Test Three-body Problem till 13:00

V každom prípade naformulujte Vašu odpoveď celou rozvinutou vetou. Zdôvodnite Vaše tvrdenia v každom kroku. Napíšte ku každému kroku a)..d) text, ktorý interpretuje výsledky minimálne 250 znakov na každý bod. Len zdrojové kódy in, out nestačia, priložte kódy a detailne okomentovane s výsledkami. Riešte v balíku Wolfram Mathematica Cloud.

Set A: Musí byť vyriešené správne aby bola hodnotená sada B.

1. Stručne vysvetlite postup animovania (napr. postavičky) pomocou techniky „Skeleton and Skinning“. Definujte pojmy „Rigging skeleton“ a „Skinning skeleton“. Odpovedajte v slovenskom jazyku minimálne 250 znakov.

Set B: Kontrolujem iba ak je správne vyriešené **A**.
See Wikipedia if needed. Let us given the three-body problem dynamics described by following set of nine second-order ODEs (notation $\ddot{x} = \frac{d^2x(t)}{dt^2}$):

$$\ddot{\mathbf{r_1}} = -Gm_2\frac{\mathbf{r_1}-\mathbf{r_2}}{|\mathbf{r_1}-\mathbf{r_2}|^3} - Gm_3\frac{\mathbf{r_1}-\mathbf{r_3}}{|\mathbf{r_1}-\mathbf{r_3}|^3}, \tag{1}$$

$$\ddot{\mathbf{r_2}} = -Gm_3\frac{\mathbf{r_2}-\mathbf{r_3}}{|\mathbf{r_2}-\mathbf{r_3}|^3} - Gm_1\frac{\mathbf{r_2}-\mathbf{r_1}}{|\mathbf{r_2}-\mathbf{r_1}|^3}, \tag{2}$$

$$\ddot{\mathbf{r_3}} = -Gm_1\frac{\mathbf{r_3}-\mathbf{r_1}}{|\mathbf{r_3}-\mathbf{r_1}|^3} - Gm_2\frac{\mathbf{r_3}-\mathbf{r_2}}{|\mathbf{r_3}-\mathbf{r_2}|^3}, \tag{3}$$

where vector $\mathbf{r_i} = (x_i, y_i, z_i)$ are the positions of three gravitationally interacting bodies with masses m_i, $G = 6.674 \times 10^{-11}\ Nm^2/kg^2$ is the gravitational constant.

1. (10 points) Set all initial conditions according to birthday.

a) Set the initial conditions for all variables and constants, and write them out for $t = 0$. Set $x_1(0) = MM$ [m], $y_2(0) = MM$ [m], $z_3(0) = MM$ [m], $z_1(0) = DD$ [m], $x_2(0) = DD$ [m], $y_3(0) = DD$ [m], where MM represents the month and DD represents the day of your birthday. Adjust the remaining conditions as you prefer. Remember that you have nine second-order ordinary differential equations (ODEs).
b) Set all initial first and second-order derivatives and write them out for $t = 0$. Derivatives must be non zero vectors.
c) Plot the initial positions of three gravitationally interacting bodies. On X, Y, Z axis [m].

2. (30 points) Solve three-body problem dynamics by Runge–Kutta numerical integration method.
a) Set the Runge–Kutta numerical integration method in your Mathematics source code.
b) Comment the source.
c) Solve the dynamics of the three-body problem using the Runge-Kutta numerical integration method. Choose a time interval that allows for clear visibility of the movement curves.
d) Create a 3D plot in XYZ coordinates depicting the changing positions of three interacting bodies over time [s] for a duration of at least 7 seconds. Choose a time interval that allows for clear visibility of the movement curves.
e) Conduct a long-time simulation. Generate a 3D plot in XYZ coordinates illustrating the evolving positions of three interacting bodies over time. Provide commentary on the solution, indicating whether it exhibits periodic or chaotic behavior.

3. (30 points) Special-case solutions, the Pythagorean three-body problem.
a) Set the initial conditions for all variables and constants so that three masses are placed at rest at the vertices of a right triangle with an edge ratio of 3:4:5, and with mass ratios of 3:4:5. Write out the initial conditions for $t = 0$.
b) Conduct a long-time simulation. Generate a 3D plot in XYZ coordinates illustrating the evolving positions of three interacting bodies over time. Provide commentary on the solution, indicating whether it exhibits periodic or chaotic behavior.
c) Set the initial conditions as in 3a), but introduce small perturbations (small noise errors).
d) Conduct a long-time simulation. Generate a 3D plot in XYZ coordinates illustrating the evolving positions of three interacting bodies over time. Provide commentary on the solution, indicating whether it exhibits periodic or chaotic behavior.
e) Determine the range of small perturbations in mass and parameters for which numerical simulations have demonstrated stability in the solution against small perturbations of mass and orbital parameters.

4. (30 points) Find the other configuration of masses and positions to form the periodic solutions.
a) Identify another configuration of masses and positions to form periodic solutions, and write them out for $t = 0$. You are free to consult Wikipedia and other media.
b) Conduct a long-time simulation. Generate a 3D plot in XYZ coordinates illustrating the evolving positions of three interacting bodies over time. Pro-

vide commentary on the solution, indicating whether it exhibits periodic or chaotic behavior.