Homework 8

1. $f(x;\theta) = (\theta+1)x^{\theta} \quad 0 \le x \le 1$

a. $E(X) = \int_0^1 x(\theta+1)x^{\theta}\,dx =$

$= (\theta+1)\dfrac{x^{\theta+2}}{\theta+2}\Bigg|_0^1 = \dfrac{\theta+1}{\theta+2}$

$\cancel{E(X) =} \; \dfrac{\theta+1}{\theta+2} = \bar{x}$

$(\theta+2)\bar{x} = \theta+1$

$\theta = \dfrac{2\bar{x}-1}{1-\bar{x}} = \dfrac{0.6}{0.2} = 3$

b. $\prod_{i=1}^{n}(\theta+1)x_i^{\theta} = (\theta+1)^n \prod_{i=1}^{n} x_i^{\theta}$

$\ln(\theta+1)^n + \ln\sum x_i^{\theta} = n\ln(\theta+1) +$

$+ \theta\sum_{i=1}^{n}\ln x_i$

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$n \cdot \dfrac{1}{\theta+1} + \sum_{i=1}^{n}\ln x_i = 0$

$$\theta = \frac{n}{\sum_{i=1}^{n} \ln x_i} - 1 = 3.11$$

$$2.\quad f(x,\theta)=\frac{1}{\sqrt{2\pi\theta}}e^{\frac{-x^2}{2\theta}},\qquad E(x)=0$$

$$\sigma^2=\theta$$

$$L=f\{x_1\cap x_2\cap\ldots x_n;\theta\}=f\{x_1;\theta\}\cdot f\{x_2;\theta\}\ldots$$

$$\ldots f\{x_n;\theta\}=\frac{1}{\sqrt{2\pi\theta}}e^{\frac{-x_1^2}{2\theta}}\cdot\frac{1}{\sqrt{2\pi\theta}}e^{\frac{-x_2^2}{2\theta}}\cdot\ldots$$

$$\cdot\frac{1}{\sqrt{2\pi\theta}}e^{\frac{-x_n^2}{2\theta}}=\left(\frac{1}{\sqrt{2\pi\theta}}\right)^n e^{-\left(\frac{x_1^2}{2\theta}+\frac{x_2^2}{2\theta}+\ldots+\frac{x_n^2}{2\theta}\right)}=$$

$$=\left(\frac{1}{\sqrt{2\pi\theta}}\right)^n e^{-\frac{1}{2\theta}\sum_{i=1}^{n}x_i^2}$$

$$\ln L=\ln\left(\left(\frac{1}{\sqrt{2\pi\theta}}\right)^n e^{-\frac{1}{2\theta}\sum_{i=1}^{n}x_i^2}\right)=$$

$$=\ln\left((2\pi\theta)^{-\frac{n}{2}}e^{-\frac{1}{2\theta}\sum_{i=1}^{n}x_i^2}\right)=$$

$$=-\frac{n}{2}\ln 2\pi\theta+\frac{1}{2\theta}\sum_{i=1}^{n}x_i^2\cdot\ln e=$$

$$=-\frac{n}{2}\ln 2\pi-\frac{n}{2}\ln\theta+\frac{1}{2\theta}\sum_{i=1}^{n}x_i^2$$

$$-\frac{n}{2}\ln\theta+\frac{1}{2\theta}\sum_{i=1}^{n}x_i^2=\frac{n}{2}\ln 2\pi$$

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$$-\frac{n}{2\theta} + \frac{1}{4\theta^2} \cdot \sum_{i=1}^{n} x_i^2 = 0$$

$$\frac{\sum_{i=1}^{n} x_i^2}{{}_2\cancel{4}\theta^{\cancel{2}}} = -\frac{n}{\cancel{2}\cancel{\theta}}$$

$$\theta = -\frac{\sum_{i=1}^{n} x_i^2}{2n}$$

4. $f(x_i) = \frac{1}{\Gamma(\alpha)\theta^{\alpha}} x^{\alpha-1} e^{-\frac{x}{\theta}}$

$L(\alpha, \theta) = \left(\frac{1}{\Gamma(\alpha)\theta^{\alpha}}\right)^{n} (x_1 \cdot x_2 \cdots x_n)^{\alpha-1} e^{-\frac{1}{\theta}\sum x_i}$

$E(X_i) = \alpha\theta$

$Var(X_i) = [(X_i - \mu)^2] = \alpha\theta^2$

$E(X) = \alpha\theta = \frac{1}{n}\sum_{i=1}^{n} X_i = \bar{X}$ (1)

$Var(X) = \alpha\theta^2 = \frac{1}{n}\sum_{i=1}^{n} (X_i - \bar{X})^2$ (2)

(1)-ру шығатыны $\alpha = \frac{\bar{X}}{\theta}$

(2)-ру $\alpha\theta^2 = \frac{\bar{X}}{\theta} \cdot \theta^2 = \bar{X}\theta = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$

$\hat{\theta} = \frac{1}{n\bar{X}}\sum_{i=1}^{n}(x_i - \bar{x})^2$

α-[illegible] [illegible] [illegible] $\bar{X}$-o [illegible]

$\hat{\Theta}$ [illegible] [illegible]

$$\hat{\lambda} = \frac{\bar{X}}{\hat{\Theta}} = \frac{\bar{X}}{\frac{1}{n\bar{X}}\sum_{i=1}^{n}(X_i-\bar{X})^2} = \frac{n\bar{X}^2}{\sum_{i=1}^{n}(X_i-\bar{X})^2}$$