Tarefa Básica - Regra de Cramer

1-a) $\begin{cases} 2x - y = 2 \\ -x + 3y = -3 \end{cases}$ $\quad D = \begin{vmatrix} 2 & -1 \\ -1 & 3 \end{vmatrix} = 6 - 1 = 5$

$D_x = \begin{vmatrix} 2 & -1 \\ -3 & 3 \end{vmatrix} = 6 - 3 = 3$ $\quad x = \frac{3}{5}$ $\quad y = \frac{-4}{5}$

$D_y = \begin{vmatrix} 2 & 2 \\ -1 & -3 \end{vmatrix} = -6 + 2 = -4$ $\quad V = \left\{ \left(\frac{3}{5}, \frac{-4}{5} \right) \right\}$

b) $\begin{cases} 3x - y + z = 1 \\ 2x + 3z = -1 \\ 4x + y - 2z = 7 \end{cases}$

$$D = \begin{vmatrix} 3 & -1 & 1 \\ 2 & 0 & 3 \\ 4 & 1 & -2 \end{vmatrix} \begin{matrix} 3 & -1 \\ 2 & 0 \\ 4 & 1 \end{matrix}$$

$0 \quad 9 \quad +4$

$0 + -12 + 2$

$D = -10 - 13 = -23$

$$D_x = \begin{vmatrix} 1 & -1 & 1 \\ -1 & 0 & 3 \\ 7 & 1 & -2 \end{vmatrix} \begin{matrix} 1 & -1 \\ -1 & 0 \\ 7 & 1 \end{matrix}$$

$0 \quad -21 \quad -1$

$D_x = -22 - 1 = -23$

$$D_y = \begin{vmatrix} 3 & 1 & 1 \\ 2 & -1 & 3 \\ 4 & 7 & -2 \end{vmatrix} \begin{matrix} 3 & 1 \\ 2 & -1 \\ 4 & 7 \end{matrix}$$

$6 \quad 12 \quad 14$

$D_y = 32 - 55 = -23$

$$D_z = \begin{vmatrix} 3 & -1 & 1 \\ 2 & 0 & -1 \\ 4 & 1 & 7 \end{vmatrix} \begin{matrix} 3 & -1 \\ 2 & 0 \\ 4 & 1 \end{matrix}$$

$0 \quad 4 \quad 2$

$D_z = 6 + 17 = 23$

$x = \frac{-23}{-23} = 1 \qquad y = \frac{-23}{-23} = 1$

$z = \frac{23}{-23} = -1$

$V = \{(1, 1, -1)\}$

2-) $\begin{cases} 3x + 4y - z = 1 \\ 4x + 5y + 2z = 12 \\ x - 2y + 3z = 8 \end{cases}$

$$D = \begin{vmatrix} 3 & 4 & -1 \\ 4 & 5 & 2 \\ 1 & -2 & 3 \end{vmatrix} \begin{matrix} 3 & 4 \\ 4 & 5 \\ 1 & -2 \end{matrix}$$

$-5 \quad -12 \quad 48$

$45 \quad 8 \quad -8$

$D = 61 - 31 = 30$

$$D_y = \begin{vmatrix} 3 & 1 & -1 \\ 4 & 12 & 2 \\ 1 & 8 & 3 \end{vmatrix} \begin{matrix} 3 & 1 \\ 4 & 12 \\ 1 & 8 \end{matrix}$$

$-12 \quad 48 \quad 12$

$108 \quad 2 \quad -32$

$D_y = 78 - 48 = 30$

$y = \frac{30}{30} = 1$

(A)

3-) $\begin{cases} x + 2y + z = 1 \\ 3x + y - 11z = -2 \\ 2x + 3y - z = 1 \end{cases}$

$$D = \begin{vmatrix} 1 & 2 & 1 \\ 3 & 1 & -11 \\ 2 & 3 & -1 \end{vmatrix} \begin{matrix} 1 & 2 \\ 3 & 1 \\ 2 & 3 \end{matrix}$$

$2 \quad -33 \quad 6$

$-1 \quad -44 \quad 3$

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$D = -36 + 37 = 1$

$$D_x = \begin{vmatrix} 1 & 2 & 1 \\ -2 & 1 & -11 \\ 1 & 3 & -1 \end{vmatrix} \begin{matrix} 1 \\ -33 \\ -1 \end{matrix} \begin{matrix} 4 \\ 2 \\ -22 \end{matrix} \begin{matrix} \\ \\ -6 \end{matrix}$$

$D_x = -29 + 28 = -1$

$x = -1 \qquad -1 + 1 + 0$

$y = 1 \qquad 0$

$z = 0$ (C)

$$D_y = \begin{vmatrix} 1 & 1 & 1 \\ 3 & -2 & -11 \\ 2 & 1 & -1 \end{vmatrix} \begin{matrix} -4 \\ -11 \\ 2 \end{matrix} \begin{matrix} -3 \\ 1 \\ -22 \end{matrix} \begin{matrix} \\ \\ 3 \end{matrix}$$

$D_y = -17 + 18 = 1$

$$D_z = \begin{vmatrix} 1 & 2 & 1 \\ 3 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix} \begin{matrix} 2 \\ -6 \\ 1 \end{matrix} \begin{matrix} 6 \\ 2 \\ -8 \end{matrix} \begin{matrix} \\ \\ 9 \end{matrix}$$

$D_z = 2 - 2 = 0$

4-) $\begin{cases} x + 2y - 3z = 29 \\ x + 3y + 2z = 4 \\ x - y - 2z = 8 \end{cases}$

$$D = \begin{vmatrix} 1 & 2 & -3 \\ 1 & 3 & 2 \\ 1 & -1 & -2 \end{vmatrix} \begin{matrix} -9 \\ 1 \\ -6 \end{matrix} \begin{matrix} -2 \\ 1 \\ 4 \end{matrix} \begin{matrix} -4 \\ 1 \\ 3 \end{matrix}$$

$D = 1 + 15 = 16$

$$D_x = \begin{vmatrix} 29 & 2 & -3 \\ 4 & 3 & 2 \\ 8 & -1 & -2 \end{vmatrix} \begin{matrix} -72 \\ 29 \\ -174 \end{matrix} \begin{matrix} -58 \\ 4 \\ 32 \end{matrix} \begin{matrix} -16 \\ 2 \\ 12 \end{matrix}$$

$D_x = -130 + 146 = 16$

$$D_y = \begin{vmatrix} 1 & 29 & -3 \\ 1 & 4 & 2 \\ 1 & 8 & -2 \end{vmatrix} \begin{matrix} -12 \\ 1 \\ -8 \end{matrix} \begin{matrix} 16 \\ 29 \\ 58 \end{matrix} \begin{matrix} -58 \\ 1 \\ -24 \end{matrix}$$

$D_y = 26 + 54 = 80$

$$D_z = \begin{vmatrix} 1 & 2 & 29 \\ 1 & 3 & 4 \\ 1 & -1 & 8 \end{vmatrix} \begin{matrix} 87 \\ 1 \\ 24 \end{matrix} \begin{matrix} -4 \\ 1 \\ 8 \end{matrix} \begin{matrix} 16 \\ 2 \\ -29 \end{matrix}$$

$D_z = 3 - 99 = -96$

$x = 1 \qquad y = 5 \qquad z = -6$

$1 + 5 + (-6) = 0$

(A)

5-) $\begin{cases} 2x + y = 5 \\ 2y + z = 3 \\ 3x + 2y + z = 7 \end{cases}$

$$D = \begin{vmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 3 & 2 & 1 \end{vmatrix} \begin{matrix} 2 & 1 \\ 0 & 2 \\ 3 & 2 \end{matrix} \quad \begin{matrix} 4 & 0 \\ 4 & 3 & 0 \end{matrix}$$

$$D_x = \begin{vmatrix} 5 & 1 & 0 \\ 3 & 2 & 1 \\ 7 & 2 & 1 \end{vmatrix} \begin{matrix} 5 & 1 \\ 3 & 2 \\ 7 & 2 \end{matrix} \quad \begin{matrix} 10 & 3 \\ 10 & 7 & 0 \end{matrix}$$

$D = 7 - 4 = 3$

$D_x = 17 - 13 = 4$

$x = \frac{4}{3}$

$2 \cdot \frac{4}{3} + y = 5$

$y = 5 - \frac{8}{3}$

$y = \frac{7}{3}$

$2 \cdot \frac{7}{3} + z = 3$

$\frac{14}{3} + z = 3$

$z = 3 - \frac{14}{3}$

$z = \frac{-5}{3}$

(D)

6-)

$$\begin{matrix} x \\ y \\ z \end{matrix}$$

$$\begin{vmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 2 & 2 \end{vmatrix} \begin{matrix} x \\ 2x + y \\ -x + 2y + 2z \end{matrix}$$

$x = 3$

$2 \cdot 3 + y = 7$

$y = 1$

$-3 + 2 + 2z = -1$

$z = 0$

(E)

Tarefa básica - Escalonamento (Gauss)

1-) $S = \begin{cases} 2x - y - 3z = -5 \\ x + 3y - z = 11 \\ x - 5z = 3 \end{cases}$ $V = \{(-2, 4, -1)\}$

$$\begin{pmatrix} 2 & -1 & -3 & -5 \\ 1 & 3 & -1 & 11 \\ 1 & 0 & -5 & 3 \end{pmatrix} \sim \begin{pmatrix} 0 & -1 & 7 & -11 \\ 0 & 3 & 4 & 8 \end{pmatrix} \sim \begin{pmatrix} 0 & 0 & 25 & -25 \\ & & & \end{pmatrix}$$

$x - 5 \cdot -1 = 3$ $\quad -2 + 3y + 1 = 11$ $\quad 25z = -25$

$x = -2$ $\quad 3y = 12$ $\quad z = -1$

$y = 4$

2-) $\begin{cases} x = 2y \\ 2y = 3z \\ x + y + z = 11 \end{cases}$ $\quad \frac{3y + 2y}{3} = 11$ $\quad x = 2 \cdot 3$

$\frac{11y}{3} = 11$ $\quad x = 6$

$2y + y + z = 11$ $\quad y = 3$ $\quad 6 = 3z$

$3y + z = 11$ $\quad z = 2$

$x + 2y + 3z$

$6 + 6 + 6$ $\quad$ (B)

18

3-) $\begin{cases} x + y + z = 0 \\ 2x - y - 2z = 1 \\ 6y + 3z = -12 \end{cases}$ $\quad -5z = -10$

$z = 2$ $\quad$ (D)

$$\begin{pmatrix} 1 & 1 & 1 & 0 \\ 2 & -1 & -2 & 1 \\ 0 & 6 & 3 & -12 \end{pmatrix} \sim \begin{pmatrix} 0 & -3 & -4 & 1 \\ 0 & 6 & 3 & -12 \end{pmatrix} \sim \begin{pmatrix} & & & \\ 0 & 0 & -5 & -10 \end{pmatrix}$$

4-) $\begin{cases} A+B+C=68 \\ B+0,2C=A \\ C+0,2A=3B \end{cases} \Rightarrow \begin{cases} A+B+C=68 \\ -5A+5B+C=0 \\ A-15B+5C=0 \end{cases}$

$B+\frac{20}{100}C=A \Rightarrow B+\frac{C}{5}=A \Rightarrow 5B+C=5A \Rightarrow$

$5B+C-5A=0$

$C+\frac{20}{100}A=3B \Rightarrow C+\frac{A}{5}=3B \Rightarrow 5C+A-15B=0$

$$\begin{pmatrix} 1 & 1 & 1 & 68 \\ -5 & 5 & 1 & 0 \\ 1 & -15 & 5 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 & 68 \\ 0 & 10 & 6 & 340 \\ 0 & -16 & 4 & -68 \end{pmatrix} \sim \begin{pmatrix} & & & \\ & & & \\ 0 & 0 & 68 & 2380 \end{pmatrix}$$

$A+13+35=68$ $\quad 10B+6.35=340$ $\quad 68C=2380$

$A=68-48$ $\quad 10B=340-210$ $\quad C=35$

$A=20$ $\quad B=\frac{130}{10}$

$B=13$ $\quad$ (A)

5-)

x	y	z	
0	3	4	$3y+4z$
1	0	5	$x+5z$
2	1	0	$2x+y$

$\begin{cases} 3y+4z=134 \\ x+5z=115 \\ 2x+y=48 \end{cases}$

$$\begin{pmatrix} 0 & 3 & 4 & 134 \\ 1 & 0 & 5 & 115 \\ 2 & 1 & 0 & 48 \end{pmatrix} \sim \begin{pmatrix} 0 & 3 & 4 & 134 \\ 0 & 1 & -10 & -182 \\ & & & \end{pmatrix} \sim \begin{pmatrix} 0 & 0 & 34 & 680 \\ & & & \\ & & & \end{pmatrix}$$

-2 $\quad$ -3

$2x + 18 = 48$

$2x = 30$

$x = 15$

$y - 10.20 = -182$

$y = 18$

$34z = 680$

$z = 20$

$15 + 18 + 20 = 53$

R$ 53,00 (A)