Homework 5

1. $f_X(x) = \begin{cases} cx^2 & |x| \le 1 \\ 0 & h.\gamma \end{cases}$

a) $\int_{-1}^{1} cx^2 dx = 1$;

$$c\frac{x^3}{3}\Big|_{-1}^{1} - 1 = 0$$

$$\frac{c}{3} + \frac{c}{3} = 1$$

$$2c = 3$$

$$c = \frac{3}{2}$$

b) $E(X) = \frac{3}{2}\int_{-1}^{1} x^3 dx = \frac{3}{2}\frac{x^4}{4}\Big|_{-1}^{1} =$

$= \frac{3}{8} - \frac{3}{8} = 0$

$var(X) = \frac{3}{2}\int_{-1}^{1} x^4 dx - 0 = \frac{3}{2}\frac{x^5}{5}\Big|_{-1}^{1} =$

$= \frac{3}{10} + \frac{3}{10} = \frac{3}{5}$

c) $P(X \geq 12) = 1 - P(X \leq 12) =$

$= \cancel{1 - \frac{3}{2}\int_{-1}^{12} x^2 \, dx = 1 - \frac{3}{2} \frac{x^3}{3} \Big|_{-1}^{12}}$ $\quad F(x) = \begin{cases} 0, & x < -1 \\ \frac{x^3}{2} & -1 \leq x \leq 1 \\ 1 & x \geq 1 \end{cases}$

$= \cancel{1 - 864 - \frac{1}{2} = -863,5}$ $1 - F(12) = 1 - 1 =$

$= 0$

2. $f_X(x) = \frac{1}{2} e^{-|x|}, \quad x \in R$

$Y = X^2$

$F_Y(y) = P(Y \leq y) = P(X^2 \leq y) =$

$= P(-\sqrt{y} \leq X \leq \sqrt{y}) = \frac{1}{2} \int_{-\sqrt{y}}^{\sqrt{y}} e^{-x} \, dx =$

$= \cancel{2} \cdot \frac{1}{\cancel{2}} \int_{0}^{\sqrt{y}} e^{-x} \, dx = -e^{-x} \Big|_{0}^{\sqrt{y}} =$

$= -e^{-\sqrt{y}} + 1$

$F_Y(y) = \begin{cases} 1 - e^{-\sqrt{y}}, & y \geq 0 \\ 0, & \text{h. z.} \end{cases}$

3. $X \sim N(\underset{\mu}{3}, \underset{\sigma^2}{9})$

a) $P(X > 0)$

$$f_x(x) = \begin{cases} \frac{1}{6} & 3 < x < 9 \\ 0 & \text{h.y} \end{cases}$$

$$P(X > 0) = 1 - \Phi\left(\frac{0-3}{3}\right) = 1 - \Phi(-1) =$$

$$= 1 - (1 - \Phi(1)) = \Phi(1) = 0.8413$$

б. $$P(-3 < x < 8) = \Phi\left(\frac{8-3}{3}\right) -$$

$$- \Phi\left(\frac{-3-3}{3}\right) = \Phi(1,(6)7) -$$

$$- \Phi(-2) \approx 0.9515 - 0.02275 =$$

$$\approx 0,928$$

d) $P(X > 5 \mid X < 3) = \cancel{P(X > 5)}$

$$= \frac{P(X > 5, X < 3)}{P(X < 3)} = \frac{0}{P(X<3)} = 0$$

9. $E(X) = \int_0^\infty P(X \geq x)\,dx$

fuklf op $1 - F_X(x) = P(X \geq x) =$

$$= \int_x^\infty f_X(t)\,dt$$

$$\int_0^\infty (1 - F_X(x))\,dx = \int_0^\infty P(X \geq x)\,dx =$$

$$= \int_0^\infty \int_x^\infty f_X(t)\,dt\,dx = \int_0^\infty \int_0^t f_X(t)\,dx\,dt =$$

$$= \int_0^\infty \left(x f_X(t) \right)\Big|_0^t dt = \int_0^\infty t f_X(t)\,dt =$$

$t = x$

$$= \int_0^\infty x f_X(x)\,dx = E(X)$$

ceuq. d