

Smoothing:

$$P(X_k | e_{1:t}) = \frac{P(X_k, e_{1:t})}{P(e_{1:t})} = \alpha P(X_k, e_{1:t})$$

$$= \alpha P(X_k, e_{1:k}, e_{k+1:t}) = \alpha P(e_{k+1:t} | X_k, e_{1:k})$$

$$P(X_k, e_{1:k}) = \alpha P(e_{k+1:t} | X_k) \cdot P(X_k | e_{1:k})$$

$$P(e_{1:k}) = \alpha' \underbrace{P(e_{k+1:t} | X_k)}_{\text{need to elaborate}} \cdot \underbrace{P(X_k | e_{1:k})}_{\text{filtering}}$$

need to elaborate
(recursion formula)

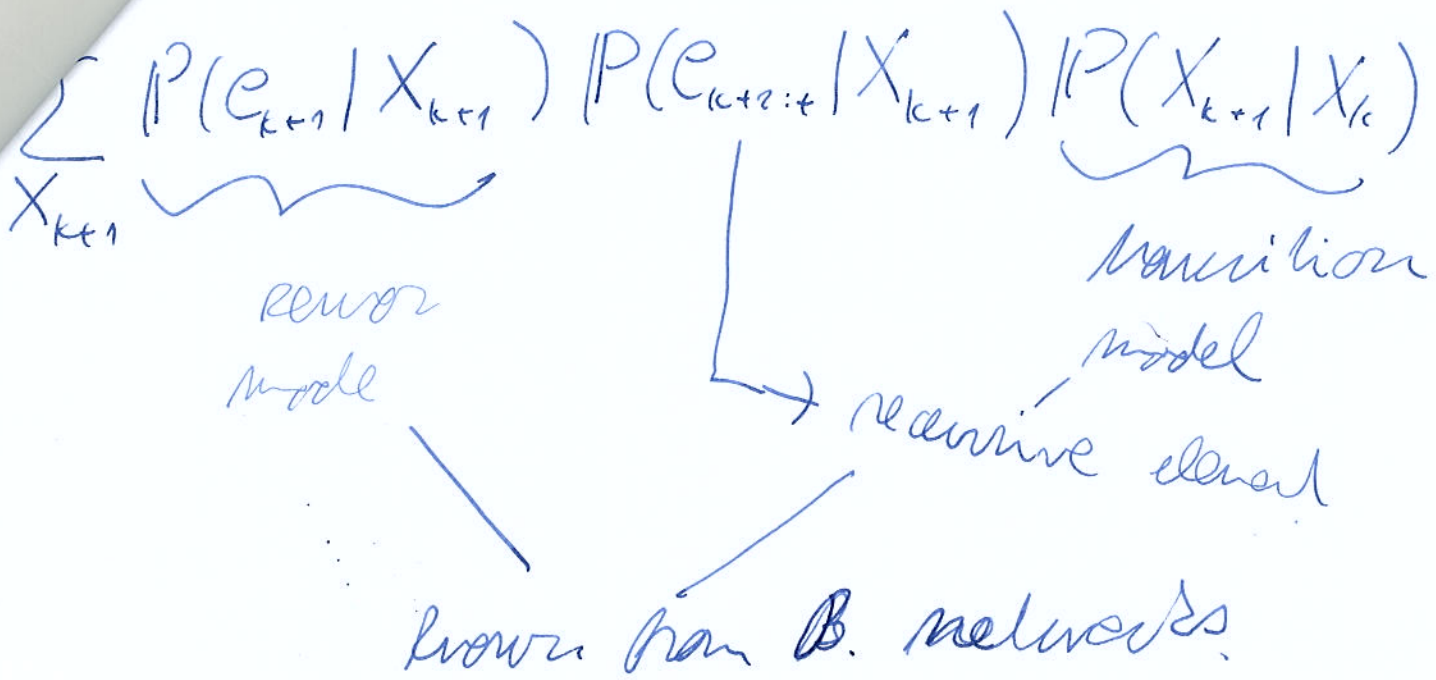
Recursive element: $P(e_{k+1:t} | X_k) = \frac{P(e_{k+1:t}, X_k)}{P(X_k)}$

$$= \frac{\sum_{X_{k+1}} P(e_{k+1:t}, X_k, X_{k+1})}{P(X_k)} = \frac{\sum_{X_{k+1}} P(e_{k+1:t} | X_k, X_{k+1}) P(X_{k+1} | X_k)}{P(X_k)}$$

$$= \frac{\sum_{X_{k+1}} P(e_{k+1:t} | X_{k+1}) P(X_{k+1} | X_k) P(X_k)}{P(X_k)} =$$

$$(e_{k+1}, e_{k+2} : t | X_{k+1}) P(X_{k+1} | X_k) = (2)$$

prob. variables



Smoothing in the Umbrella world:

$$P(R_1 | u_1, u_2) = \alpha' \underbrace{P(R_1 | u_1)}_{\text{billing}} \cdot \underbrace{P(u_2 | R_1)}_{\text{carrier process}}$$

$$\langle 0.818, 0.182 \rangle$$

$$R_1 = r_1$$

$$R_1 = \neg r_1$$

$$P(R_2) | P(R_1)$$

$$\begin{aligned} P(u_2 | R_1) &= \sum_{R_2} P(u_2 | R_2) P(R_2 | R_1) P(R_2 | R_1) = \\ &= P(u_2 | r_2) \cdot 1 \cdot P(r_2 | R_1) + P(u_2 | \neg r_2) \cdot P(\neg r_2 | R_1) \end{aligned}$$

(3)

$$\begin{aligned}
 P(u_2 | r_1) &= P(u_2 | v_2) \cdot 1 \cdot P(v_2 | r_1) + P(u_2 | \neg v_2) \cdot P(\neg v_2 | r_1) \\
 &= 0.9 \times 1 \times 0.7 + 0.2 \times 1 \times 0.3 = 0.6
 \end{aligned}$$

\uparrow ^{error} model (tbl.) \uparrow naive model

$$R_1 = \neg v_1:$$

$$\begin{aligned}
 P(u_2 | \neg v_1) &= P(u_2 | v_2) \cdot 1 \cdot P(v_2 | \neg v_1) + P(u_2 | \neg v_2) \cdot P(\neg v_2 | \neg v_1) \\
 &= 0.9 \times 1 \times 0.3 + 0.2 \times 1 \times 0.7 = 0.27 + 0.14 = \underline{\underline{0.41}}
 \end{aligned}$$

$$P(u_2 | R_1) = \langle 0.69, 0.41 \rangle \rightarrow \text{naive}$$

Unrolled sum:

$$\begin{aligned}
 P(R_1 | u_1, u_2) &= \sum P(R_1 | u_1) \cdot P(u_2 | R_1) = \\
 &= \langle \underset{R_1 = v_1}{\uparrow} 0.883, \underset{R_1 = \neg v_1}{\uparrow} 0.417 \rangle
 \end{aligned}$$