(1) Prove that for all integers n and m, if $n - m$ is even, then n^3 - m^3 is even.

Let $m, n \in \mathbb{Z}$ and suppose $n - m$ is even. Then $n - m$=$2k$ for some k∈ $\mathbb{Z}$. To prove this let's factor n^3 - m^3. Upon factoring we get:

$$\begin{aligned} n^3 - m^3 &= (n - m)(n^3 + nm + m^3) \\ &= 2k(n^2 + nm + m^2). \end{aligned}$$

This shows that n^3 - m^3 is even.

(2) Use an infinite geometric series to write 320.54924924924... as a fraction of integers.

First let's convert this to an infinite geometric series.

$$320.5\overline{492} = 320.5 + 0.0492 + 0.0000492 + 0000000492.....$$

convert everything to fractions and we get the series to look like this:

$$\frac{3205}{10} + \frac{123}{2500} + \frac{123}{2500}\left(\frac{1}{1000}\right) + \frac{123}{2500}\left(\frac{1}{1000}\right)^2 ...$$

The sum of the geometric series is equal to $\frac{a}{1-r}$ where r represents the common ratio and a representing the first term.This only works when $|r|$<1.

$$\sum_{k=1}^{\infty} ar^{k-1} = \frac{a}{1-r}.$$

$$a = \frac{123}{2500}, \quad r = \frac{1}{1000}.$$

$$\begin{aligned} \frac{a}{1-r} &= \frac{\frac{123}{2500}}{1 - \frac{1}{1000}} \\ &= \frac{82}{1665}. \end{aligned}$$

$$\begin{aligned} 320.5\overline{492} &= \left(\frac{3205}{10} + \frac{82}{1665}\right) \\ &= \left(\frac{1067429}{3330}\right). \end{aligned}$$