Universidad Nacional Autónoma de México

Facultad de Ingeniería

División de Ciencias Básicas

Álgebra (1120)

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SERIE 3

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Grupo: 28

Serie 3

lun, 14 de diciembre 2020

"Números complejos" — Sólo impares —
(21 al 43)

21. Determinar el módulo del argumento, la forma polar y la representación en el plano de Argand de los siguientes números

Número	Módulo	Argumento	Forma Polar
$1+i$	$\sqrt{2}$	$45°$	$z=\sqrt{2}\ cis\, 45°$
$1-i$	$\sqrt{2}$	$-45°$ $315°$	$z=\sqrt{2}\ cis\, 315°$
$2\sqrt{3}+2$	$2\sqrt{3}+2$	$0°$	$z=2\sqrt{3}+2\ cis\ 0°$
$4-4\sqrt{3}$	$4\sqrt{3}-4$	$0°$ $180°$	$z=4\sqrt{3}-4\ cis\, 180°$
-2	2	$180°$	$z=2\, cis\, 180°$
$4i$	4	$90°$	$z=4\, cis\, 90°$

Número:

$z=1+i$ Módulo $r=|z|=\sqrt{a^2+b^2}$

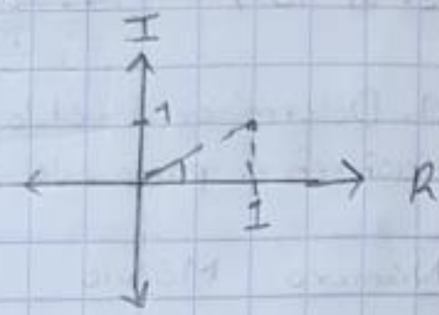
$r=|z|=\sqrt{1^2+1^2}$; $r=|z|=\sqrt{2}$; $|z|=\sqrt{2}$

Argumento:

$\tan\alpha=\frac{b}{a}$; $\alpha=\arctan\left(\frac{b}{a}\right)$; $\alpha=\arctan\left(\frac{1}{1}\right)$

$\alpha=45°$

Binomica $z = 1 + i$ a Polar:



$z = r_\alpha$ $z = \sqrt{2}_{45}$

Trigonometrica:

$z = \sqrt{2}(\cos 45° + i\,\text{sen}\, 45°)$

Exponencial: $z = \sqrt{2}\, e^{45i}$

$z = 1 - i$

Modulo: $r = |z| = \sqrt{a^2 + b^2}$; $r = |z| = \sqrt{1^2 + (-1)^2}$

$r = |z| = \sqrt{2}$

Argumento:

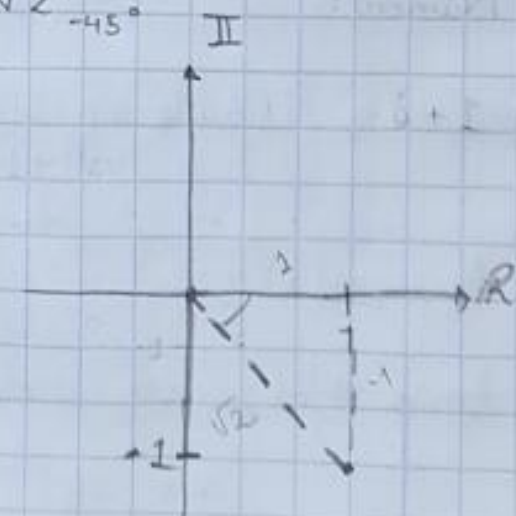
$\alpha = \arctan\left(\frac{b}{a}\right)$; $\alpha = \arctan\left(\frac{-1}{1}\right)$; $\alpha = \arctan(-1)$

$\alpha = -45°$; $\alpha = 315$

Binomica $z = 1 - i$ a polar $z = \sqrt{2}_{-45°}$

Trigonometrica: $z = \sqrt{2}\ \text{cis}\ -45°$

Exponencial: $z = \sqrt{2}\, e^{45i}$



$z = 2\sqrt{3} + 2$

Modulo: $r = |z| = \sqrt{(2\sqrt{3}+2)^2 + (0)^2}$; $r = |z| = 5.46$

Argumento: $\alpha = \arctan\left(\frac{0}{2\sqrt{3}+2}\right)$ $\alpha = \arctan(1) = 0°$

Binomica: $z = 2\sqrt{3} + 2$

Polar: $z = (5.46)_{0}$

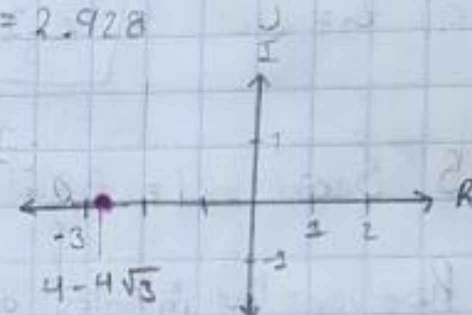
$z = 4 - 4\sqrt{3}$

Modulo: $r = |z| = \sqrt{(4-4\sqrt{3})^2}$; $r = |z| = \sqrt{64 - 32\sqrt{3}}$

$r = |z| = 2.928$

Argumento:

$\alpha = \arctan\left(\frac{0}{4-4\sqrt{3}}\right)$; $\alpha = 0$

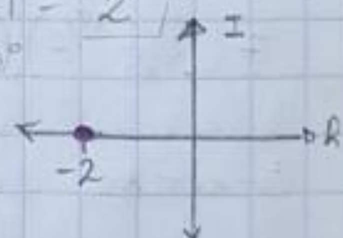


Polar = $z = 2.928_{0°}$

$z = -2 + 0$

Modulo: $r \cdot |z| = \sqrt{(-2)^2 - (0)^2}$, $|z| = \sqrt{4}$, $|z| = 2$

Argumento: $\alpha = \arctan\left(\frac{0}{-2}\right) = 0°$ $\alpha = 0°$



Polar = $z = 2_{0°}$

$z = 0 + 4i$

Modulo: $r = |z| = \sqrt{(0)^2 + (4)^2}$, $|z| = \sqrt{16}$, $|z| = 4$

Argumento: $\alpha = \arctan\left(\frac{4}{0}\right)$

$\alpha = 90°$

23. Expresar los siguientes números complejos en forma exponencial

a) $z_1 = (\sqrt{2})_{45°}$ $\boxed{z_1 = \sqrt{2}\ e^{\frac{\pi}{4} i}}$

b) $z_2 = 2(\cos 60° + i \operatorname{sen} 60°)$

$\boxed{z_2 = 2e^{\frac{\pi}{3} i}}$

c) $z_3 = i$ $\boxed{z_3 = e^{\frac{\pi}{2} i}}$

25. Sean $z_1 = 3e^{i\frac{\pi}{2}}$; $z_2 = \sqrt{2}\ e^{i\frac{7\pi}{4}}$

Resolver las siguientes operaciones

a) $z_1 \cdot z_2$

$= (3e^{i\pi/2})(\sqrt{2}\,e^{i\frac{7\pi}{4}})$

$= 3\sqrt{2}\ e^{\frac{9}{4}\pi i}$

$= 3\sqrt{2}\ e^{(9/4\pi - 2\pi)i}$

$= 3\sqrt{2}\ e^{\frac{\pi}{4} i}$

$$z_1 \cdot z_2 = 3\sqrt{2}\ e^{\pi/4 i}$$

b) $\frac{z_1}{z_2} = \frac{3e^{i\pi/2}}{\sqrt{2}\ e^{i\frac{7\pi}{4}}} = \frac{3}{\sqrt{2}}\ e^{(\frac{-5}{4}\pi + 2\pi)i}$

$= \frac{3}{\sqrt{2}}\ e^{(-\frac{5}{4}\pi)i}$; $= \frac{3\sqrt{2}}{2}\ e^{3/4\pi i}$

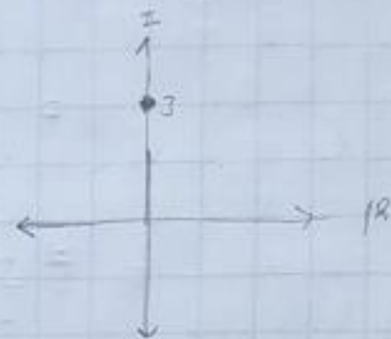
$$\frac{z_1}{z_2} = \frac{3\sqrt{2}}{2}\ e^{\frac{3}{4}\pi i}$$

c) z_1^{10} $\quad z_1 = 3i \; : \; 0+3i$

Teorema de Moivre

Si $z = r(\cos\theta + i\,\text{Sen}\,\theta)$

$z^n = r^n(\cos n\theta + i\,\text{Sen}\,n\theta)$



$z = a + ib \qquad a = 0 \;,\; b = 3$

Modulo o r

$r = \sqrt{(0)^2 + (3)^2} \;;\; = \sqrt{9} \;;\; \underline{3}$

Argumento

$\alpha = \arctan\left(\frac{3}{0}\right) \;;\; = \underline{90°}$

$z = 3(\cos 90° + i\,\text{Sen}\,90°)$

$z^{10} = (3)^{10}(\cos 10(90°) + i\,\text{Sen}\,10(90°))$

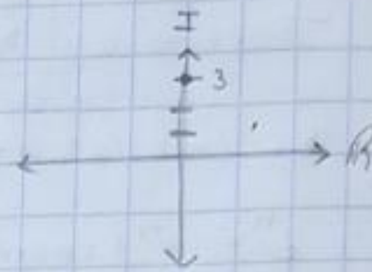
$z^{10} = 3^{10}(\cos 900° + i\,\text{Sen}\,900°$

$z_1^{10} = 3^3\, e^{[5\pi - 2(2\pi)]i}$

$$\boxed{z = 3^{10}\, e^{\pi i}}$$

27. Encontrar el modúlo y el argumento de las siguientes expresiones ; expresar el resultado en forma exponencial

a) $$\frac{3i^{30} - i^{19}}{2i-1} = \frac{3(i^2)^{15} - i^{18} \cdot i}{2i-1}$$



$$= \frac{3(-1)^{15} - (i^2)^9 i}{2i-1}$$

$$= \frac{-3+i}{-1+2i} \cdot \frac{-1-2}{-1-2i}$$

$$= \frac{(3+2)+(6-1)i}{5} = \frac{5+5i}{5}$$

$z = 1+i$

$r = \sqrt{1^2+1^2}$; $= \sqrt{2}$ $\theta = \text{angtan}\left(\frac{1}{1}\right)$

$\theta = 45^\circ$

$\boxed{r = \sqrt{2}}$

$\boxed{\theta = \frac{\pi}{4}}$

$\therefore z = \sqrt{2}\, e^{\frac{\pi}{4} i}$

b) $\left(\frac{1+i}{1-i}\right)^7 = \left(\frac{\sqrt{2}\ cis\ 45^\circ}{\sqrt{2}\ cis\ (-45^\circ)}\right)^7$

$= \left[1 cis\ (45-(-45^\circ))\right]^7$

$= [cis\ 90^\circ]^7$

$= \left[cis\ \frac{\pi}{2}\right]^7 \quad ; \quad = \left[e^{\frac{\pi}{2}i}\right]^7$

$= e^{7(\frac{\pi}{2})i} \quad ; \quad = e^{(\frac{7}{2}\pi - 2\pi)i}$

$\boxed{e^{\frac{3}{2}\pi i}}$

$\theta = \frac{3}{2}\pi$

$r = 1$

c) $[\sqrt{2}\,(\cos 315^\circ + i \operatorname{sen} 315^\circ)]^4$

$(\sqrt{2}\ cis\ 315^\circ)^4$

$= (\sqrt{2})^4\ cis\ (315^\circ)(4)$

$= 4\ cis\ (-180^\circ)$

$= 4\ cis\ \pi$

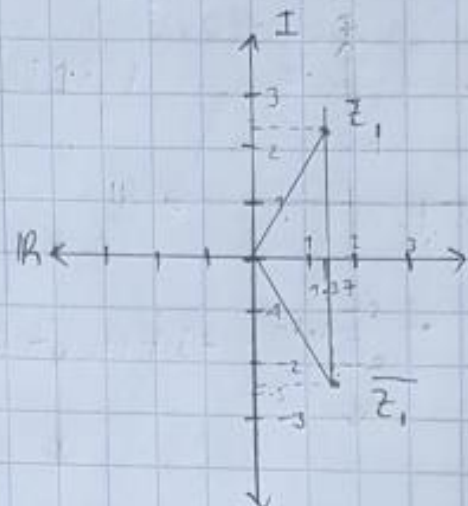
$\theta = \pi \qquad r = 4$

29. Representar en el diagrama de Argand el conjugado de cada uno de los siguientes números:

a) $z_1 = \sqrt{8}$ cis $61°$ $r = \sqrt{8}$ $\theta = 61°$

$\overline{z}_1 = \sqrt{8}$ cis $(360° - 61°)$

$\overline{z}_1 = \sqrt{8}$ cis $299°$

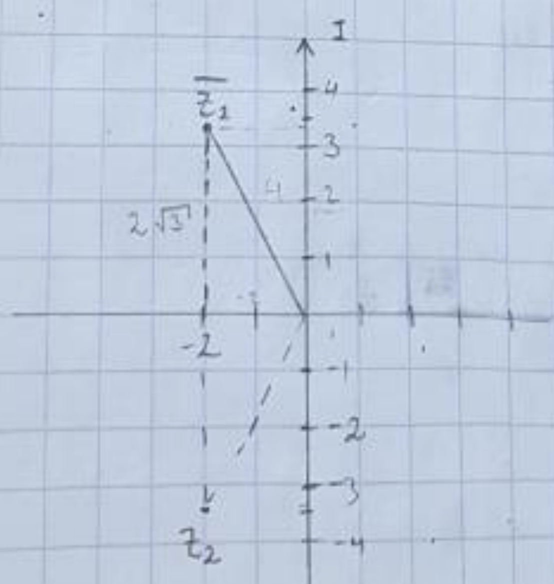


b) $z_2 = 4$ cis $240°$ $r = 4$ $\theta = 240°$

$a = 4 \cos 240° = -2$ $z_2 = -2 - 2\sqrt{3}\, i$

$b = 4 \operatorname{sen} 240° = -2\sqrt{3}$

$\overline{z_2} = -2 + 2\sqrt{3}\, i$



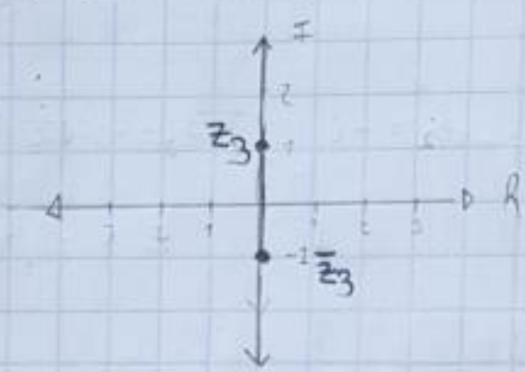
$\overline{z}_2 = -2 + 2\sqrt{3}\, i$

c) $z_3 = e^{-\frac{\pi}{2}i}$ $\quad z_3 = \cos\frac{-\pi}{-2} + i\,\text{sen} -\frac{\pi}{2}$

$r = 1 \quad \theta = -90° = 270$

$a = \cos(-90°) = 0 \qquad z_3 = -i \;;\; \bar{z}_3 = i$

$b = \text{Sen}(-90°) = -1$

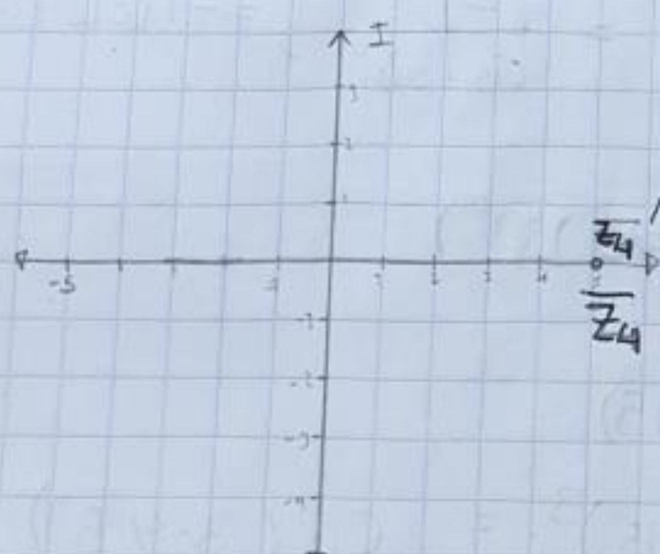


d) $z_4 = 5e^{2\pi i}$ $\quad z_4 = 5(\cos 2\pi + i\,\text{sen}\; 2\pi)$

$r = 5 \quad \theta = 2\pi = 360°$

$a = 5\cos(360°) = 5 \qquad z_4 = 5 +$

$b = 5\,\text{Sen}(360°) = 0 \qquad \overline{z_4} = 5 \text{ cis } 0°$



$= 5 \text{cis } 0°$

$\overline{z_4} = 5 \text{ cis } 0°$

31. Obtener el valor o los valores de $z \in \mathbb{C}$ que satisfacen la ecuación

$$(\sqrt[3]{z})(z_1) = \frac{3i(\overline{z_2}) - \overline{z_3}}{(z_4)\, i^{21}} \qquad i^{21} = i$$

donde

$$z_1 = \text{cis } 360° \; ; \quad z_2 = -i \quad ; \quad z_3 = -3 - \sqrt{2}\, i$$

$$y \quad z_4 = \sqrt{2}\; e^{\frac{3}{4}\pi i}$$

$$\sqrt[3]{z} = \frac{-3 + 3 - \sqrt{2}\, i}{(\sqrt{2}\, e^{3/4 \pi i})i}$$

$$\sqrt[3]{z} = \frac{-\sqrt{2}\, i}{(\sqrt{2}\, e^{3/4\pi i})i} \; ; \; = \frac{-1}{e^{3/4\pi i}} \; ; \; = \frac{-1}{\text{cis } 135°}$$

$$= -1(\text{cis } 135°)^{-1} \; ; \; = \text{cis}(-135° + 180°)$$

$$\sqrt[3]{z} = \text{cis}(45°)$$

$$z = \text{cis}(45°)^3 \; , \; z = \text{cis}(3 \cdot 45°)$$

$$\boxed{z = \text{cis } 135°}$$

35. Determinar $z \in \mathbb{C}$, que satisfacen la ecuación

$\text{Real}(z) = \frac{z+\bar{z}}{2}$ $\quad \text{Im}(z) = \frac{z-\bar{z}}{2}$

$$2\bar{z} = (3-3i)^2 \left(\frac{1}{9} \text{ cis } 300°\right)\left(e^{\frac{\pi}{6}i}\right) + z$$

$$2\bar{z} - z = (9 + 18i - 9)\ \frac{1}{9} \text{ cis } 330°$$

$$2\bar{z} - z = 18i \ (1/9 \text{ cis } 330°)$$

$$2\bar{z} - z = 18 \text{ cis } 90° \ (1/9 \text{ cis } 330°)$$

$$2\bar{z} - z = 2 \text{ cis } 60°$$

$z = x + iy$
$\bar{z} = x - iy$

$$2(x-iy) - (x+iy) = 2 \text{ cis } 60°$$

$$2x - 2yi - x - iy = 2 \text{ cis } 60°$$

$$x - 3yi = 1 + \sqrt{3}\, i$$

$$x = -1 \ ; \ -3y = \sqrt{3}$$

$$y = +\frac{\sqrt{3}}{3}$$

$$\boxed{z = -1 + \frac{\sqrt{3}}{3} i}$$

$z = 1 - \frac{\sqrt{3}}{3} i$

$\bar{z} = 1 + \frac{\sqrt{3}}{3}$

$R = \frac{1+1}{-2} = -1$ ✓

$I = \frac{-\frac{\sqrt{3}}{3} - \frac{\sqrt{3}}{3}}{-2} = \frac{-\frac{2\sqrt{3}}{3}}{-2}$

$I = +\frac{\sqrt{3}}{3}$

37. Obtener el valor o los valores de $w \in \mathbb{C}$ que satisfacen la ecuación

$$\frac{w^4 \cdot \left(3\sqrt{2} \text{ cis } 15^\circ\right)}{4e^{\frac{\pi}{2}i}} = (8 \text{ cis } 60^\circ)^2 (3+3i)$$

$$w^4 \left(3\sqrt{2} \text{ cis } 15^\circ\right) = \left(8^2 \text{ cis } ((2)60^\circ)\right) \left(3\sqrt{2} \text{ cis } 45^\circ\right)$$

$$w^4 = \frac{(4 \text{ cis } 90^\circ)(64 \text{ cis } 120^\circ)(\text{cis } 45^\circ)}{\text{cis } 15^\circ}$$

$$w^4 = \frac{256 \text{ cis}(90^\circ + 120^\circ + 45^\circ)}{\text{cis } 15^\circ}$$

$$w^4 = \frac{256 \text{ cis}(255^\circ)}{\text{cis } 15^\circ} \; ; \; = 256 \text{ cis}(255 - 15)$$

$$w^4 = 256 \text{ cis } 240^\circ$$

$W = \sqrt[4]{256 \quad \text{cis } 240^\circ}$

1. $K = 0$

$w = 4 \text{ cis}(60^\circ + 90^\circ K) = 4 \text{ cis}(60^\circ)$

$W_1 = 4 \text{ cis } 60^\circ$

2. $K = 1$

$W_2 = 4 \text{ cis } 150^\circ$

3. $K = 2$

$W = 4 \text{ cis } 240^\circ$

4. $K = 3$

$W = 4 \text{ cis } 330^\circ$

39. Sean $z_1 = 20\, e^{\pi i}$, $z_2 = 5 \text{cis } 45°$; $z_3 = 8 + 8\sqrt{3}\, i$

$z_4 = 4 \text{cis} 135°$. Obtener los valores de $z \in \mathbb{C}$, en forma polar, que satisfacen la ecuación

$$z^4 z_1 = z_2 z_3 z_4$$

$z_1 = 20 \text{ cis } 180$, $z_3 = 16 \text{ cis } 60°$

$z^4 z_1 = (5 \text{cis} 45°)(256 \text{cis } 60°)(4 \text{ cis } 135°)$

$z^4 z_1 = 20 \cdot 256 \text{ cis } 240°$

$z^4 \cdot 20 \text{ cis } 180° = 20 \cdot 256 \text{ cis } 240°$

$z^4 = 256 \text{ cis}(240°) / \text{cis } 180°$

$z^4 = 256 \text{ cis } 60°$; $z = \sqrt[4]{256 \text{ cis } 60°}$

$\sqrt[4]{r} \quad \dfrac{\alpha + 360 K}{n} \qquad K = 0, 1, 2, \ldots, n-1$

1 $\sqrt[4]{256}\ \dfrac{60° + 360(0)}{4} = 4_{15°}$

2 $\dfrac{60 + 360(1)}{4} = 4_{105°}$

3 $(2) = 4_{195}$

4 $(3) = 4_{285}$

$z_1 = 4 \text{ cis } 15°$

$z_2 = 4 \text{ cis } 105°$

$z_3 = 4 \text{ cis } 195°$

$z_4 = 4 \text{ cis } 285°$

41. Obtener $z \in \mathbb{C}$, en forma polar, que satisfacen la ecuación.

$$(4+3i)z^{3/2} - \sqrt{2}\ \text{cis}\ 45°(-1-i) = -e^{\frac{\pi}{2}i}\ z^{3/2}$$

$$(4+3i)z^{3/2} - \sqrt{2}\ \text{cis}\ 45°(\sqrt{2}\ \text{cis}\ 225°) = -\text{cis}\ 90°\ z^{3/2}$$

$$-2\ \text{cis}\ 270 = -\text{cis}\ 90°\ z^{3/2}$$

$$+\text{cis}\ 90\ z^{3/2} = 2\ \text{cis}\ 270°$$

$$z^{3/2}(4+3i+i) = 2\ \text{cis}\ 270°$$

$$z^{3/2}(4+4i) = 2\ \text{cis}\ 270°$$

$$z^{3/2} = \frac{2\ \text{cis}\ 270°}{\sqrt{32}\ \text{cis}\ 45°} = \frac{2\ \text{cis}\ 270°}{4\sqrt{2}\ \text{cis}\ 45°} = \frac{\text{cis}\ 225°}{2\sqrt{2}}$$

$$z^{3/2} = \frac{1}{2\sqrt{2}}\ \text{cis}\ 225°$$

$$z^{3} = \left(\frac{1}{2\sqrt{2}}\ \text{cis}\ 225°\right)^{2} = \frac{1}{8}\ \text{cis}\ 90°$$

$$z^{3} = \frac{1}{8}\ \text{cis}\ (450-360) = \frac{1}{8}\ \text{cis}\ 90°$$

$$z^{3} = \frac{1}{8}\ \text{cis}\ 90° \qquad z = \sqrt[3]{\frac{1}{8}\ \text{cis}\ 90°}$$

$$= \sqrt[3]{\frac{1}{8}}\ \text{cis}\ \left(\frac{90° + 360° k}{3}\right)$$

Con: $K = 0, 1, 2$

$$z = \frac{1}{2} \operatorname{cis} (30^\circ + 120 k)$$

Entre ellas 120°

$$z_1 = \frac{1}{2} \operatorname{cis} 30^\circ$$

$$z_2 = \frac{1}{2} \operatorname{cis} 150^\circ$$

$$z_3 = \frac{1}{2} \operatorname{cis} 270^\circ$$

43. Obtener $w \in \mathbb{C}$, en forma polar, que satisfacen la ecuación

$$\frac{z_1 w^{\frac{3}{2}} - 2z_3}{4z_2} = 2z_1 + 3z_2$$

donde $z_1 = 3-2i$, $z_2 = 4 \operatorname{cis} \frac{\pi}{6}$ y $z_3 = 2e^{\pi i}$.

$$\frac{z_1 w^{3/2} - 2z_3}{4z_2} = 2(3-2i) + 3(2\sqrt{3} + 2i)$$

$$\frac{3-2i(w^{3/2}) - 2(-2)}{4(2\sqrt{3}+2i)} = 6-4i + 6\sqrt{3} + 6i$$

$$\frac{3w^{3/2} - 2iw^{3/2} + 4}{8\sqrt{3}+8i} = 6 + 6\sqrt{3} + 2i$$

$$3w^{3/2} - 2iw^{3/2} + 4 = (8\sqrt{3} + 8i)(6 + 6\sqrt{3} + 2i)$$

$$w^{3/2}(3-2i) + 4 = 16 \operatorname{cis} 30° \,(16.514 \operatorname{cis} 6.956°)$$

$$= 264.224 \operatorname{cis} 36.956 + 4 \operatorname{cis} 180°$$

$$w^{3/2}(3-2i) = 207.127 + 158.85i$$

$$= 261.021 \operatorname{cis} 37.483°$$

$$w^{3/2} = \frac{261.021 \text{ cis } 37.483^\circ}{\sqrt{13} \text{ cis } 326.309^\circ}$$

$$w^{3/2} = 72.394 \text{ cis } (-288.826^\circ)$$

$$w^{3} = [(72.394 \text{ cis } (71.174)]^{2}$$

$$w^{3} = 5240.891 \text{ cis } 142.348^\circ$$

$$w = \sqrt[3]{5240.891 \text{ cis } 142.348^\circ}$$

$$w = 17.370 \text{ cis } \left(\frac{142.348^\circ + 360^\circ K}{3}\right) \qquad K = 0, 1, 2$$

$K=0$

① $17.370 \text{ cis } 47.449^\circ$

$K=1$

② $17.370 \text{ cis } 167.449^\circ$

$K=2$

③ $17.370 \text{ cis } 287.449^\circ$

$$w_1 = 17.370 \text{ cis } 47.449^\circ$$

$$w_2 = 17.370 \text{ cis } 167.449^\circ$$

$$w_3 = 17.370 \text{ cis } 287.449^\circ$$