Decibel

$1\ \underline{\underline{Bel}} = \times 10$

$X_0 \rightarrow \boxed{g_p = \div 10} \xrightarrow{X_1} \boxed{g_p = \div 10} \rightarrow X_2$

$$\frac{X_B}{10} = \log\left(\frac{X}{X_0}\right)$$

$$X_{dB} = 10 \log\left(\frac{X}{X_0}\right)$$

$1mW \rightarrow \boxed{g_p = 100} \rightarrow 100mW$

$-1\ Bel = \div 10 = \frac{1}{10} = 10^{-1}$

$1\ Bel = \times 10 = 10^{1}$

$2\ Bels = \times 10 \times 10 = 100 = 10^{2}$

$3\ Bels = \times 10 \times 10 \times 10 = 1000 = 10^{3}$

$$0_{dB} = 10 \log\left(\frac{X}{X_0}\right)$$

$$0 = \log\left(\frac{X}{X_0}\right)$$

$$10^0 = \boxed{\frac{X}{X_0} = 1}$$

$1mW \rightarrow \boxed{g_p = 30} \rightarrow 30mW$

$$10^x = 30$$

$$\log_{10} 10^x = \log_{10} 30 \rightarrow x = \log 30 = 1{,}47\ Bels$$

$$P_R = \frac{V^2}{R}$$

$-2\ Bels = \text{ganho} \div 100$

$-3\ Bel = \text{ganho} \div 1000$

$$V = R \cdot i \rightarrow i = \frac{V}{R}$$

$$P_R = V \cdot i = R i^2 = \frac{V^2}{R}$$

$$E = mc^2$$

$$E_c = \frac{1}{2} C V^2$$

$$E_i = \frac{1}{2} L \cdot i^2$$

$$E_{X(t)} = \int_{-\infty}^{+\infty} |X(t)|^2 \, dt$$

$$X_{dB} = 10 \log\left(\frac{X}{X_0}\right)^2$$

$$\boxed{X_{dB} = 20 \log\left(\frac{X}{X_0}\right)}$$

$$F.T(z) = \frac{X(z)}{X_0(z)} \cancel{=} \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{a_0 + a_1 z^{-1} + a_2 z^{-2}}$$

$$a_0 X(z) + a_1 z^{-1} X(z) + a_2 z^{-2} X(z) = b_0 X_0 + b_1 z^{-1} X_0 + b_2 z^{-2} X_0$$

$$a_0 X[n] + a_1 x[n-1] + a_2 x[n-2] = b_0 X_0[n] + b_1 X_0[n-1] + b_2 X[n-2]$$

$$Z\{x[n]\} = X(z) \qquad Z\{x[n-1]\} = z^{-1} X(z)$$

$$Z\{a_0 x[n]\} = a_0 X(z)$$

$$a_0 X[n] + a_1 x[n-1] + a_2 x[n-2] = b_0 X_0[n] + b_1 X_0[n-1] + b_2 X[n-2]$$

$$X_0 \rightarrow \boxed{H(z)} \xrightarrow{X}$$

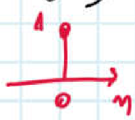
$a_0 = 1$

$$\boxed{x[n] = -a_1 x[n-1] - a_2 x[n-2] + b_0 X_0[n] + b_1 X_0[n-1] + b_2 X[n-2]}$$

Entrada $\quad$ Saída

$$x[n] \rightarrow \boxed{A} \rightarrow y[n] \overset{?}{=} x[n] * h[n] = \boxed{\sum_{k=-\infty}^{+\infty} x[k]\,h[n-k]}$$

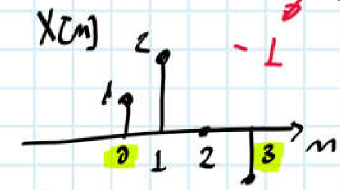
$$\delta[n] \rightarrow h[n]$$



$$y[n] = h[n] * x[n] = \sum_{k=-\infty}^{+\infty} h[k]\,x[n-k]$$

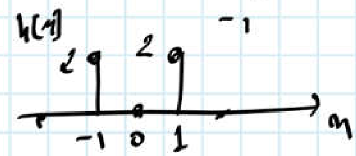
$$y[n] = \cancelto{0}{\ldots} + \overbrace{\cancelto{0}{x[-1]}\,\underbrace{h[n+1]}_{c^{te}}}^{k=-1} + \overbrace{\cancelto{1}{x[0]}\,\underbrace{h[n]}_{c^{te}}}^{k=0} + \overbrace{\cancelto{2}{x[1]}\,\underbrace{h[n-1]}_{c^{te}}}^{k=1} + \overbrace{\cancelto{0}{x[2]}\,\underbrace{h[n-2]}_{c^{te}}}^{k=2} + \cdots$$

$$\cancelto{-1}{x[3]}\,h[n-3] +$$



$$y[n] = h[n] + 2\,h[n-1] - h[n-3]$$

$$* \; h[n] = 2\,\delta[n-1] + 2\,\delta[n+1] \quad (1)$$

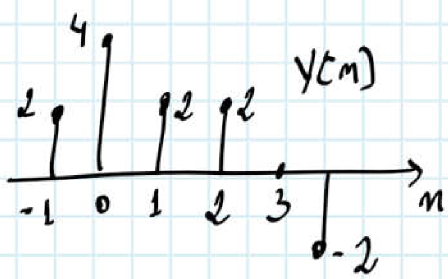


$$h[n-1] = 2\,\delta[n-2] + 2\,\delta[n]$$

$$2\,h[n-1] = 4\,\delta[n-2] + 4\,\delta[n] \quad (2)$$

$$* \; h[n-3] = 2\,\delta[n-4] + 2\,\delta[n-2] \quad (3)$$

$$y[n] = 2\,\delta[n-1] + 2\,\delta[n+1] + \boxed{4\,\delta[n-2]} + 4\,\delta[n] - 2\,\delta[n-4] \boxed{- 2\,\delta[n-2]}$$



$$Z\{\alpha^n u[n]\} = \sum_{n=-\infty}^{+\infty} \alpha^n u[n]\, z^{-n}$$

$$u[n] = \begin{cases} 0 \; ; \; n<0 \\ 1 \; ; \; n \geq 0 \end{cases}$$

$$X_1(z) = \sum_{n=0}^{+\infty} \alpha^n z^{-n} = \frac{z}{z-\alpha} = \frac{1}{1-\frac{\alpha}{z}} = \frac{1}{1-\alpha z^{-1}}$$

$$Z\{x_2[n]\} = Z\{u[n]-u[n-2]\} = \underbrace{Z\{u[n]\}}_{U(z)} - \underbrace{Z\{u[n-2]\}}_{z^{-2}\,U(z)}$$

$$U_1(z) = \frac{z}{z-1} \qquad U_2(z) = \frac{z^{-1}}{z-1}$$

$$z^{-2} \cdot \frac{z}{z-1}$$

$$Z\{u[n]-u[n-2]\} = \frac{z}{z-1} - \frac{z^{-1}}{z-1} = \frac{z-z^{-1}}{z-1} = \frac{z-\frac{1}{z}}{z-1}$$

$$= \frac{z^2-1}{z(z-1)}$$

$$\mathcal{Z}\{\cos[nT]\} =$$

$$e^{j\theta} = \cos\theta + j\,\text{sen}\,\theta \qquad \theta = nT$$

$$+\ e^{-j\theta} = \cos\theta - j\,\text{sen}\,\theta$$

$$e^{j\theta} + e^{-j\theta} = 2\cos\theta$$

$$\cos\theta = \frac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right)$$

$$\cos[nT] = \frac{1}{2}\left(\big(\underbrace{e^{jT}}_{\alpha_1}\big)^n + \big(\underbrace{e^{-jT}}_{\alpha_2}\big)^n\right)$$

$$\cos[nT] = \frac{1}{2}\left(\alpha_1^n + \alpha_2^n\right) = \frac{1}{2}\mathcal{Z}\{\alpha_1^n\} + \frac{1}{2}\mathcal{Z}\{\alpha_2^n\}$$

$$\mathcal{Z}\{\alpha^n\} = \frac{z}{z-\alpha}$$

$$\frac{1}{2}\left\{\frac{z}{z-\alpha_1} + \frac{z}{z-\alpha_2}\right\} = \frac{1}{2}\left\{\frac{z}{z-e^{jT}} + \frac{z}{z-e^{-jT}}\right\}$$

$$= \frac{1}{2}\left\{\frac{z(z-e^{-jT}) + z(z-e^{jT})}{(z-e^{jT})(z-e^{-jT})}\right\} = \frac{1}{2}\,\frac{z\left[z-e^{-jT}+z-e^{jT}\right]}{z^2 - ze^{-jT} - ze^{jT} + e^{(jT-jT)}}$$

$$= \frac{1}{2} \frac{z\left[z - e^{-jT} + z - e^{jT}\right]}{z^2 - z e^{-jT} - z e^{jT} + e^{(jT - jT)}}$$

$$= \frac{z\left(z - \boxed{\frac{1}{2}\left(e^{jT} + e^{-jT}\right)}\right]}{z^2 - 2z\cos T + 1}$$

$$= \frac{z\,(z - \cos T)}{z^2 - 2z\cos T + 1}$$

a) $X(z) = \frac{Y(z)}{R(z)} \cancel{=} \frac{0.3z^{-1}+0.1z^{-2}}{1-0.7z^{-1}+0.1z^{-2}}$

$$\begin{cases} \mathcal{Z}\{x[n]\} = X(z) \\ \mathcal{Z}\{x[n-k]\} = z^{-k} X(z) \end{cases}$$

b) $\cancel{X(z)} = \frac{2}{z-0.8} \cancel{=} \frac{Y(z)}{X(z)}$

$$\boxed{X(z) \longrightarrow \boxed{\frac{2}{z-0{,}8}} \longrightarrow Y(z)}$$

$$Y(z) - 0{,}7\, z^{-1} Y(z) + 0{,}1\, z^{-2} Y(z) = 0{,}3\, z^{-1} R(z) + 0{,}1\, z^{-2} R(z)$$

$$y[n] - 0{,}7\, y[n-1] + 0{,}1\, y[n-2] = 0{,}3\, r[n-1] + 0{,}1\, r[n-2]$$

$$\boxed{y[n] = +0{,}7\, y[n-1] - 0{,}1\, y[n-2] + 0{,}3\, r[n-1] + 0{,}1\, r[n-2]}$$

(b) $z^{1} Y(z) - 0{,}8\, Y(z) = 2 X(z)$

* $y[n+1] - 0{,}8\, y[n] = 2 x[n]$

$$y[n] = \frac{1}{0{,}8} y[n+1] - \frac{2}{0{,}8} x[n]$$

$$y[n] - 0{,}8\, y[n-1] = 2 x[n-1]$$

* $y[n] = 0{,}8\, y[n-1] + 2 x[n-1]$

Questão 4

$$G(s) = \frac{s+2}{s^2 + 2 \cdot s + 5}$$

Forward $s = \frac{z-1}{T}$ $\qquad \frac{10.0 \cdot z - 8.0}{100.0 \cdot z^2 - 180.0 \cdot z + 85.0}$

Backward $s = \frac{z-1}{T \cdot z}$ $\qquad -\frac{10.0 \cdot z - 12.0 \cdot z^2}{125.0 \cdot z^2 - 220.0 \cdot z + 100.0}$

Trapezoidal $s = \frac{2 \cdot (z-1)}{T \cdot (z+1)}$ $\qquad \frac{22.0 \cdot z^2 + 4.0 \cdot z - 18.0}{445.0 \cdot z^2 - 790.0 \cdot z + 365.0}$

$$U(z) = \underbrace{K_p E(z)}_{U_P(z)} + \underbrace{K_I \frac{1}{1-z^{-1}} E(z)}_{U_I(z)} + \underbrace{K_D\left(1-z^{-1}\right)E(z)}_{U_D(z)} = \frac{U(z)}{E(z)}$$

a) $u_p(k) = K_p\, e[n]$	(5)
b) $u_i(k)$	(5)
c) $u_d(k)$	(5)

$$U_p(z) = K_p E(z) \rightarrow \frac{U_p(z)}{E(z)} \cancel{=} \frac{K_p}{1} \qquad \frac{U_p(z) = K_p E(z)}{\boxed{u_p[n] = K_p e[n]}}$$

$$U_I(z) = K_I \frac{1}{(1-z^{-1})} E(z)$$

$$U_I(z) - z^{-1} U_I(z) = K_I E(z)$$

$$u_I[n] - u_I[n-1] = K_I e[n]$$

$$\boxed{u_I[n] = u_I[n-1] + K_I e[n]}$$

$$Z\{x[n]\} = X(z)$$

$$Z\{f[n]\} = F(z)$$

$$Z\{u_I[n]\} = U_I(z)$$

$$Z\{e[n]\} = E(z)$$

$$U_D(z) = K_d (1 - z^{-1}) E(z)$$

$$U_D(z) = K_d E(z) - K_d z^{-1} E(z)$$

$$u_D[n] = K_d e[n] - K_d e[n-1] = K_d \left(\underline{e[n] - e[n-1]} \right)$$