Projectile Motion with and without Drag

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Abstract

This report investigates the motion of a projectile under the influence of gravity, both with and without aerodynamic drag. The effects of quadratic drag on the trajectory are analyzed and compared to the idealized no-drag case. Numerical simulations are performed using MATLAB, and the results are visualized through trajectory plots.

1 Introduction

Projectile motion is a classical problem in mechanics. In the simplest case, a projectile moves under gravity alone. However, in real-world scenarios, air resistance (drag) significantly affects the trajectory. This report compares the no-drag and quadratic-drag cases, highlighting the differences in range and flight time.

2 Theory

2.1 Projectile Motion without Drag

For a projectile launched with initial speed V_0 at an angle θ to the horizontal, under gravity g, the equations of motion are:

$$x(t) = V_0 \cos\theta \, t \tag{1}$$

$$y(t) = V_0 \sin\theta \, t - \frac{1}{2} g t^2 \tag{2}$$

The time of flight is:

$$T_{\text{no-drag}} = \frac{2V_0 \sin\theta}{g}$$

2.2 Projectile Motion with Quadratic Drag

Quadratic drag force is proportional to the square of the velocity:

$$\vec{F}_D = -\frac{1}{2}\rho C_D A \, v^2 \, \hat{v}$$

where:

- ρ = air density
- C_D = drag coefficient
- A = cross-sectional area
- v = instantaneous speed
- $\hat{v}$ = unit vector of velocity

The equations of motion become coupled ODEs:

$$\frac{dv_x}{dt} = -\frac{F_D}{m}\frac{v_x}{v} \tag{3}$$
$$\frac{dv_y}{dt} = -g - \frac{F_D}{m}\frac{v_y}{v} \tag{4}$$

where $v = \sqrt{v_x^2 + v_y^2}$.

3 Methodology

Numerical integration is performed using MATLAB's `ode45` solver with an event function to stop the simulation when the projectile hits the ground ($y = 0$). The initial conditions are:

- $V_0 = 50\,\mathrm{m/s}$
- Launch angle $\theta = 35°$
- Mass $m = 0.2\,\mathrm{kg}$
- Drag coefficient $C_D = 0.1$
- Air density $\rho = 1.225\,\mathrm{kg/m^3}$
- Projectile radius $r = 0.03\,\mathrm{m}$

The no-drag trajectory is computed analytically using the standard projectile equations.

4 Results

Figure 1 shows the trajectories with and without drag.

4.1 Observations

- Maximum range decreases from 238 m (no drag) to 146 m (with quadratic drag).
- Maximum height is also reduced.
- Flight time slightly decreases due to drag.

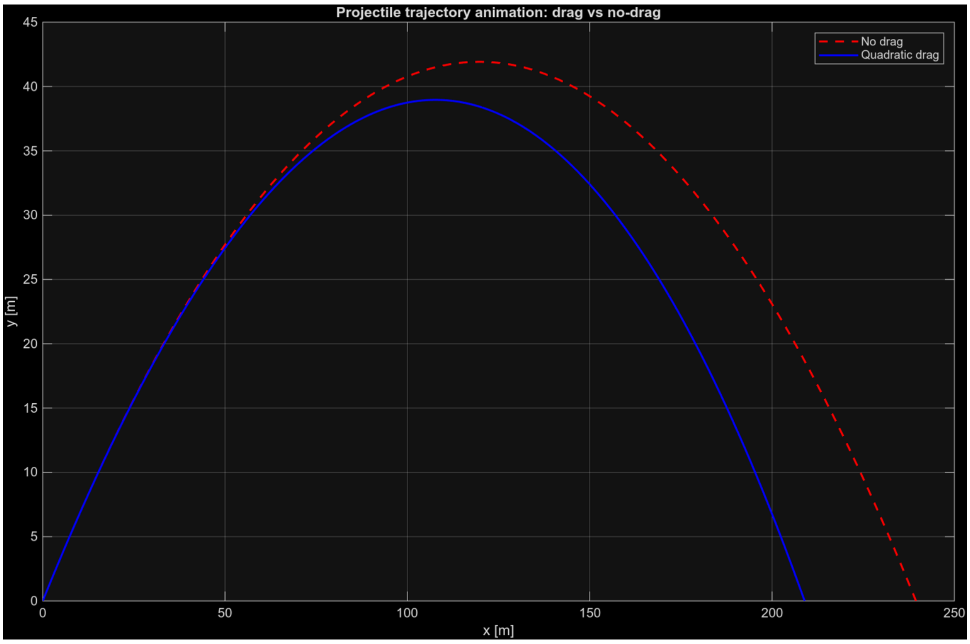

Figure 1: Comparison of projectile trajectories: red dashed = no drag, blue solid = quadratic drag. Drag reduces the range and maximum height.

5 Discussion

The results highlight the importance of drag in real-world projectile motion. Even a modest drag coefficient ($C_D = 0.1$) significantly shortens the range and reduces height. This demonstrates why neglecting air resistance in practical applications can lead to overestimations.

6 Conclusion

Numerical simulations confirm the theoretical expectations: drag reduces the projectile's range and height. The MATLAB simulation provides a flexible tool to visualize and quantify these effects.

7 References

- Morin, D., *Introduction to Classical Mechanics*, Cambridge University Press, 2008.
- Kreyszig, E., *Advanced Engineering Mathematics*, 10th Edition, Wiley, 2011.