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Parcial 1: Señales y sistemas.

1. La distancia media entre 2 señales periódicas $x_1(t) \in \mathbb{R}, \mathbb{C}$ y $x_2(t) \in \mathbb{R}, \mathbb{C}$; se puede expresar a partir de la potencia media de la diferencia entre ellas:

$$d^2(x_1, x_2) = \bar{P}_{x_1 - x_2} = \lim_{T \to \infty} \frac{1}{T}\int_T |x_1(t) - x_2(t)|^2 \, dt$$

Sea $x_1(t)$ y $x_2(t)$ 2 señales definidas como:

$$x_1(t) = Ae^{-jn\omega_0 t} \qquad x_2(t) = Be^{jm\omega_0 t}$$

Con $\omega_0 = 2\pi/T$; $T, A, B \in \mathbb{R}^+$ y $n, m \in \mathbb{Z}$. Determine la distancia entre las 2 señales. Compruebe sus resultados con Python.

$$\bar{P}_{x_1-x_2} = \frac{1}{T}\int_T |x_1(t) - x_2(t)|^2 \, dt = \frac{1}{T}\left(\int_T (x_1(t) - x_2(t))(x_1(t) - x_2(t))^* \right) dt$$

$$= \frac{1}{T}\left(\int_T (x_1(t) - x_2(t))(x_1^*(t) - x_2^*(t)) \, dt\right.$$

$$= \frac{1}{T}\left(\int_T x_1(t)\, x_1^*(t) \, dt - \int_T x_1(t)\, x_2^*(t) \, dt - \int_T x_2(t)\, x_1^*(t) \, dt + \int_T x_2(t)\, x_2^*(t) \, dt\right)$$

$$= \underbrace{\frac{1}{T}\int_T |x_1(t)|^2 \, dt}_{\bar{P}_{x_1}} - \underbrace{\frac{2}{T}\int_T x_1(t)\, x_2^*(t) \, dt}_{C_{12}} + \underbrace{\frac{1}{T}\int_T |x_2(t)|^2 \, dt}_{\bar{P}_{x_2}}.$$

$$\bar{P}_{x_1} = \frac{1}{T}\int_T x_1(t)\, x_1^*(t) \, dt = \frac{1}{T}\int_0^T Ae^{-jn\omega_0 t}\left(Ae^{jn\omega_0 t}\right) dt$$

$$= \frac{A^2}{T}\int_0^T \underbrace{e^{j\overset{0}{(n-n)}\omega_0 t}}_{1} \, dt = \frac{A^2}{T}\int_0^T dt = \frac{A^2}{T} t \Big|_0^T = \frac{A^2}{\cancel{T}}(\cancel{T}) = A^2$$

$$\bar{P}_{x_2} = \frac{1}{T}\int_T x_2(t)\, x_2^*(t) \, dt = \frac{1}{T}\int_0^T Be^{jm\omega_0 t}\left(Be^{-jm\omega_0 t}\right) dt$$

$$= \frac{B^2}{T}\int_0^T \underbrace{e^{j\overset{0}{(m-m)}\omega_0 t}}_{1} \, dt = \frac{B^2}{T}\int_0^T dt = \frac{B^2}{T} t \Big|_0^T = \frac{B^2}{\cancel{T}}(\cancel{T}) = B^2$$

$$C_{12} = -\frac{2}{T}\int_T x_1(t)\, x_2^*(t)\, dt = -\frac{2}{T}\int_0^T (Ae^{-jnw_0t})(Be^{-jmw_0t})\, dt$$

$$= -\frac{2AB}{T}\int_0^T e^{-jnw_0t - jmw_0t}\, dt = -\frac{2AB}{T}\int_0^T e^{-j\overbrace{(n+m)}^{K}w_0t}\, dt$$

$\rightarrow$ Sí $n+m=0$, $n=-m$:

$$C_{12} = -\frac{2AB}{T}\int_0^T \underbrace{e^{\cancel{-j(0)w_0t}}}_{1}\, dt = -\frac{2AB}{T}\int_0^T dt = -\frac{2AB}{T}\, t\Big|_0^T = -\frac{2AB\cancel{T}}{\cancel{T}}$$

$\rightarrow$ Sí $n+m \neq 0$, $n \neq -m$; $n+m = K \in \mathbb{Z}$:

$$C_{12} = -\frac{2AB}{T}\int_0^T e^{-jKw_0t}\, dt = \cancel{-}\frac{2AB}{T}\left(\frac{e^{-jKw_0t}}{\cancel{+}jKw_0}\right)\Big|_0^T$$

$$= \frac{2AB}{jKw_0T}\left(e^{-jKw_0T} - \underbrace{e^{\cancel{-jKw_0(0)}}}_{1}\right) \qquad = \cos(Kw_0T) - j\,\text{sen}(Kw_0T) = 1$$
$$\qquad = \underbrace{\cos(\cancel{K2\pi})}_{1} - j\,\underbrace{\text{sen}(\cancel{K2\pi})}_{0}$$

$$= \frac{2AB}{jKw_0T}(1-1) = 0$$

NOTA: $d^2 = \overline{P}_{x_1-x_2} \rightarrow d = \sqrt{\overline{P}_{x_1-x_2}}$

$$\sqrt{\overline{P}_{x_1-x_2}} = \begin{cases} \sqrt{A^2-2AB+B^2}; & n=m \\ \sqrt{A^2+B^2}; & n \neq m \end{cases} \qquad \overline{P}_{x_1-x_2} = \overline{P}_{x_1} + C_{12} + \overline{P}_{x_2}$$

2. Encuentre la señal en tiempo discreto al utilizar un conversor análogo digital con frecuencia de muestreo de 5KHz y 4 bits de capacidad de representación, aplicado a la señal continua:

$x(t) = 3\cos(1000\pi t) + 5\,\text{sen}(3000\pi t) + 10\cos(11000\pi t)$

Realizar la simulación del proceso de discretización (incluyendo al menos 3 períodos de $x(t)$). En caso de que la discretización no sea apropiada, diseñe e implemente un conversor adecuado para la señal estudiada.

$F_s = 5KHz$ Estados $= 2^{\#\text{bits}} = 2^4 = 16$ niveles de cuantización

$t = nT_s \rightarrow F_s = \frac{1}{T_s}$; $F = \frac{1}{T}$; $T = \frac{2\pi}{w}$

$A\cos(\Omega n) = A\cos(2\pi f n) \rightarrow \Omega = 2\pi f \rightarrow f = \frac{T_s}{T} = \frac{F}{F_s}$

$A\,\text{sen}(\Omega n) = A\,\text{sen}(2\pi f n) \rightarrow \Omega = 2\pi f \rightarrow f = \frac{T_s}{T} = \frac{F}{F_s}$

Para $x_1(t) = 3\cos(1000\pi t)$

$$\omega_1 = 1000\pi \rightarrow T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{1000\pi} = \frac{1}{500} \rightarrow F_1 = \frac{1}{T_1} = \frac{1}{1/500} = 500\ Hz$$

$$x_1[n] = A\cos\left(2\pi n \frac{F_1}{F_s}\right) = 3\cos\left(2\pi n \frac{500}{5000}\right) = 3\cos\left(\frac{n\pi}{5}\right)$$

Para $x_2(t) \rightarrow 5\,sen(3000\pi t)$

$$\omega_2 = 3000\pi \rightarrow T_2 = \frac{2\pi}{\omega_2} = \frac{2\pi}{3000\pi} = \frac{1}{1500} \rightarrow F_2 = \frac{1}{T_2} = \frac{1}{1/1500} = 1500\ Hz$$

$$x_2[n] = A\,sen\left(2\pi n \frac{F_2}{F_s}\right) = 5\,sen\left(2\pi n \frac{1500}{5000}\right) = 3\,sen\left(\frac{3n\pi}{5}\right)$$

Para $x_3(t) \rightarrow 10\cos(11000\pi t)$

$$\omega_3 = 11000\pi \rightarrow T_3 = \frac{2\pi}{11000\pi} = \frac{1}{5500} \rightarrow F_3 = \frac{1}{T_3} = \frac{1}{1/5500} = 5500\ Hz$$

$$x_3[n] = A\cos\left(2\pi n \frac{F_3}{F_s}\right) = 10\cos\left(2\pi n \frac{5500}{5000}\right) = 10\cos\left(\frac{11n\pi}{5}\right)$$

$$x[n] = 3\cos\left(\frac{n\pi}{5}\right) + 3\,sen\left(\frac{3n\pi}{5}\right) + 10\cos\left(\frac{11n\pi}{5}\right)$$

$$\frac{\omega_1}{\omega_2} = \frac{1000\pi}{3000\pi} = \frac{1}{3} \in \mathbb{Q} \qquad \frac{\omega_2}{\omega_3} = \frac{3000\pi}{11000\pi} = \frac{3}{11} \in \mathbb{Q}$$

$$\frac{\omega_1}{\omega_3} = \frac{1000\pi}{11000\pi} = \frac{1}{11} \in \mathbb{Q}$$

$\therefore$ la señal es cuasiperiódica

NO cumple Nyquist.

Si $F_s \geq 2F_{max} \rightarrow 2F_{max} = 2(5500\ Hz) = 11000\ Hz$ pero $F_s = 5000\ Hz$

Comparando con las originales $\rightarrow -\pi \leq \Omega \leq \pi$

copia o aliasing

$\Omega_1 = \frac{1}{5}\pi$ cumple, $\Omega_2 = \frac{3}{5}\pi$ cumple, $\Omega_3 = \frac{11}{5}\pi$ no cumple $> \pi$

Ω_3 original $= \frac{11}{5}\pi - 2\pi = \frac{1}{5}\pi$ ahora si cumple.

Se supone una frecuencia de muestreo más grande

$F_s = 3F_{max} = 3(5500\,Hz) = 16500\,Hz \longrightarrow T_s = \frac{1}{F_s} = \frac{1}{16500}$

Para $x_1(t) = 3\cos(1000\pi t)$

$x_1[n] = 3\cos\left(2\pi n \frac{500}{16500}\right) = 3\cos\left(\frac{2}{33} n\pi\right) \quad \Omega_1 = \frac{2\pi}{33} \approx 0{,}06\pi$

Para $x_2(t) = 5\,\text{sen}(3000\pi t)$

$x_2[n] = 5\,\text{sen}\left(2\pi n \frac{1500}{16500}\right) = 5\,\text{sen}\left(\frac{2}{11} n\pi\right) \quad \Omega_2 = \frac{2\pi}{11} \approx 0{,}18\pi$

Para $x_3(t) = 10\cos(11000\pi t)$

$x_3[n] = 10\cos\left(2\pi n \frac{5500}{16500}\right) = 10\cos\left(\frac{2n\pi}{3}\right) \quad \Omega_3 = \frac{2\pi}{3} \approx 0{,}67\pi$

$-\pi \leq \Omega_1, \Omega_2, \Omega_3 < \pi \longrightarrow$ las 3 frecuencias digitales están en las originales.

$w_1 = 1000\pi$
$w_2 = 3000\pi$
$w_3 = 11000\pi$

$x[n] = 3\cos\left(\frac{2n\pi}{33}\right) + 5\,\text{sen}\left(\frac{2n\pi}{11}\right) + 10\cos\left(\frac{2n\pi}{3}\right)$

Hallamos el periodo T para graficar las señales:

$T = \frac{r2\pi}{w_1} = \frac{l2\pi}{w_2} = \frac{k2\pi}{w_3} \longrightarrow T = \frac{r\,2\cancel{\pi}}{1000\cancel{\pi}} = \frac{l\,2\cancel{\pi}}{3000\cancel{\pi}} = \frac{k\,2\cancel{\pi}}{11000\cancel{\pi}}$

$T = \frac{r}{500} = \frac{l}{1500} = \frac{k}{5500} \longrightarrow 16500\,T = 33r = 11l = 3k$

$MCM\{33, 11, 3\} = 33 \longrightarrow r = 1, \quad l = 3, \quad k = 11$

$16.500\,T = 33 \longrightarrow T = \frac{33}{16500} = \frac{1}{500}$

3. Sea $x''(t)$ la segunda derivada de la señal $x(t)$, donde $t \in [t_i, t_f]$.
Demuestre que los coeficientes de la serie exponencial de Fourier se pueden calcular según:

$$C_n = \frac{1}{(t_i - t_f)\, n^2 w_0^2} \int_{t_i}^{t_f} x''(t)\, e^{-jnw_0 t}\, dt \;;\; n \in \mathbb{Z}.$$

¿Cómo se pueden calcular los coeficientes a_n y b_n desde $x''(t)$ en la serie trigonométrica de Fourier?

Sabemos que $x(t) = \sum_n C_n e^{jnw_0 t}$

$$\rightarrow x'(t) = \frac{d}{dt} \sum_n C_n e^{jnw_0 t} = \boxed{\sum_n C_n \frac{d}{dt} e^{jnw_0 t}} \longrightarrow \frac{d}{dt} e^{jnw_0 t} = jnw_0\, e^{jnw_0 t}$$

$$\rightarrow x''(t) = \frac{d\, x'(t)}{dt} = \frac{d}{dt} \sum_n C_n \frac{d e^{jnw_0 t}}{dt} = \boxed{\sum_n C_n \frac{d^2}{dt^2} e^{jnw_0 t}}$$

$$\frac{d^2}{dt^2} e^{jnw_0 t} = \frac{d}{dt}\, jnw_0\, e^{jnw_0 t} = (jnw_0)^2 e^{jnw_0 t} = j^2 n^2 w_0^2\, e^{jnw_0 t}$$
$$= -n^2 w_0^2\, e^{jnw_0 t}$$

$$x''(t) = \sum_n \underbrace{-C_n n^2 w_0^2}_{\tilde{C}_n} e^{jnw_0t} = \sum_n \tilde{C}_n e^{jnw_0t} \; ; \text{ con } \tilde{C}_n = -C_n n^2 w_0^2$$

Si $C_n = \frac{1}{T}\int_T x(t) e^{-jnw_0t}\, dt$ y $\tilde{C}_n = \frac{1}{T}\int_T x''(t) e^{-jnw_0t}\, dt$

Se reemplaza el $\tilde{C}_n \rightarrow C_n n^2 w_0^2 = \frac{1}{T}\int_T x''(t) e^{-jnw_0t}\, dt.$

$$C_n = \frac{1}{-(t_f - t_i) n^2 w_0^2}\int_T x''(t) e^{-jnw_0t}\, dt = \boxed{\frac{1}{(t_i - t_f) n^2 w_0^2}\int_T x''(t) e^{-jnw_0t}\, dt}$$

$$C_n = \frac{1}{(t_i - t_f) n^2 w_0^2}\int_T x''(t) e^{-jnw_0t}\, dt = \frac{1}{(t_i - t_f) n^2 w_0^2}\int x''(t)\left[\cos(nw_0t) - j\,\text{sen}(nw_0t)\right] dt$$

$$= \frac{1}{(t_i - t_f) n^2 w_0^2}\int_T x''(t)\cos(nw_0t)\, dt - j\frac{1}{(t_i - t_f) n^2 w_0^2}\int_T x''(t)\,\text{sen}(nw_0t)\, dt.$$

Sabemos que $a_n = \frac{2}{T}\int_T x(t)\cos(nw_0t)\, dt$; $b_n = \frac{2}{T}\int_T x(t)\,\text{sen}(nw_0t)\, dt.$

y que $a_n = 2\text{Re}\{C_n\}$; $b_n = -2\text{Im}\{C_n\}$

$$\rightarrow a_n = 2\text{Re}\{C_n\} = 2\frac{1}{(t_i - t_f) n^2 w_0^2}\int_T x''\cos(nw_0t)\, dt$$

$$\rightarrow b_n = \cancel{-}2\text{Im}\{C_n\} = \cancel{-}2\frac{1}{(t_i - t_f) n^2 w_0^2}\int_T x''\,\text{sen}(nw_0t)\, dt$$

O también con $x(t) = \sum_n a_n\cos(nw_0t) + b_n\,\text{sen}(nw_0t)$

$$x(t) = \sum_n a_n\cos(nw_0t) + b_n\,\text{sen}(nw_0t)$$

$$x'(t) = \sum_n -a_n\,\text{sen}(nw_0t)\, nw_0 + b_n\cos(nw_0t)\, nw_0$$

$$x''(t) = \sum_n -a_n\cos(nw_0t)(nw_0)^2 - b_n\,\text{sen}(nw_0t)(nw_0)^2$$

$$= \sum_n \underbrace{-a_n n^2 w_0^2}_{\tilde{a}_n}\cos(nw_0t) \underbrace{- b_n n^2 w_0^2}_{\tilde{b}_n}\,\text{sen}(nw_0t)$$

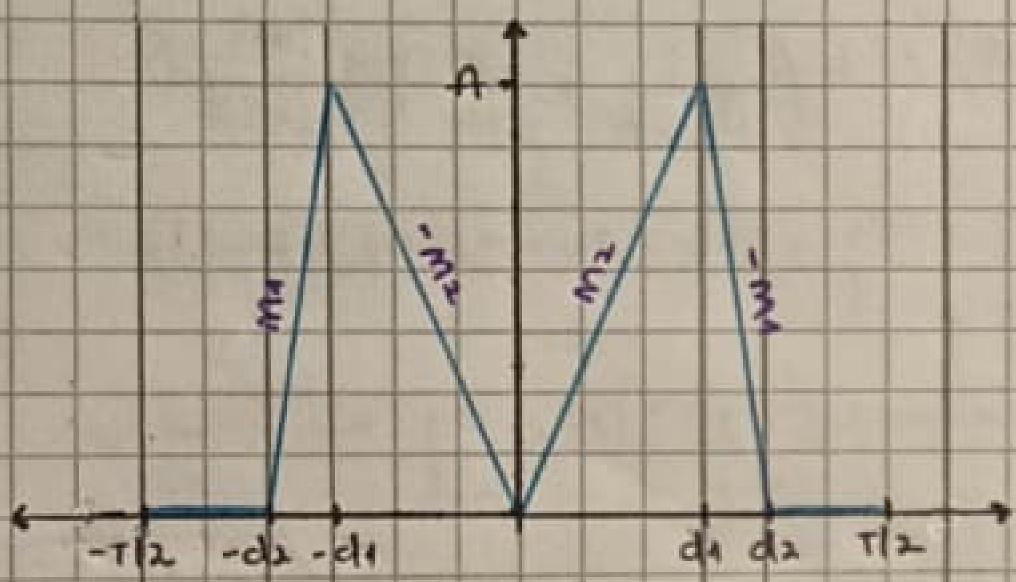
$$a_n = \frac{2}{T}\int_T x(t)\cos(nw_0t)\, dt \longrightarrow \tilde{a}_n = \frac{2}{T}\int_T x''(t)\cos(nw_0t)\, dt.$$

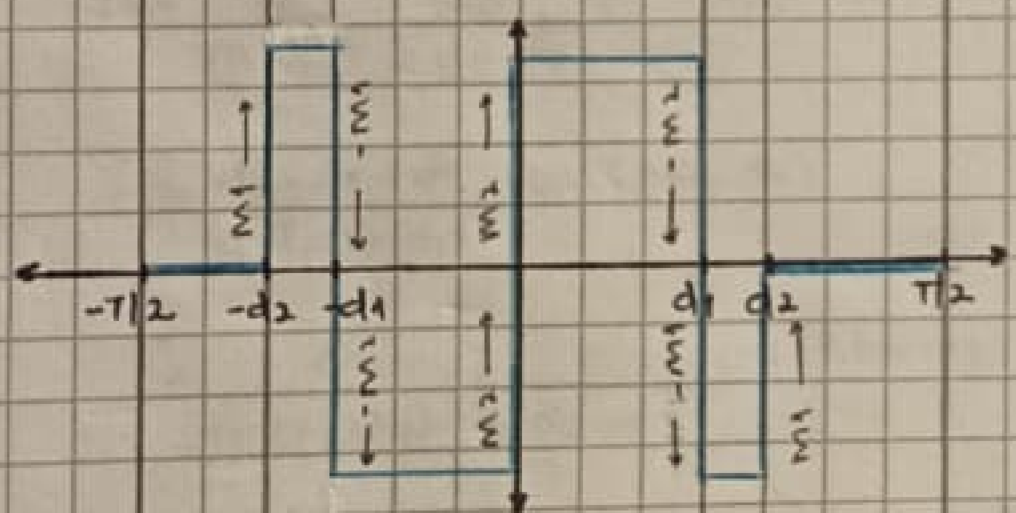
$$-a_n n^2 w_0^2 = \frac{2}{T}\int_T x''(t)\cos(nw_0t)\, dt \longrightarrow \boxed{a_n = \frac{2}{\underbrace{-(t_f - t_i)}_{t_i - t_f} n^2 w_0^2}\int x''(t)\cos(nw_0t)\, dt.}$$

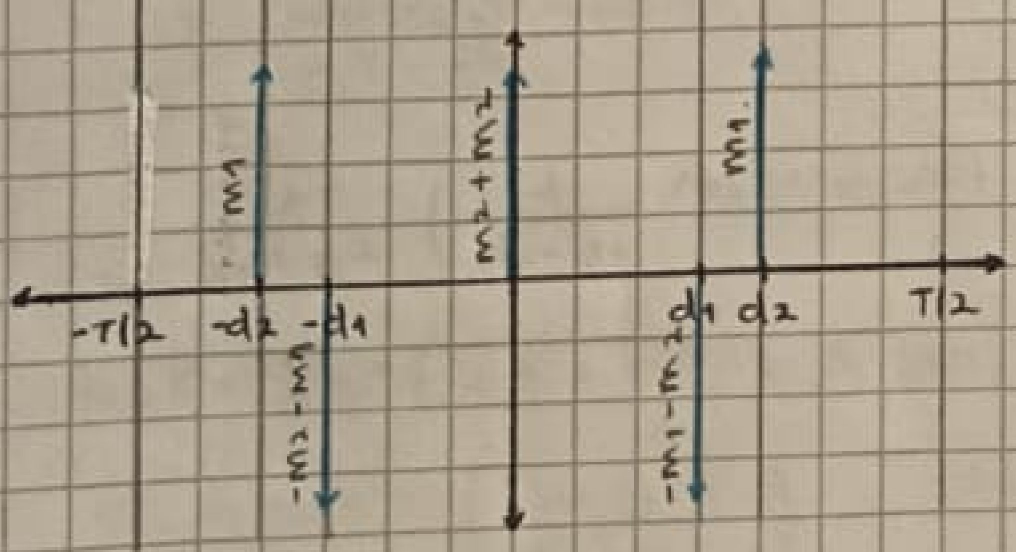
$$b_n = \frac{2}{T}\int_T x(t)\,\text{sen}(n\omega_0 t)\,dt \rightarrow \tilde{b}_n = \frac{2}{T}\int_T x'(t)\,\text{sen}(n\omega_0 t)\,dt$$

$$-b_n n^2 \omega_0^2 = \frac{2}{T}\int_T x''(t)\,\text{sen}(n\omega_0 t)\,dt \rightarrow \boxed{b_n = \frac{2}{\frac{-(t_f - t_i)\,n^2\omega_0^2}{t_i - t_f}}\int_T x''(t)\,\text{sen}(n\omega_0 t)\,dt}$$

4. Encuentre el espectro de Fourier, su parte real, imaginaria, magnitud, fase y el error relativo para $n \in \{0, \pm1, \pm2, \pm3, \pm4, \pm5\}$, a partir de $x''(t)$ para la señal $x(t)$ en la figura. Compruebe el espectro obtenido con la estimación a partir de $x(t)$. Presente las simulaciones de Python respectivas.







$$m_1 = \frac{y_{max} - y_{min}}{x_{max} - x_{min}} = \frac{A - 0}{-d_1 - (-d_2)} = \frac{A}{d_2 - d_1}$$

$$m_2 = \frac{y_{max} - y_{min}}{x_{max} - x_{min}} = \frac{A - 0}{d_1 - 0} = \frac{A}{d_1}$$

$x_1(t) = m_1 x_1 + b_1$

$\rightarrow b_1 = x_1(t) - m_1 x_1$

$b_1 = 0 - \frac{A}{d_2 - d_1}(-d_2)$

$b_1 = \frac{A d_2}{d_2 - d_1}$

$x_2(t) = m_2 x_2 + b_2$

$\rightarrow b_2 = x_2(t) - m_2 x_2$

$b_2 = 0 - \frac{A}{d_1}(0)$

$b_2 = 0$

$$x(t) = \begin{cases} 0, & d_2 \le |t| \le T/2 \\ \frac{A|t|}{d_2 - d_1} + \frac{A d_2}{d_2 - d_1} = \frac{A}{d_2 - d_1}(|t| + d_2), & d_2 \le |t| \le d_1 \\ \frac{A|t|}{d_1} & 0 \le |t| \le d_1 \end{cases}$$

$x''(t) = 0$ en cada tramo y en los puntos donde hay cambios tiende a $\pm\infty$ y estos cambios los podemos representar como $m_k\delta(t + t_k)$ donde m_k es la diferencia de la derivada por la derecha y la izquierda y t_k donde ocurre el salto (cambio instantáneo)

$$x''(t) = \overbrace{m_{-d_2}}^{m_1}\delta(t + d_2) + \overbrace{m_{-d_1}}^{-m_2 - m_1}\delta(t + d_1) + \overbrace{m_0}^{2m_2}\delta(t + 0) + \overbrace{m_{d_1}}^{-m_1 - m_2}\delta(t - d_1) + \overbrace{m_{d_2}}^{m_1}\delta(t - d_2)$$

$$x''(t) = m_1[\delta(t + d_2) - \delta(t - d_2)] - (m_1 + m_2)[\delta(t + d_1) + \delta(t - d_1)] + 2m_2\,\delta(t + 0)$$

Sabemos que $\int_{-\infty}^{\infty} \delta(t + T) = 1$

$$C_n = \frac{1}{\frac{(t_i - t_f) n^2 w_0^2}{T}} \int_T x''(t) e^{-jnw_0 t} \, dt \qquad T = t_f - t_i = \frac{T}{2} - \left(-\frac{T}{2}\right) = T \; ; \; w_0 = \frac{2\pi}{T}$$

$$= \frac{1}{T n^2 w_0^2} \int_{-T/2}^{T/2} \left[\frac{A}{d_2 - d_1} (\delta(t+d_2) + \delta(t-d_2)) - \left(\frac{A}{d_1} + \frac{A}{d_2 - d_1}\right) (\delta(t+d_1) + \delta(t-d_1)) + \right.$$

$$\left. + \frac{2A}{d_1} \delta(t) \right] e^{-jnw_0 t} \, dt \longrightarrow \text{Como } x(t) \text{ con simetría par}$$

$$\text{sen}(\theta) = 0$$

$$= \frac{1}{T n^2 w_0^2} \left[\int_{-T/2}^{T/2} \frac{A}{d_2 - d_1} \delta(t+d_2) \cos(nw_0 t) \, dt + \int_{-T/2}^{T/2} \frac{A}{d_2 - d_1} \delta(t - d_2) \cos(nw_0 t) \, dt \right.$$

$$- \int_{-T/2}^{T/2} A \left(\frac{1}{d_1} + \frac{1}{d_2 - d_1}\right) \delta(t+d_1) \cos(nw_0 t) \, dt - \int_{-T/2}^{T/2} A \left(\frac{1}{d_1} + \frac{1}{d_2 - d_1}\right)$$

$$\left. \delta(t - d_1) \cos(nw_0 t) \, dt + \int_{-T/2}^{T/2} \frac{2A}{d_1} \delta(t) \cos(nw_0 t) \, dt \right]$$

$$\text{usamos} \longrightarrow \int_{-\infty}^{\infty} x(t) \delta(t \pm t_0) \, dt = x(\mp t_0) \qquad \boxed{\cos(x) = \cos(-x)}$$

$$C_n = \frac{A}{T n^2 w_0^2} \left(\frac{1}{d_2 - d_1} \cos(nw_0(-d_2)) + \frac{1}{d_2 - d_1} \cos(nw_0 d_2) - \frac{d_2 \cancel{- d_1} + \cancel{d_1}}{d_1(d_2 - d_1)} \cos(nw_0(-d_1)) \right.$$

$$\left. - \frac{d_2 \cancel{- d_1} + \cancel{d_1}}{d_1(d_2 - d_1)} \cos(nw_0 d_1) + \frac{2}{d_1} \overset{1}{\cancel{\cos(nw_0 0)}} \right)$$

$$= \frac{A}{(t_i - t_f) n^2 w_0^2} \left(\frac{2}{d_2 - d_1} \cos(nw_0 d_2) - \frac{2 d_2}{d_1(d_2 - d_1)} \cos(nw_0 d_1) + \frac{2}{d_1} \right)$$

$$= \frac{\cancel{2}A}{\left(\frac{T}{2} - \frac{T}{2}\right) \frac{n^2 2^{\cancel{2}} \pi^2}{T^{\cancel{2}}}} \left(\frac{1}{d_2 - d_1} \cos(nw_0 d_2) - \frac{d_2}{d_1(d_2 - d_1)} \cos(nw_0 d_1) + \frac{1}{d_1} \right)$$

$$= \frac{-AT}{2 n^2 \pi^2} \left(\frac{1}{d_2 - d_1} \cos(nw_0 d_2) - \frac{d_2}{d_1(d_2 - d_1)} \cos(nw_0 d_1) + \frac{1}{d_1} \right) \rightarrow Re\{C_n\}$$

$$|C_n| = \sqrt{Re^2\{C_n\} + 0} = Re\{C_n\}$$

$$\theta_{C_n} = \tan^{-1}\left(\frac{Im\{C_n\}}{Re\{C_n\}}\right) = \tan^{-1}\left(\frac{0}{Re\{C_n\}}\right) = 0$$

$$e_r[\%] = \left(1 - \sum |C_n|^2 \frac{P_n}{P_x}\right) \cdot 100 \, [\%] \; , \; P_n = \frac{1}{T} E_n = \frac{1}{T} \int_T \left| e^{-jnw_0 t} \right|^2 dT = 1$$

Señal par → Se integra de 0 a d_2 → $x(t) = x(-t)$

$$P_x = \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2 \, dt = \frac{2}{T}\int_{0}^{T/2} |x(t)|^2 \, dt$$ De d_2 a $T/2 = 0$

$$P_x = \frac{2}{T}\int_{0}^{d_2} |x(t)|^2 \, dt = \frac{2}{T}\left[\int_{0}^{d_1}\left(\frac{A}{d_1}t\right)^2 dt + \int_{d_1}^{d_2}\left(\frac{A}{d_2-d_1}(d_2-t)\right)^2 dt\right]$$

$$P_x = \frac{2}{T}\left[\frac{A^2}{d_1^2}\underbrace{\int_{0}^{d_1} t^2 \, dt}_{I_1} + \frac{A^2}{(d_2-d_1)^2}\underbrace{\int_{d_1}^{d_2}(d_2-t)^2 \, dt}_{I_2}\right]$$

$$I_1 = \int_{0}^{d_1} t^2 \, dt = \left.\frac{t^3}{3}\right|_{0}^{d_1} = \frac{d_1^3}{3}$$

$$I_2 = \int_{d_1}^{d_2}(d_2-t)^2 \, dt = \int_{0}^{d_2} u^2 \, du = \left.\frac{u^3}{3}\right|_{0}^{d_2-d_1} = \frac{(d_2-d_1)^3}{3}$$

$u = d_2 - t$ Si $t = d_1 \rightarrow u = d_2 - d_1$
$du = dt$ Si $t = d_2 \rightarrow u = d_2 - d_2 = 0$ } $u \in [0, d_2 - d_1]$

$$P_x = \frac{2}{T}\left(\frac{A^2}{\cancel{d_1^2}}\,\frac{d_1^{\cancel{3}}}{3} + \frac{A^2}{\cancel{(d_2-d_1)^2}}\,\frac{(d_2-d_1)^{\cancel{3}}}{3}\right) = \frac{2}{T}\left(\cancel{\frac{A^2 d_1}{3}} + \frac{A^2 d_2}{3} - \cancel{\frac{A^2 d_1}{3}}\right)$$

$$P_x = \frac{2}{T}\left(\frac{A^2 d_2}{3}\right) = \frac{2A^2 d_2}{3T}$$

$$|c_n|^2 = \left(\cancel{\sqrt{}}\left(-\frac{AT}{2n^2\pi^2}\left(\frac{1}{d_2-d_1}\cos(n\omega_0 d_2) - \frac{d_2}{d_1(d_2-d_1)}\cos(n\omega_0 d_1) + \frac{1}{d_1}\right)\right)^2\right)^{\cancel{2}}$$

$$|c_n|^2 = \left(-\frac{AT}{2n^2\pi^2}\left(\frac{1}{d_2-d_1}\cos(n\omega_0 d_2) - \frac{d_2}{d_1(d_2-d_1)}\cos(n\omega_0 d_1) + \frac{1}{d_1}\right)\right)^2$$

$$e_r[\%] = \left(\frac{1 - \sum_{n}\left(-\frac{AT}{2n^2\pi^2}\left(\frac{1}{d_2-d_1}\cos(n\omega_0 d_2) - \frac{d_2}{d_1(d_2-d_1)}\cos(n\omega_0 d_1) + \frac{1}{d_1}\right)\right)^2}{\frac{2A^2 d_2}{3T}}\right)\cdot 100$$

Confirmamos con $x(t)$ → se tiene en cuenta que la señal es par
∴ el espectro solo depende de a_n.

$$a_n = \frac{2}{T}\int_T x(t)\cos(n w_0 t)\,dt$$

Como es par → $-T/2$ a 0 = 0 a $T/2$
Función = 0 → $-T/2$ a $-d_2$ y d_2 a $T/2$

$$a_n = \frac{2}{T}(2)\left[\int_0^{d_1}\frac{A}{d_1}\cos(n w_0 t)\,dt + \int_{d_1}^{d_2}\frac{A}{d_2-d_1}(d_2-t)\cos(n w_0 t)\,dt\right.$$

$$a_n = \frac{4A}{T}\left[\frac{1}{d_1}\overbrace{\int_0^{d_1}\cos(n w_0 t)\,dt}^{I_1} + \frac{1}{d_2-d_1}\left(\overbrace{\int_{d_1}^{d_2} d_2\cos(n w_0 t)\,dt}^{I_2} - \overbrace{\int_{d_2}^{d_1} t\cos(n w_0 t)\,dt}^{I_3}\right)\right]$$

$$I_1 = \int_0^{d_1} t\cos(n w_0 t)\,dt \rightarrow \begin{matrix} u = t & dv = \cos(n w_0 t) \\ du = dt & v = \operatorname{sen}(n w_0 t)/n w_0 \end{matrix} \Bigg\}\ uv - \int v\,du$$

$$= \frac{t\operatorname{sen}(n w_0 t)}{n w_0}\Big|_0^{d_1} - \frac{\cos(n w_0 t)}{n^2 w_0^2}\Big|_0^{d_1} = \left(\frac{d_1\operatorname{sen}(n w_0 d_1)}{n w_0} - 0\right) - \left(\frac{-\cos(n w_0 d_1)}{n^2 w_0^2} - \frac{-\cancelto{1}{\cos(0)}}{n^2 w_0^2}\right)$$

$$= \frac{d_1\operatorname{sen}(n w_0 d_1)}{n w_0} + \frac{\cos(n w_0 d_1)}{n^2 w_0^2} - \frac{1}{n^2 w_0^2}$$

$$I_2 = \int_{d_1}^{d_2} d_2\cos(n w_0 t)\,dt = \frac{d_2\operatorname{sen}(n w_0 t)}{n w_0}\Big|_{d_1}^{d_2} = \frac{d_2\operatorname{sen}(n w_0 d_2)}{n w_0} - \frac{d_2\operatorname{sen}(n w_0 d_1)}{n w_0}$$

$$I_3 = \int_{d_1}^{d_2} t\cos(n w_0 t)\,dt = \frac{t\operatorname{sen}(n w_0 t)}{n w_0}\Big|_{d_1}^{d_2} - \frac{-\cos(n w_0 t)}{n^2 w_0^2}\Big|_{d_1}^{d_2}$$

$$= \left(\frac{d_2\operatorname{sen}(n w_0 d_2)}{n w_0} - \frac{d_1\operatorname{sen}(n w_0 d_1)}{n w_0}\right) - \left(-\frac{\cos(n w_0 d_1)}{n^2 w_0^2} - \frac{\cos(n w_0 d_1)}{n^2 w_0^2}\right)$$

$$= \frac{d_2\operatorname{sen}(n w_0 d_2)}{n w_0} - \frac{d_1\operatorname{sen}(n w_0 d_1)}{n w_0} + \frac{\cos(n w_0 d_2)}{n^2 w_0^2} - \frac{\cos(n w_0 d_1)}{n^2 w_0^2}$$

$$a_n = \frac{4A}{T}\left[\frac{1}{d_1}\left(\frac{d_1\operatorname{sen}(n w_0 t)}{n w_0} + \frac{\cos(n w_0 d_1)}{n^2 w_0^2} - \frac{1}{n^2 w_0^2}\right) + \frac{1}{d_2-d_1}\left(\frac{d_2\operatorname{sen}(n w_0 d_2)}{n w_0} - \right.\right.$$

$$\left.\left. - \frac{d_2\operatorname{sen}(n w_0 d_1)}{n w_0}\right) - \left(\frac{d_2\operatorname{sen}(n w_0 d_2)}{n w_0} - \frac{d_1\operatorname{sen}(n w_0 d_1)}{n w_0} + \frac{\cos(n w_0 d_2)}{n^2 w_0^2} - \frac{\cos(n w_0 d_1)}{n^2 w_0^2}\right)\right]$$

$$a_n = \frac{4A}{T}\left[\frac{1}{d_2-d_1}\frac{\cos(n w_0 d_1)}{n^2 w_0^2} - \frac{1}{d_2-d_1}\frac{\cos(n w_0 d_2)}{n^2 w_0^2} - \frac{1}{d_1}\frac{1}{n^2 w_0^2} + \frac{1}{d_1}\frac{\cos(n w_0 d_1)}{n^2 w_0^2}\right]$$