BEGY 6503 Biomedical Instrumentation Lab 02 Signal Sampling and Frequency Analysis

(Ameya Srivastava, Mariam Zoair, Qing Xiang)

Lab 02 Report Checklist, Due on Mar 3, 2025

[1] Laboratory Exercises 1.1 – Include the required computational process, fill out relevant results and observations in the table in Table 1, answer to the discussion questions, and attach the output figures.

Figure 1

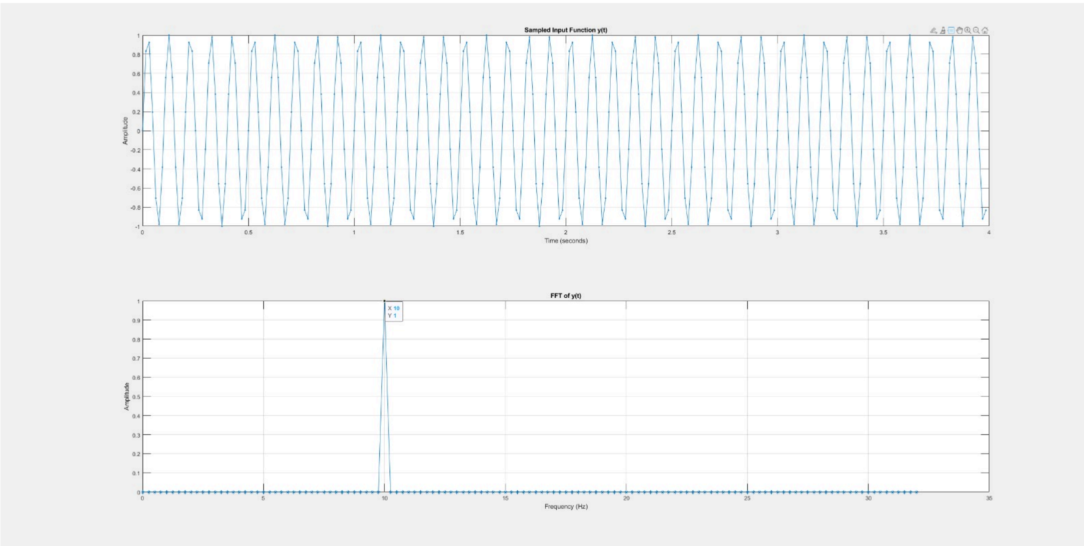


Figure 2

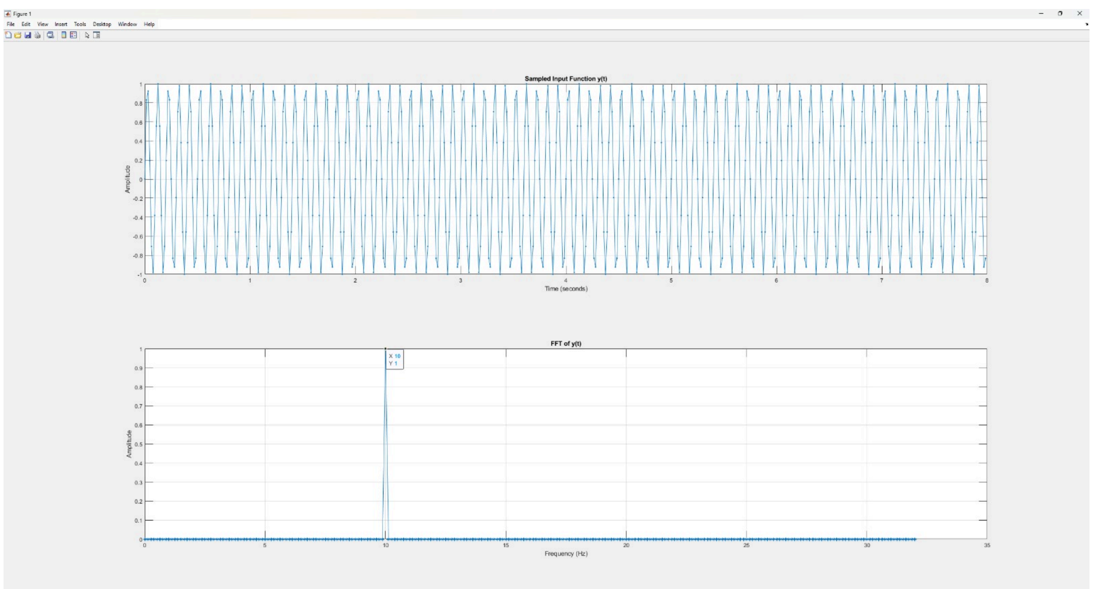


Figure 3

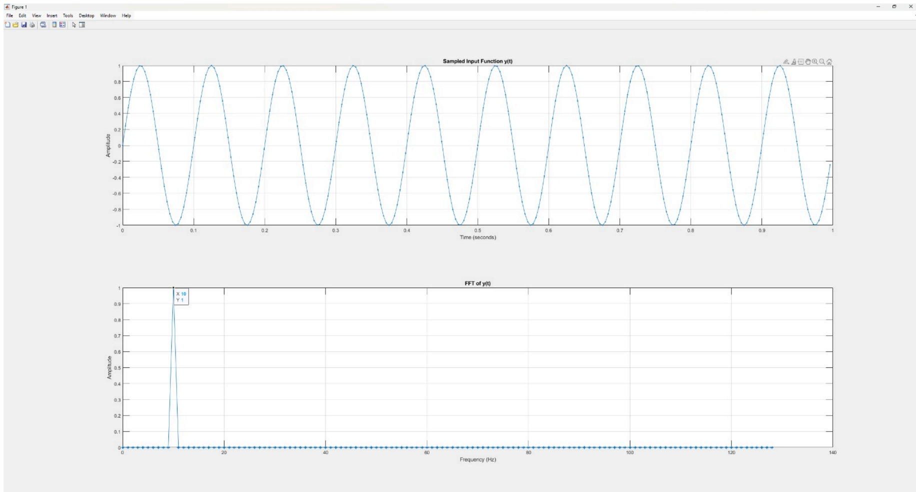


Figure 4

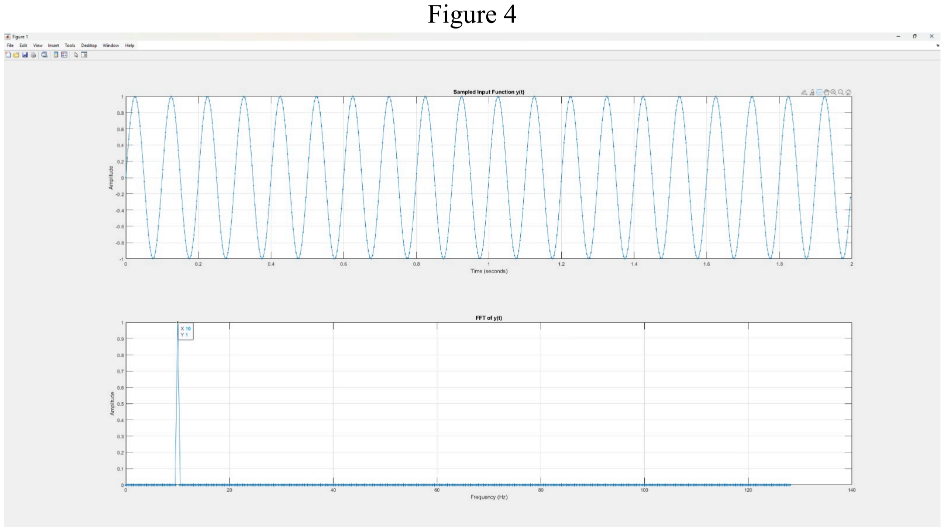


Figure 5

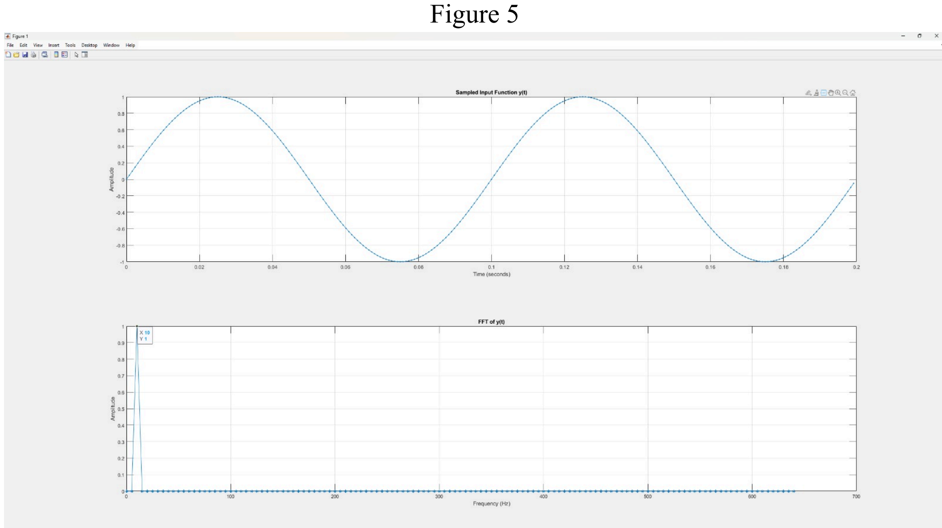
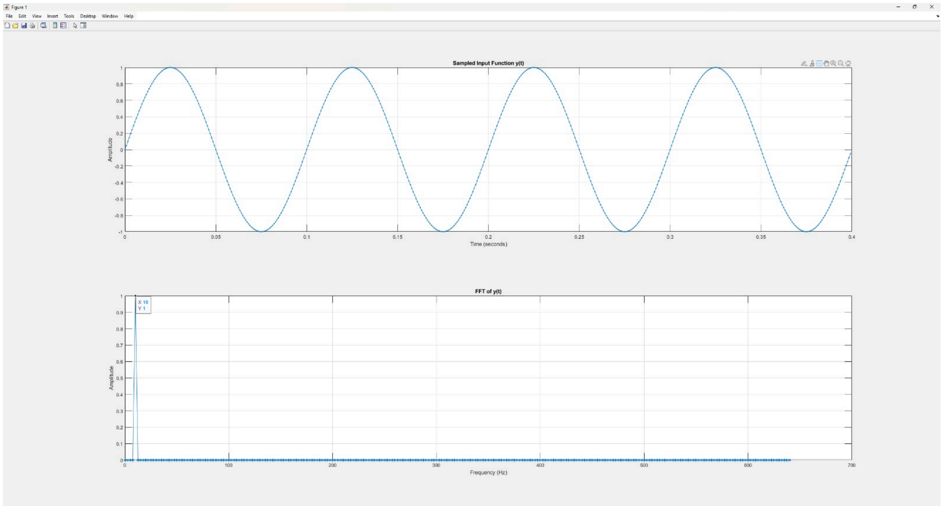


Figure 6



Observation Table 1.1

Figure Number	Input freq fi (Hz)	Sampling Rate fs (Hz)	N	Compute δt, s	Compute δf (Hz)	FFT output Freq. (Hz)	FFT output amplitude	Compute Nyquist Freq., fn (Hz)	Compare fi and fn	Is there Aliasing?	Compute fi/δf	Is there Leakage?	Perfect sine wave?
1	10	64	256	0.0156	0.25	10	1	32	fi< fn	No Aliasing	40	No Leakage	No
2	10	64	512	0.0156	0.125	10	1	32	fi< fn	No Aliasing	80	No Leakage	No
3	10	256	256	0.0039	1	10	1	128	fi< fn	No Aliasing	10	No Leakage	Yes
4	10	256	512	0.0039	0.5	10	1	128	fi< fn	No Aliasing	20	No Leakage	Yes
5	10	1280	256	7.81E-04	5	10	1	640	fi< fn	No Aliasing	2	No Leakage	Yes
6	10	1280	512	7.81E-04	2.5	10	1	640	fi< fn	No Aliasing	4	No Leakage	Yes

Compute δt, s Calculation: 1 / fs (Hz)

- fi (Hz)= 10, fs (Hz)= 64, N= 256 —-------> 1 / 64 = 0.0156 s
- fi (Hz)= 10, fs (Hz)= 64, N= 512 —-------> 1 / 64 = 0.0156 s
- fi (Hz)= 10, fs (Hz)= 256, N= 256 —-------> 1 / 256 = 0.0039 s
- fi (Hz)= 10, fs (Hz)= 256, N= 512 —-------> 1/ 256 = 0.0039 s
- fi (Hz)= 10, fs (Hz)= 1280, N= 256 —-------> 1 / 1280 = 7.81E-04
- fi (Hz)= 10, fs (Hz)= 1280, N= 512 —-------> 1 / 1280 = 7.81E-04

Compute δf (Hz) Calculation: fs (Hz) / N

- fi (Hz)= 10, fs (Hz)= 64, N= 256 —--------> 64 / 256 = 0.25 HZ
- fi (Hz)= 10, fs (Hz)= 64, N= 512 —--------> 64 / 512= 0.125 HZ
- fi (Hz)= 10, fs (Hz)= 256, N= 256 —--------> 256 / 256 = 1 HZ
- fi (Hz)= 10, fs (Hz)= 256, N= 512 —--------> 256 / 512 = 0.5 HZ
- fi (Hz)= 10, fs (Hz)= 1280, N= 256 —--------> 1280 / 256 = 5 HZ
- fi (Hz)= 10, fs (Hz)= 1280, N= 512 —--------> 1280 / 512 = 2.5 HZ

Compute Nyquist Freq., fn (Hz) calculation: fs (Hz) / 2

- fi (Hz)= 10, fs (Hz)= 64, N= 256 —--------> 64 / 2 = 32 HZ
- fi (Hz)= 10, fs (Hz)= 64, N= 512 —--------> 64 / 2= 32 HZ
- fi (Hz)= 10, fs (Hz)= 256, N= 256 —--------> 256 / 2 = 128 HZ
- fi (Hz)= 10, fs (Hz)= 256, N= 512 —--------> 256 / 2 = 128 HZ
- fi (Hz)= 10, fs (Hz)= 1280, N= 256 —--------> 1280 / 2 = 640 HZ
- fi (Hz)= 10, fs (Hz)= 1280, N= 512 —--------> 1280 / 2 = 640 HZ

Compute fi/δf calculation:

- fi (Hz)= 10, fs (Hz)= 64, N= 256 —--------> 10/ 0.25 = 40
- fi (Hz)= 10, fs (Hz)= 64, N= 512 —--------> 10 / 0.125 = 80
- fi (Hz)= 10, fs (Hz)= 256, N= 256 —--------> 10 / 1 = 10
- fi (Hz)= 10, fs (Hz)= 256, N= 512 —--------> 10 / 0.5 = 20
- fi (Hz)= 10, fs (Hz)= 1280, N= 256 —--------> 10 /5 = 2
- fi (Hz)= 10, fs (Hz)= 1280, N= 512 —--------> 10 / 2.5 = 4
-

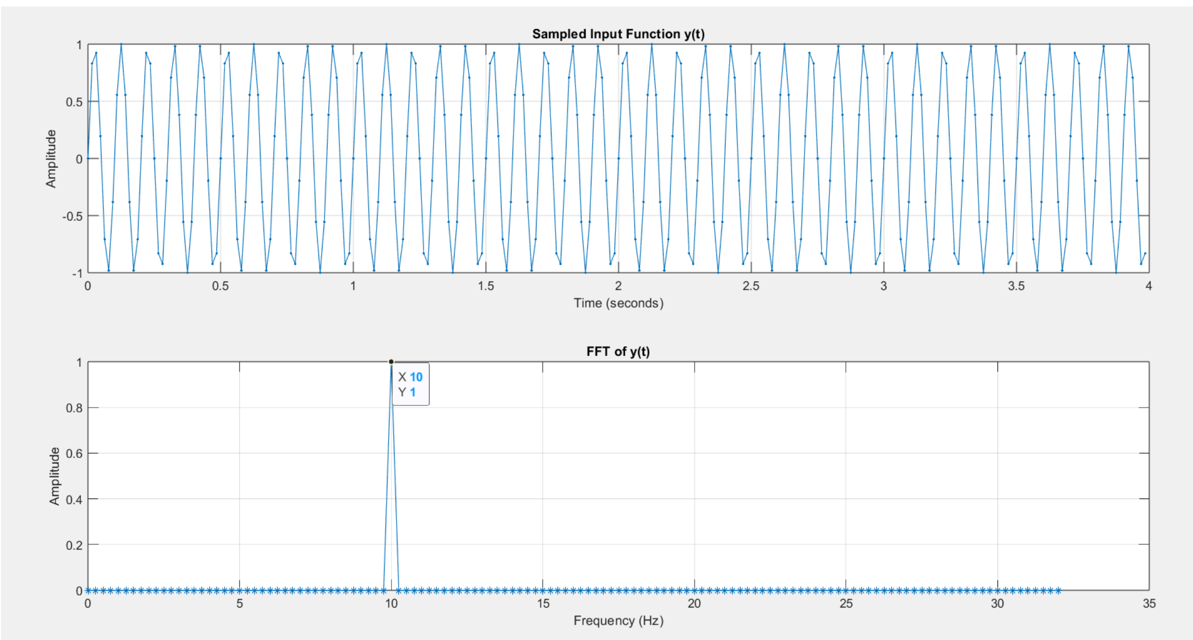
Leakage occurs if f_i/δf is not an integer:

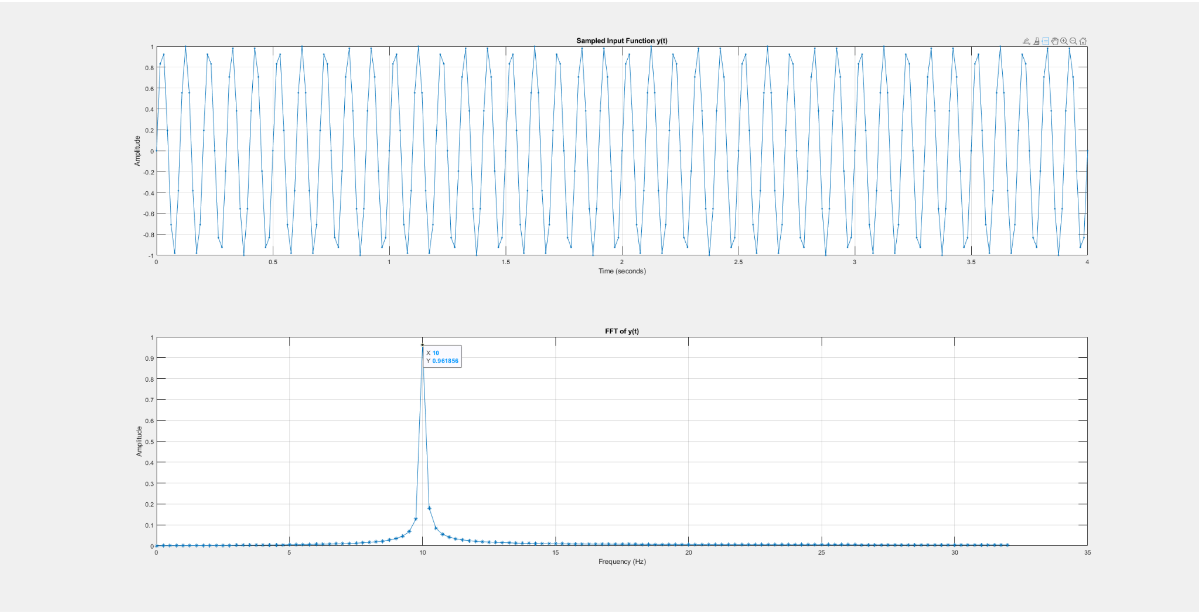
- fi (Hz)= 10, fs (Hz)= 64, N= 256 —--------> 10/ 0.25 = 40   —-----> integer —-----> No leakage
- fi (Hz)= 10, fs (Hz)= 64, N= 512 —--------> 10 / 0.125 = 80   —-----> integer —-----> No leakage
- fi (Hz)= 10, fs (Hz)= 256, N= 256 —--------> 10 / 1 = 10   —-----> integer —-----> No leakage
- fi (Hz)= 10, fs (Hz)= 256, N= 512 —--------> 10 / 0.5 = 20   —-----> integer —-----> No leakage
- fi (Hz)= 10, fs (Hz)= 1280, N= 256 —--------> 10 /5 = 2   —-----> integer —-----> No leakage
- fi (Hz)= 10, fs (Hz)= 1280, N= 512 —--------> 10 / 2.5 = 4   —-----> integer —-----> No leakage

Discuss the reason leading to distinct appearance of sinusoidal waves associated with different combinations of parameter sets *fi*, *fs* and *N*.

- Different choices of fi, fs, and N lead to distinct sinusoidal wave appearances because these parameters determine how the continuous sine wave is sampled and displayed: fi and fs set the normalized frequency (fi/fs), which governs whether the waveform repeats periodically or not, while fs controls how many samples per cycle are taken affecting smoothness versus aliasing and N specifies the total number of samples, which can cause discontinuities if the captured cycles aren't complete.

[2] Laboratory Exercises 1.2a – Change Matlab script as instructed and run it, answer the associated questions, and attach the output figures.

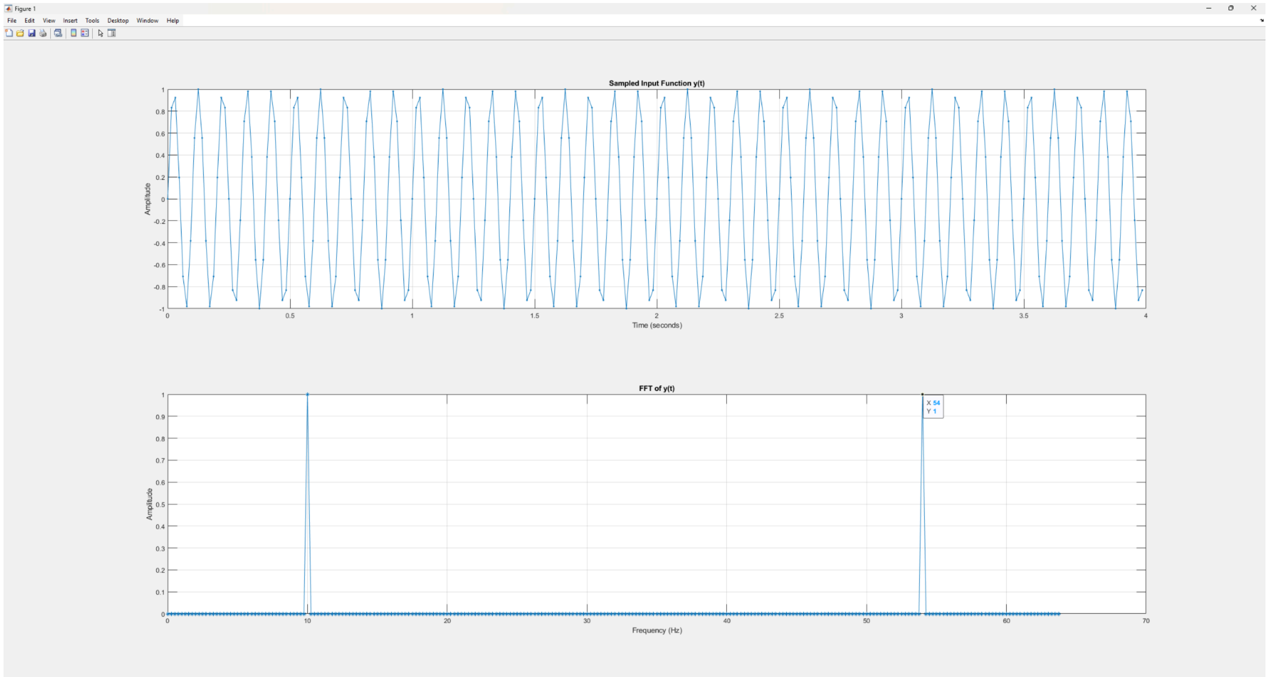




What change of the output figure do you observe? And why is that?

- The main change in the second output figure is the widening of the frequency peak in the FFT plot. In the first figure, the 10 Hz peak is sharp and well-defined, while in the second figure, it appears broader and less distinct. This change is likely due to spectral leakage, which occurs when the signal is not perfectly periodic within the time window or when a different windowing function is applied. If the signal is truncated at a non-integer number of periods or if a smoothing window is used, the energy spreads around the main frequency, causing the peak to widen.
- There is frequency leakage. This is because the end sampling point is not on the integer period.

[3] Laboratory Exercises 1.2b – Change Matlab script as instructed and run it, answer the associated questions, and attach the output figures.



What phenomena do you observe?

- The figure shows a periodic signal in the time domain (top plot) and its frequency components using FFT (bottom plot). The FFT reveals two main frequencies at 10 Hz and 54 Hz, meaning the signal is made up of these two components. There are no unexpected frequencies, so the signal was sampled correctly. The second frequency could be a harmonic or another independent signal.
- There are now two peaks due to Fourier Transform's symmetric property.

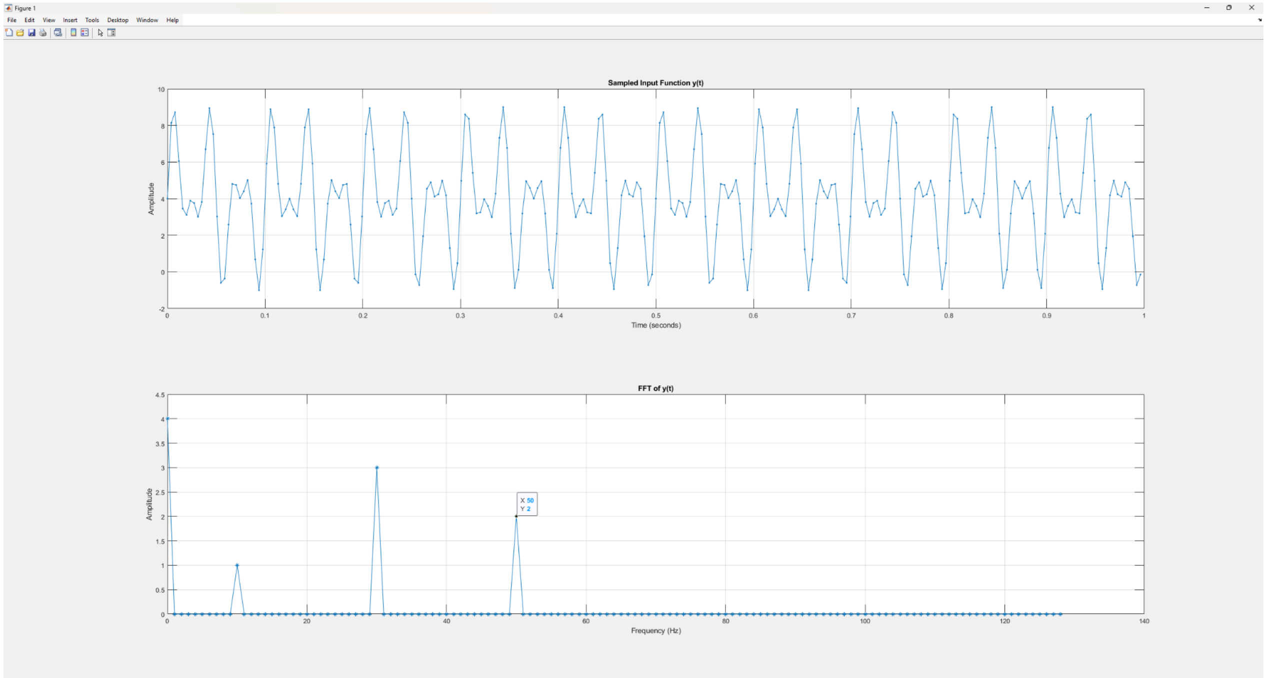
[4] Laboratory Exercises 1.3 – Change Matlab script on specified lines to perform FFT analysis on combined sine waves, <u>include your rewritten code on specific lines 5 and 13</u>, and attach the output figure.

Line 5:

```
fi0 = 0;fi1 = 10;fi2 = 30;fi3 = 50;A0 = 4;A1 = 1;A2 = 3;A3 = 2;
```

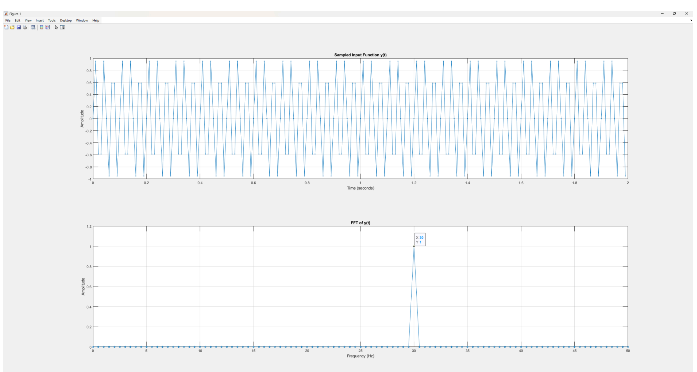
Line 13:

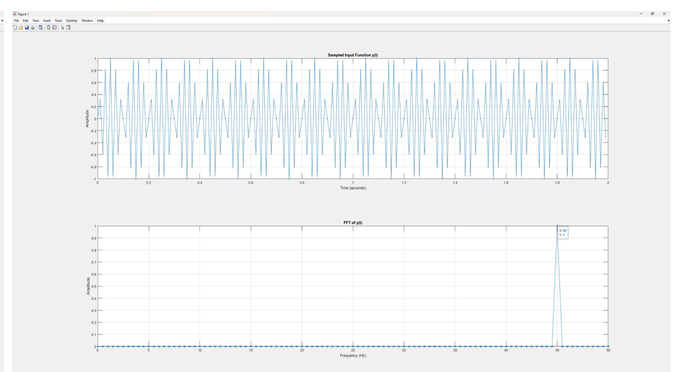
```
y = A0+A1*sin(2*pi*fi1*t)+A2*sin(2*pi*fi2*t)+A3*sin(2*pi*fi3*t);
```



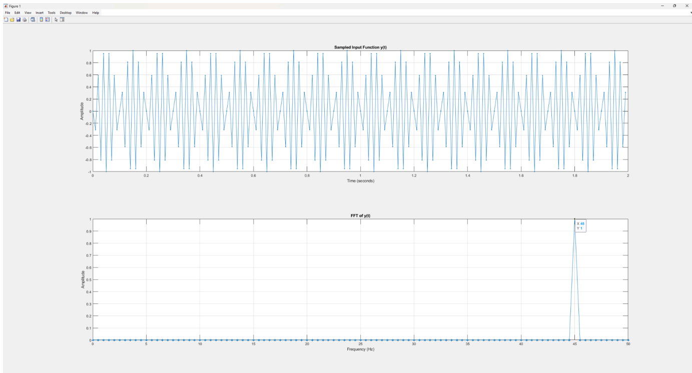
[5] Laboratory Exercises 2.1 – Include the required <u>computational process</u>, fill out relevant results and observations in Table 2 and attach the output figures.

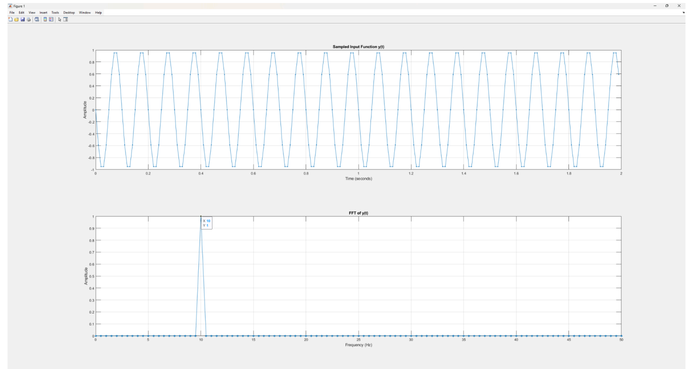
Rows 1-2:



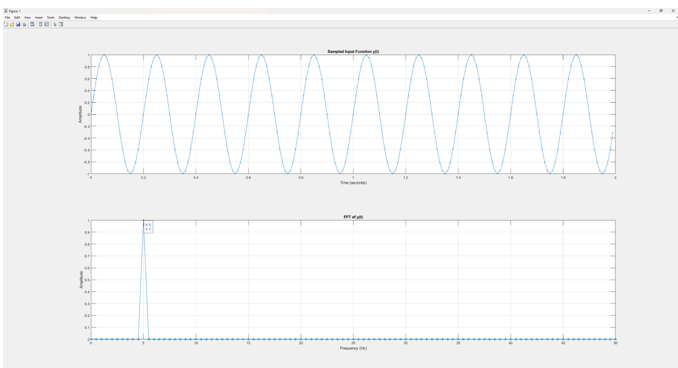


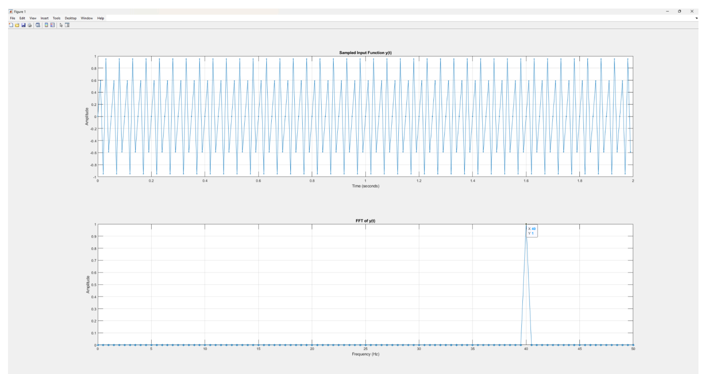
Rows 3-4:



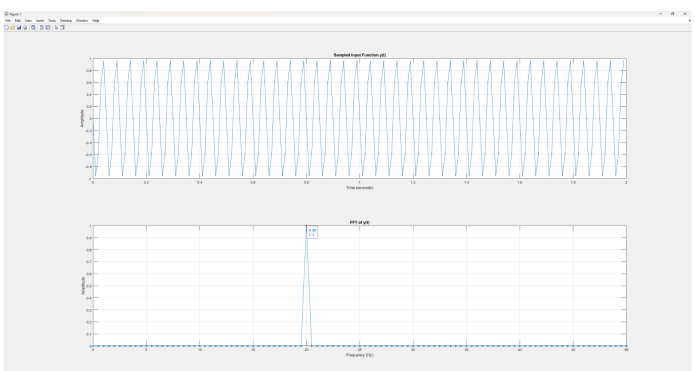


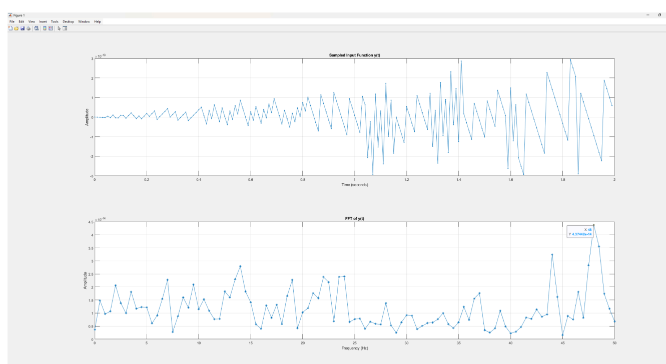
Rows 5-6:





Rows 7-8:





Observation Table 2.1:

Table 2: Nyquist criterion and alising			Ex 2.1			
Input frequency fi, (Hz)	**Sampling Rate fs (Hz)**	**N**	**Compute Nyquist frequency fn (Hz)**	**FFT output Frequency (Hz)**	**FFT output Amplitude**	**If aliasing emerges, Calculated alias frequency (Hz)**
30	100	200	50	30	1	No Aliasing
45	100	200	50	45	1	No Aliasing
55	100	200	50	45	1	45
90	100	200	50	10	1	10
105	100	200	50	5	1	5
140	100	200	50	40	1	40
180	100	200	50	20	1	20
200	100	200	50	48	4.37E-14	0

Compute Nyquist frequency fn (Hz) calculation: fs (Hz) / 2

- fi (Hz)= 30, fs (Hz)= 100, N= 200 —-------> 100 / 2 = 50 HZ
- fi (Hz)= 45, fs (Hz)= 100, N= 200 —-------> 100 / 2 = 50 HZ
- fi (Hz)= 55, fs (Hz)= 100, N= 200 —-------> 100 / 2 = 50 HZ
- fi (Hz)= 90, fs (Hz)= 100, N= 200 —-------> 100 / 2 = 50 HZ
- fi (Hz)= 105, fs (Hz)= 100, N= 200 —-------> 100 / 2 = 50 HZ
- fi (Hz)= 140, fs (Hz)= 100, N= 200 —-------> 100 / 2 = 50 HZ
- fi (Hz)= 180, fs (Hz)= 100, N= 200 —-------> 100 / 2 = 50 HZ
- fi (Hz)= 200, fs (Hz)= 100, N= 200 —-------> 100 / 2 = 50 HZ
- The Computing of Nyquist frequency fn (Hz) will be 50 HZ for all since fs (HZ) is 100 HZ for all

Compute the aliasing frequency:

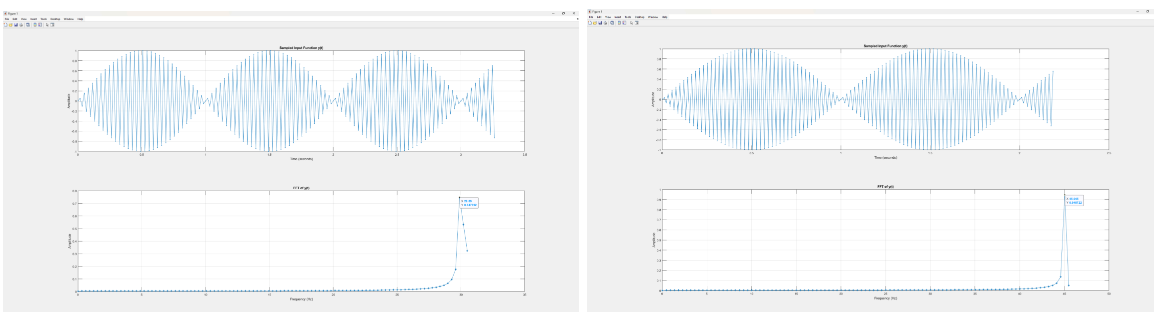
observe whether this computed *fa* is equal to FFT output frequency:

[6] Laboratory Exercises 2.2 – Include the required computational process for minimum sampling rate to eliminate aliasing, change Matlab script as instructed and run it, fill out relevant results and observations in Table 2, and attach the output figures.

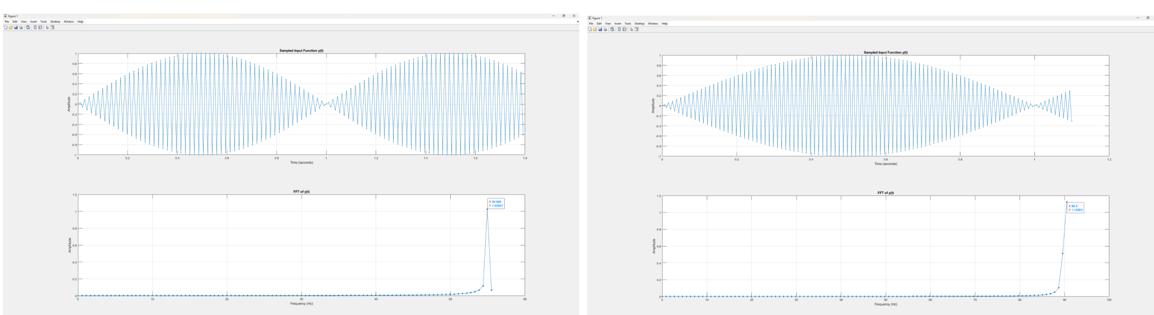
Observation Table 2.2:

Table 2: Nyquist criterion and alising		Ex 2.2			
Input frequency fi, (Hz)	Sampling Rate fs (Hz)	N	The required minimum Sampling rate to prevent aliasing (Hz)	FFT output Frequency for non-aliasing condition by taking the required minimum samplin rate (Hz)	FFT output Amplitude for non-aliasing condition by taking the required minimum samplin rate
30	100	200	61	29.89	0.748
45	100	200	91	45.045	0.946
55	100	200	111	54.945	1.025
90	100	200	181	90.5	1.123
105	100	200	211	105.5	1.333
140	100	200	281	140.5	1.443
180	100	200	361	180.5	1.338
200	100	200	401	200.5	1.266

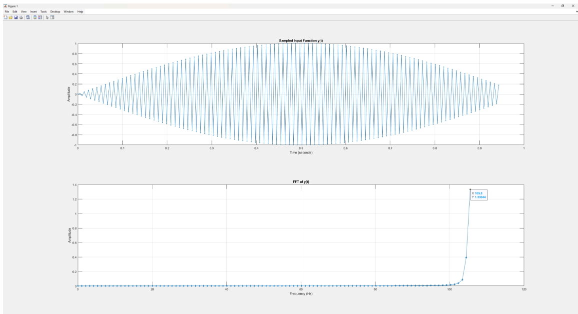
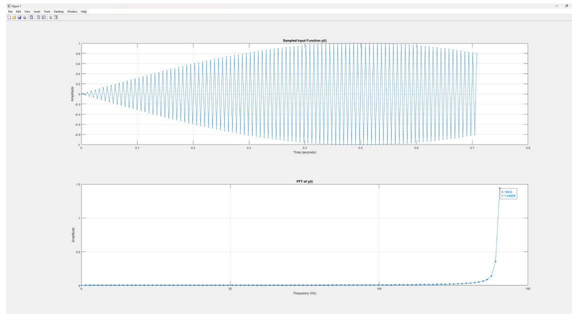
Rows 1-2:



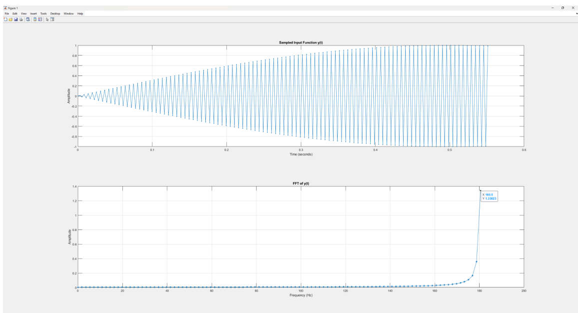
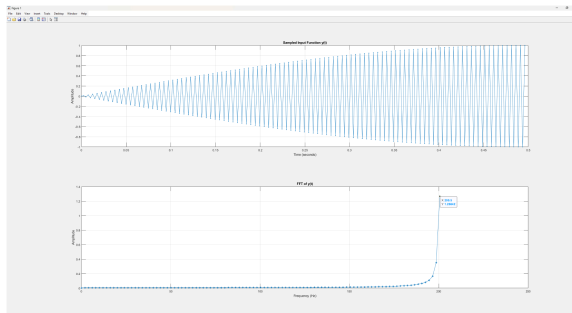
Rows 3-4:



Rows 5-6:

Rows 7-8:

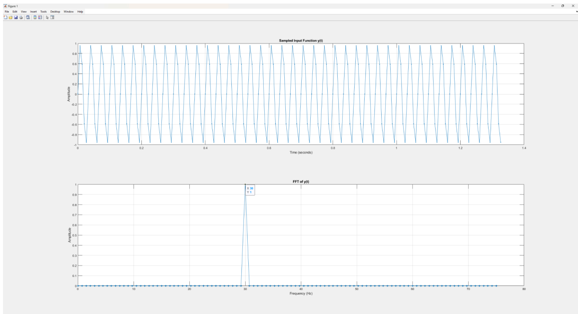
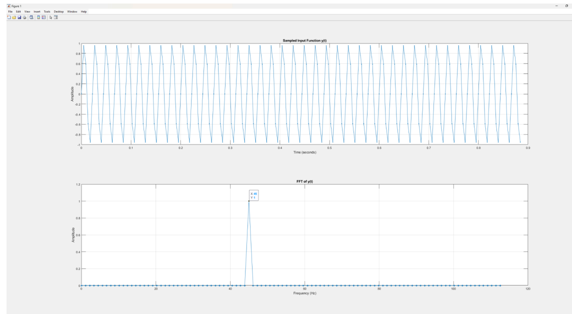
Determine the required theoretical sampling rates for eliminating aliasing

[7] Laboratory Exercises 2.3 – Change each *fs* to 5 times of each *fi*, fill out relevant results and observations in Table 2, and attach the output figures.

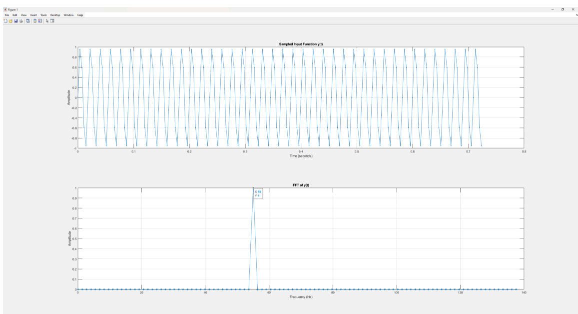
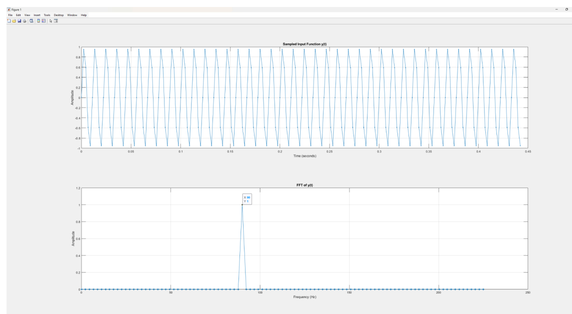
Observation Table 2.3:

Table 2: Nyquist criterion and alising			Ex 2.3		
Input frequency fi, (Hz)	**Sampling Rate fs (Hz)**	**N**	**Sampling rate that is 5X of fi (Hz)**	**FFT output Frequency for fs = 5fi (Hz)**	**FFT output Amplitude for fs = 5fi**
30	100	200	150	30	1
45	100	200	225	45	1
55	100	200	275	55	1
90	100	200	450	90	1
105	100	200	525	105	1
140	100	200	700	140	1
180	100	200	900	180	1
200	100	200	1000	200	1

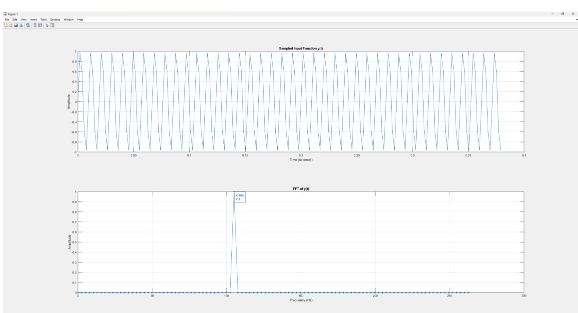
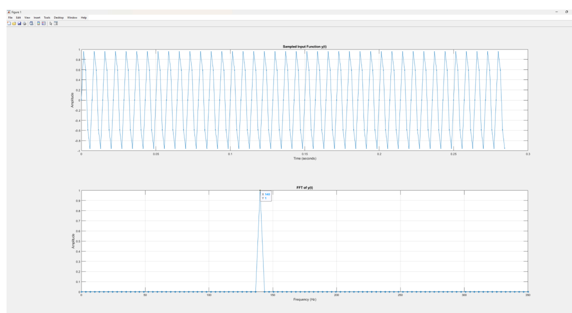
Rows 1-2:

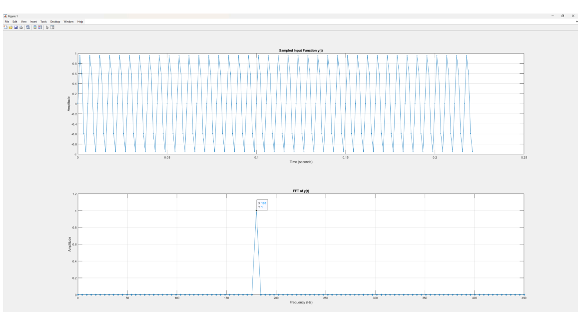
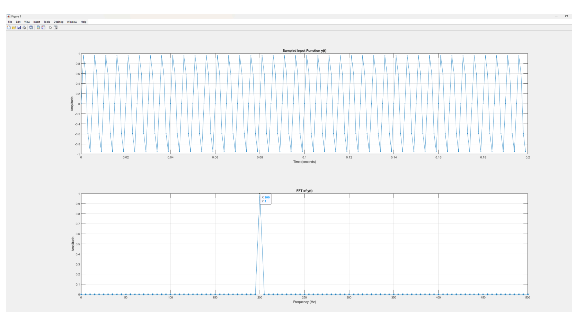
Rows 3-4:

Rows 5-6:

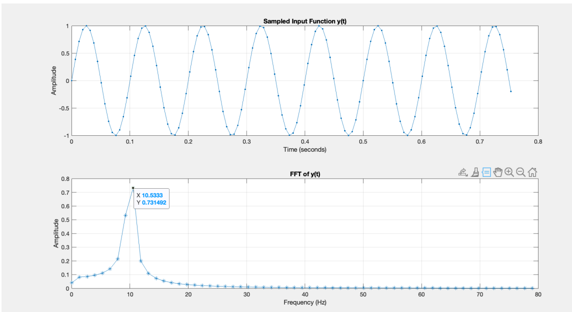
Rows 7-8:

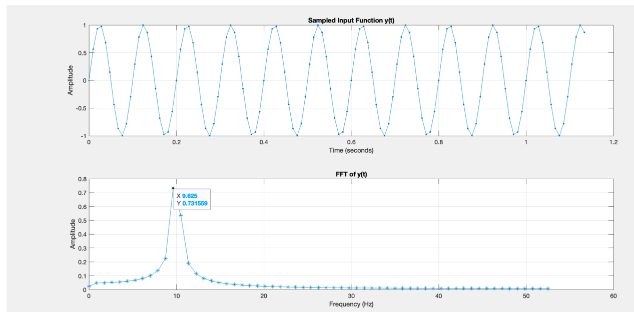



Q) In comparison with the results and observations in Ex 2.1 – Ex 2.3, what conclusion or experience you may draw accordingly for signal sampling (Laboratory Exercises 2.3)?

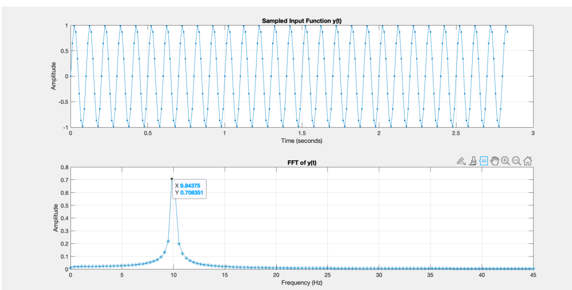
[8] Laboratory Exercises 3.1 – Include the required <u>computational process</u>, evaluate leakage condition, and fill out relevant results and observations in Table 3.

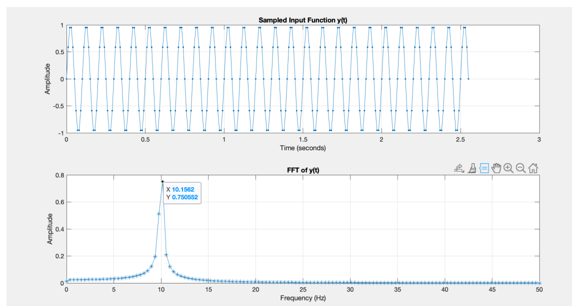
Rows 1-2:





Rows 3-4:





Row 5:

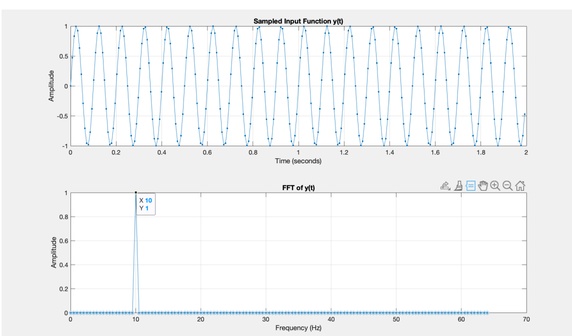


Table 3: Leakage and windowing	Note: include the table, computational processes, figures and answers to the discussion questions in your lab 02 report word file		Ex 3.1				
Input frequency fi, Hz	**Sampling Rate fs, Hz**	**N**	**Compute δf (Hz)**	**Compute fi/δf**	**Leakage?**	**FFT output Freq. (Hz)**	**FFT output amplitude**
10	105	120	0.875	11.43	Yes	9.625	0.731559
10	158	120	1.3167	7.5947	Yes	10.5333	0.731492
10	90	256	0.3516	28.441	Yes	9.84375	0.708351
10	100	256	0.3906	25.602	Yes	10.1562	0.75055
10	128	256	0.5	20	No	10	1

Compute δf (Hz) Calculation: fs (Hz) / N

- fi (Hz)= 10, fs (Hz)= 105, N= 120 —-------> 105 / 120= 0.875 HZ
- fi (Hz)= 10, fs (Hz)= 158, N= 120 —-------> 158 / 120= 1.3167 HZ
- fi (Hz)= 10, fs (Hz)= 90, N= 256 —-------> 90 / 256 = 0.3516 HZ
- fi (Hz)= 10, fs (Hz)= 100, N= 256 —-------> 100 / 256 = 0.3906 HZ
- fi (Hz)= 10, fs (Hz)= 128, N= 256 —-------> 128 / 256 = 0.5 HZ

Compute fi/δf calculation:

- fi (Hz)= 10, fs (Hz)= 105, N= 120 —-------> 10 /0.875 = 11.43
- fi (Hz)= 10, fs (Hz)= 158, N= 120 —-------> 10 / 1.3167 = 7.5947
- fi (Hz)= 10, fs (Hz)= 90, N= 256 —-------> 10 / 0.3516 = 28.441
- fi (Hz)= 10, fs (Hz)= 100, N= 256 —-------> 10 / 0.3906 = 25.602
- fi (Hz)= 10, fs (Hz)= 128, N= 256 —-------> 10 / 0.5 =20

Leakage Check: fi/δf = fi . N / fs

fi (Hz)= 10, fs (Hz)= 105, N= 120
- 11.43 = (10 . 120)/ 105 —-----------> No Integer —-------------> Leakage

fi (Hz)= 10, fs (Hz)= 158, N= 120
- 7.5947 = (10 . 120)/ 158 —-----------> No Integer —-------------> Leakage

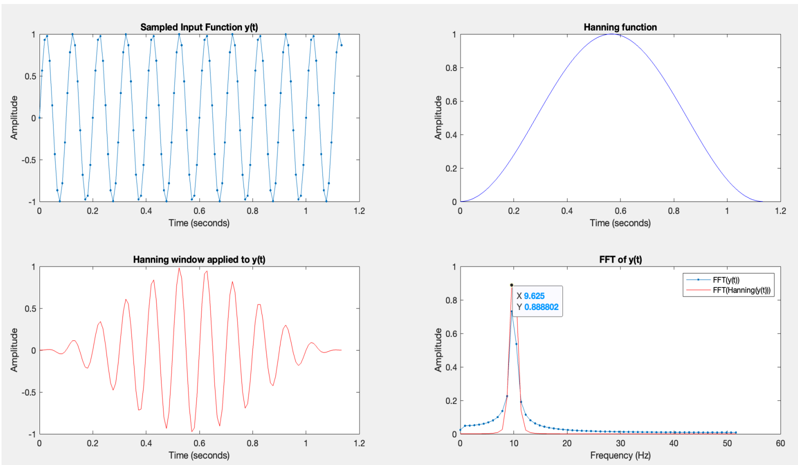
fi (Hz)= 10, fs (Hz)= 90, N= 256
- 28.441 = (10 . 256)/ 90 —-----------> No Integer —-------------> Leakage

fi (Hz)= 10, fs (Hz)= 100, N= 256
- 25.602 = (10 . 256)/ 100 —-----------> No Integer —-------------> Leakage

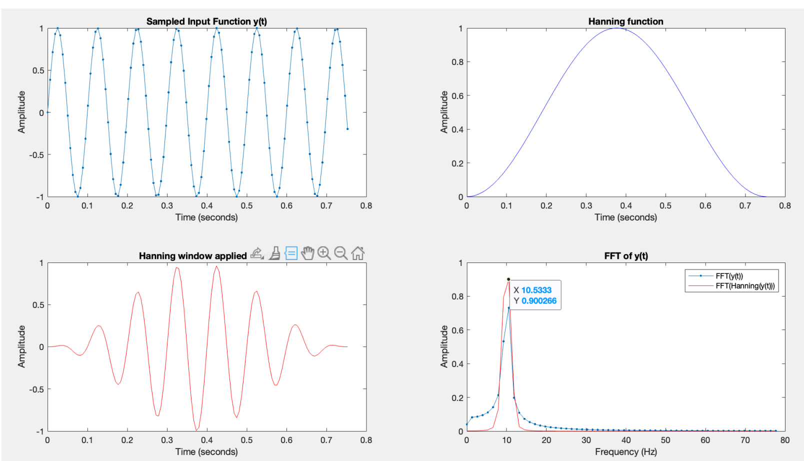
fi (Hz)= 10, fs (Hz)= 128, N= 256
- 20 = (10 . 256)/ 128 —-----------> Integer —-------------> No Leakage

[9] Laboratory Exercises 3.2 – Run Matlab code with the application of Hanning window, fill FFT output frequency and amplitude after applying Hanning window in Table 3, and save figures.

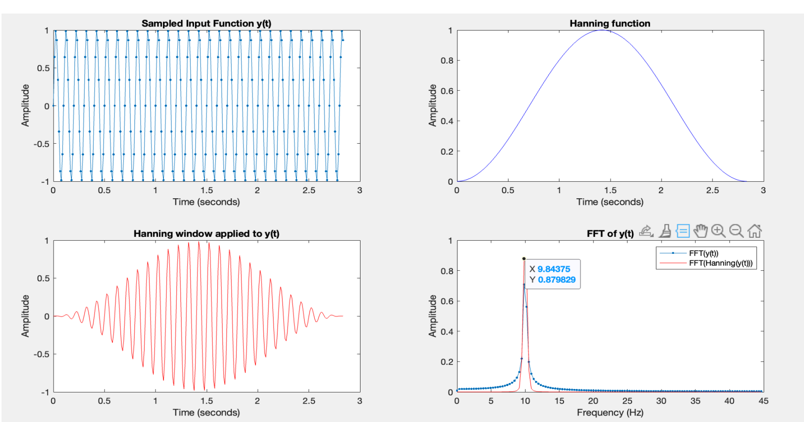
Row 1:



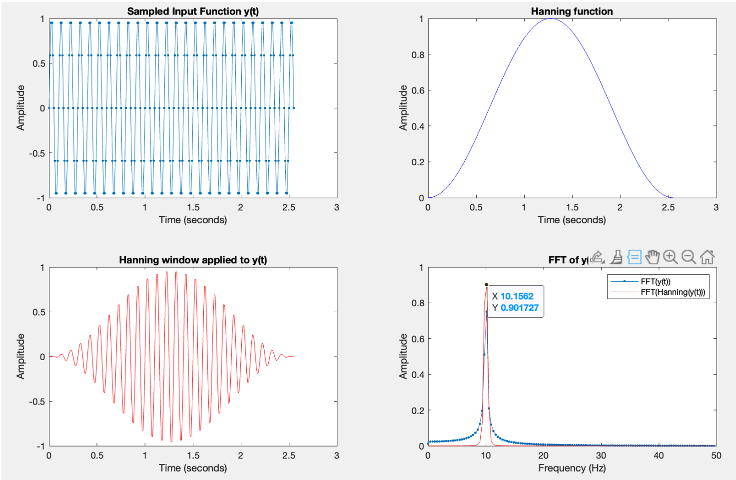
Row 2:



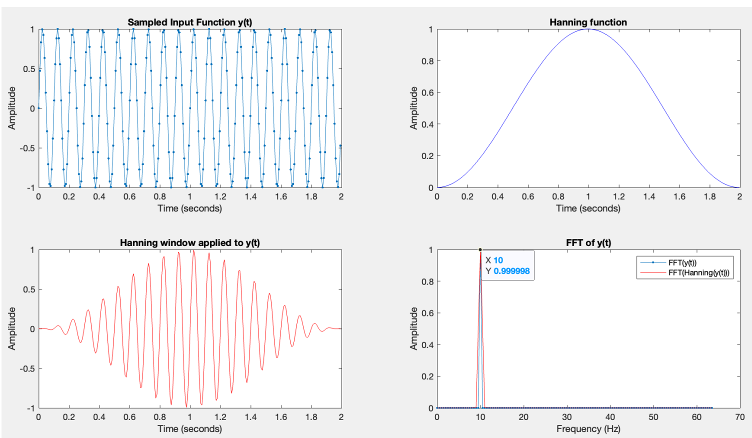
Row 3:



Row 4:



Row 5:

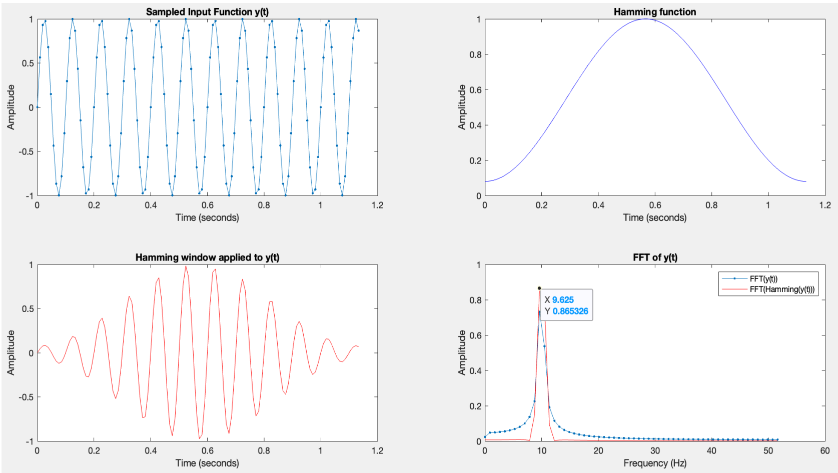


Ex 3.2	
FFT output Freq. after applying Hanning window (Hz)	**FFT output amplitude after applying Hanning window**
9.625	0.888802
10.5333	0.900266
9.84375	0.879829
10.1562	0.901727
10	0.999998

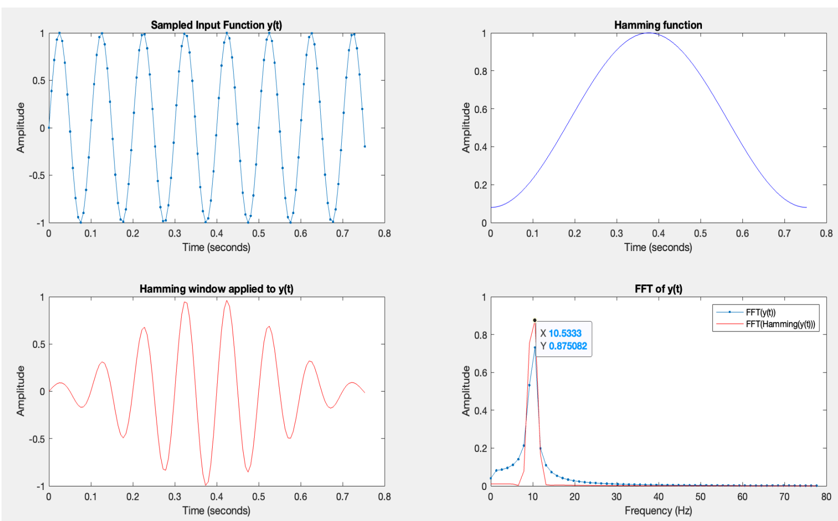
[10] Laboratory Exercises 3.3 – Run Matlab code with the application of Hamming window, fill FFT output frequency and amplitude after applying Hamming window in Table 3, and save figures. Answer the two discussion questions: 1) compare and point out the primary difference in the Matlab scripts

“Lab2_Ex3p2.m” and “Lab2_Ex3p3.m”; 2) <u>compare the effects</u> of two window functions.

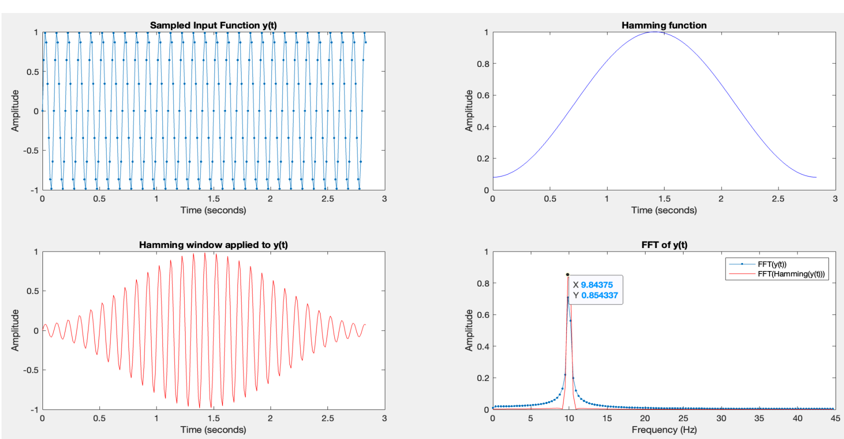
Row 1:



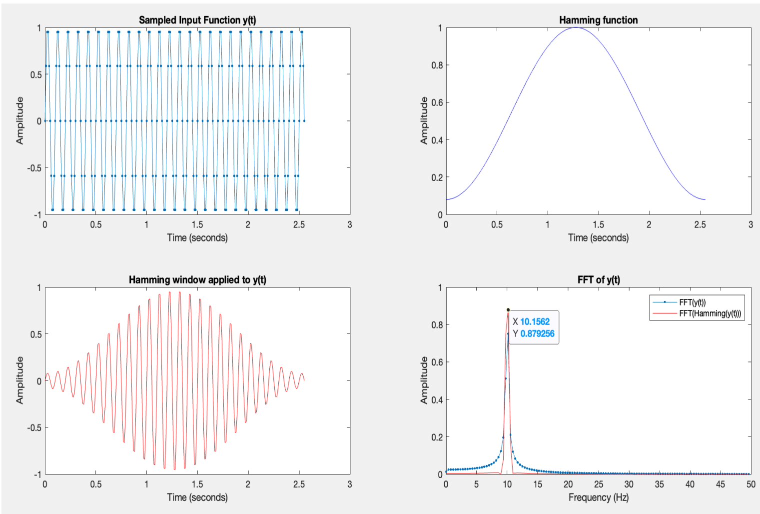
Row 2:



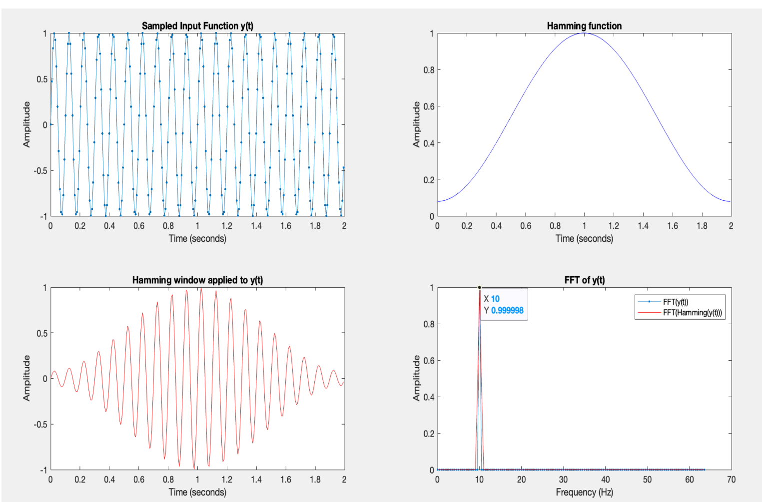
Row 3:



Row 4:



Row 5:



Ex 3.3	
FFT output Freq. after applying Hamming window (Hz)	**FFT output amplitude after applying Hamming window**
9.625	0.865326
10.5333	0.875082
9.84375	0.854337
10.1562	0.879256
10	0.999998

Discussion question: Compare the Matlab scripts “Lab2_Ex3p2.m” and “Lab2_Ex3p3.m”. Specify the essential change(s) in “Lab2_Ex3p3.m” from “Lab2_Ex3p2.m” by pointing out the specific line number(s) and corresponding codes in script, so as to show it is the application of Hamming window, but not Hanning window.

There are some differences in the script between **Lab2_Ex3p2.m** and **Lab2_Ex3p3.m** because they compare two different window functions. The key difference is that **Lab2_Ex3p2.m** applies a **Hanning window**, while **Lab2_Ex3p3.m** applies a **Hamming window**. Below are the specific lines of code that differ between the two scripts:

In line 26:

- Lab2_Ex3p2.m —---------------> H = hann(N); %creates a Hanning window
- Lab2_Ex3p3.m —---------------> H = hamming(N); %creates a Hamming window

In line 45:

- Lab2_Ex3p2.m —---------------> title('Hanning function');
- Lab2_Ex3p3.m —---------------> title('Hamming function');

In line 51:

- Lab2_Ex3p2.m —---------------> title('Hanning window applied to y(t)');
- Lab2_Ex3p3.m —---------------> title('Hamming window applied to y(t)');

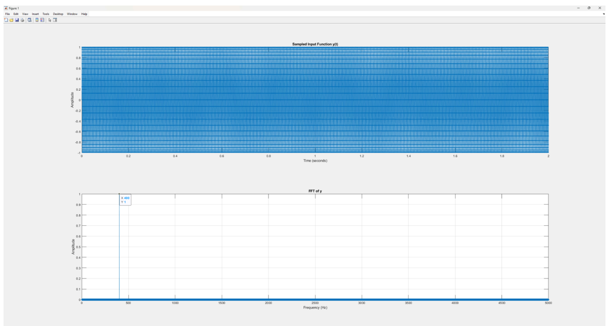
In line 58:

- Lab2_Ex3p2.m —---------------> legend('FFT(y(t))','FFT(Hanning(y(t)))');
- Lab2_Ex3p3.m —---------------> legend('FFT(y(t))','FFT(Hamming(y(t)))');

Discussion question: Observe and compare the effect of Hanning window and Hamming window that are applied to the original signal, what difference do you find in the FFT spectra? What is the magnitude ranking (from high to low) about the amplitude of the output frequency in FFT spectra for original signal, that for signal after applying Hanning window, and that for signal after applying Hamming window? Would you expect we can totally eliminate leakage using window functions (Laboratory Exercises 3.3)?

[11] Laboratory Exercises 4.1- Fill out sound output feature, FFT spectrum feature, FFT output frequency and amplitude of the original signal in Table 4, and save the figures.

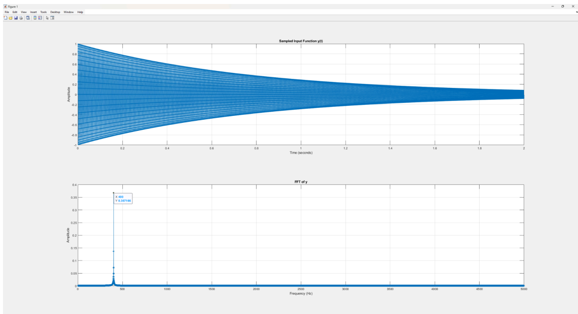
Input frequency fi (Hz)	Sampling Rate fs (Hz)	N	Time constant, tc, s	Sound feature	FFT spectrum feature	FFT output Freq. (Hz)	FFT output amplitude
400	10000	20000	N/A	Homogenous	Constant, with only one spike	400	1

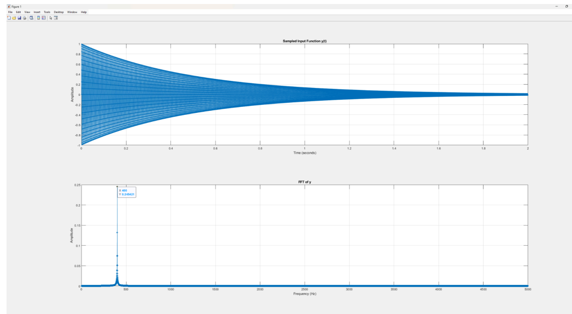


[12] Laboratory Exercises 4.2- Fill out sound output feature, FFT spectrum feature, FFT output frequency and amplitude of the decayed signal in Table 4, and save the figures.

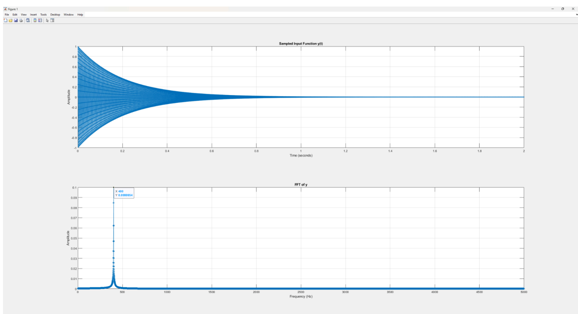
Input frequency fi (Hz)	Sampling Rate fs (Hz)	N	Time constant, tc, s	Sound feature	FFT spectrum feature	FFT output Freq. (Hz)	FFT output amplitude
400	10000	20000	0.8	Non Homogeneous since more faint	The height of the spike decreased	400	0.367166
400	10000	20000	0.5	Non Homogenous, weaker than first two sounds	The height of the spike was shorter than above sounds	400	0.245421
400	10000	20000	0.2	Non Homogenous, weakest sound	The height of the spike was shortest	400	0.0999954

Rows 1-2:





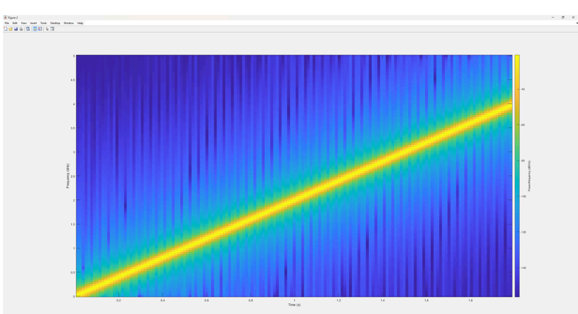
Row 3:

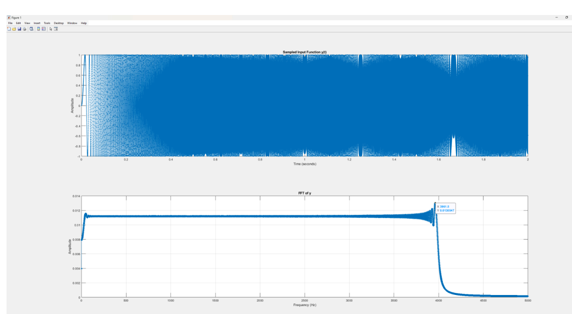


[13] Laboratory Exercises 4.3 - Fill out sound output feature of the signal presenting linearly changed frequency, and observations upon time domain signal, FFT spectra, and spectrogram in Table 4, and save the figures.

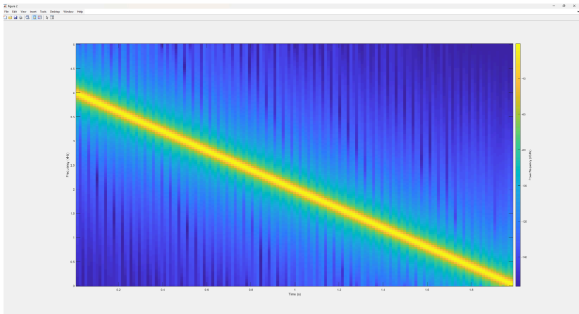
Input frequency fi (Hz)	Sampling Rate fs (Hz)	N	Freq change	Sound feature	Change in time domain signal?	Change in FFT spectrum ?	Change in spectrogram?
1000	10000	20000	linear increase	High pitched, like a horn (Increasing)			
1000	10000	20000	linear decrease	The pitch of the sound decreases	The time domain signal is reversed	No change	The graph is mirrored about vertical axis

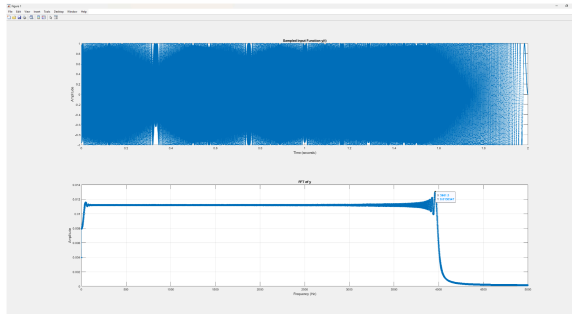
Row 1:





Row 2:

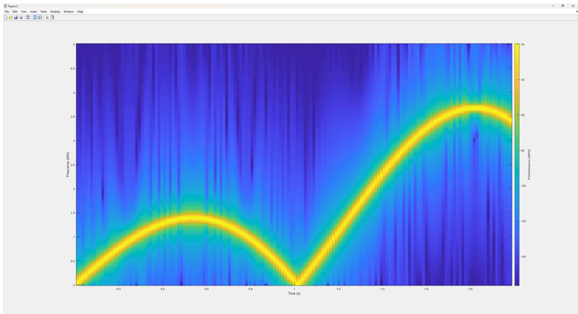


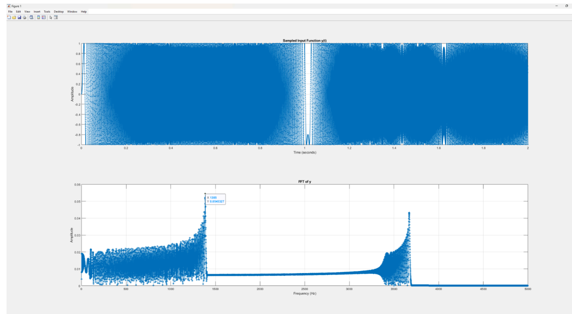


[14] Laboratory Exercises 4.4 - Fill out sound output feature of the signal presenting nonlinearly changed frequency, and observations upon time domain signal, FFT spectra, and spectrogram in Table 4, and save the figures.

Input frequency fi (Hz)	Sampling Rate fs (Hz)	N	Freq change	Sound feature	Change in time domain signal?	Change in FFT spectrum?	Change in spectrogram?
1000	10000	20000	non-linear sin	Two High pitch points, first sound is weaker than second one (uneven double peak)			
1000	10000	20000	non-linear sin reverse	Two High pitch points, second sound is weaker than first one (uneven double peak)	Yes, The signal is reversed	No change	The graph is flipped about the vertical axis (X-axis)

Row 1:





Row 2: