Universidad Nacional Autónoma de México

Facultad de Ingeniería

División de Ciencias Básicas

Álgebra (1120)

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Semestre 2021-1

SERIE 3

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Grupo: 28

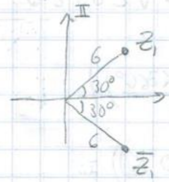
Cd. Universitaria a 21 de noviembre de 2020.

SERIE TEMA 3: "NÚMEROS COMPLEJOS"

19. Por simple inspección determinar el conjugado de los siguientes números.

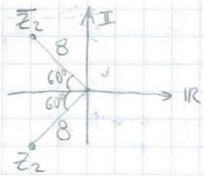
a) $z_1 = 6 \text{ cis } 30°$

$\boxed{\bar{z}_1 = 6 \text{ cis } 330°}$



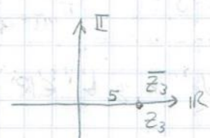
b) $z_2 = 8 \text{ cis } 240°$

$\boxed{\bar{z}_2 = 8 \text{ cis } 120°}$



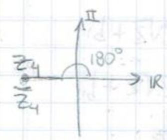
c) $z_3 = 5 \cos 0°$

$\boxed{\bar{z}_3 = 5 \text{ cis } 0°}$



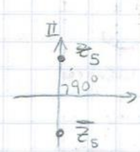
d) $z_4 = 25 \text{ cis } 180°$

$\boxed{\bar{z}_4 = 25 \text{ cis } 180°}$



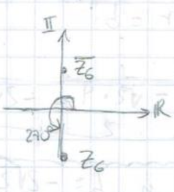
e) $z_5 = 100 \text{ cis } 90°$

$\boxed{\bar{z}_5 = 100 \text{ cis } 270°}$



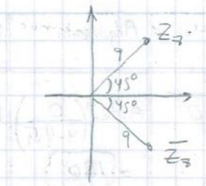
f) $z_6 = 40 \text{ cis } 270°$

$\boxed{\bar{z}_6 = 40 \text{ cis } 90°}$



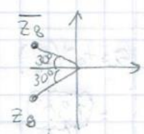
g) $z_7 = 9 \text{ cis } 45°$

$\overline{z_7} = 9 \text{ cis } 315°$

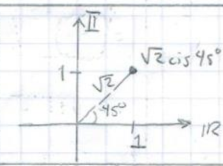
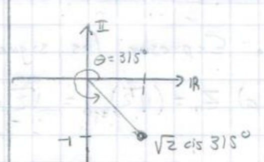
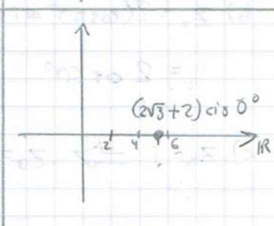


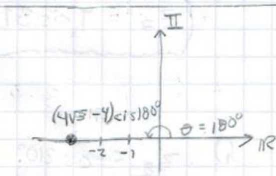
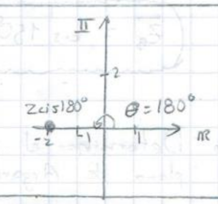
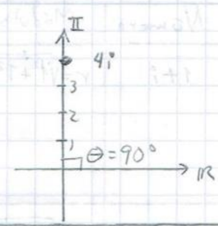
h) $z_8 = \text{cis } 210°$

$\overline{z_8} = \text{cis } 150°$



21.- Determinar el módulo del argumento, la forma polar y la representación en el plano de Argand de los siguientes números.

Número	Módulo	Argumento	Forma polar	Gráfica
$1+i$	$r=\sqrt{1^2+1^2}=\sqrt{2}$	$\theta=\tan^{-1}\left(\frac{1}{1}\right)$ $=45°$	$\sqrt{2} \text{ cis } 45°$	
$1-i$	$r=\sqrt{1^2+(-1)^2}=\sqrt{2}$	$\theta=\tan^{-1}\left(\frac{-1}{1}\right)$ $=\tan^{-1}(-1)$ $=-45°$ $=315°$	$\sqrt{2} \text{ cis } 315°$	
$2\sqrt{3}+2$	$r=\sqrt{(2\sqrt{3}+2)^2+0}$ $=2\sqrt{3}+2$	$\theta=\tan^{-1}\left(\frac{0}{2\sqrt{3}+2}\right)$ $=\tan^{-1}(0)$ $=0°$	$(2\sqrt{3}+2) \text{ cis } 0°$	

Número	Módulo	Argumento	Forma polar	Gráfica
$4-4\sqrt{3}$	$r=\sqrt{(4-4\sqrt{3})^2+0^2}$ $=\lvert 4-4\sqrt{3}\rvert$ $=4\sqrt{3}-4$	$\theta=\tan^{-1}\left(\frac{0}{4-4\sqrt{3}}\right)$ $=180^\circ$	$(4\sqrt{3}-4)\,\text{cis}\,180^\circ$	
-2	$r=\sqrt{(-2)^2+0^2}=$ $=2$	$\theta=\tan^{-1}\left(\frac{0}{-2}\right)$ $=180^\circ$	$2\,\text{cis}\,180^\circ$	
$4i$	$r=\sqrt{0^2+4^2}$ $=4$	$\theta=\tan^{-1}\left(\frac{4}{0}\right)$ $\theta=90^\circ$	$4\,\text{cis}\,90^\circ$	

23.- Expresar los siguientes números complejos en forma exponencial.

a) $z_1=(\sqrt{2})_{45^\circ}=\sqrt{2}\,\text{cis}\,45^\circ \Longrightarrow \boxed{z_1=\sqrt{2}\,e^{\frac{\pi}{4}i}}$

b) $z_2=2(\cos 60^\circ + i\,\text{sen}\,60^\circ)$

$= 2\,\text{cis}\,60^\circ \Longrightarrow \boxed{z_2=2\,e^{\frac{\pi}{3}i}}$

c) $z_3=i \Longrightarrow z_3=\text{cis}\,90^\circ \Longrightarrow \boxed{z_3=e^{\frac{\pi}{2}i}}$

25. - Sean $z_1 = 3e^{i\frac{\pi}{2}}$, $z_2 = \sqrt{2}\, e^{i\frac{7\pi}{4}}$. Resolver las siguientes operaciones en forma exponencial

a) $z_1 \cdot z_2 = (3e^{i\frac{\pi}{2}})(\sqrt{2}\, e^{i\frac{7\pi}{4}})$

$= 3\sqrt{2}\; e^{(\frac{\pi}{2} + \frac{7}{4}\pi)i}$

$= 3\sqrt{2}\; e^{\frac{9}{4}\pi i}$

$= 3\sqrt{2}\; e^{(\frac{9}{4}\pi - 2\pi)i}$

$$\boxed{z_1 \cdot z_2 = 3\sqrt{2}\; e^{\frac{\pi}{4}i}}$$

b) $\dfrac{z_1}{z_2} = \dfrac{3e^{i\frac{\pi}{2}}}{\sqrt{2}\, e^{i\frac{7\pi}{4}}}$

$= \dfrac{3}{\sqrt{2}}\; e^{(\frac{\pi}{2} - \frac{7}{4}\pi)i}$

$= \dfrac{3}{\sqrt{2}}\; e^{(-\frac{5}{4}\pi)i}$

$= \dfrac{3}{\sqrt{2}}\; e^{(-\frac{5}{4}\pi + 2\pi)i}$

$$\boxed{\frac{z_1}{z_2} = \frac{3\sqrt{2}}{2}\; e^{\frac{3}{4}\pi i}}$$

c) $z_1^{10} = (3e^{i\frac{\pi}{2}})^{10}$

$= 3^{10}\, e^{10(\frac{\pi}{2})i}$

$= 3^{10}\, e^{5\pi i}$

$= 3^{10}\, e^{[5\pi - 2(2\pi)]i}$

$$\boxed{z_1^{10} = 3^{10}\, e^{\pi i}}$$

27.- Encontrar el módulo y el argumento de las siguientes expresiones y expresar el resultado en forma exponencial.

a) $\frac{3i^{30} - i^{19}}{2i - 1} = \frac{3(i^2)^{15} - i^{18} \cdot i}{2i - 1}$

$= \frac{3(-1)^{15} - (i^2)^9 i}{2i - 1}$

$= \frac{3(-1) - (-1)^9 i}{2i - 1}$

$= \frac{-3 + i}{-1 + 2i} \cdot \frac{-1 - 2i}{-1 - 2i}$

$= \frac{(3 + 2) + (6 - 1)i}{1 + 4}$

$= \frac{5 + 5i}{5} = 1 + i = z$

$r = \sqrt{1^2 + 1^2} = \sqrt{2}$

$\boxed{r = \sqrt{2}}$

$\theta = \text{angtan}\left(\frac{1}{1}\right)$

$\theta = 45°$

$\boxed{\theta = \frac{\pi}{4}}$

$\boxed{\therefore \; z = \sqrt{2}\, e^{\frac{\pi}{4} i}}$

b) $\left(\frac{1+i}{1-i}\right)^7 = \left(\frac{\sqrt{2}\, cis\, 45^\circ}{\sqrt{2}\, cis(-45^\circ)}\right)^7$

$= \left[1\, cis(45^\circ - (-45^\circ))\right]^7$

$= \left[cis\, 90^\circ\right]^7$

$= \left[cis\, \frac{\pi}{2}\right]^7$

$= \left[e^{\frac{\pi}{2} i}\right]^7$

$= e^{7(\frac{\pi}{2})i}$

$= e^{\frac{7\pi}{2} i}$

$= e^{(\frac{7}{2}\pi - 2\pi)i}$

$\left(\frac{1+i}{1-i}\right)^7 = e^{\frac{3}{2}\pi i}$ $\quad \theta = \frac{3}{2}\pi$ $\quad r = 1$

c) $\left[\sqrt{2}\,(\cos 315^\circ + i\, sen\, 315^\circ)\right]^4 = \left[\sqrt{2}\, cis\, 315^\circ\right]^4$

$= \left[\sqrt{2}\, cis(-45^\circ)\right]^4$

$= (\sqrt{2})^4\, cis(4(-45^\circ))$

$= 4\, cis(-180^\circ)$

$= 4\, cis\, 180^\circ$

$= 4\, cis\, \pi$

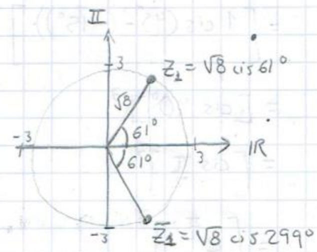
$\left[\sqrt{2}(\cos 315^\circ + i\, sen\, 315^\circ)\right]^4 = 4\, e^{\pi i}$ $\quad \theta = \pi$ $\quad r = 4$

29.- Representar en el diagrama de Argan el conjugado de cada uno de los siguientes números:

a) $z_1 = \sqrt{8}\,\text{cis}\,61^\circ$

$\bar{z}_1 = \sqrt{8}\,\text{cis}\,(360^\circ - 61^\circ)$

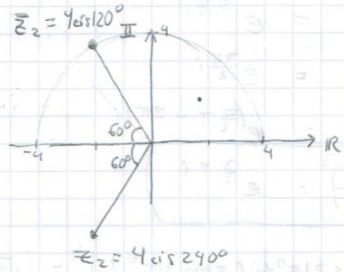
$\boxed{\bar{z}_1 = \sqrt{8}\,\text{cis}\,299^\circ}$



b) $z_2 = 4\,\text{cis}\,240^\circ$

$\bar{z}_2 = 4\,\text{cis}\,(-240^\circ + 360^\circ)$

$\boxed{\bar{z}_2 = 4\,\text{cis}\,(120^\circ)}$



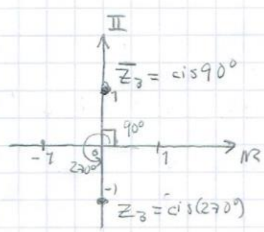
c) $z_3 = e^{-\frac{\pi}{2} i} = \text{cis}(-\frac{\pi}{2})$

$= \text{cis}\,(-90^\circ)$

$= \text{cis}\,(270^\circ)$

$\bar{z}_3 = \text{cis}\,(-270^\circ + 360^\circ)$

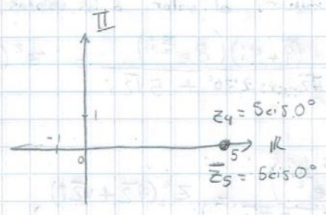
$\boxed{\bar{z}_3 = \text{cis}\,90^\circ}$



d) $z_4 = 5e^{2\pi i}$

$$= 5e^{0i} = 5e^{0}$$
$$= 5$$
$$= 5\,\text{cis}\,0°$$

$$\boxed{\overline{z}_4 = 5\,\text{cis}\,0°}$$



31.- Obtener el valor o los valores de $z \in \mathbb{C}$ que satisfacen la ecuación

$$\left(\sqrt[3]{z}\right)(z_1) = \frac{3i(\overline{z_2}) - \overline{z_3}}{(z_4)\, i^{21}}$$

donde

$$z_1 = \text{cis}\,360°,\quad z_2 = -i,\quad z_3 = -3 - \sqrt{2}i \quad y \quad z_4 = \sqrt{2}\, e^{3/4\pi i}$$

Resolución:

$$\left(\sqrt[3]{z}\right)(z_1) = \frac{3i(\overline{z_2}) - \overline{z_3}}{(z_4)\, i^{21}}$$

$$\rightarrow \sqrt[3]{z} = \frac{3i(\overline{z_2}) - \overline{z_3}}{z_1 \cdot z_4 \cdot i^{21}}$$

$$\rightarrow \sqrt[3]{z} = \frac{3i(\overline{-i}) - (\overline{-3 - \sqrt{2}i})}{(\text{cis}\,360°)(\sqrt{2}\, e^{3/4\pi i}) \cdot (i^2)^{10}\, i}$$

$$\rightarrow = \frac{3i(i) - (-3 + \sqrt{2}i)}{(\sqrt{2}\, e^{3/4\pi i})\,(-1)^{10}\, i}$$

$$\rightarrow = \frac{3(-1) + 3 - \sqrt{2}i}{(\sqrt{2}\, e^{3/4\pi i})\, i}$$

$$\rightarrow = \frac{-3 + 3 - \sqrt{2}i}{(\sqrt{2}\, e^{3/4\pi i})\, i}$$

$$\rightarrow = \frac{-\sqrt{2}i}{(\sqrt{2}\, e^{3/4\pi i})\, i}$$

$$\rightarrow = \frac{-1}{e^{3/4\pi i}}$$

$$\rightarrow = \frac{-1}{(\text{cis}\,135°)}$$

$$= -1\,(\text{cis}\,135°)^{-1}$$

$$= -1\,(\text{cis}(-135°))$$

$$= \text{cis}(-135° + 180°)$$

$$\sqrt[3]{z} = \text{cis}(45°)$$

$$z = (\text{cis}\,45°)^3 = \text{cis}(3 \cdot 45°)$$

$$\boxed{z = \text{cis}\,135°}$$

33. - Determinar el valor o los valores de $z \in \mathbb{C}$ que satisfaga(n) la ecuación

$$\frac{(4+4i)\left(8e^{\frac{\pi}{2}i}\right)}{\sqrt{2}\operatorname{cis}270^\circ + 5\sqrt{2}i} = z^2(\sqrt{2}+\sqrt{2}i)$$

Resolución:

$$\frac{(4+4i)\left(8e^{\frac{\pi}{2}i}\right)}{\sqrt{2}\operatorname{cis}270^\circ + 5\sqrt{2}i} = z^2(\sqrt{2}+\sqrt{2}i)$$

$$\leftrightarrow \frac{(\sqrt{32}\operatorname{cis}45^\circ)(8\operatorname{cis}90^\circ)}{-\sqrt{2}i + 5\sqrt{2}i} = z^2(2\operatorname{cis}45^\circ)$$

$$\leftrightarrow \frac{(4\sqrt{2}\operatorname{cis}\cancel{45^\circ})(8\operatorname{cis}90^\circ)}{4\sqrt{2}i} = z^2(2\operatorname{cis}\cancel{45^\circ})$$

$$\leftrightarrow \frac{8\operatorname{cis}90^\circ}{i} = 2z^2$$

$$\leftrightarrow \frac{8\cancel{\operatorname{cis}90^\circ}}{2\cancel{\operatorname{cis}90^\circ}} = z^2$$

$$\leftrightarrow 4 = z^2$$

$$\leftrightarrow z^2 = 4$$

$$\leftrightarrow z = \pm\sqrt{4} = \pm 2$$

$$\boxed{\begin{aligned} z_1 &= 2+0i = 2\operatorname{cis}0^\circ = 2e^{0i} \\ z_2 &= -2+0i = 2\operatorname{cis}180^\circ = 2e^{\pi i} \end{aligned}}$$

35.- Determinar $z \in \mathbb{C}$, que satisfacen la ecuación

$$2\bar{z} = (3-3i)^2\left(\frac{1}{9}\operatorname{cis} 300^\circ\right)\left(e^{\frac{\pi}{6}i}\right) + z$$

Resolución:

$$2\bar{z} = (3-3i)^2\left(\frac{1}{9}\operatorname{cis} 300^\circ\right)\left(e^{\frac{\pi}{6}i}\right) + z$$

$$\leftrightarrow\ 2\bar{z} = \left(\sqrt{18}\operatorname{cis} 315^\circ\right)^2\left(\frac{1}{9}\operatorname{cis} 300^\circ\right)\left(\operatorname{cis} 30^\circ\right) + z$$

$$\leftrightarrow\ 2\bar{z} = \left(\sqrt{18}\operatorname{cis}(-45^\circ)\right)^2\left(\frac{1}{9}\operatorname{cis}(-60^\circ)\right)\left(\operatorname{cis} 30^\circ\right) + z$$

$$\leftrightarrow\ 2\bar{z} = \left[(\sqrt{18})^2\operatorname{cis}(-90^\circ)\right]\left(\frac{1}{9}\operatorname{cis}(-30^\circ)\right) + z$$

$$\leftrightarrow\ 2\bar{z} = 18\operatorname{cis}(-90^\circ)\cdot\frac{1}{9}\operatorname{cis}(-30^\circ) + z$$

$$\leftrightarrow\ 2\bar{z} = 2\operatorname{cis}(-90^\circ - 30^\circ) + z$$

$$\leftrightarrow\ 2\bar{z} = 2\operatorname{cis}(-120^\circ) + z$$

$$\leftrightarrow\ 2\bar{z} - z = 2\operatorname{cis}(240^\circ)$$

$$\leftrightarrow\ 2(a-bi) - (a+bi) = 2\operatorname{cis}(240^\circ)$$

$$\leftrightarrow\ 2a - 2bi - a - bi = 2\operatorname{cis}(240^\circ)$$

$$\leftrightarrow\ a - 3bi = 2\cos 240^\circ + 2\operatorname{sen} 240^\circ i$$

$$\leftrightarrow\ a - 3bi = -2\left(\frac{1}{2}\right) + 2\left(-\frac{\sqrt{3}}{2}\right)i$$

$$\leftrightarrow\ a - 3bi = -1 - \sqrt{3}i$$

$$\Rightarrow\ a = -1 \quad y \quad -3b = -\sqrt{3}$$

$$b = \frac{-\sqrt{3}}{-3} = \frac{\sqrt{3}}{3}$$

$$b = \frac{\sqrt{3}}{3}$$

$$\therefore\ z = -1 + \frac{\sqrt{3}}{3}i$$

37.- Obtener el valor o los valores de $w \in \mathbb{C}$ que satisfacen la ecuación:

$$\frac{w^4(3\sqrt{2}\operatorname{cis}15^\circ)}{4e^{\pi/2 i}} = (8\operatorname{cis}60^\circ)^2(3+3i)$$

Resolución:

$$\frac{w^4(3\sqrt{2}\operatorname{cis}15^\circ)}{4e^{\pi/2 i}} = (8\operatorname{cis}60^\circ)^2(3+3i)$$

$$\longleftrightarrow \frac{w^4(3\sqrt{2}\operatorname{cis}15^\circ)}{4\operatorname{cis}90^\circ} = (64\operatorname{cis}120^\circ)(\sqrt{18}\operatorname{cis}45^\circ)$$

$$\longleftrightarrow w^4 = \frac{(4\operatorname{cis}90^\circ)(64\operatorname{cis}120^\circ)(\sqrt{18}\operatorname{cis}45^\circ)}{(3\sqrt{2}\operatorname{cis}15^\circ)}$$

$$\longleftrightarrow w^4 = \frac{(4\operatorname{cis}90^\circ)(64\operatorname{cis}120^\circ)(\cancel{3\sqrt{2}}\operatorname{cis}45^\circ)}{(\cancel{3\sqrt{2}}\operatorname{cis}15^\circ)}$$

$$\longleftrightarrow w^4 = \frac{256\operatorname{cis}(90^\circ+120^\circ+45^\circ)}{\operatorname{cis}15^\circ}$$

$$\longleftrightarrow w^4 = \frac{256\operatorname{cis}(255^\circ)}{\operatorname{cis}15^\circ}$$

$$\longleftrightarrow w^4 = 256\operatorname{cis}(255^\circ - 15^\circ)$$

$$\longleftrightarrow w^4 = 256\operatorname{cis}(240^\circ)$$

$$\longleftrightarrow w = \sqrt[4]{256\operatorname{cis}240^\circ} = \sqrt[4]{256}\operatorname{cis}\left(\frac{240^\circ + K\cdot 360^\circ}{4}\right) \; ; \text{ con } K = 0,1,2,3$$

$$w_1 = 4\operatorname{cis}(60^\circ + 90^\circ K) = 4\operatorname{cis}(60^\circ)$$

$w_1 = 4\operatorname{cis}60^\circ$ Nota: Cada raíz está espaciada 90°, AST:

$w_2 = 4\operatorname{cis}150^\circ$

$w_3 = 4\operatorname{cis}240^\circ$

$w_4 = 4\operatorname{cis}330^\circ$

39.- Sean $z_1 = 20e^{\pi i}$, $z_2 = 5 \operatorname{cis} 45°$, $z_3 = 8 + 8\sqrt{3}\,i$ y $z_4 = 4 \operatorname{cis} 135°$. Obtener los valores de $z \in \mathbb{C}$, en forma polar, que satisfacen la ecuación

$$z^4 z_1 = z_2 z_3 z_4$$

Resolución:

$$z^4 \cdot z_1 = z_2 \cdot z_3 \cdot z_4$$

$$\leftrightarrow z^4 \cdot z_1 = (5 \operatorname{cis} 45°)(8 + 8\sqrt{3}i)(4 \operatorname{cis} 135°)$$

$$\leftrightarrow z^4 z_1 = (5 \operatorname{cis} 45°)(256 \operatorname{cis} 60°)(4 \operatorname{cis} 135°)$$

$$\leftrightarrow z^4 \cdot z_1 = 4 \cdot 5 \cdot 256 \operatorname{cis}(45° + 60° + 135°)$$

$$\leftrightarrow z^4 \cdot z_1 = 4 \cdot 5 \cdot 256 \operatorname{cis}(240°)$$

$$\leftrightarrow z^4 \cdot 20e^{\pi i} = 20 \cdot 256 \operatorname{cis}(240°)$$

$$\leftrightarrow z^4 \cdot e^{\pi i} = 256 \operatorname{cis}(240°)$$

$$\leftrightarrow z^4 \cdot \operatorname{cis} 180° = 256 \operatorname{cis}(240°)$$

$$\leftrightarrow z^4 = \frac{256 \operatorname{cis}(240°)}{\operatorname{cis} 180°}$$

$$\leftrightarrow z^4 = 256 \operatorname{cis} 60°$$

$$\leftrightarrow z = \sqrt[4]{256 \operatorname{cis} 60°} = \sqrt[4]{256} \operatorname{cis}\left(\frac{60° + k360°}{4}\right); \text{ con } K = 0, 1, 2, 3.$$

$$\leftrightarrow z = 4 \operatorname{cis}(15° + 90°K); \; K = 0, 1, 2, 3.$$

Nota: Cada raíz está espaciada 90°

$\boxed{z_1 = 4 \operatorname{cis} 15°}$

$\boxed{z_2 = 4 \operatorname{cis} 105°}$

$\boxed{z_3 = 4 \operatorname{cis} 195°}$

$\boxed{z_4 = 285°}$

41.- Obtener $z \in \mathbb{C}$, en forma polar, que satisfacen la ecuación

$$(4+3i)z^{3/2} - \sqrt{2}\operatorname{cis}45°(-1-i) = -e^{\frac{\pi}{2}i} z^{3/2}$$

Resolución

$$(4+3i)z^{3/2} - \sqrt{2}\operatorname{cis}45°(-1-i) = -e^{\frac{\pi}{2}i} z^{3/2}$$

$$\leftrightarrow (4+3i)z^{3/2} - \sqrt{2}\operatorname{cis}45°(\sqrt{2}\operatorname{cis}225°) = -\operatorname{cis}90° z^{3/2}$$

$$\leftrightarrow (4+3i)z^{3/2} - 2\operatorname{cis}270° = -\operatorname{cis}90° z^{3/2}$$

$$\leftrightarrow (4+3i)z^{3/2} + \operatorname{cis}90° z^{3/2} = 2\operatorname{cis}270°$$

$$\leftrightarrow (4+3i)z^{3/2} + i\cdot z^{3/2} = 2\operatorname{cis}270°$$

$$\leftrightarrow z^{3/2}(4+3i+i) = 2\operatorname{cis}270°$$

$$\leftrightarrow z^{3/2}(4+4i) = 2\operatorname{cis}270°$$

$$\leftrightarrow z^{3/2}\cdot(\sqrt{32}\operatorname{cis}45°) = 2\operatorname{cis}270°$$

$$\leftrightarrow z^{3/2} = \frac{2\operatorname{cis}270°}{\sqrt{32}\operatorname{cis}45°} = \frac{2\operatorname{cis}270°}{4\sqrt{2}\operatorname{cis}45°} = \frac{\operatorname{cis}225°}{2\sqrt{2}}$$

$$\leftrightarrow z^{3/2} = \frac{1}{2\sqrt{2}}\operatorname{cis}225°$$

$$\leftrightarrow z^{3} = \left(\frac{1}{2\sqrt{2}}\operatorname{cis}225°\right)^2 = \frac{1}{8}\operatorname{cis}(2\cdot 225°) = \frac{1}{8}\operatorname{cis}(450°)$$

$$\leftrightarrow z^{3} = \frac{1}{8}\operatorname{cis}(450° - 360°) = \frac{1}{8}\operatorname{cis}90°$$

$$\leftrightarrow z = \sqrt[3]{\frac{1}{8}\operatorname{cis}90°} = \sqrt[3]{\frac{1}{8}}\operatorname{cis}\left(\frac{90° + 360°K}{3}\right);\ \text{con } K = 0, 1, 2$$

$$\longleftrightarrow z = \frac{1}{2}\operatorname{cis}(30° + 120°K);\ \text{con } K = 0, 1, 2.$$

Nota: Cada raíz está espaciada 120°. Así:

$$\boxed{z_1 = \frac{1}{2}\operatorname{cis}30°} \qquad \boxed{z_2 = \frac{1}{2}\operatorname{cis}150°} \qquad \boxed{z_3 = \frac{1}{2}\operatorname{cis}270°}$$

43.- Obtener $w \in \mathbb{C}$, en forma polar, que satisfacen la ecuación

$$\frac{z_1 w^{3/2} - 2z_3}{4z_2} = 2z_1 + 3z_2$$

donde. $z_1 = 3-2i$, $z_2 = 4 \operatorname{cis} \frac{\pi}{6}$, y $z_3 = 2e^{\pi i}$. (Utilizar calculadora).

Resolución:

$$\frac{z_1 w^{3/2} - 2z_3}{4z_2} = 2z_1 + 3z_2$$

$$\leftrightarrow \frac{(3-2i)\, w^{3/2} - 2(2e^{\pi i})}{4(4 \operatorname{cis} \frac{\pi}{6})} = 2(3-2i) + 3(4 \operatorname{cis} \frac{\pi}{6})$$

$$\leftrightarrow \frac{(3-2i)\, w^{3/2} - 4e^{\pi i}}{16 \operatorname{cis} \frac{\pi}{6}} = 6 - 4i + 12 \operatorname{cis} 30°$$

$$\leftrightarrow \frac{(3-2i)\, w^{3/2} - 4e^{\pi i}}{16 \operatorname{cis} 30°} = 6 - 4i + 12\left(\frac{\sqrt{3}}{2}\right) + 12\left(\frac{1}{2}\right)i$$

$$\leftrightarrow \frac{(3-2i)\, w^{3/2} - 4e^{\pi i}}{16 \operatorname{cis} 30°} = 6 - 4i + 6\sqrt{3} + 6i$$

$$\leftrightarrow (3-2i)\, w^{3/2} - 4e^{\pi i} = 16 \operatorname{cis} 30° \left(6 + 6\sqrt{3} + 2i\right)$$

$$\leftrightarrow (3-2i)\, w^{3/2} - 4e^{\pi i} = 16 \operatorname{cis} 30° \left(16.513 \operatorname{cis} 6.956°\right)$$

$$\leftrightarrow (3-2i)\, w^{3/2} - 4e^{\pi i} = 264.208 \operatorname{cis}(36.956°)$$

$$\leftrightarrow (3-2i)\, w^{3/2} = 264.208 \operatorname{cis} 36.956° + 4e^{\pi i}$$

$$\leftrightarrow (3-2i)\, w^{3/2} = 264.208 \operatorname{cis} 36.956° + 4 \operatorname{cis} 180°$$

$$\leftrightarrow (3-2i)\, w^{3/2} = 211.127 + 158.842i - 4$$

$$\leftrightarrow (3-2i)\, w^{3/2} = 207.127 + 158.842i$$

$$\leftrightarrow (3-2i)\, w^{3/2} = 261.021 \operatorname{cis} 37.483°$$

$\longleftrightarrow (3-2i)\,w^{3/2} = 261.021 \operatorname{cis} 37.483^\circ$

$\longrightarrow w^{3/2} = \dfrac{261.021 \operatorname{cis} 37.483^\circ}{3-2i}$

$\longrightarrow w^{3/2} = \dfrac{261.021 \operatorname{cis} 37.483^\circ}{\sqrt{13} \operatorname{cis} 326.309^\circ}$

$\longrightarrow w^{3/2} = 72.394 \operatorname{cis} (-288.826^\circ)$

$\longrightarrow w^{3/2} = 72.394 \operatorname{cis} (71.174^\circ)$

$\longrightarrow w^{3} = [72.394 \operatorname{cis} (71.174^\circ)]^2$

$\longrightarrow w^{3} = 5240.891 \operatorname{cis} 142.348^\circ$

$\longrightarrow w = \sqrt[3]{5240.891 \operatorname{cis} 142.348^\circ}$

$\longrightarrow w = \sqrt[3]{5240.891} \operatorname{cis} \left(\dfrac{142.348^\circ + 360^\circ K}{3}\right)$; con $K = 0, 1, 2$.

$\longrightarrow w = 17.370 \operatorname{cis} \left(47.449^\circ + 120^\circ K\right)$; con $k = 0, 1, 2$

Nota: Cada raíz está espaciada 120°. Así:

$$\boxed{\begin{aligned} w_1 &= 17.370 \operatorname{cis} (47.449^\circ) \\ w_2 &= 17.370 \operatorname{cis} (167.449^\circ) \\ w_3 &= 17.370 \operatorname{cis} (287.449^\circ) \end{aligned}}$$