

XI) Existencial/disyunción: $\exists X. (P(X) \vee Q(X)) \Leftrightarrow (\exists X. P(X) \vee \exists X. Q(X))$

\Rightarrow

$$\begin{array}{c}
 \begin{array}{c} \checkmark \\ \hline \Gamma \vdash \exists X (P(X) \vee Q(X)) \quad Ax \\ \hline \Gamma \vdash \exists X (P(X) \vee Q(X)) \quad \exists i \end{array} \\
 \begin{array}{c} \checkmark \\ \hline \Gamma \vdash P(X) \vdash P(X) \quad Ax \\ \hline \Gamma \vdash P(X) \vdash \exists X P(X) \quad \exists i \end{array} \\
 \begin{array}{c} \checkmark \\ \hline \Gamma \vdash Q(X) \vdash Q(X) \quad Ax \\ \hline \Gamma \vdash Q(X) \vdash \exists X Q(X) \quad \exists i \end{array} \\
 \hline
 \Gamma \vdash P(X) \vdash \exists X P(X) \vee \exists X Q(X) \quad \vee i_1 \\
 \Gamma \vdash Q(X) \vdash \exists X P(X) \vee \exists X Q(X) \quad \vee i_2 \\
 \hline
 \Gamma \vdash \exists X (P(X) \vee Q(X)) \vdash \exists X P(X) \vee \exists X Q(X) \quad \vee e \\
 \hline
 \Gamma \vdash \exists X (P(X) \vee Q(X)) \vdash (\exists X. P(X) \vee \exists X. Q(X)) \quad \Rightarrow i
 \end{array}$$

\Leftarrow

$$\begin{array}{c}
 \begin{array}{c} \checkmark \\ \hline \Gamma \vdash \exists X P(X) \vdash \exists X P(X) \quad Ax \\ \hline \Gamma \vdash \exists X P(X) \vdash P(X) \quad \exists e \\ \hline \Gamma \vdash \exists X P(X) \vdash P(X) \vee Q(X) \quad \vee i_1 \end{array} \\
 \begin{array}{c} \checkmark \\ \hline \Gamma \vdash \exists X Q(X) \vdash \exists X Q(X) \quad Ax \\ \hline \Gamma \vdash \exists X Q(X) \vdash Q(X) \quad \exists e \\ \hline \Gamma \vdash \exists X Q(X) \vdash P(X) \vee Q(X) \quad \vee i_2 \end{array} \\
 \hline
 \Gamma \vdash \exists X P(X) \vee \exists X Q(X) \vdash \exists X (P(X) \vee Q(X)) \quad \exists i \\
 \hline
 \Gamma \vdash \exists X (P(X) \vee Q(X)) \vdash \exists X. P(X) \vee \exists X. Q(X) \quad \Rightarrow i
 \end{array}$$