Evaluación de Matemáticas par Ingeniería II

Ingeniería en Software

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1. Resuelve la ecuación por el método de separación de variables

5. $y' = \frac{5x}{y}$

$\frac{dy}{dx} = \frac{5x}{y}$

$dy = \left(\frac{5x}{y}\right)dx$

$\int y\,dy = \int\left(\frac{5x}{y}\right)dx \qquad \frac{y^2}{2} = 5\int x\,dx \qquad = \frac{y^2}{2} = \frac{5x^2}{2}$

$y^2 = 5x^2 + C$ //

6. $y' = \frac{\sqrt{x}}{3y}$

$\frac{dy}{dx} = \frac{\sqrt{x}}{3y}$

$dy = \left(\frac{\sqrt{x}}{3y}\right)dx \qquad \frac{1}{3}\int \sqrt{x}\,dx$

$\int y\,dy = \int \frac{\sqrt{x}}{3}\,dx \rightarrow \frac{1}{3}\int x^{\frac{1}{2}}\,dx$

$\frac{1}{3} \cdot \frac{2x\sqrt{x}}{3}$

$\frac{y^2}{2} = \frac{2x\sqrt{x}}{9} + C$ //

7. $y' = \sqrt{x}\, y$

$$\frac{dy}{dx} = \sqrt{x}\, y$$

$$\int \frac{dy}{y} = \int \sqrt{x}\, dx$$

$$\ln y + c = \frac{2}{3} \sqrt{x^2} + c$$

$$\ln y = \frac{2}{3} \sqrt[2]{x^3}$$

$$\ln y = e^{\frac{2}{3}} \sqrt[2]{x^3} + C$$

$$y = e^{\frac{2}{3}} \sqrt[2]{x^3} + e^c$$

$$\underline{y = Ce^{\frac{2}{3}} \sqrt[2]{x^3}}\ //$$

8. $y' = x(1+y)$

$$\frac{dy}{dx} = x(1+y)$$

$$dy = x(1+y)\,dx$$

$$\int \frac{dy}{(1+y)} = \int x\,dx \qquad u = 1+y \qquad du = dy$$

$$= \int \frac{1}{u}\,du = \frac{x^2}{2}dx$$

$$= \ln u = \frac{x^2}{2}dx$$

$$= \cancel{e^{\ln}}(1+y) = e\frac{x^2}{2}dx$$

$$1+y = e^{\frac{x^2}{2}} + C$$

$$1+y = e^{\frac{x^2}{2}} \cdot e^{c}$$

$$1+y = e^{\frac{x^2}{2}}\,C$$

$$\underline{1+y = Ce^{\frac{x^2}{2}}} \;//$$

9. $(1+x^2)y' - 2xy = 0$

$$(1+x^2)\frac{dy}{dx} = 2xy$$

$$\frac{dy}{dx} = \frac{2xy}{(1+x^2)}$$

$$dy = \frac{2xy}{(1+x^2)}dx$$

$$\int \frac{dy}{y} = \int \frac{2x}{(1+x^2)}dx$$

CV

$$\ln y = \int \frac{du}{u}$$

$u = (1+x^2)$

$du = 2x$

$$\ln y + c = \ln(1+x^2) + c$$

$$\ln y = \ln(1+x^2) + \ln C$$

$$\cancel{e^{\ln y}} = \cancel{e^{\ln}} C(1+x^2)$$

$$y = C(1+x^2)$$

//

10. $xy + y' = 100x$

$$xy + \frac{dy}{dx} = 100x$$

$$\frac{dy}{dx} = 100x - xy$$

$$dy = 100x - xy\,dx$$

$$\int \frac{dy}{y} = \int 100x - x\,dx$$

$$\ln y = 100\int x\,dx$$

$$\ln y = 100\int \frac{x^2}{2}\,dx$$

$$\cancel{e}^{\ln y} = e^{-\frac{x}{2}+c} + 100$$

$$y = Ce^{-\frac{x}{2}} + 100 \quad //$$

2. Resuelve por el método de ecuación diferencial de primer orden

$$5.\quad \frac{dy}{dx} + \overbrace{\left(\frac{1}{x}\right)}^{P(x)} y = \overbrace{3x+4}^{Q(x)}$$

$$u(x) = e^{\int \left(\frac{1}{x}\right) dx} = \cancel{e^{\ln x}} = x$$

$$y = \frac{1}{x}\int (3x+4)(x)\,dx = \frac{1}{x}\int (3x^2+4x)\,dx$$

$$y = \frac{1}{x}\left[\int 3x^2\,dx + \int 4x\,dx\right]$$

$$y = \frac{1}{x}\left[\frac{\cancel{3}x^3}{\cancel{3}} + \frac{4x^2}{2} + C\right]$$

$$y = \frac{1}{x}\left[x^3 + 2x^2 + C\right]$$

$$y = x^2 + 2x + \frac{C}{x}$$

7. $y' - y = 10$

$$\frac{dy}{dx} - \overset{P(x)}{y} = \overset{Q(x)}{10}$$

$$u(x) = e^{\int y\,dx} = e^{-x}$$

$$y = \frac{1}{e^{-x}} \int 10 e^{-x}\,dx$$

$$y = \frac{1}{\cancel{e^{-x}}} \left[-10 e^{-x} + C \right]$$

$$y = -10 + \frac{C}{e^{-x}}$$

9. $(y+1)\cos x\,dx - dy = 0$

$$\left((y+1)\cos x\,dx - dy = 0\right)\frac{1}{dx}$$

$$(y+1)\cos x\ \frac{dy}{dx} = 0$$

$$-\frac{dy}{dx} + (y+1)\cos x = 0$$

$$-\frac{dy}{dx} + y\cos x + \cos x = 0$$

$$\left(-\frac{dy}{dx} + y\cos x = \cos x\right)(-1)$$

$$\frac{dy}{dx} + \overset{P(x)}{y\cos x} = \overset{Q(x)}{\cos x}$$

$$e^{\int -\cos x\,dx} = e^{\operatorname{sen} x}$$

$u(x) = e^{\operatorname{sen} x}$

$$y = \frac{1}{e^{\operatorname{sen} x}}\int -\cos x\ e^{\operatorname{sen} x}\ dx$$

CV

$u = \operatorname{sen} x \quad du = -\cos x$

$$y = \frac{1}{e^{\operatorname{sen} x}}\int e^{u}\,du$$

$$y = \frac{1}{e^{\operatorname{sen} x}}\left[e^{u} + c\right]$$

$$y = \frac{1}{\cancel{e^{senx}}}\left[\cancel{e^{senx}} + c\right]$$

$$y = 1 + \frac{c}{e^{senx}}$$

11. $(x-1)y' + y = x^2-1$

$$(x-1)\frac{dy}{dx} + y = x^2-1$$

$$\frac{dy}{dx} + \overset{P(x)}{\frac{1}{x-1}} = \overset{Q(x)}{x^2-1}$$

$$u(x) = e^{\int \frac{1}{x-1}dx} = e^{\ln(x-1)} = x-1$$

$$y = \frac{1}{x-1}\int (x-1)x\,dx$$

$$y = \frac{1}{x-1} = \int (x^2-1)\,dx$$

$$y = \frac{1}{x-1}\left[\frac{x^3}{3} + C\right]$$

$$y = \frac{x^3}{3(x-1)} - \frac{x}{x-1} + \frac{C}{x-1}$$ //

13. $y' - \underset{P(x)}{3x^2} y = \underset{Q(x)}{e^{x^3}}$

$$\frac{dy}{dx} - 3x^2 y = e^{x^3}$$

$$e^{\int -3x^2} = e^{\frac{\cancel{3}x^3}{\cancel{3}}} = e^{-x^3}$$

$$y = \frac{1}{e^{x^3}}\left[e^{x^3} + c\right]$$

$$y = 1 + \frac{c}{e^{x^3}}$$