**(1)** For $n \in \mathbb{N}$, let $P(n)$ be the predicate $\sum_{k=1}^{n-1} k(k+1) = \frac{n(n-1)(n+1)}{3}$. Show for all $n \geq 2$:

$$LHS = 1(1+1) = 2.$$
$$RHS = \frac{2(2-1)(2+1)}{3}$$
$$= 2.$$

$\therefore$P(1)is true.
**Inductive Hypothesis**
Assume for some arbitrary $n \geq 2$ that $P(n)$, that is $\sum_{k=1}^{n-1} k(k+1) = \frac{n(n-1)(n+1)}{3}$ holds.
**Inductive Step:**

$$\begin{aligned}
\sum_{k=1}^{n} k(k+1) &= \sum_{k=1}^{n-1} k(k+1) + n(n+1) \\
&= \frac{n(n-1)(n+1)}{3} + n(n+1) \\
&= \frac{n(n-1)(n+1) + 3(n+1)n}{3} \\
&= \frac{(n+1)(n)(n-1+3)}{3} \\
&= \frac{n(n-1)(n+1)}{3}.
\end{aligned}$$

Thus, $P(n+1)$ is true and so we have proved the inductive step.
Therefore by the principle of Mathmatical Induction, $\sum_{k=1}^{n-1} k(k+1) = \frac{n(n-1)(n+1)}{3}$ is true for all $n \in \mathbb{N}$

**(2)**

For $n \in \mathbb{N}$, let $P(n)$ be the predicate $\sum_{k=1}^{n+1} k \cdot 2^k = n \cdot 2^{n+2} + 2$.
Show for all $n \geq 0$:

$$LHS = 1 * 2^1 = 2,$$
$$RHS = 0 * 2^{0+2} + 2 = 2.$$

**Inductive Hypothesis**
Assume for some arbitrary $n \geq 0$ that $P(n)$ that is $\sum_{k=1}^{n+1} k \cdot 2^k = n \cdot 2^{n+2} + 2$ holds.
**Inductive Step:**

$$
\begin{aligned}
\sum_{k=1}^{n} k \cdot 2^k &= \sum_{k=1}^{n+1} k \cdot 2^k + (n+2) \cdot 2^{n+2} \\
&= n \cdot 2^{n+2} + 2 + (n+2) \cdot 2^{n+2} \\
&= (2m+2) \cdot 2^{n+2} + 2 \\
&= 2(n+1) \cdot 2^{n+2} + 2 \\
&= (n+1) \cdot 2^{n+2} + 2
\end{aligned}
$$

Thus, $P(n+1)$ is true and so we have proved the inductive step.
Therefore by the principles of Mathmatical Induction $\sum_{k=1}^{n+1} k{\cdot}2^k = n{\cdot}2^{n+2}{+}2$ is true for all $n \in \mathbb{N}$.