

Dimensionally Extended Nine-Intersection Model (DE-9IM)

Christian Strobl,
German Remote Sensing Data Center (DFD)
German Aerospace Center (DLR)

SYNONYMS

Dimensionally Extended Nine-Intersection Model (DE-9IM), Nine-Intersection Model (9IM), Four-Intersection Model (4IM), Egenhofer Operators, Clementini Operators; Topological Operators

DEFINITION

The Dimensionally Extended Nine-Intersection Model (DE-9IM) or Clementini-Matrix is specified by the OGC “Simple Features for SQL” specification for computing the spatial relationships between geometries. It is based on the Nine-Intersection Model (9IM) or Egenhofer-Matrix which in turn is an extension of the Four-Intersection Model (4IM).

The Dimensionally Extended Nine-Intersection Model considers the two objects’ interiors, boundaries and exteriors and analyzes the intersections of these nine objects parts for their relationships (maximum dimension (-1, 0, 1, or 2) of the intersection geometries with a numeric value of -1 corresponding to no intersection).

The spatial relationships described by the DE-9IM are “Equals”, “Disjoint”, “Intersects”, “Touches”, “Crosses”, “Within”, “Contains” and “Overlaps”.

MAIN TEXT

For the description of topological relationships of geodata there exist three common and accepted approaches. Each of these systems describes the relationship between two objects based on an intersection matrix.

- Four-Intersection Model (4IM): Boolean set of operations (considering intersections between boundary and exterior) (7), (4)
- Nine-Intersection Model (9IM) Egenhofer operators (taking into account exterior, interior and boundary of objects) (6), (5)
- **Dimensionally Extended Nine-Intersection Model (DE-9IM):** Clementini operators using the same topological primitives as Egenhofer but considering the dimension type of the intersection.(1), (2)

The **Dimensionally Extended Nine-Intersection Model (DE-9IM)** is accepted by the ISO/TC 211 (8) and by the Open Geospatial Consortium (9) and will be described in the following paragraphs.

Each of the mentioned intersection models is based on the accepted definitions of the boundaries, interiors and exteriors for the basic geometry types which are considered. Therefore the first step is the definition of the interior, boundary and exterior of the involved geometry types. The domain of geometric objects considered is those that are topologically closed.

- Boundary: The boundary of a geometry object is a set of geometries of the next lower dimension.
- The interior of a geometry object consists of those points that are left (inside) when the boundary points are removed.
- The exterior of a geometry object consists of points not in the interior or boundary.

Geometric Subtypes	Interior (I)	Boundary (B)	Exterior (E)
<i>Point, MultiPoint</i>	Point, Points	Empty set	Points not in the interior or boundary
<i>LineString, Line</i>	Points that are left when the boundary points are removed.	Two end Points	Points not in the interior or boundary
<i>LinearRing</i>	All Points along the LinearRing	Empty set	Points not in the interior or boundary
<i>MultiLineString</i>	Points that are left when the boundary points are removed	Those Points that are in the boundaries of an odd number of its element Curves	Points not in the interior or boundary
<i>Polygon</i>	Points within the Rings	Set of Rings	Points not in the interior or boundary
<i>MultiPolygon</i>	Points within the Rings	Set of Rings of its Polygons	Points not in the interior or boundary

Table 1: Definition of the Interior, Boundary and Exterior for the main geometry types which are described by the Open Geospatial Consortium (9).

Next we consider the topological relationship of two geometry objects. Each geometry is represented by its Interior (I), Boundary (B) and Exterior (E) and so all possible relationships of two geometry objects can be described by a 3x3-matrix. If the values of the matrix are the dimension of the respective relationship of the two geometry objects, e.g. between the interior of geometry object A and the boundary of geometry object B, the result is the dimensionally extended nine-intersection matrix (DE-9IM) after Clementini (2). This matrix has the form

$$DE-9IM(A, B) = \begin{bmatrix} \dim(I(A) \cap I(B)) & \dim(I(A) \cap B(B)) & \dim(I(A) \cap E(B)) \\ \dim(B(A) \cap I(B)) & \dim(B(A) \cap B(B)) & \dim(B(A) \cap E(B)) \\ \dim(E(A) \cap I(B)) & \dim(E(A) \cap B(B)) & \dim(E(A) \cap E(B)) \end{bmatrix}$$

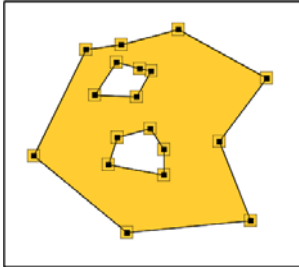
Topological predicates are Boolean functions that are used to test the spatial relationships between two geometry objects. The Dimensionally Extended Nine-Intersection Model provides eight such spatial relationships between points, lines and polygons (q.v. (9) and **Table 2**).

Topological Predicate	Meaning
Equals	The Geometries are topologically equal
Disjoint	The Geometries have no point in common
Intersects	The Geometries have at least one point in common (the inverse of Disjoint)
Touches	The Geometries have at least one boundary point in common, but no interior points
Crosses	The Geometries share some but not all interior points, and the dimension of the intersection is less than that of at least one of the Geometries.
Overlaps	The Geometries share some but not all points in common, and the intersection has the same dimension as the Geometries themselves
Within	Geometry A lies in the interior of Geometry B
Contains	Geometry B lies in the interior of Geometry A (the inverse of Within)

Table 2: Topological predicates and their corresponding meanings after the Dimensionally Extended Nine-Intersection Model, from (3).

In the following each topological predicate is described by an example:

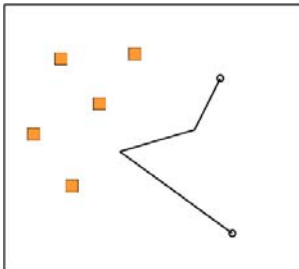
“Equals”: Example DE-9IM for the case where A is a Polygon which is equal to a Polygon B.



	Interior (B)	Boundary (B)	Exterior (B)
Interior(A)	2	-1	-1
Boundary (A)	-1	1	-1
Exterior (A)	-1	-1	2

Figure 1: Example for an “Equals”-relationship between a Polygon A and a Polygon B.

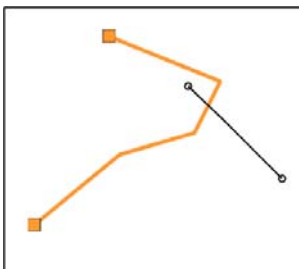
“Disjoint”: Example DE-9IM for the case where A is a Line which is disjoint to a MultiPoint object B. NB: The boundary of a Point is per definition empty (-1).



	Interior (B)	Boundary (B)	Exterior (B)
Interior(A)	-1	-1	1
Boundary (A)	-1	-1	0
Exterior (A)	0	-1	2

Figure 2: Example for a “Disjoint”-relationship between a Line A and a MultiPoint B.

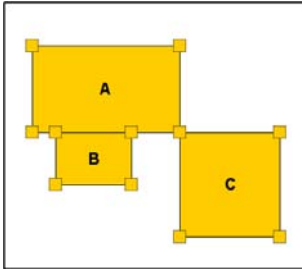
“Intersects”: Example DE-9IM for the case where A is a Line which intersects a Line B. NB: The “Intersects”-relationship is the inverse of Disjoint. The Geometry objects have at least one point in common, so the “Intersects” relationship includes all other topological predicates. The example in **Figure 3** is therefore also an example for a “Crosses”-relationship.



	Interior (B)	Boundary (B)	Exterior (B)
Interior(A)	0	-1	1
Boundary (A)	-1	-1	0
Exterior (A)	1	0	2

Figure 3: Example for a “Disjoint”-relationship between a Line A and a MultiPoint B.

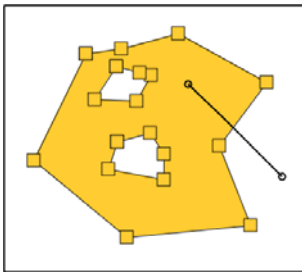
“Touches”: Example DE-9IM for the case where A is a Polygon that touches two other Polygons B and C. The DE-9IM for both relationships differs only in the dimension of the boundary-boundary-intersection which has the value 1 for the relationship A/B and the value 0 for the relationship A/C.



	Interior (B)	Boundary (B)	Exterior (B)
Interior(A)	-1	-1	2
Boundary (A)	-1	1/0	1
Exterior (A)	2	1	2

Figure 4: Example for a “Touches”-relationship between three Polygons A, B and C.

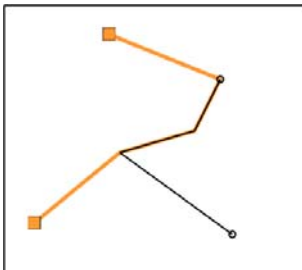
“Crosses”: Example DE-9IM for the case where A is a Polygon and B is a Line that crosses line A.



	Interior (B)	Boundary (B)	Exterior (B)
Interior(A)	1	0	2
Boundary (A)	0	-1	1
Exterior (A)	1	0	2

Figure 5: Example for a “Crosses”-relationship between a Polygon A and a Line B.

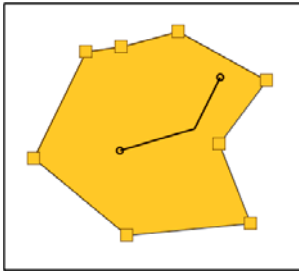
“Overlaps”: Example DE-9IM for the case where A is a Line which overlaps the Line B. The overlaps-relationship is not commutative. Line A overlaps Line B is different from Line B overlaps Line A. The DE-9IM differs yet in the interior-boundary- respectively in the boundary-interior-relationship (bold printed).



	Interior (B)	Boundary (B)	Exterior (B)
Interior(A)	1	-1/0	1
Boundary (A)	0/-1	-1	0
Exterior (A)	1	0	2

Figure 6: Example for an “Overlaps”-relationship between two Lines A and B.

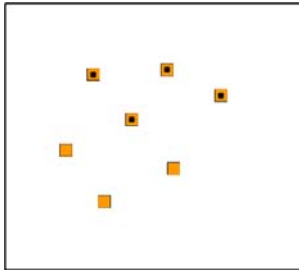
“Within”: Example DE-9IM for the case where A is a Line which lies within the Polygon B.



	Interior (B)	Boundary (B)	Exterior (B)
Interior(A)	1	-1	-1
Boundary (A)	0	-1	-1
Exterior (A)	2	1	2

Figure 7: Example for a “Within”-relationship between a Line A and a Polygon B.

“Contains”: Example DE-9IM for the case where A is a MultiPoint Object (squares) which contains another MultiPoint B (circles).



	Interior (B)	Boundary (B)	Exterior (B)
Interior(A)	0	-1	0
Boundary (A)	-1	-1	-1
Exterior (A)	-1	-1	2

Figure 8: Example for a “Contains”-relationship between two MultiPoints A and B.

The pattern matrix represents the DE-9IM set of all acceptable values for a topological predicate of two geometries.

The pattern matrix consists of a set of 9 pattern-values, one for each cell in the matrix.

The possible pattern values p are (T, F, *, 0, 1, 2) and their meanings for any cell where x is the intersection set for the cell are as follows:

$p = T \Rightarrow \dim(x) \in (0, 1, 2)$, i.e. $x = \emptyset$

$p = F \Rightarrow \dim(x) = -1$, i.e. $x = \emptyset$

$p = * \Rightarrow \dim(x) \in (-1, 0, 1, 2)$, i.e. Don't Care

$p = 0 \Rightarrow \dim(x) = 0$

$p = 1 \Rightarrow \dim(x) = 1$

$p = 2 \Rightarrow \dim(x) = 2$

The Relate predicate based on the pattern matrix has the advantage that clients can test for a large number of spatial relationships the appropriate topological predicate. For the eight topological predicates of the DE-9IM the pattern matrices are described in **Table 3**.

Topological Predicate	Pattern Matrix
A.Equals(B)	$\begin{bmatrix} T & * & F \\ * & * & F \\ F & F & * \end{bmatrix}$
A.Disjoint(B)	$\begin{bmatrix} F & F & * \\ F & F & * \\ * & * & * \end{bmatrix}$
A.Intersects(B)	$\begin{bmatrix} T & * & * \\ * & * & * \\ * & * & * \end{bmatrix} \text{ or } \begin{bmatrix} * & T & * \\ * & * & * \\ * & * & * \end{bmatrix} \text{ or } \begin{bmatrix} * & * & * \\ T & * & * \\ * & * & * \end{bmatrix} \text{ or } \begin{bmatrix} * & * & * \\ * & T & * \\ * & * & * \end{bmatrix}$
A.Touches(B)	$\begin{bmatrix} F & T & * \\ * & * & * \\ * & * & * \end{bmatrix} \text{ or } \begin{bmatrix} F & * & * \\ * & T & * \\ * & * & * \end{bmatrix} \text{ or } \begin{bmatrix} F & * & * \\ T & * & * \\ * & * & * \end{bmatrix}$
A.Crosses(B)	$\begin{bmatrix} T & * & T \\ * & * & * \\ * & * & * \end{bmatrix} \text{ or } \begin{bmatrix} 0 & * & * \\ * & * & * \\ * & * & * \end{bmatrix}$
A.Overlaps(B)	$\begin{bmatrix} T & * & T \\ * & * & * \\ T & * & * \end{bmatrix} \text{ or } \begin{bmatrix} 1 & * & T \\ * & * & * \\ T & * & * \end{bmatrix}$
A.Within(B)	$\begin{bmatrix} T & * & F \\ * & * & F \\ * & * & * \end{bmatrix}$
A.Contains(B)	$\begin{bmatrix} T & * & * \\ * & * & * \\ F & F & * \end{bmatrix}$

Table 3: Topological predicates and the corresponding pattern matrices after the Dimensionally Extended Nine-Intersection Model (DE-9IM).

With the relate method defined by (9) the pattern matrix after the DE-9IM can be determined, e.g. in PostGIS

```
SELECT RELATE(a.geom,b.geom)
      FROM country a, river b
      WHERE a.country_name='Bavaria'
      AND b.river_name='Isar';
-----
1020F1102
```

The comparison with the pattern matrices from **Table 3** shows the “Crosses”-predicate as result for the topological relationship between the country “Bavaria” and the river “Isar”.

CROSS REFERENCES

1. Geometries in Oracle Spatial (Entry 00061)
2. Microsoft Spatial Databases (Entry 00120)
3. Open Standards for GeoSpatial Interoperability (Entry 00147)
4. Standards and Spatial Database Modeling (Entry 00212)
5. Mathematical Foundations of GIS (Entry 00252)
6. Topology (Entry 261)
7. PostGIS (Entry XXX)

REFERENCES

1. Clementini E and Di Felice PA (1994): Comparison of Methods for Representing Topological Relationships.- Information Sciences 80:1-34
2. Clementini E and Di Felice PA (1996): Model for Representing Topological Relationships Between Complex Geometric Features in Spatial Databases.- Information Sciences 90(1-4):121-136
3. Davis M and Aquino J (2003): JTS Topology Suite - Technical Specifications.- Vivid Solutions Victoria, British Columbia
4. Egenhofer M, Sharma J and Mark D (1993): A Critical Comparison of the 4-Intersection and 9-Intersection Models for Spatial Relations: Formal Analysis.- In: McMaster R, Armstrong M (eds) Proceedings of AutoCarto 11 Minneapolis
5. Egenhofer MF and Franzosa R (1991): Point Set Topological Spatial Relations.- International Journal of Geographical Information Systems 5(2):161-174
6. Egenhofer MJ, Clementini E and di Felice PA (1994): Topological relations between regions with holes.- International Journal of Geographical Information Systems 8(2):129-142
7. Egenhofer MJ and Herring J (1990): A mathematical framework for the definition of topological relationships.- Fourth International Symposium on Spatial Data Handling Zürich, Switzerland, pp 803-813
8. ISO/TC211 (ed) (2003): ISO 19107: Geographic information — Spatial schema.-
9. OGC (ed) (1999): OpenGIS® Simple Features Specification for SQL (Revision 1.1).- OGC getr. Zähl.