

Homework 8

$1. x$	25	40	65
$2. P(x)$	0.2	0.5	0.3

$$\mu = 44.5$$

$$s^2 = \frac{1}{n} \sum_{i=1}^n (x_i - \mu)^2 = \frac{1}{3} [(25-44.5)^2 + (40-44.5)^2 + (65-44.5)^2] = 800$$

x_1	x_2	$P(x_1, x_2)$	\bar{x}	s^2
25	25	$0.2 \cdot 0.2 = 0.04$	25	0
25	40	$0.2 \cdot 0.5 = 0.1$	32.5	112.5
25	65	$0.2 \cdot 0.3 = 0.06$	45	800
40	40	$0.5 \cdot 0.5 = 0.25$	40	0
40	25	$0.5 \cdot 0.2 = 0.1$	32.5	112.5
40	65	$0.5 \cdot 0.3 = 0.15$	52.5	312.5
65	65	$0.3 \cdot 0.3 = 0.09$	65	0
65	25	$0.3 \cdot 0.2 = 0.06$	45	800
65	40	$0.3 \cdot 0.5 = 0.15$	52.5	312.5

$\bar{x} = 45$ - 2 outcomes & hypergeometric \Rightarrow

$$P(\bar{x} = 45) = 0.04 + 0.1 + 0.06 + 0.06 = 0.26$$

$s^2 = 800$ - 2 outcomes & hypergeometric \Rightarrow

$$P(s^2 = 800) = 0.06 + 0.06 = 0.12$$

\bar{x}	25	32,5	40	45	52,5	65
$P(\bar{x})$	0,04	0,2	0,25	0,12	0,3	0,09

$$E(\bar{x}) = 25 \cdot 0,04 + 32,5 \cdot 0,2 + 40 \cdot 0,25 + 45 \cdot 0,12 + 52,5 \cdot 0,3 + 65 \cdot 0,09 = 44,5$$

$$\mu = E(\bar{x})$$

1.2

s^2	0	112,5	312,5	800
$P(s^2)$	0,38	0,2	0,3	0,12

$$E(s^2) = 0 \cdot 0,38 + 112,5 \cdot 0,2 + 312,5 \cdot 0,3 + 800 \cdot 0,12 = 212,25$$

$$\sigma^2 = (25^2 \cdot 0,2 + 40^2 \cdot 0,5 + 65^2 \cdot 0,3) - 44,5^2 = 212,25$$

$$\sigma^2 = E(s^2)$$

$$4. \mu = 11,5$$

$$\sigma = 4$$

$$X_1, X_2, \dots, X_{50}$$

$$P_2 \quad E(T_0) = n\mu = 50 \cdot 11,5 = 575$$

$$E(\bar{X}) = \mu = 11,5$$

$$\sigma_{T_0} = \sqrt{n} \sigma = \sqrt{50} \cdot 4 = 20\sqrt{2}$$

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{4}{\sqrt{50}} = \frac{2}{5}\sqrt{2}$$

$$X_0 \sim N(11,5, 4) \Rightarrow T_0 \sim N(575, 20\sqrt{2})$$

$$P(T_0 \geq 12) = 1 - P(T_0 < 12) =$$

$$\approx 1 - \Phi\left(\frac{12 - 575}{20\sqrt{2}}\right) \approx 1 - \Phi(-19,9) \approx$$

$$1) P(\bar{X} \geq 12) = 1 - P(\bar{X} < 12) =$$

$$\approx 1 - \Phi\left(\frac{12 - 11,5}{\frac{2\sqrt{2}}{5}}\right) \approx 1 - \Phi(0,88) \approx$$

$$\approx 1 - 0,8106 =$$

$$\approx 0,1894$$

$$2) P(T_0 \leq 600) = \Phi\left(\frac{600 - 575}{20\sqrt{2}}\right) \approx$$

$$\approx \Phi(0,88) = 0,8106$$

$$3) P(T_0 \geq X) = \Phi\left(\frac{X - 575}{20\sqrt{2}}\right) = 0,95$$

$$\frac{X - 575}{20\sqrt{2}} = 1,645$$

$$X = 1,645 \cdot 20\sqrt{2} + 575 \approx 625$$