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PROJECT

Course Title

Numerical Analysis

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CHOLESKY DECOMPOSITION

INTRODUCTION

Cholesky decomposition method is a numerical method used to decompose a symmetric positive definite matrix into a product of a lower triangular matrix and its transpose.

INVENTOR

Cholesky decomposition method is named after the **French Mathematician Andre-Louis Cholesky**. Cholesky's work on the decomposition of symmetric positive definite matrices was published in his 1910 doctoral thesis titled **"Sur la decomposition matrices,"** which introduced the method that bears his name.

EXPLANATION

Cholesky decomposition solves a system of linear equations by breaking it down into two steps: forward substitution and back substitution.

Cholesky decomposition:

- Cholesky decomposition factorizes the original matrix A into a lower triangular matrix L and its conjugate transpose L^T, such that A = LL^T.

- This decomposition is possible because A is symmetric positive definite, meaning it is symmetric (A = A^T)

- Once the Cholesky decomposition is obtained (L matrix), the original system of linear equations AX = B can be rewritten as LL^TX = B.

Forward substitution:

- The goal is to find the intermediate vector Y such that LY = B. This can be done through forward substitution.

Back substitution:

- After obtaining the intermediate vector Y, the goal is to find the final solution X such that L^TX= Y. This can be done through backward substitution.

ADVANTAGES

Efficiency:

The Cholesky method requires fewer operations compared to other methods such as Gaussian elimination or LU decomposition. This is because the Cholesky decomposition takes advantage of the symmetry of the matrix and only calculates and stores the lower triangular matrix, resulting in reduced computational complexity.

Numerical stability:
The Cholesky method is numerically stable for solving systems of linear equations when the coefficient matrix is symmetric positive definite. It avoids issues such as negative value of square root.

Symmetric positive definite matrices:
The Cholesky method is specifically designed for symmetric positive definite matrices. If the coefficient matrix is known to have these properties, using the Cholesky method can provide a more tailored and efficient solution compared to other general methods.

DISADVANTAGES

Limited applicability:
The Cholesky method can only be applied to systems of linear equations where the coefficient matrix is symmetric positive definite. If the matrix does not possess these properties, the Cholesky decomposition is not possible, and an alternative method must be used.

Storage requirements:
While the Cholesky method has reduced storage requirements compared to LU decomposition, it still requires storing the product of lower triangular matrix L and upper triangular matrix L^T.

Matrix Decomposition

The Cholesky method requires the matrix to be decomposed into the product of a lower triangular matrix and its conjugate transpose. This decomposition process involves additional computations, which may add complexity and overhead in some cases.

Computational complexity:
Although the Cholesky method is generally more computationally efficient than other methods, it still requires computational effort, especially for larger matrices. The decomposition itself involves calculations such as square roots and sums of squares, which can be time-consuming for large matrices.

QUESTION

Solve by using Cholesky Method
| 2 4 -6 | | x | | -4 |
| 1 5 3 | | y | = | 10 |
| 1 3 2 | | z | | 5 |

To solve the given system of linear equations using the Cholesky method, we first represent the system in matrix form as AX = B:

2 4 -6		x		-4
1 5 3		y	=	10
1 3 2		z		5

Now, let's apply the Cholesky method step by step:
Compute the Cholesky Decomposition:

A=LL^T

| 2 4 -6 | | l11 0 0 | | u11 u12 u13 |

| 1 5 3 | = | l21 l22 0 | x | 0 u22 u23 |

| 1 3 2| | l31 l32 l33 | | 0 0 u33 |

Solving these equations, we get:

l11 =1.4142 , l21 = 0.7071 , l31 = 0.7071

u11 =1.4142 , u12 =2.8284 , u13 =-4.2426

l22 =1.7321 , l32 = 0.5774 , u22 =1.7321

l33 = 1.7321 , u23 =3.4641, u33 = 1.7321

Therefore, the L and U matrices are:

L=

| 1.4142 0 0 |

| 0.7071 1.7321 0 |

| 0.7071 0.5774 1.7321 |

L^T=

| 1.4142 2.8284 -4.2426 |

| 0 1.7321 3.4641 |

| 0 0 1.7321 |

Solve LY = B:

Next, we solve the equation LY = B to find the intermediate vector Y.

| 1.4142 0 0 | y1 = | -4 |

| 0.7071 1.7321 0 | y2 = | 10 |

| 0.7071 0.5774 1.7321| | y3 =|5 |

Using forward substitution:

Equation 1: 1.4142(Y1) =-4
Y1= -2.8284

Equation 2:0.7071(Y1) +1.7321(Y2)=10
Y2=6.9282

Equation 3: 0.7071(Y1)+ 0.5774(Y2) + 1.7321(Y3)=5
Y3=1.7321
So, Y = |-2.8284 |
|6.9282 |
| 1.7321|

Solve L^TX = Y:

Finally, we solve the equation L^TX = Y to find the solution vector X.

| 1.4142 2.8284 -4.2426 | | x1 | = |-2.8284 |

| 0 1.7321 3.4641 | |x2|= | 6.9282 |

| 0 0 1.7321 | | x3 | = | 1.7321|

Using back substitution:

Equation 3: 1.7321(x3) =1.7321
X3=1

Equation 2:
1.73219x2) +3.4641(x3) =6.9282
X2=2

Equation 1:

1.4142(x1) +2.8284(x2)-4.2426(X3) =-2.8284

X1=-3

So, X = | -3 |
| 2 |
| 1 |

Therefore, the solution to the system of linear equations is x = -3, y = 2, z = √3.

MATLAB

Solve by using Cholesky Method

| 2 4 -6 | | x | | -4 |
| 1 5 3 | | y | = | 10 |
| 1 3 2 | | z | | 5 |

cholesky.m

/MATLAB Drive/cholesky.m

```
A=input('enter coefficient matrix:');
b=input('enter source vector:');
N=length(b);
L=zeros(N,N);
U=zeros(N,N);
L(1,1)=sqrt(A(1,1));
U(1,1)=L(1,1);
for a=2:N
    L(a) = A(a,1)/L(1,1);
    U(a,1) = A(a,1)/L(1,1);
end
for i=2:N
    forj = i:N
    if i==j
         L(j,i) = sqrt(A(j,i) -L(j,1:i-1)*U(1:i-1,i));
         U(j,1) =L(j,1);
    else
        L(j,i) = (A(j,i) -L(j,1:i-1)*U(1:i-1,i))/L(i,i);
    end
end
    for K=i+1:N
        U(i,K) = (A(i,K) - L(i,1:i-1)*U(1:i-1,K))/L(i,i);
    end
```

```
L U
Y=zeros(N,1);
Y(1)=b(1)/L(1,1);
for k=2:N
    Y(K) = (b(K) -L(K,1:K-1)*Y(1:K-1))/L(K,K);
end
Y
X= zeros(N,1);
X(N)=Y(N)/U(N,N);
for K=N-1:-1:1
    x(K)=(Y(K) - U(K,K+1:N)*X(K+1:N))/U(K,K);
end
X
```

OUTPUT

```
>> cholesky
enter coefficient matrix:
[2 4 -6;1 5 3;1 3 2]
enter source vector:
[-4;10;5]
```

Value of L

```
L =

   1.4142        0        0
   0.7071   1.7321        0
   0.7071   0.5774   1.7321
```

Value of U

```
U =

   1.4142   2.8284  -4.2426
        0   1.7321   3.4641
        0        0   1.7321
```

Value of Y

```
y =

  -2.8284
   6.9282
   1.7321
```

Value of X

```
x =

    -3
     2
     1
```