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Parcial 1: Señales y Sistemas.

1. La distancia media entre 2 señales periódicas $x_1(t) \in \mathbb{R}, \mathbb{C}$ y $x_2(t) \in \mathbb{R}, \mathbb{C}$; se puede expresar a partir de la potencia media de la diferencia entre ellas:

$$d^2(x_1, x_2) = \bar{P}_{x_1 - x_2} = \lim_{T \to \infty} \frac{1}{T} \int_T |x_1(t) - x_2(t)|^2 \, dt$$

Sea $x_1(t)$ y $x_2(t)$ 2 señales definidas como:

$$x_1(t) = Ae^{jn\omega_0 t} \qquad x_2(t) = Be^{jm\omega_0 t}$$

Con $\omega_0 = 2\pi/T$; $T, A, B \in \mathbb{R}^+$ y $n, m \in \mathbb{Z}$. Determine la distancia entre las 2 señales. Compruebe sus resultados con Python.

$$\bar{P}_{x_1 - x_2} = \frac{1}{T} \int_T |x_1(t) - x_2(t)|^2 \, dt = \frac{1}{T} \left(\int_T (x_1(t) - x_2(t))(x_1(t) - x_2(t))^* \right) dt$$

$$= \frac{1}{T} \left(\int_T (x_1(t) - x_2(t))(x_1^*(t) - x_2^*(t)) \, dt \right.$$

$$= \frac{1}{T} \left(\int_T x_1(t) x_1^*(t) \, dt - \int_T x_1(t) x_2^*(t) \, dt - \int_T x_2(t) x_1^*(t) \, dt + \int_T x_2(t) x_2^*(t) \, dt \right)$$

$$= \underbrace{\frac{1}{T} \int_T |x_1(t)|^2 \, dt}_{\bar{P}_{x_1}} - \underbrace{\frac{2}{T} \int_T x_1(t) x_2^*(t) \, dt}_{C_{12}} + \underbrace{\frac{1}{T} \int_T |x_2(t)|^2 \, dt}_{\bar{P}_{x_2}}.$$

$$\bar{P}_{x_1} = \frac{1}{T} \int_T x_1(t) x_1^*(t) \, dt = \frac{1}{T} \int_0^T Ae^{-jn\omega_0 t} (Ae^{jn\omega_0 t}) \, dt.$$

$$= \frac{A^2}{T} \int_0^T \underbrace{e^{j(n-n)\omega_0 t}}_{1} \, dt = \frac{A^2}{T} \int_0^T dt = \frac{A^2}{T} t \Big|_0^T = \frac{A^2}{T}(T) = A^2$$

$$\bar{P}_{x_2} = \frac{1}{T} \int_T x_2(t) x_2^*(t) \, dt = \frac{1}{T} \int_0^T Be^{jm\omega_0 t} (Be^{-jm\omega_0 t}) \, dt.$$

$$= \frac{B^2}{T} \int_0^T \underbrace{e^{j(m-m)\omega_0 t}}_{1} \, dt = \frac{B^2}{T} \int_0^T dt = \frac{B^2}{T} t \Big|_0^T = \frac{B^2}{T}(T) = B^2$$

$$C_{12} = -\frac{2}{T}\int_T x_1(t)\,x_2^*(t)\,dt = -\frac{2}{T}\int_0^T (Ae^{jn\omega_0 t})(Be^{-jm\omega_0 t})\,dt$$

$$= -\frac{2AB}{T}\int_0^T e^{-jn\omega_0 t - jm\omega_0 t}\,dt = -\frac{2AB}{T}\int_0^T e^{-j\overbrace{(n+m)}^{k}\omega_0 t}\,dt$$

→ Si $n+m=0$, $n=-m$:

$$C_{12} = -\frac{2AB}{T}\int_0^T \underbrace{e^{-jk\omega_0 t^0}}_{1}\,dt = -\frac{2AB}{T}\int_0^T dt = -\frac{2AB}{T}\,t\Big|_0^T = -\frac{2AB}{\cancel{T}}\cancel{T}$$

→ Si $n+m\neq 0$, $n\neq -m$; $n+m=k\in\mathbb{Z}$:

$$C_{12} = -\frac{2AB}{T}\int_0^T e^{-jk\omega_0 t}\,dt = -\frac{2AB}{T}\left(\frac{e^{-jk\omega_0 t}}{-jk\omega_0}\right)\Big|_0^T$$

$$= \frac{2AB}{jk\omega_0 T}\left(\underbrace{e^{-jk\omega_0 T}}_{1} - \underbrace{e^{-jk\omega_0(0)}}_{1}\right)$$

$$= \cos(k\omega_0 T) - j\,\mathrm{sen}(k\omega_0 T) = 1$$

$$= \cancel{\cos(k2\pi T)} - j\,\cancel{\mathrm{sen}(k2\pi T)}$$

$$1 \quad - \quad 0$$

$$= \frac{2AB}{jk\omega_0 T}(1-1) = 0$$

NOTA: $d^2 = \overline{P}_{x_1-x_2} \rightarrow d = \sqrt{\overline{P}_{x_1-x_2}}$

$$\sqrt{\overline{P}_{x_1-x_2}} = \begin{cases} \sqrt{A^2-2AB+B^2} & ; n=m \\ \sqrt{A^2+B^2} & ; n\neq m \end{cases} \qquad \overline{P}_{x_1-x_2} = \overline{P}_{x_1} + C_{12} + \overline{P}_{x_2}$$

2. Encuentre la señal en tiempo discreto al utilizar un conversor análogo digital con frecuencia de muestreo de 5kHz y 4 bits de capacidad de representación, aplicado a la señal continua:

$x(t) = 3\cos(1000\pi t) + 5\,\mathrm{sen}(3000\pi t) + 10\cos(11000\pi t)$

Realizar la simulación del proceso de discretización (incluyendo al menos 3 períodos de $x(t)$). En caso de que la discretización no sea apropiada, diseñe e implemente un conversor adecuado para la señal estudiada.

$F_s = 5kHz$ Estados $= 2^{\#bits} = 2^4 = 16$ niveles de cuantización

$t = nT_s \rightarrow F_s = \frac{1}{T_s}$; $F = \frac{1}{T}$; $T = \frac{2\pi}{\omega}$

$A\cos(\Omega n) = A\cos(2\pi f n) \rightarrow \Omega = 2\pi f \rightarrow f = \frac{T_s}{T} = \frac{F}{F_s}$

$A\,\mathrm{sen}(\Omega n) = A\,\mathrm{sen}(2\pi f n) \rightarrow \Omega = 2\pi f \rightarrow f = \frac{T_s}{T} = \frac{F}{F_s}$

Para $x_1(t) = 3\cos(1000\pi t)$

$\omega_1 = 1000\pi \longrightarrow T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{1000\pi} = \frac{1}{500} \longrightarrow F_1 = \frac{1}{T_1} = \frac{1}{1/500} = 500\ Hz$

$x_1[n] = A\cos\left(2\pi n \frac{F_1}{F_s}\right) = 3\cos\left(2\pi n \frac{500}{5000}\right) = 3\cos\left(\frac{n\pi}{5}\right)$

Para $x_2(t) \rightarrow 5\operatorname{sen}(3000\pi t)$

$\omega_2 = 3000\pi \longrightarrow T_2 = \frac{2\pi}{\omega_2} = \frac{2\pi}{3000\pi} = \frac{1}{1500} \longrightarrow F_2 = \frac{1}{T_2} = \frac{1}{1/1500} = 1500\ Hz$

$x_2[n] = A\operatorname{sen}\left(2\pi n \frac{F_2}{F_s}\right) = 5\operatorname{sen}\left(2\pi n \frac{1500}{5000}\right) = 3\operatorname{sen}\left(\frac{3n\pi}{5}\right)$

Para $x_3(t) \rightarrow 10\cos(11000\pi t)$

$\omega_3 = 11000\pi \longrightarrow T_3 = \frac{2\pi}{11000\pi} = \frac{1}{5500} \longrightarrow F_3 = \frac{1}{T_3} = \frac{1}{1/5500} = 5500\ Hz$

$x_3[n] = A\cos\left(2\pi n \frac{F_3}{F_s}\right) = 10\cos\left(2\pi n \frac{5500}{5000}\right) = 10\cos\left(\frac{11n\pi}{5}\right)$

$x[n] = 3\cos\left(\frac{n\pi}{5}\right) + 3\operatorname{sen}\left(\frac{3n\pi}{5}\right) + 10\cos\left(\frac{11n\pi}{5}\right)$

$\frac{\omega_1}{\omega_2} = \frac{1000\pi}{3000\pi} = \frac{1}{3} \in \mathbb{Q}$ $\quad \frac{\omega_2}{\omega_3} = \frac{3000\pi}{11000\pi} = \frac{3}{11} \in \mathbb{Q}$

$\frac{\omega_1}{\omega_3} = \frac{1000\pi}{11000\pi} = \frac{1}{11} \in \mathbb{Q}$ $\quad \therefore$ la señal es cuasiperiódica

NO cumple Nyquist.

Si $F_s \geq 2F_{max} \longrightarrow 2F_{max} = 2(5500\ Hz) = 11000\ Hz$ pero $F_s = 5000\ Hz$

comparando con las originales $\longrightarrow -\pi \leq \Omega < 4\pi$ copia o aliasing

$\Omega_1 = \frac{1}{5}\pi$ cumple, $\Omega_2 = \frac{3}{5}\pi$ cumple, $\Omega_3 = \frac{11}{5}\pi$ no cumple $> \pi$

$\Omega_{3\,original} = \frac{11}{5}\pi - 2\pi = \frac{1}{5}\pi$ ahora sí cumple.

Se supone una frecuencia de muestreo más grande

$F_s = 3F_{max} = 3(5500\,Hz) = 16500\,Hz \longrightarrow T_s = \frac{1}{F_s} = \frac{1}{16500}$

Para $x_1(t) = 3\cos(1000\pi t)$

$x_1[n] = 3\cos\left(2\pi n \frac{500}{16500}\right) = 3\cos\left(\frac{2n\pi}{33}\right) \quad \Omega_1 = \frac{2\pi}{33} \approx 0,06\pi$

Para $x_2(t) = 5\,sen(3000\pi t)$

$x_2[n] = 5\,sen\left(2\pi n \frac{1500}{16500}\right) = 5\,sen\left(\frac{2n\pi}{11}\right) \quad \Omega_2 = \frac{2\pi}{11} \approx 0,18\pi$

Para $x_3(t) = 10\cos(11000\pi t)$

$x_3[n] = 10\cos\left(2\pi n \frac{5500}{16500}\right) = 10\cos\left(\frac{2n\pi}{3}\right) \quad \Omega_3 = \frac{2\pi}{3} \approx 0,67\pi$

$-\pi \leq \Omega_1, \Omega_2, \Omega_3 < \pi$ → las 3 frecuencias digitales están en los originales.

$x[n] = 3\cos\left(\frac{2n\pi}{33}\right) + 5\,sen\left(\frac{2n\pi}{11}\right) + 10\cos\left(\frac{2n\pi}{3}\right)$

$\omega_1 = 1000\pi$
$\omega_2 = 3000\pi$
$\omega_3 = 11000\pi$

Hallamos el período T para graficar las señales:

$T = \frac{r2\pi}{\omega_1} = \frac{l2\pi}{\omega_2} = \frac{k2\pi}{\omega_3} \longrightarrow T = \frac{r2\pi}{1000\pi} = \frac{l2\pi}{3000\pi} = \frac{k2\pi}{11000\pi}$

$T = \frac{r}{500} = \frac{l}{1500} = \frac{k}{5500} \longrightarrow 16500\,T = 33r = 11l = 3k$

$MCM\{33, 11, 3\} = 33 \longrightarrow r = 1, \; l = 3, \; k = 11.$

$16500\,T = 33 \longrightarrow T = \frac{33}{16500} = \frac{1}{500}$

3. Sea $x''(t)$ la segunda derivada de la señal $x(t)$, donde $t \in [t_i, t_f]$. Demuestre que los coeficientes de la serie exponencial de Fourier se pueden calcular según:

$$c_n = \frac{1}{(t_i - t_f)\, n^2 w_0^2} \int_{t_i}^{t_f} x''(t)\, e^{-jnw_0 t}\, dt \; ; \; n \in \mathbb{Z}.$$

¿Cómo se pueden calcular los coeficientes a_n y b_n desde $x''(t)$ en la serie trigonométrica de Fourier?

Sabemos que $x(t) = \sum_n c_n e^{jnw_0 t}$

$$\rightarrow x'(t) = \frac{d}{dt} \sum_n c_n e^{jnw_0 t} = \sum_n c_n \boxed{\frac{d}{dt} e^{jnw_0 t}} \longrightarrow \frac{d}{dt} e^{jnw_0 t} = jnw_0 e^{jnw_0 t}$$

$$\rightarrow x''(t) = \frac{d}{dt} x'(t) = \frac{d}{dt} \sum_n c_n \frac{d e^{jnw_0 t}}{dt} = \sum_n c_n \boxed{\frac{d^2 e^{jnw_0 t}}{dt^2}}$$

$$\frac{d^2}{dt^2} e^{jnw_0 t} = \frac{d}{dt} jnw_0 e^{jnw_0 t} = (jnw_0)^2 e^{jnw_0 t} = j^2 n^2 w_0^2 e^{jnw_0 t} = -n^2 w_0^2 e^{jnw_0 t}$$

$$x''(t) = \sum_n \underbrace{-C_n n^2 w_0^2}_{\tilde{C}_n} e^{jnw_0t} = \sum_n \tilde{C}_n e^{jnw_0t} \quad ; \text{ con } \tilde{C}_n = -C_n n^2 w_0^2$$

Si $C_n = \frac{1}{T}\int_T x(t) e^{-jnw_0t}\,dt$ y $\tilde{C}_n = \frac{1}{T}\int_T x''(t) e^{-jnw_0t}\,dt$

Se reemplaza el $\tilde{C}_n \rightarrow C_n n^2 w_0^2 = \frac{1}{T}\int_T x''(t) e^{-jnw_0t}\,dt$.

$$C_n = \frac{1}{-(t_f - t_i) n^2 w_0^2}\int_T x''(t) e^{-jnw_0t}\,dt = \boxed{\frac{1}{(t_i - t_f) n^2 w_0^2}\int_T x''(t) e^{-jnw_0t}\,dt}$$

$$C_n = \frac{1}{(t_i - t_f) n^2 w_0^2}\int_T x''(t) e^{-jnw_0t}\,dt = \frac{1}{(t_i - t_f) n^2 w_0^2}\int x''(t)\left[\cos(nw_0t) - j\,\text{sen}(nw_0t)\right]dt$$

$$= \frac{1}{(t_i - t_f) n^2 w_0^2}\int_T x''(t)\cos(nw_0t)\,dt - j\,\frac{1}{(t_i - t_f) n^2 w_0^2}\int_T x''(t)\,\text{sen}(nw_0t)\,dt.$$

Sabemos que $a_n = \frac{2}{T}\int_T x(t)\cos(nw_0t)\,dt$; $b_n = \frac{2}{T}\int_T x(t)\,\text{sen}(nw_0t)\,dt$.

y que $a_n = 2\,\text{Re}\{C_n\}$; $b_n = -2\,\text{Im}\{C_n\}$

$$\rightarrow a_n = 2\,\text{Re}\{C_n\} = \boxed{2\,\frac{1}{(t_i - t_f) n^2 w_0^2}\int_T x''\cos(nw_0t)\,dt}$$

$$\rightarrow b_n = -2\,\text{Im}\{C_n\} = -2\,\frac{1}{(t_i - t_f) n^2 w_0^2}\int_T x''\,\text{sen}(nw_0t)\,dt$$

O también con $x(t) = \sum_n a_n\cos(nw_0t) + b_n\,\text{sen}(nw_0t)$

$$x(t) = \sum_n a_n\cos(nw_0t) + b_n\,\text{sen}(nw_0t)$$

$$x'(t) = \sum_n -a_n\,\text{sen}(nw_0t)\,nw_0 + b_n\cos(nw_0t)\,nw_0.$$

$$x''(t) = \sum_n -a_n\cos(nw_0t)(nw_0)^2 - b_n\,\text{sen}(nw_0t)(nw_0)^2$$

$$= \sum_n \underbrace{-a_n n^2 w_0^2}_{\tilde{a}_n}\cos(nw_0t) \underbrace{- b_n n^2 w_0^2}_{\tilde{b}_n}\,\text{sen}(nw_0t)$$

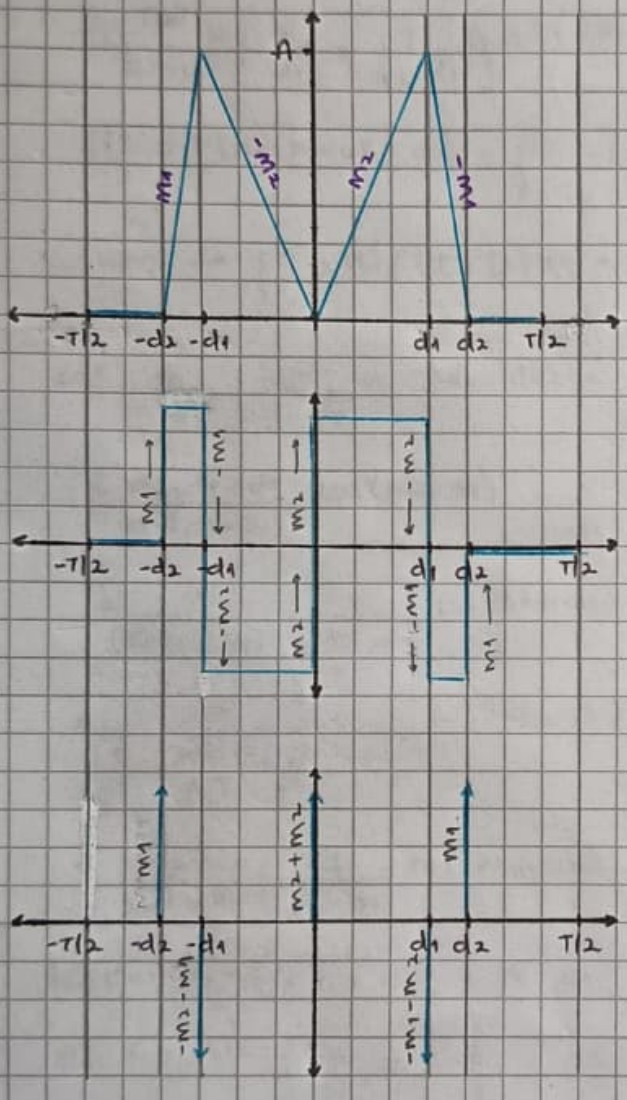
$$a_n = \frac{2}{T}\int_T x(t)\cos(nw_0t)\,dt \rightarrow \tilde{a}_n = \frac{2}{T}\int_T x''(t)\cos(nw_0t)\,dt.$$

$$-a_n n^2 w_0^2 = \frac{2}{T}\int_T x''(t)\cos(nw_0t)\,dt \rightarrow \boxed{a_n = \frac{2}{\frac{-(t_f - t_i)}{t_i - t_f} n^2 w_0^2}\int_T x''(t)\cos(nw_0t)\,dt.}$$

$$b_n = \frac{2}{T}\int_T x(t)\,\text{sen}(n\omega_0 t)\,dt \rightarrow \tilde{b_n} = \frac{2}{T}\int_T x''(t)\,\text{sen}(n\omega_0 t)\,dt$$

$$-b_n n^2\omega_0^2 = \frac{2}{T}\int_T x''(t)\,\text{sen}(n\omega_0 t)\,dt \rightarrow \boxed{b_n = \frac{2}{-(t_f-t_i)n^2\omega_0^2}\int_T x''(t)\,\text{sen}(n\omega_0 t)\,dt}$$

4. Encuentre el espectro de Fourier, su parte real, imaginaria, magnitud, fase y el error relativo para $n \in \{0, \pm1, \pm2, \pm3, \pm4, \pm5\}$, a partir de $x''(t)$ para la señal $x(t)$ en la figura. Compruebe el espectro obtenido con la estimación a partir de $x(t)$. Presente las simulaciones de Python respectivas.



$$m_1 = \frac{y_{max} - y_{min}}{x_{max} - x_{min}} = \frac{A-0}{-d_1-(-d_2)} = \frac{A}{d_2-d_1}$$

$$m_2 = \frac{y_{max} - y_{min}}{x_{max} - x_{min}} = \frac{A-0}{d_1-0} = \frac{A}{d_1}$$

$$x_1(t) = m_1 x_1 + b_1 \qquad x_2(t) = m_2 x_2 + b_2$$

$$\rightarrow b_1 = x_1(t) - m_1 x_1 \qquad \rightarrow b_2 = x_2(t) - m_2 x_2$$

$$b_1 = 0 - \frac{A}{d_2-d_1}(-d_2) \qquad b_2 = 0 - \frac{A}{d_1}(0)$$

$$b_1 = \frac{Ad_2}{d_2-d_1} \qquad b_2 = 0$$

$$x(t) = \begin{cases} 0, & d_2 \le |t| \le T/2 \\ \frac{At}{d_2-d_1} + \frac{Ad_2}{d_2-d_1} = \frac{At+Ad_2}{d_2-d_1}, & d_2 \le |t| \le d_1 \\ \frac{At}{d_1}, & 0 \le |t| \le d_1 \end{cases}$$

$x''(t) = 0$ en cada tramo y en los puntos donde hay cambios tiende a $\pm\infty$ y estos cambios los podemos representar como $m_k\delta(t+t_k)$ donde m_k es la diferencia de la derivada por la derecha y la derivada por la izquierda y t_k donde ocurre el salto (cambio instantáneo)

$$x''(t) = \frac{m_1}{m_1-d_2}\delta(t+d_2) + \frac{-m_2-m_1}{m_1-d_1}\delta(t+d_1) + \frac{2m_2}{m_0}\delta(t+0) + \frac{-m_1-m_2}{m_1 d_1}\delta(t-d_1) + \frac{m_1}{m_1 d_2}\delta(t-d_2)$$

$$x''(t) = m_1[\delta(t+d_2) - \delta(t-d_2)] - (m_1+m_2)[\delta(t+d_1) + \delta(t-d_1)] + 2m_2\delta(t+0)$$

Sabemos que $\int_{-\infty}^{\infty}\delta(t+T) = 1$

$$C_n = \frac{1}{\frac{(t_i - t_f)}{T} n^2 w_0^2} \int_T x''(t) e^{-jnw_0 t}\, dt \qquad T = t_f - t_i = \frac{T}{2} - \left(-\frac{T}{2}\right) = T \;;\; w_0 = \frac{2\pi}{T}$$

$$= \frac{1}{Tn^2 w_0^2} \int_{-T/2}^{T/2} \left[\frac{A}{d_2 - d_1} \left(\delta(t+d_2) + \delta(t-d_2)\right) - \left(\frac{A}{d_1} + \frac{A}{d_2 - d_1}\right)\left(\delta(t+d_1) + \delta(t-d_1)\right) + \right.$$

$$\left. + \frac{2A}{d_1}\delta(t) \right] e^{-jnw_0 t}\, dt \longrightarrow \text{como } x(t) \text{ con simetría par} \quad \operatorname{sen}(\theta) = 0$$

$$= \frac{1}{Tn^2 w_0^2} \left[\int_{-T/2}^{T/2} \frac{A}{d_2 - d_1} \delta(t+d_2) \cos(nw_0 t)\, dt + \int_{-T/2}^{T/2} \frac{A}{d_2 - d_1} \delta(t-d_2) \cos(nw_0 t)\, dt \right.$$

$$- \int_{-T/2}^{T/2} A\left(\frac{1}{d_1} + \frac{1}{d_2 - d_1}\right) \delta(t+d_1) \cos(nw_0 t)\, dt - \int_{-T/2}^{T/2} A\left(\frac{1}{d_1} + \frac{1}{d_2 - d_1}\right)$$

$$\left. \delta(t-d_1) \cos(nw_0 t)\, dt + \int_{-T/2}^{T/2} \frac{2A}{d_1} \delta(t) \cos(nw_0 t)\, dt \right]$$

usamos $\rightarrow \int_{-\infty}^{\infty} x(t)\delta(t \mp t_0)\, dt = x(\mp t_0)$ $\qquad \boxed{\cos(x) = \cos(-x)}$

$$C_n = \frac{A}{Tn^2 w_0^2} \left(\frac{A}{d_2 - d_1} \cos(nw_0(-d_2)) + \frac{1}{d_2 - d_1} \cos(nw_0 d_2) - \frac{d_2 - d_1 + d_1}{d_1(d_2 - d_1)} \cos(nw_0(-d_1)) \right.$$

$$\left. - \frac{d_2 - d_1 + d_1}{d_1(d_2 - d_1)} \cos(nw_0 d_1) + \frac{2}{d_1} \underbrace{\cos(nw_0 \cdot 0)}_{1} \right)$$

$$= \frac{A}{(t_i - t_f) n^2 w_0^2} \left(\frac{2}{d_2 - d_1} \cos(nw_0 d_2) - \frac{2d_2}{d_1(d_2 - d_1)} \cos(nw_0 d_1) + \frac{2}{d_1} \right)$$

$$= \frac{2A}{\left(\frac{T}{2} - \frac{T}{2}\right) n^2 \frac{4\pi^2}{T^2}} \left(\frac{1}{d_2 - d_1} \cos(nw_0 d_2) - \frac{d_2}{d_1(d_2 - d_1)} \cos(nw_0 d_1) + \frac{1}{d_1} \right)$$

$$= \frac{-AT}{2n^2\pi^2} \left(\frac{1}{d_2 - d_1} \cos(nw_0 d_2) - \frac{1}{d_1(d_2 - d_1)} \cos(nw_0 d_1) + \frac{1}{d_1} \right) \rightarrow \operatorname{Re}\{C_n\}$$

$$|C_n| = \sqrt{\operatorname{Re}^2\{C_n\} + 0^2} = \operatorname{Re}\{C_n\}$$

$$\theta_{C_n} = \tan^{-1}\left(\frac{\operatorname{Im}\{C_n\}}{\operatorname{Re}\{C_n\}}\right) = \tan^{-1}\left(\frac{0}{\operatorname{Re}\{C_n\}}\right) = 0$$

$$e_r[\%] = \left(1 - \sum |C_n|^2 \frac{P_n}{P_x}\right) \cdot 100\,[\%] \;,\; P_n = \frac{1}{T} E_n = \frac{1}{T} \int_T \left| e^{-jnw_0 t} \right|^2 dT = 1.$$

Confirmamos con x(t)

$$C_n = \frac{1}{T}\int_T x(t)e^{-jn\omega_0 t}\,dt \;;\; T = t_4 - t_1 = \frac{T}{2} - \left(-\frac{T}{2}\right) = T \;,\; \omega_0 = \frac{2\pi}{T}$$

$$C_n = \frac{1}{T}\left(\int_{-T/2}^{-d_2} 0\cdot e^{-jn\omega_0 t}\,dt + \int_{-d_2}^{-d_1}\frac{A}{d_2-d_1}(t+d_2)e^{-jn\omega_0 t}\,dt + \int_{-d_1}^{0} -\frac{A}{d_1}e^{-jn\omega_0 t}\,dt\right.$$

$$\left. + \int_0^{d_1}\frac{A}{d_1}e^{-jn\omega_0 t}\,dt + \int_{d_1}^{d_2} -\frac{A}{d_2-d_1}(t+d_2)e^{-jn\omega_0 t}\,dt + \int_{d_2}^{T/2} 0\,e^{-jn\omega_0 t}\,dt\right)$$

$$C_n = \frac{1}{T}\left(\frac{A}{d_2-d_1}\int_{-d_2}^{-d_1}(t+d_2)e^{-jn\omega_0 t}\,dt - \frac{A}{d_1}\int_{-d_1}^{0} te^{-jn\omega_0 t} + \frac{A}{d_1}\int_0^{d_1} te^{-jn\omega_0 t}\,dt\right.$$

$$\left. - \frac{A}{d_2-d_1}\int_{d_1}^{d_2}(t+d_2)e^{-jn\omega_0 t}\,dt\right)$$

$$C_n = \frac{1}{T}\left(\frac{A}{d_2-d_1}\left(\overbrace{\int_{-d_2}^{-d_1} te^{-jn\omega_0 t}\,dt}^{I_1} + d_2\overbrace{\int_{-d_2}^{-d_1} e^{-jn\omega_0 t}\,dt}^{I_2}\right) - \frac{A}{d_1}\overbrace{\int_{-d_1}^{0} te^{-jn\omega_0 t}\,dt}^{I_3}\right.$$

$$\left. + \frac{A}{d_1}\underbrace{\int_0^{d_1} te^{-jn\omega_0 t}\,dt}_{I_4} - \frac{A}{d_2-d_1}\left(\underbrace{\int_{d_1}^{d_2} te^{-jn\omega_0 t}\,dt}_{I_5} + d_2\underbrace{\int_{d_1}^{d_2} e^{-jn\omega_0 t}\,dt}_{I_6}\right)\right)$$

Hecho en calculadora:

$$I_1 = e^{jn\omega_0 d_1}\left(\frac{d_1}{jn\omega_0} - \frac{1}{n^2\omega_0^2}\right) + e^{jn\omega_0 d_2}\left(\frac{1}{n^2\omega_0^2} - \frac{d_2}{jn\omega_0}\right)$$

$$I_2 = \frac{d_2}{jn\omega_0}\left(e^{jn\omega_0 d_2} - e^{jn\omega_0 d_1}\right)$$

$$I_3 = -d_1\frac{e^{jn\omega_0 d_1}}{jn\omega_0} - \left(\frac{1}{n^2\omega_0^2} - \frac{e^{jn\omega_0 d_1}}{n^2\omega_0^2}\right)$$

$$I_4 = -d_1\frac{e^{-jn\omega_0 d_1}}{jn\omega_0}\left(\frac{e^{-jn\omega_0 d_1}}{n^2\omega_0^2} - \frac{1}{n^2\omega_0^2}\right)$$

$$I_5 = \left(d_1\frac{e^{-jn\omega_0 d_1}}{jn\omega_0} - d_2\frac{e^{-jn\omega_0 d_2}}{jn\omega_0}\right) - \left(\frac{e^{-jn\omega_0 d_2}}{n^2\omega_0^2} - \frac{e^{-jn\omega_0 d_1}}{n^2\omega_0^2}\right)$$

$$I_6 = d_2\frac{e^{-jn\omega_0 d_1}}{jn\omega_0} - d_2\frac{e^{-jn\omega_0 d_2}}{jn\omega_0}$$

$$C_n = \frac{1}{T}\left(\frac{d_1 A e^{jn\omega_0 d_1}}{jn\omega_0(d_2-d_1)} - \frac{d_2 A e^{jn\omega_0 d_2}}{jn\omega_0(d_2-d_1)} - \frac{A e^{jn\omega_0 d_1}}{n^2\omega_0^2(d_2-d_1)} + \frac{A e^{jn\omega_0 d_2}}{n^2\omega_0^2(d_2-d_1)} + \right.$$

$$+ \frac{d_2 A e^{jn\omega_0 d_2}}{jn\omega_0(d_2-d_1)} + \frac{A e^{jn\omega_0 d_1}}{jn\omega_0} + \frac{A}{d_1 n^2\omega_0^2} - \frac{A e^{jn\omega_0 d_1}}{d_1 n^2\omega_0^2} - \frac{A e^{-jn\omega_0 d_1}}{jn\omega_0} -$$

$$- \frac{A e^{-jn\omega_0 d_1}}{d_1 n^2\omega_0^2} + \frac{A}{d_1 n^2\omega_0^2} - \frac{d_1 A e^{-jn\omega_0 d_1}}{jn\omega_0(d_2-d_1)} + \frac{d_2 A e^{-jn\omega_0 d_2}}{jn\omega_0(d_2-d_1)} +$$

$$\left. + \frac{A e^{-jn\omega_0 d_2}}{n^2\omega_0^2(d_2-d_1)} - \frac{A e^{-jn\omega_0 d_1}}{n^2\omega_0^2(d_2-d_1)} - \frac{d_2 A e^{-jn\omega_0 d_1}}{jn\omega_0(d_2-d_1)} + \frac{d_2 A e^{jn\omega_0 d_2}}{jn\omega_0(d_2-d_1)} \right)$$

Pelando $e^{j\theta} = \cos(\theta) + j\,\mathrm{sen}(\theta)$

$$C_n = \frac{1}{T}\left(\frac{2A\,\mathrm{sen}(n\omega_0 d_1)}{n\omega_0}\left(\frac{d_1}{d_2-d_1} + 1\right) + \frac{2A\cos(n\omega_0 d_2)}{n\omega_0(d_2-d_1)}\left(\frac{d_2}{j} + \frac{1}{n\omega_0}\right) - \right.$$

$$- \frac{2A\cos(n\omega_0 d_2)}{n\omega_0(d_2-d_1)}\left(\frac{d_2}{j} + \frac{1}{n\omega_0}\right) - \frac{2A\cos(n\omega_0 d_1)}{n\omega_0}\left(\frac{1}{d_1 n\omega_0} + \frac{d_2}{d_2-d_1}\right) -$$

$$\left. - \frac{2d_2 A}{n\omega_0(d_2-d_1)}\,\mathrm{sen}(n\omega_0 d_2) + \frac{2A}{d_1 n^2\omega_0^2}\right)$$

Sendo par → Se integra de 0 a d_2 → $x(t) = x(-t)$.

$$P_x = \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2\,dt = \frac{2}{T}\int_0^{T/2} |x(t)|^2\,dt \quad \text{— De } d_2 \text{ a } T/2 = 0$$

$$= \frac{2}{T}\int_0^{d_2} |x(t)|^2\,dt = \frac{2}{T}\left[\int_0^{d_1}\left(\frac{A}{d_1}t\right)^2 dt + \int_{d_1}^{d_2}\left(-\frac{A}{d_2-d_1}(t+d_2)\right)^2 dt\right]$$

$$= \frac{2}{T}\left[\frac{A^2}{d_1^2}\int_0^{d_2} t^2\,dt - \frac{A^2}{(d_2-d_1)^2}\int_{d_1}^{d_2}(t-d_2)^2 dt\right]$$

$$= \frac{2}{T}\left[\frac{A^2}{d_1^2}\frac{t^3}{3}\Big|_0^{d_1} - \frac{A}{(d_2-d_1)^2}\int_{d_1}^{d_2}(t^2 + 2td_2 + d_2^2)\,dt\right]$$

$$= \frac{2}{T}\left[\frac{A^2}{d_1^2}\left(\frac{d_1^3}{3} - 0\right) - \frac{A^2}{(d_2-d_1)^2}\left(\frac{t^3}{3} + \frac{2t^2d_2}{2} + d_2^2 t\right)\Big|_{d_1}^{d_2}\right]$$

$$= \frac{2A^2}{T}\left[\frac{d_1}{3} - \frac{1}{(d_2-d_1)^2}\left(\frac{d_2^3}{3} + \frac{2d_2^2d_2}{2} + d_2^2d_2 - \frac{d_1^3}{3} - \frac{2d_1^2d_2}{2} + d_2^2d_1\right)\right]$$

$$= \frac{2A^2}{T}\left[\frac{d_1}{3} - \frac{1}{(d_2-d_1)^2}\left(\frac{d_2^3}{3} + \frac{2d_2^2d_2}{2} + d_2^2d_2 - \frac{d_1^3}{3} - \frac{2d_1^2d_2}{2} + d_2d_1\right)\right]$$

$$= \frac{2A^2d_1}{3T} + \frac{2A^2}{3T(d_2-d_1)^2}\left(8d_2^3 - (d_1+d_2)^3\right)$$

$$|C_n|^2 = \left|\left(-\frac{AT}{2n^2\pi^2}\left(\frac{1}{(d_2-d_1)}\cos(n\omega_0 d_2) - \left(\frac{1}{d_1} + \frac{1}{d_2-d_1}\right)\cos(n\omega_0 d_1) + \frac{1}{d_1}\right)\right|^2$$

$$= \left(-\frac{AT}{2n^2\pi^2}\left(\frac{1}{d_2-d_1}\cos(n\omega_0 d_1) - \left(\frac{1}{d_1} + \frac{1}{d_2-d_1}\right)\cos(n\omega_0 d_1) + \frac{1}{d_1}\right)\right)^2$$

$$e_1(\text{l.l.}) = \left(1 - \frac{\sum_n\left(-\frac{AT}{2n^2\pi^2}\left(\frac{1}{d_2-d_1}\cos(n\omega_0 d_2) - \left(\frac{1}{d_1} + \frac{1}{d_2-d_1}\right)\cos(n\omega_0 d_1) + \frac{1}{d_1}\right)\right)^2}{\frac{2A^2d_1}{3T} + \frac{2A^2}{3T(d_2-d_1)^2}\left(8d_2^3 - (d_1+d_2)^3\right)}\right)$$