Dynamic Econometric Models: a state-space formulation

Abstract

In the area of econometrics, the investigation and characterization of processes that retain memory for the past is often of interest. This work overcomes colinearity problems that arise in distributed lag formulations by modeling these effects as structural elements within non-linear dynamic models using transfer functions. Our main contribution lies in performing sequential Bayesian inference for nonlinear dynamic models, providing an efficient computational solution based on analytical approximations. The scalability offered by the proposed sequential method is particularly relevant in the econometric context, where long time series or multiple levels of disaggregation are often encountered. The models that are proposed take into account stochastic volatility, which is carried out by using discount factors. An extensive simulation investigation validates the inferential approximation. An illustrative application to real world data on consumption, accessible at the supplementary material, compares the results of the proposed sequential and analytical approximation with the inference produced via Hamiltonian Monte Carlo, demonstrating broadly comparable outcomes achieved by the sequential approach, with significantly reduced computational time. A case study is developed, based on a comprehensive analysis of the Phillips curve, applying the proposed Bayesian state space formulation and focusing on the relationship between inflation and the output gap within the Brazilian context. We conclude with a substantial contribution, based on an innovative approach that preserves Bayesian sequential inference and offers a joint model for inflation and the output gap, with dynamic predictive structures assigned to the means, precisions and correlation between both economic indicators.

Keywords: Non-linear dynamic models, Transfer function, Sequential inference, Stochastic volatility, Discount factor, Phillips curve.

A Supplementary Material

Consumption function: a comparison between analytical sequential approximation and HMC

In this appendix, we illustrate the advantages of employing analytical approximations over stochastic simulation techniques, utilizing previously published data. This serves as a complement to the simulation studies conducted, demonstrating that the analytical approximation provides reliable estimates in practical applications.

An illustrative example showcasing the utility of the transfer function model is presented utilizing the dataset from Griliches et al. (1962). This dataset comprises U.S. quarterly price-deflated, seasonally adjusted data on personal disposable income (x_t) and personal consumption expenditure (y_t), spanning from the first quarter of 1947 to the fourth quarter of 1960. Both time series are depicted in Figure A.1.

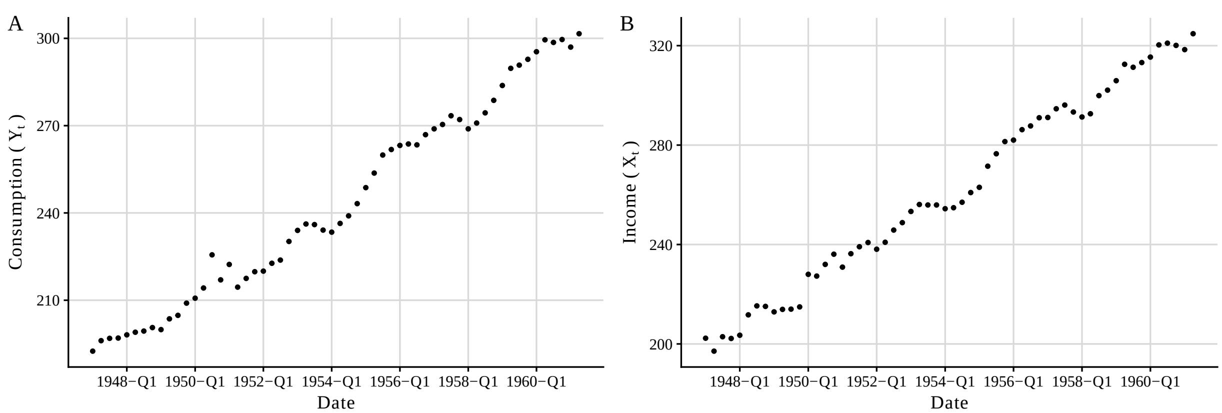

Figure A.1: USA quarterly price-deflated, seasonally adjusted data on personal disposable income (x_t) and personal consumption expenditure (y_t).

The primary objective of this analysis is to estimate consumption as a function of income, comparing our sequential analytical approach to stochastic simulation techniques. It is evident that income exerts a long-term impact on consumption, as x_t affects y_t not only instantaneously but also through its lags. Consistent with the findings of Ravines et al.

(2006), we adopt a first-order transfer function representation to preserve the memory of consumption with respect to income. The model is defined as a particular specification of (7), with $r = 1$, $b = 0$, $s = 0$, $\phi_1 = \ldots = \phi_p = \psi_1 = \ldots = \psi_s = 0$. Unlike Ravines et al. (2006), who employed MCMC techniques for Bayesian inference, we take advantage of the analytic approximation described in Section 3. Our method offers several advantages over MCMC approaches, including computational speed, sequential inference, elimination of MCMC convergence issues, and production of accurate outcomes comparable to MCMC results. Additionally, it is simple to extend the specified model while preserving computational efficiency. By introducing a random walk structure to both parameters, $\psi_t = \psi_{t-1} + \omega_{\psi,t}$ and $\lambda_t = \lambda_{t-1} + \omega_{\lambda,t}$, we could incorporate temporal dynamics. Readers interested in the subject may validate the significance of introducing discount factors to account for information loss over time in Migon et al. (2005), which reviews inference within this class of models. We compare the outcomes of the analytical approximation with those derived from a Hamiltonian Monte Carlo (HMC) procedure, executed in Stan (Stan Development Team, 2022)..

A key issue is the estimation of the evolution variance $\mathbf{W}_t$ of ψ_t given the past information. In MCMC, this variance is typically assumed to be static, following an Inverse Wishart prior distribution. Sequential inference becomes more manageable with the application of the discount factor technique alongside analytical approximation. If λ remains constant over time but is sequentially updated in its estimation, then δ_λ, its discount factor, is set to 1. For ψ_t, a discount factor of $\delta_\psi = 0.98$ is employed, allowing smooth temporal variations.

To ensure robustness and minimize the effects of stochastic variations, each fitting method (analytical approximation and HMC) was replicated ten times. The HMC approach required 36.47 seconds, while the analytical approximation took only 0.004 seconds.

Figure A.2 provides a summary of the comparison between the HMC approach and the analytical approximation. Panel **A** compares the mean fitted values and their 95% credible intervals for both methods. Since the analytical approximation uses sequential inference, we employed the mean smoothed response distribution (refer to West and Harrison (1997), Sec. 4.7) to ensure a fair comparison with the HMC technique. The predicted outcomes and uncertainties, represented by the 95% credible intervals, are largely comparable between the two methodologies. However, notable differences are observed in the log-likelihood values, with the HMC-Stan and analytical approximations yielding -122.98 and -97.40, respectively. This discrepancy strongly favors the sequential approach, likely due to the different model employed in the sequential method, which utilized time-varying evolution variances determined by discount factors. Panel **B** depicts the filtering distribution for λ, reflecting the temporal evolution of the inference on this parameter. It is evident that the density becomes more dispersed between 1942-Q2 and 1951-Q4, which can be attributed to the model's inability to account for a slight variation in the observed value in 1950. Subsequently, the uncertainty surrounding λ naturally diminishes with the accumulation of new data.

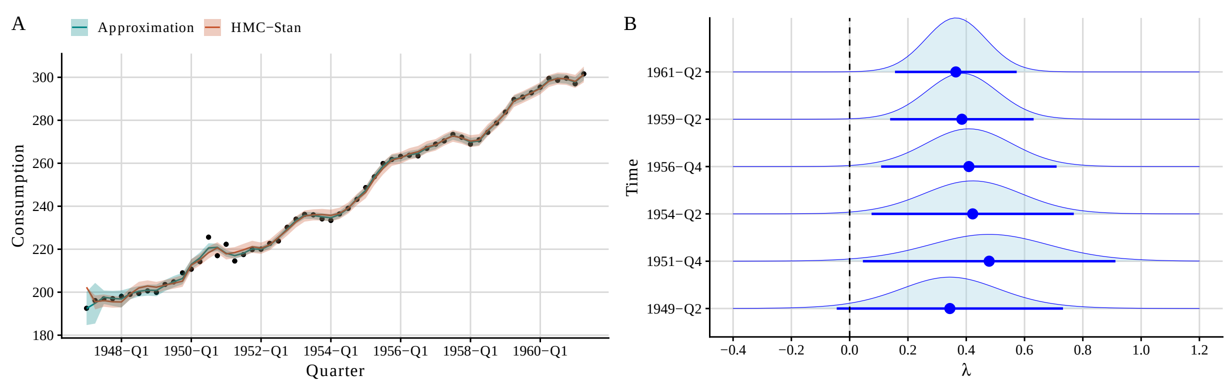

Figure A.2: Comparison of Analytical Approximation and HMC-Stan Methods. **A:** The mean fitted values $[E_t|D_n]$ (solid line), obtained through HMC-Stan and analytical approximations, are depicted with 95% credible intervals (shaded region). **B:** The sequential posterior distribution of λ from the linearized DLM is illustrated, including the 95% credible interval (lines below the density) and the posterior mean (colored dot).

Multi-process analysis: selection of r and b

In this appendix, we provide a detailed description of the multi-process analysis performed in order to evaluate the proper orders r of the transfer function component, as well as elements on the choice of the lag b to be applied to the output gap.

Specifically, we use the multi-process type I describe in West and Harrison (1997) to derive a discrete approximation of the posterior probability of the model. The lag used for the output gap and the order of the transfer function define individual processes. We take into account the combination of $b = 0, \ldots, 3$ lags for the output gap and order $r = 1, \ldots, 3$ for the transfer function, resulting in $j = 1, \ldots, 12$ distinct models. As mentioned in Section 3.1, the Bayesian sequential analysis is performed for each model using first order Taylor approximation.

Let $p_t(r, b)$ be the posterior for time t, that is, $p_t(r, b) = P[R = r, B = b \mid D_t]$, $r = 1, \ldots, 3$; $b = 0 \ldots, 3$, with a specified initial prior $p_0(r, b)$. Following Bayes' Theorem, inference about the vector (R, B) is sequentially updated via $p_t(r, b) = c_t\, p_{t-1}(r, b)\, l_t(r, b)$, $r = 1, 2, 3$; $b = 0 \ldots, 3$, where $l_t(r, b)$ denotes the predictive likelihood $l_t(r, b) = p[Y_t \mid r, b, D_{t-1}]$ for $R = r, B = b$, for each t, which is a t-Student, and the normalized constant is approximated by: $c_t^{-1} = \sum_{r,b} p[Y_t \mid r, b, D_{t-1}]\, p_{t-1}(r, b)$. Table A.1 displays the resulting posterior probabilities for the fitted models.

Table A.1: Posterior probabilities of different models for the last observed time according to the output gap lag (b) and the transfer function order (r).

b	r		
	1	2	3
0	0.0003	0.0005	0.0003
1	**0.5903**	0.1359	0.0339
2	0.1192	0.0335	0.0152
3	0.0340	0.0367	0.0002

References

Griliches, Z., Maddala, G.S., Lucas, R., Wallace, N., 1962. Notes on estimated aggregate quarterly consumption functions. Econometrica 30, 491–500.

Migon, H.S., Gamerman, D., Lopes, H.F., Ferreira, M.A., 2005. Dynamic models. Handbook of Statistics 25, 557–592.

Ravines, R.R., Schmidt, A.M., Migon, H.S., 2006. Revisiting distributed lag models through a bayesian perspective. Appl. Stochastic Models Bus. Ind. , 193–210.

Stan Development Team, 2022. Stan Modeling Language Users Guide and Reference Manual. 2.33.

West, M., Harrison, P.J., 1997. Bayesian Forecasting and Dynamic Models. 2nd ed., Springer, New York.