Homework 8

1.

x	25	40	65
$P(x)$	0.2	0.5	0.3

$\mu = 44.5$

1.1

X_1	X_2	$P(x_1, x_2)$	$\bar{x}$	s^2
25	25	$0.2 \cdot 0.2 = 0.04$	25	0
25	40	$0.2 \cdot 40.5 = 0.1$	32.5	112.5
25	65	$0,2 \cdot 0,3 = 0.06$	45	800
40	40	$0.5 \cdot 0,5 = 0.25$	40	0
40	25	$0.5 \cdot 0,2 = 0.1$	32.5	112.5
40	65	$0,5 \cdot 0,3 = 0.15$	52.5	312.5
65	65	$0.3 \cdot 0,3 = 0.09$	65	0
65	25	$0.3 \cdot 0.2 = 0.06$	45	800
65	40	$0.3 \cdot 0.5 = 0.15$	52.5	312.5

$\bar{x} = 45$ - в 2 [illegible] в [illegible] $\Rightarrow$

$P(\bar{x} = 45) = \cancel{0,4 + 0,1 +}\ 0.06 + 0.06 =$

$= 0,12$

$s^2 = 800$ - в 2 [illegible] в [illegible] $\Rightarrow$

$P(S^2 = 800) = 0.06 + 0.06 = 0,12$

$\bar{x}$	25	32,5	40	45	52,5	65
$P(\bar{x})$	0,04	0,2	0,25	0,12	0,3	0,09

$E(\bar{x}) = 25 \cdot 0,04 + 32,5 \cdot 0,2 + 40 \cdot 0,25 +$
$+ 45 \cdot 0,12 + 52,5 \cdot 0,3 + 65 \cdot 0,09 =$
$= 44,5$

$\mu = E(\bar{x})$

1.2

s^2	0	112,5	312,5	800
$P(s^2)$	0,38	0,2	0,3	0,12

$E(s^2) = 0 \cdot 0,38 + 112,5 \cdot 0,2 +$
$+ 312,5 \cdot 0,3 + 800 \cdot 0,12 = 212,25$

$\sigma^2 = (25^2 \cdot 0,2 + 40^2 \cdot 0,5 + 65^2 \cdot 0,3) -$
$- 44,5^2 = 212,25$

$\sigma^2 = E(s^2)$

4. $\mu = 11{,}5$

$\sigma = 4$

$X_1, X_2 \ldots X_{50}$

$E(T_0) = n\mu = 50 \cdot 11{,}5 = 575$

$E(\bar{X}) = \mu = 11{,}5$

$\sigma_{T_0} = \sqrt{n}\,\sigma = \sqrt{50} \cdot 4 = 20\sqrt{2}$

$\sigma_{\bar{X}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{4}{\sqrt{50}} = \dfrac{2}{5}\sqrt{2}$

$X_i \sim N(11{,}5,\ 4) \Rightarrow T_0 \sim N(575,\ 20\sqrt{2})$

$P(T_0 \geq 12) = 1 - P(T_0 < 12) =$

$= 1 - \Phi\left(\dfrac{12 - 575}{20\sqrt{2}}\right) \approx 1 - \Phi(-19{,}9) \approx$

1) $P(\bar{X} \geq 12) = 1 - P(X < 12) =$

$= 1 - \Phi\left(\dfrac{12 - 11{,}5}{\frac{2\sqrt{2}}{5}}\right) \approx 1 - \Phi(0{,}88) \approx$

$\approx 1 - 0{,}8106 =$

$= 0{,}1894$

2) $P(T_0 \leq 600) = \Phi\left(\frac{600-575}{20\sqrt{2}}\right) =$

$\approx \Phi(0{,}88) = 0{,}8106$

3) $P(T_0 = X) = \Phi\left(\frac{X-575}{20\sqrt{2}}\right) = 0{,}95$

$\frac{X-575}{20\sqrt{2}} = 1{,}645$

$X = 1{,}645 \cdot 20\sqrt{2} + 575 \approx 622$