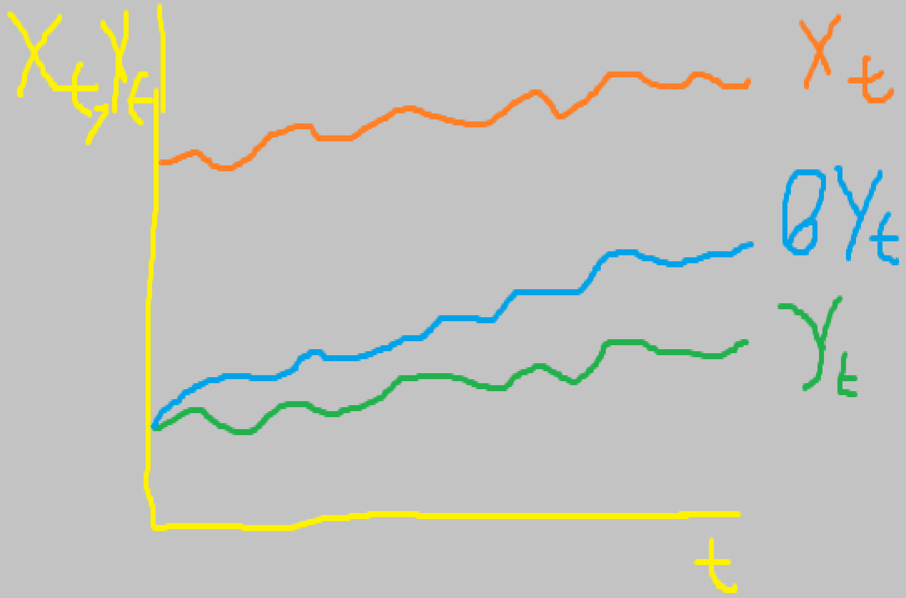
Are called COINTEGRATING relationships

- Cointegration is an econometric technique for testing the correlation between non-stationary time series variables. If two or more series are themselves non-stationary, but a linear combination of them of them is stationary, then the series are said to be cointegrated.

- In more technical terms, if we have two non-stationary time series X and Y that become stationary when differenced (these are called integrated of order one series, or I(1) series; random walks are one example) such that some linear combination of X and Y is stationary (aka, I(0)), then we say that X and Y are cointegrated. In other words, while neither X nor Y alone hovers around a constant value, some combination of them does, so we can think of cointegration as describing a particular kind of *long-run equilibrium relationship. (*The definition of cointegration can be extended to multiple time series, with higher orders of integration.)

- Income and consumption: as income increases/decreases, so does consumption.
- Size of police force and amount of criminal activity
- A book and its movie adaptation: while the book and the movie may differ in small details, the overall plot will remain the same.
- Number of patients entering or leaving a hospital

In economics we observe many non-stationary variables that are dependent to each other, and so, cointegration is an important aspect as it comes to find the effect of the independent variable onto the dependent one.



Yt is I(1) and Xt is also I(1), but there is a β, such that Xt-βYt is I(0). This is called COINTEGRATION and Xt,Yt are cointegrated.

Testing for Cointegration

- Eangle-Granger procedure

1. Test the order of integration.
2. If all of them, say I(1), specify the cointegrating relationship :

   $Y_t=\beta_0+\beta_1 X_t+\varepsilon_t$

2.1. Estimate the parameters β_0 , β_1 via OLS.

2.2 . Compute the residuals (ε_t)

3. Test the residuals for stationarity, if stationary, then the variables are cointegrated.

Notice the difference between the unit root and cointegration tests. Tests for unit roots are performed on single time series, whereas simple cointegration deals with the relationship among a group of variables, each having a unit root

Error Correction Model- setting up the model

Lets assume now that we have Y_t , X_t , both (1) and they are cointegrated.

So a possible relationship between them could be :

$$Y_t = c + \delta_1 X_t + \delta_2 X_{t-1} + \mu Y_{t-1} + v_t$$

By including lagged values of both X and Y this specification allows for a wide variety of dynamic patterns in the data.

Notice that, because X and Y are cointegrated, this means, that there is some short of equilibrium between them :

$$Y^E=\alpha+\beta X^E$$

Also, since they are I(1), their first differences will be I(0)

So, $\Delta Y_t=\delta_0+\delta_1 X_t+v_t$ which as we mentioned previously is the **short run**

It turns out now, that we can do something much more powerful than regressing first differences here.

Perhaps the Y_t might be different than the equilibrium value.

$$Y_t = c + \delta_1 X_t + \delta_2 X_{t-1} + \mu Y_{t-1} + v_t$$

Y_t might take some time to react to changes in X_t

has some sort of dependance in the lagged value of Y_t

$$Y_t = c + \delta_1 X_t + \delta_2 X_{t-1} + \mu Y_{t-1} + v_t$$

1) Can't tell us anything for the dynamic of Y_t ,X_t

No real economic content

2) We may have a Spurious regression

Error Correction Model is our way of getting past these

- $Y_t - Y_{t-1} = c + \delta_1 X_t + \delta_2 X_{t-1} - (1-\mu) Y_{t-1} + v_t$

- $\Delta Y_t = c + \delta_1 X_t - \delta_1 X_{t-1} + \delta_1 X_{t-1} + \delta_2 X_{t-1} - (1-\mu) Y_{t-1} + v_t$

- $\Delta Y_t = c + \delta_1 \Delta X_t - \lambda(Y_{t-1} - \alpha - \beta X_{t-1}) + v_t$

$\lambda = 1 - \mu$ and $\beta = (\delta_1 + \delta_2)/1 - \mu$

$$\underbrace{\Delta Y_t}_{I(0)} = c' + \delta_1 \underbrace{\Delta X_t}_{I(0)} - \lambda\underline{(Y_{t-1} - \alpha - \beta X_{t-1})} + v_t$$

If $Y^E=\alpha+\beta X^E$ equilibrium exists,
then $Y_{t-1} - \alpha - \beta X_{t-1}$ will be cointegrated. I(0)

$Y_{t-1} > \alpha - \beta X_{t-1}$ Y is above its equilibrium value. So we take off a little bit of Y ($-\lambda$), so the change in Y_t will be slightly negative, hence we **correct** the error in the last period to adjust further towards the equilibrium value of Y.

And this sort of error correction mechanism is why we call it ***error correction model.***

$$\underbrace{\Delta Y_t = c' + \delta_1 \Delta X_t}_{\text{Short run dynamics}} - \lambda(\underbrace{Y_{t-1} - \alpha - \beta X_{t-1}}_{\text{long run dynamics}}) + v_t$$

$0<\lambda<1$

$\lambda<1$ because we want time to adjust

It allows to be some sort of long run cointegrated relationship between Y,X

The parameter λ tells us the speed that which our variable adjust to any sort of disequilibrium.

These models allows of both interaction between short run and long run dynamics.

Our last step now is to estimate the model, and for doing so we need α,β estimators. Engel and Granger showed that, we can OLS

$Y_t = \alpha + \beta X_t + v_t$ and so rewritting the equation we have :

$$\Delta Y_t = c' + \delta_1 \Delta X_t + \gamma_0 \hat{v}_t$$

Now we can use OLS to estimate the model.

VAR

- To understand what VAR is, we need to know AR (AutoRegression) model firstly. AR means that the value of variable Y at time t is decided by the values of previous Ys.
- As we know ,an autoregressive procedure *p-th order has the general the form :*

- $Y_t = \alpha_0 + \alpha_1 Y_{t-1} + \alpha_2 Y_{t-2} + ... + \alpha_p Y_{t-p} + \varepsilon_t$,
- where 'α' is constant and 'ε_t ' is white noise.

- Instead of a single variable ,we could have more than one variables.Generally, we could have K different variables which means a column vector of Kx1 dimensions .
- This vector is a function of the values of its previous years.For the aforementioned reason we have the term Vector Autoregression.
- For instance, if K=2 and p=1 then we have the following system of two equations:

- $Y_{1t} = \delta_1 + \alpha_{11} Y_{1,t-1} + \alpha_{12} Y_{2,t-1} + \varepsilon_{1t}$
- $Y_{2t} = \delta_2 + \alpha_{21} Y_{1,t-1} + \alpha_{22} Y_{2,t-1} + \varepsilon_{2t}$

- Each variable in the aforementioned model has one equation. This example has a maximum lag *p equal to 1 .The structure is that each variable is a linear function of past lags of itself and past lags of the other variables. In our case is a model of VAR(1) with lagged values for every variable.*

Johansen's methology

Assume Y_t a vector of I(1) variables which is expressed as VAR(p)

$$\mathbf{Y_t = A_1 Y_{t-1} + A_2 Y_{t-2} + \ldots + A_p Y_{t-p} + \varepsilon_t}$$

and, we can write it as a vector error correction model (VECM)

$$\mathbf{\Delta Y_t = \Pi Y_{t-1} + \sum \Gamma_i \, \Delta Y_{t-i} + B X_t + u_t} \qquad \text{for } i=1,\ldots,p-1$$

$\Pi = \sum A_i - I$ I = unit matrix

The degree of matrix **Π** defines the existence or no, of the cointegration between the variables.

- If r(Π) is degree of 0 then, the model becomes VAR as all the variables are stationary and there is no cointegration
- If r(Π) is full degree, this can happen only if $\mathbf{Y_t}$ is stationary and so, VECM has no point.
- If r(Π) is , this means that the columns of vector Π are not all linear independent and that allows variables to be cointegrated. In the case when **Π** has lower degree than the order of p which is the number of the endogenous variables, then we can say that the variables are cointegrated. In that case we should use VECM and not VAR because of the short run and the long run relationship.

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Βήμα 1: Έλεγχος μοναδιαίας ρίζας στο SALESa

Augmented Dickey-Fuller test for SALESa
συμπεριλαμβανομένης μίας υστέρησης του (1-L)SALESa
μέγεθος δείγματος 60
μηδενική υπόθεση μοναδιαίας ρίζας: a = 1

   έλεγχος με σταθερό όρο
   υπόδειγμα: (1-L)y = b0 + (a-1)*y(-1) + ... + e
   εκτιμώμενη τιμή του (a - 1): -0,0126368
   στατιστική ελέγχου: tau_c(1) = -0,851078
   ασυμπτωτική p-τιμή 0,8039
   συντελεστής αυτοσυσχέτισης 1ης τάξης για e: -0,067
```

I(1)

```
Βήμα 2: Έλεγχος μοναδιαίας ρίζας στο EQUITYa

Augmented Dickey-Fuller test for EQUITYa
συμπεριλαμβανομένης μίας υστέρησης του (1-L)EQUITYa
μέγεθος δείγματος 60
μηδενική υπόθεση μοναδιαίας ρίζας: a = 1

   έλεγχος με σταθερό όρο
   υπόδειγμα: (1-L)y = b0 + (a-1)*y(-1) + ... + e
   εκτιμώμενη τιμή του (a - 1): -0,0311986
   στατιστική ελέγχου: tau_c(1) = -1,637
   ασυμπτωτική p-τιμή 0,4636
   συντελεστής αυτοσυσχέτισης 1ης τάξης για e: 0,017
```

I(1)

```
Βήμα 3: παλινδρόμηση συνολοκλήρωσης

Παλινδρόμηση συνολοκλήρωσης -
OLS, χρήση των παρατηρήσεων 1960-2021 (T = 62)
Εξαρτημένη μεταβλητή: SALESa

              συντελεστής     τυπ. σφάλμα      t-λόγος    p-τιμή
   ---------------------------------------------------------------
   const      -2,42127e+06    557931           -4,340     5,56e-05   ***
   EQUITYa     2,06194          0,0893740      23,07      1,59e-031  ***

   Μέσος εξαρτημένης μεταβλητής              9434185
   Τυπική Απόκλιση εξαρτημένης μεταβλητής    5331752
   Άθροισμα Τετραγώνων Καταλοίπων           1,76e+14
   Τυπικό Σφάλμα παλινδρόμησης               1711104
   R-τετράγωνο                              [illegible]
   Προσαρμοσμένο R-τετράγωνο                0,897006
   Λογαριθμική πιθανοφάνεια                 [illegible]
   Akaike κριτήριο                          1957,644
   Schwarz κριτήριο                         1961,898
   Hannan-Quinn                             1959,314
   rho                                      [illegible]
   Durbin-Watson                            0,151582
σημειώσεις σχετικά με τις συντμήσεις των στ[illegible]ν του υποδείγματος, θα βρείτε εδώ

Βήμα 4: Έλεγχος μοναδιαίας ρίζας στο uhat

Augmented Dickey-Fuller test for uhat
συμπεριλαμβανομένης μίας υστέρησης του (1-L)uhat
μέγεθος δείγματος 60
μηδενική υπόθεση μοναδιαίας ρίζας: a = 1

   υπόδειγμα: (1-L)y = (a-1)*y(-1) + ... + e
   εκτιμώμενη τιμή του (a - 1): -0,0888177
   στατιστική ελέγχου: tau_c(2) = -1,74313
   ασυμπτωτική p-τιμή 0,6579
   συντελεστής αυτοσυσχέτισης 1ης τάξης για e: -0,040
```

I(0) ???

```
There is evidence for a cointegrating relationship if:
(a) The unit-root hypothesis is not rejected for the individual variables, and
(b) the unit-root hypothesis is rejected for the residuals (uhat) from the
```

Example using ECM

We use the data of the private consumption and the GDP in Greece, for the period 1960-1994, in PPT of 1980.

$Y_t = \delta_0 + \delta_1 X_t + \delta_2 t + \varepsilon_t$ and in equilibrium it will be $\varepsilon_t = 0$ and so $Y^*_t = \alpha + \beta X^*_t + \gamma t^*$

After using OLS we have :

- $Y_t = 1{,}818 + 0{,}821X_t + 0{,}007t$
- (8,75) (47,88) (10,34) in parenthesis the t statistic of each respected coefficient
- $R^2 = 0{,}99$
- d = 1,41

  where
- Y = logarithm of private consumption
- X =logarithm of GDP
- t = time (1,2,...,35)

The regression will not be spurious if the variables are cointegrated. After using DF tests we found that they are both I(1).

Now we will test whether they are cointegrated.

ADF regression for the residuals of the above cointegrating regression is :

- $\Delta\hat{u}_t = -\ 1{,}141u_{t-1} + 0{,}168\Delta u_{t-1}$

  $(-4{,}28) \qquad (0{,}89)$

- $|t^*| = 4{,}06 < |t| = 4{,}28$ we reject the null hypothesis that is no stationary. So our variables are cointegrated.

Engle-Granger

- $\Delta Y_t = \delta_0 + \delta_1 \Delta X_t + \mu Y_{t-1} + \delta_3 X_{t-1} + \delta_2 Z_{t-1}$
- where Z = time (1,2,...,35)

  The results are the following:
- $\Delta Y_t = \delta_0 + \delta_1 \Delta X_t - \lambda Y_{t-1} + \delta_3 X_{t-1} - \delta_2 Z_{t-1}$

- $\Delta Y_t = 0{,}9951 + 0{,}5484 \Delta X_t - 0{,}6226 Y_{t-1} + 0{,}5242 X_{t-1} - 0{,}0035 Z_{t-1}$

  (3,31) (7,98) (-4,48) (4,57) (2,87)

  We could also use this in order to estimate our model.
- $\Delta Y_t = \delta_1 \Delta X_t - \lambda(Y_{t-1} - \alpha - \beta X_{t-1} + \gamma Z_{t-1})$

$\lambda = 1 - \mu$

$\beta = (\delta_1 + \delta_2)/(1-\mu)$

We can also estimate α, β via OLS on our equilibrium function.

- $\Delta Y_t = 0{,}5484 \Delta X_t - 0{,}6226(Y_{t-1} - 1{,}5983 - 0{,}8419 X_{t-1} + 0{,}0056 Z_{t-1})$

So, the estimation of the short run is 0,5484, and of the long run is 0,8419. the estimation of λ is 0,6226, which means that Y steps 6/10 of the way towards the equilibrium level.