Homework Assignment

October 16, 2023

1 Problem 1

We have the PD controller for the hypothetical electric-motor. We control the input voltage $V(t)$ for the motor and measure the torque $\tau(t)$ of the motor. $K_m = 0.xNm/V$ is the motor torque constant. Variable x after the decimal separator is computed using the equation $x = d \times m$, where d is the number of the day and m is the number of the month in your date of birth. $K_p = 0.1$ is the proportional gain of the PD controller. $K_d = 0.01$ is the derivative gain of the PD controller. Let $A_p = K_p$ and $A_d = K_d$.

The error between the process variable and the setpoint s_p is

$$e(t) = s_p - \tau(t). \tag{1}$$

The equation for a PD controller is as follows:

$$V(t) = A_p e(t) + A_d \frac{de(t)}{dt}. \tag{2}$$

The differential equation for the closed-loop transfer function is given by:

$$\frac{d\tau(t)}{dt} = -K_p \tau(t) - K_d \frac{d\tau(t)}{dt} + K_m V(t). \tag{3}$$

1. (15 points) Derive the zero initial conditions for all variables.
a) Derive $V(0)$, $e(0)$ and set setpoint $s_p = 100Nm$. Initial torque is $\tau(0) = 0Nm$.
b) Draw the plot of input voltage. On X axis time [s] and on Y axis input voltage [V].
c) Draw the plot of torque. On X axis time [s] and on Y axis torque [Nm].

2. (35 points) Test your motor.
a) Find out the voltage at the time $t = 10s$.
b) Does the motor reach the setpoint torque $s_p = 100Nm$? If does not, what is the maximum torque?
c) At what time t does it reach the maximum torque?
d) Find out the voltage and torque at the time $t = 1hour$.

3. (50 points) Find the coefficients K_p, K_d and initial conditions such that the motor torque is the periodic (or chaotic) function for the entire time interval $< 0, 1 >$ sec.
a) For the periodic function. Draw the plot of input voltage. On X axis time [s] and on Y axis input voltage [V].
b) Draw the plot of torque. On X axis time [s] and on Y axis torque [Nm].
e) For the chaotic function. Draw the plot of input voltage. On X axis time [s] and on Y axis input voltage [V].
f) For the chaotic function. Draw the plot of torque. On X axis time [s] and on Y axis torque [Nm].

g) Have two solutions of the problem using zero initial contidions and $\tau_1(0) = 0.000000001$ a $\tau_2(0) = 0.000000002$. Calculate the difference $d_n = \tau_2(10000) - \tau_1(10000)$. Calculate the coefficient λ using $d_n = 0.000000001e^{10000\lambda}$. λ and derive the chaotic sensitivity that is called Lyapunov exponent.