(1) Prove that $3^{2n} - 1$ is divisible by 8 for each integer $n \geq 0$.

For $n \in \mathbb{N}$, let $P(n)$ be divisible by 8 for each integer $n \geq 0$.
Basis:
$3^{2(0)} - 1 = 1 - 1 = 0$, which is divisible by 8.
Induction:
Let $n \in \mathbb{N}$, and assume $P(n)$ is true, that is, $3^{2n} - 1$ is divisible by 8. Thus, by the definition of divisibility, there exists an integer p, such that $3^{2k} - 1 = 8p$
Now we must prove that $P(n+1)$ is true.
Consider:

$$\begin{aligned} 3^{2(n+1)} - 1 &= 3^{2n+2} - 1 \\ &= 3^{2n} \cdot 3^2 - 1 \\ &= 9(8p+1) - 1 \\ &= 72p + 9 - 1 \\ &= 72p + 8 \\ &= 8(9p+1) \end{aligned}$$

$3^{2n+1} - 1$ is divisible by 8. Thus, $3^{2n} - 1$ is divisible by 8 for each integer $n \geq 0$, and the value of $(9p+1)$ is also an integer.

(2) Prove that $1 + 4n < 2^n$ for all integers $n \geq 5$.
For $n \in \mathbb{W}$, assume $P(n)$ is greater than or equal to 5.
Basis:

$$1 + 4(5) < 2^5$$

$\therefore P(5)$ is true.
Let $n \in \mathbb{N}, n \geq 5$, and assume P(n) is true
Induction:

$$\begin{aligned} 1 + 4n + 4 &< 2^n + 2^n \\ 1 + 4(n+1) &< 2^n(1+1) = 2^n \cdot 2 = 2^{n+1} \\ 1 + 4(n+1) &< 2^{n+1} \end{aligned}$$

$\therefore 1 + 4n < 2^n$ is true for all integers $n \geq 5$.