DEPARTAMENTO DE ELETRÓNICA, TELECOMUNICAÇÕES
E INFORMÁTICA

MESTRADO INTEGRADO EM ENG. DE COMPUTADORES E TELEMÁTICA

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DESEMPENHO E DIMENSIONAMENTO DE REDES

ASSIGNMENT GUIDE NO. 1

APPLICATION EXAMPLES OF PROBABILITIES, RANDOM VARIABLES AND MARKOV CHAINS

Assignment Guide No. 1

Assignment Description

Implement the following tasks using MATLAB to obtain the requested numerical solutions and conclusions. At the end, submit a report with the answers to the questions of the tasks requested for reporting including the numerical results, the MATLAB codes duly explained and the requested conclusions.

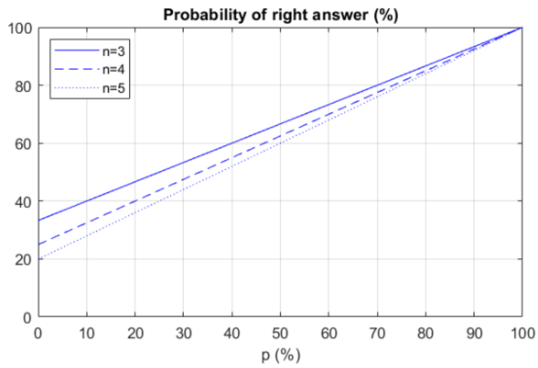
Task 1

Consider a multiple choice test such that each question has n possible answers and only one is correct. Assume that the student has studied a percentage p (with $0\% \leq p \leq 100\%$) of the test content. When a question addresses the content the student has studied, he selects the right answer with 100% of probability. Otherwise, he selects randomly one of the n answers with a uniform distribution.

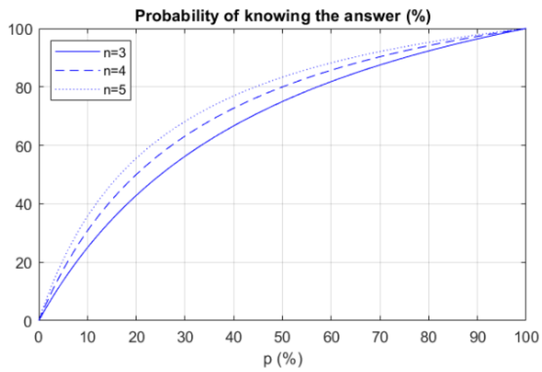
1.a. When $p = 60\%$ and $n = 4$, determine the probability of the student to select the right answer. Answer: 70%

1.b. When $p = 70\%$ and $n = 5$, determine the probability of the student to known the answer when he selects the right answer. Answer: 92.1%

1.c. Draw a plot with the same look as the answer below of the probability of the student to select the right answer as a function of the probability p (consider $n = 3, 4$ and 5). Answer:



1.d. Draw a plot with the same look as the answer below of the probability of the student to know the answer when he selects the right answer as a function of the probability p (consider $n = 3, 4$ and 5). Answer:



Task 2

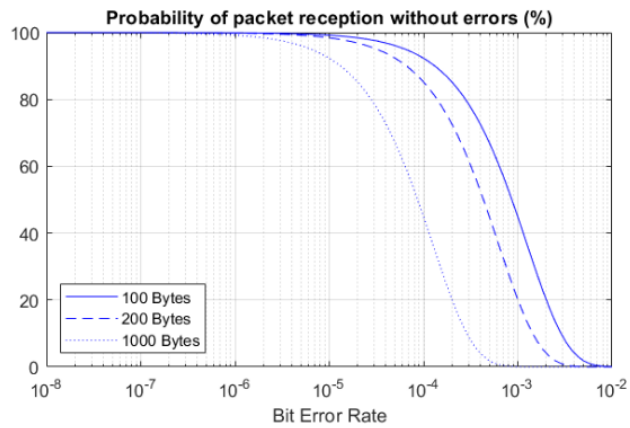
Consider a wireless link between multiple stations for data communications with a bit error rate (*ber*) of p. Assume that errors in the different bits of a data packet are statistically independent (i.e., the number of errors of a data packet is a binomial random variable).

2.a. Determine the probability of a data packet of 100 Bytes to be received without errors when $p = 10^{-2}$. Answer: 0.0322%

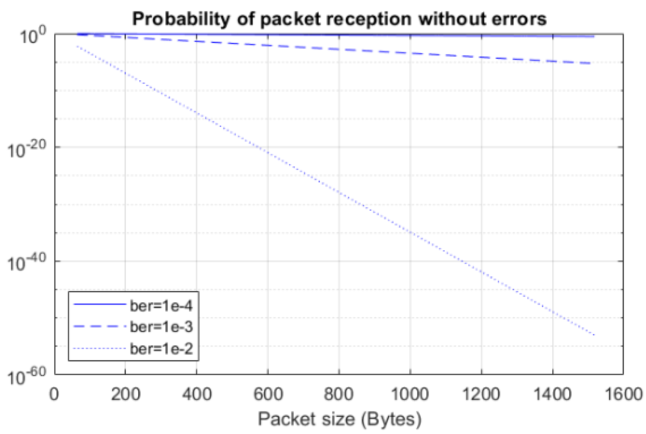
2.b. Determine the probability of a data packet of 1000 Bytes to be received with exactly one error when $p = 10^{-3}$. Answer: 0.2676%

2.c. Determine the probability of a data packet of 200 Bytes to be received with one or more errors when $p = 10^{-4}$. Answer: 14.7863%

2.d. Draw a plot with the same look as the answer below of the probability of a data packet (of size 100 Bytes, 200 Bytes or 1000 Bytes) being received without errors as a function of the *ber* (from $p = 10^{-8}$ up to $p = 10^{-2}$). Answer:

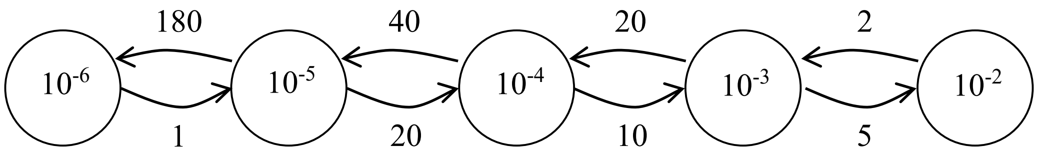


2.e. Draw a plot with the same look as the answer below of the probability of a data packet being received without errors (for $p = 10^{-4}$, 10^{-3} and 10^{-2}) as a function of the packet size (from 64 Bytes up to 1518 Bytes). Answer:



Task 3

Consider a wireless link between multiple stations for data communications. The bit error rate (*ber*) introduced by the wireless link (due to the variation of the propagation and interference factors along with time) is approximately given by the following Markov chain:



where the state transition rates are in number of transitions per hour. Consider that the link is in an interference state when its *ber* is at least 10^{-3} and in a normal state, otherwise. Determine:

3.a. the probability of the link being in one of the five states; answer: 9.87×10^{-1} (10^{-6}), 5.48×10^{-3} (10^{-5}), 2.74×10^{-3} (10^{-4}), 1.37×10^{-3} (10^{-3}), 3.43×10^{-3} (10^{-2})

3.b. the average percentage of time the link is in each of the five states; answer: 9.87×10^{-1} (10^{-6}), 5.48×10^{-3} (10^{-5}), 2.74×10^{-3} (10^{-4}), 1.37×10^{-3} (10^{-3}), 3.43×10^{-3} (10^{-2})

3.c. the average *ber* of the link; answer: 3.70×10^{-5}

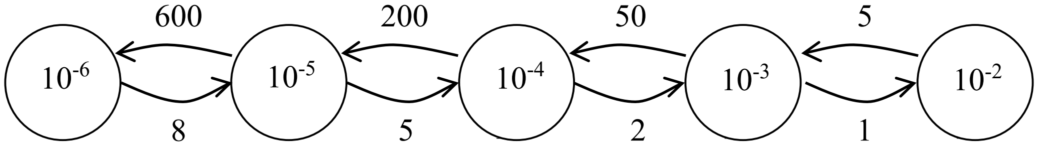
3.d. the average time duration (in minutes) that the link stays in each of the five states; answer: 60.0 (10^{-6}), 0.30 (10^{-5}), 1.20 (10^{-4}), 2.40 (10^{-3}), 30.0 (10^{-2})

3.e. the probability of the link being in interference state; answer: 4.80×10^{-3}

3.f. the average *ber* of the link when it is in the interference state. Answer: 7.43×10^{-3}

Task 4 – for reporting (evaluation weight = 50%)

Consider a wireless link between two stations for data communications. The bit error rate (*ber*) introduced by the wireless link is approximately given by the following Markov chain:



where the state transition rates are in number of transitions per hour. Consider that the link is in the interference state when its *ber* is at least 10^{-3} and is in the normal state, otherwise. Assume that both stations detect with a probability of 100% when the data frames sent by the other station are received with errors.

4.a. Determine the probability of the link being in the normal state and in the interference state.

4.b. Determine the average *ber* of the link when it is in the normal state and when it is in the interference state.

4.c. Consider that a data frame of size B (in Bytes) sent by one of the stations is received with errors by the other station. Draw a plot of the probability of the link being in the normal state as a function of the packet size (from 64 Bytes up to 200 Bytes). Analyze and justify the results.

4.d. Consider now that a data frame of size B (in Bytes) sent by one of the stations is received without errors by the other station. Draw a plot of the probability of the link being in the interference state as a function of the packet size (from 64 Bytes up to 200 Bytes). Analyze and justify the results.

Task 5 – for reporting (evaluation weight = 50%)

Consider again the wireless link between two stations of the previous Task 4. Consider now that the two stations run an interference detection system that works as follows: the stations exchange from time to time a set of n consecutive control frames (each frame with a size of B = 64 Bytes) and the stations decide that the link:

- is in the interference state when all n control frames are received with errors and
- is in the normal station if at least one of the n control frames is received without errors.

Consider the following definitions:

- a false positive is when a station decides wrongly that the link is in interference state (i.e., it receives the n control frames with error and the link is in the normal state)
- a false negative is when a station decides wrongly that the link is in the normal state (i.e., at least one of the n control frames is received without errors and the link is in the interference state)

5.a. Draw a plot using a logarithmic scale for the Y-axis (use the MATLAB function `semilogy`) with the probability of false positives for n = 2, 3, 4 and 5. Analyze and justify the results.

5.b. Draw a plot with the probability of false negatives for n = 2, 3, 4 and 5. Analyze and justify the results.

5.c. Assume that the probabilities of false positives and false negatives are equally important in the accuracy of the interference detection system. From the plots obtained in **5.a** and **5.b**, determine the best value of n to be used by the system. Justify your answer.