(1) Use induction to prove that for all integers $n \geq 0$, $F_{n+2}F_n - F_{n+1}^2 = (-1)^n$.
Basis of Induction
For $n \in \mathbb{N}$, let $P(n)$ be the predicate $F_{n+2}F_n - F_{n+1}^2 = (-1)^n$.

$$\begin{aligned}
&F_2F_0 - F_1^2 \\
&= 1 \cdot 2 - 1^2 \\
&= (-1)^0
\end{aligned}$$

So $P(0)$ is true.
Inductive Hypothesis
Let $F_{n+2}F_n - F_{n+1}^2 = (-1)^n$ be true for some $n \in \mathbb{W}$.
Inductive Step

$$\begin{aligned}
&F_{n+3}F_{n+1} - F_{n+2}^2 \\
&= F_{n+3} = F_{n+1} + F_{n+2} \\
&= (F_{n+1} + F_{n+2})F_{n+1} - F_{n+2}^2 \\
&= F_{n+1}^2 + F_{n+1}F_{n+2} - F_{n+2}^2 \\
&= F_{n+2}^2 = F_{n+2}F_n - (-1)^n \\
&= F_{n+2}F_n - (-1)^n + F_{n+1}F_{n+2} - F_{n+2}^2 \\
&= F_{n+2}(F_n + F_{n+1}) - (-1)^n - F_{n+2}^2 \\
&= F_n + F_{n+1} = F_{n+2} \\
&= F_{n+2}^2 - (-1)^n - F_{n+2}^2 \\
&= -(-1)^n \\
&= (-1)^{n+1}.
\end{aligned}$$

We finally have $F_{n+3}F_{n+1} - F_{n+2}^2 = (-1)^{n+1}$. $\therefore F_{n+2}F_n - F_{n+1}^2 = (-1)^n$