Black Holes

Some generalities (both theoretical and observational)

Ignacio F. Ranea-Sandoval (Argentina)

Facultad de Ciencias Astronómicas y Geofísicas
UNIVERSIDAD NACIONAL DE LA PLATA

CONICET

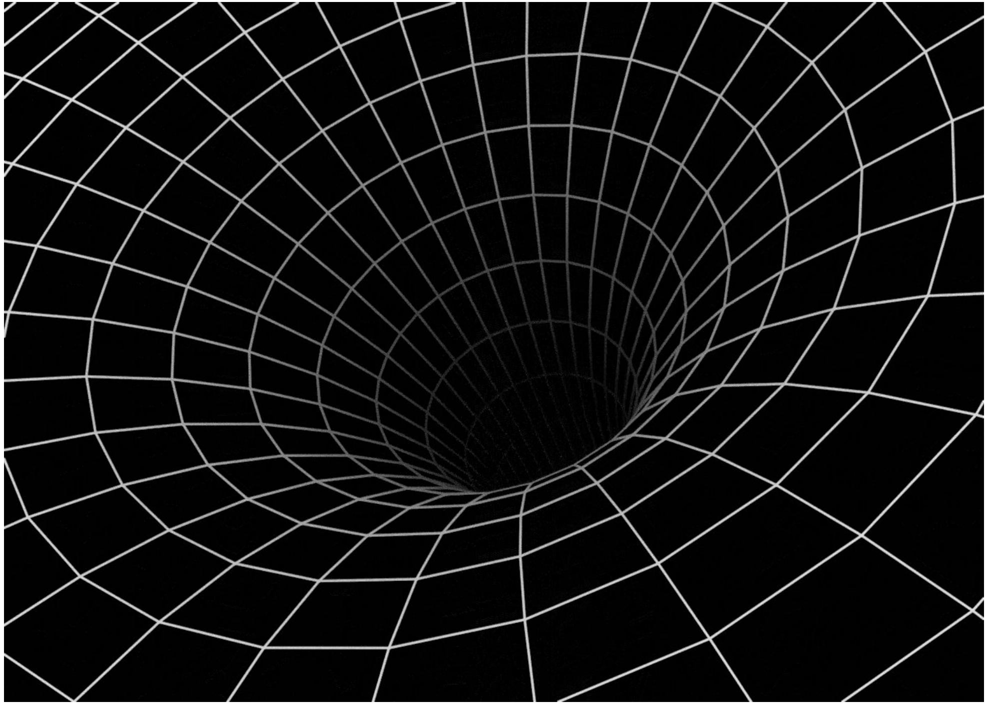
ISYA INTERNATIONAL SCHOOL FOR YOUNG ASTRONOMERS
UNIVERSIDAD YACHAY TECH
IAU
OYA OFFICE FOR YOUNG ASTRONOMERS
DET NORSKE VIDENSKAPS-AKADEMI
ESCUELA POLITÉCNICA NACIONAL

What are Black Holes?

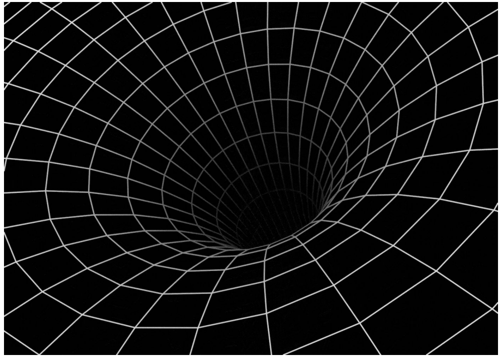
A formal definition for a BH needs solid formation on General Relativity.
I will present general ideas that are (kind of) sufficient to have a clear picture.

What are Black Holes?

A formal definition for a BH needs solid formation on General Relativity.
I will present general ideas that are (kind of) sufficient to have a clear picture.



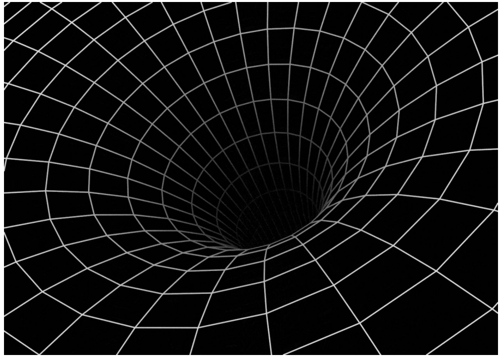
What are Black Holes?



A formal definition for a BH needs solid formation on General Relativity.
I will present general ideas that are (kind of) sufficient to have a clear picture.

“an object whose gravitational field is so strong not even light can escape from it”

What are Black Holes?

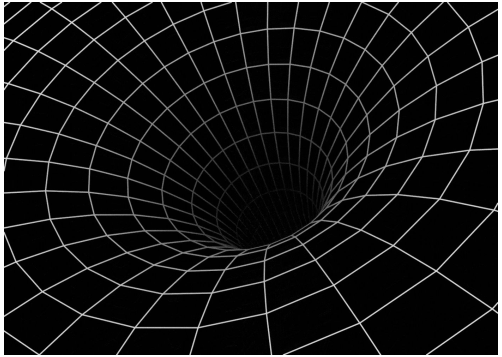


A formal definition for a BH needs solid formation on General Relativity.
I will present general ideas that are (kind of) sufficient to have a clear picture.

“The region of spacetime inside an event horizon”

“an object whose gravitational field is so strong not even light can escape from it”

What are Black Holes?



A formal definition for a BH needs solid formation on General Relativity.
I will present general ideas that are (kind of) sufficient to have a clear picture.

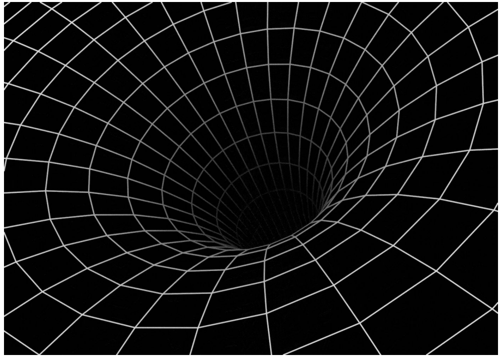
“a region of spacetime that is not in the causal past of future null infinity”

“The region of spacetime inside an event horizon”

“an object whose gravitational field is so strong not even light can escape from it”

What are Black Holes?

A formal definition for a BH needs solid formation on General Relativity.
I will present general ideas that are (kind of) sufficient to have a clear picture.



“a region of spacetime that is not in the causal past of future null infinity”

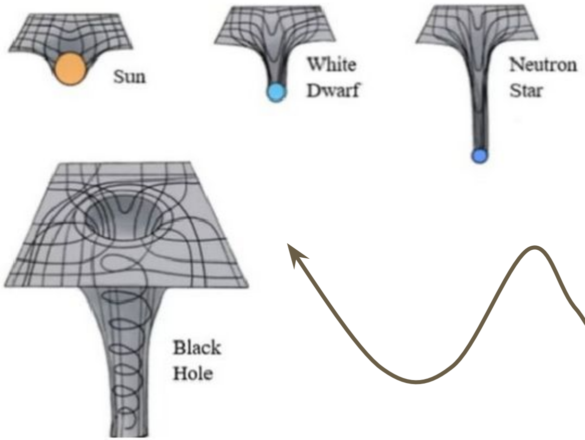
This can be understood as observers inside the black hole region cannot send causal messages to infinity and thus they cannot send causal messages to observers outside the black hole either

“The region of spacetime inside an event horizon”

“an object whose gravitational field is so strong not even light can escape from it”

What are Black Holes?

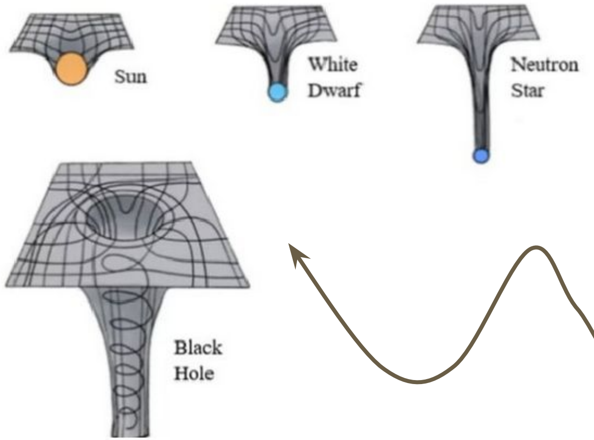
A formal definition for a BH needs solid formation on General Relativity.
I will present general ideas that are (kind of) sufficient to have a clear picture.



You add mass to a certain region of spacetime it gets more and more curved

What are Black Holes?

A formal definition for a BH needs solid formation on General Relativity.
I will present general ideas that are (kind of) sufficient to have a clear picture.

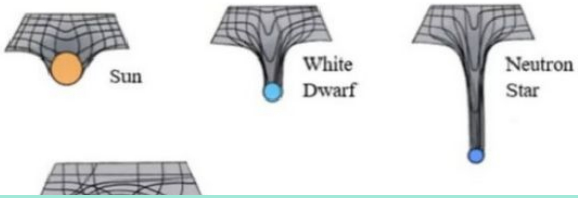


You add mass to a certain region of spacetime it gets more and more curved

When this curvature reaches a limit and not even light can escape gravitational attraction, a black hole is formed

What are Black Holes?

A formal definition for a BH needs solid formation on General Relativity.
I will present general ideas that are (kin
sufficient to have
clear picture.



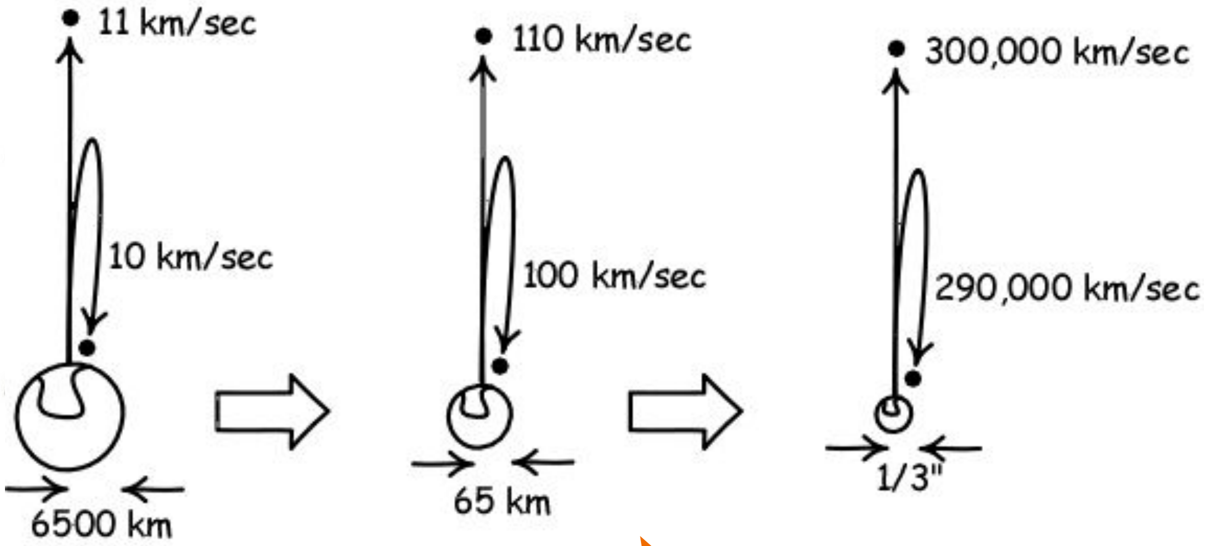
Perfect environment to test General Relativity.
Many alternative theories to GR can be tested with astronomical observations of black holes

escape gravitational attraction, a black hole is formed

You add mass to a certain region of spacetime it gets more and more curved

What are Black Holes?

A formal definition for a BH needs solid formation on General Relativity.
I will present general ideas that are (kind of) sufficient to have a clear picture.

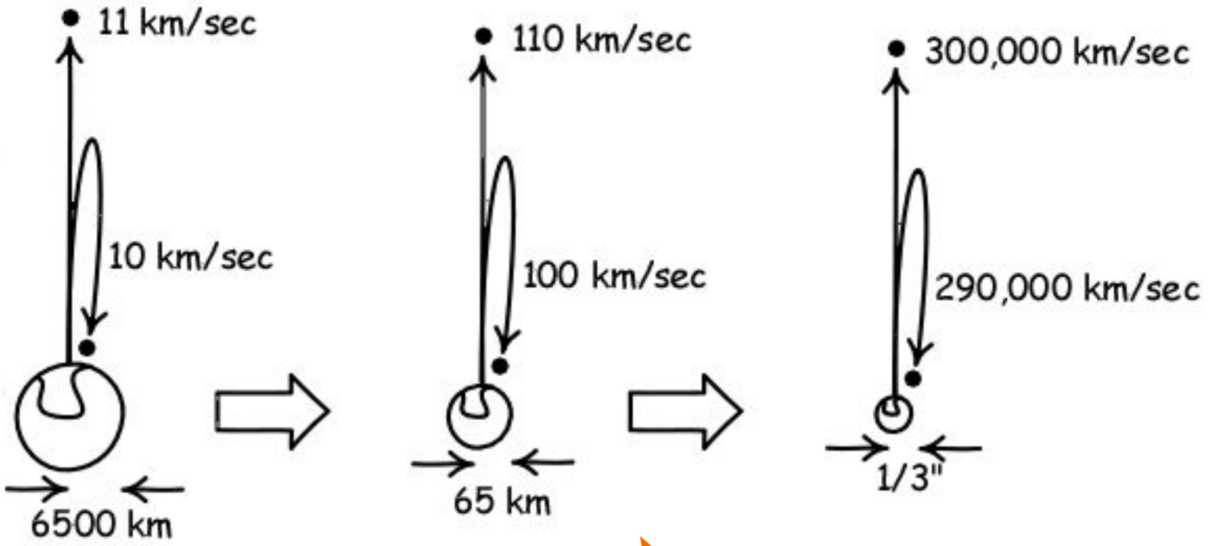


When this curvature reaches a limit and not even light can escape gravitational attraction, a black hole is formed

Classical picture using the escape velocity from a given object

What are Black Holes?

A formal definition for a BH needs solid formation on General Relativity.
I will present general ideas that are (kind of) sufficient to have a clear picture.

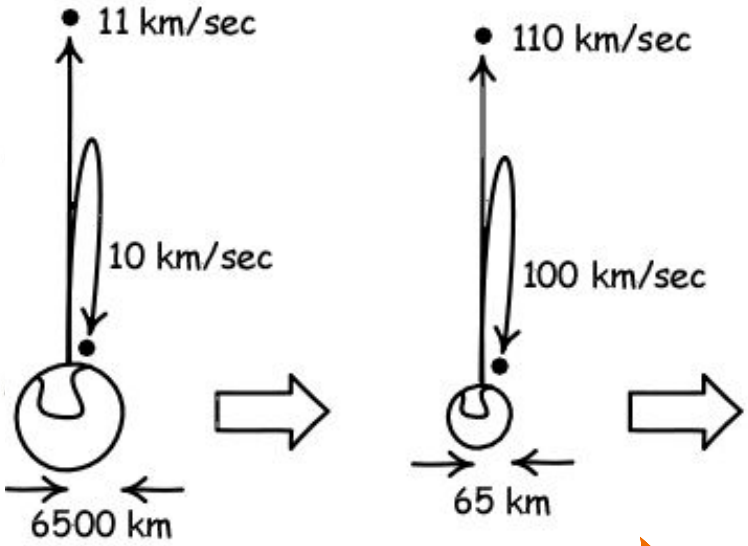


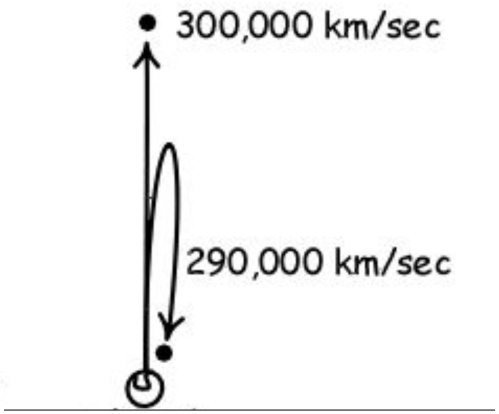
Remember the classical definition of escape velocity?

Classical picture using the escape velocity from a given object

What are Black Holes?

A formal definition for a BH needs solid formation on General Relativity.
I will present general ideas that are (kind of) sufficient to have a clear picture.





$$\frac{1}{2}mv_1^2 = G\frac{M_E m}{R_E}$$

$$v_1 = \sqrt{2G\frac{M_E}{R_E}}$$

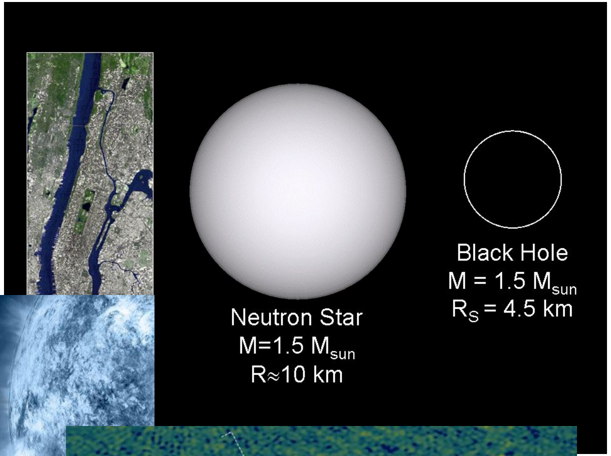
Remember the classical definition of escape velocity?

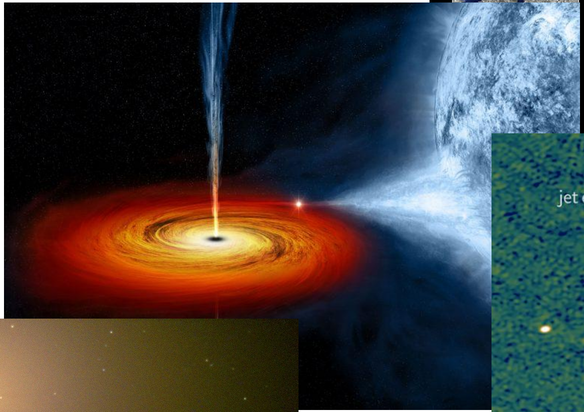
What are Black Holes?

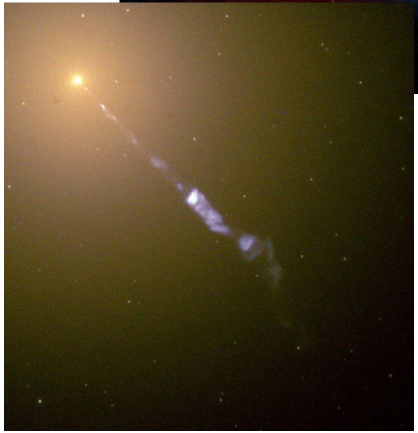
They can be tiny or huge, we focus in those with astronomical relevance

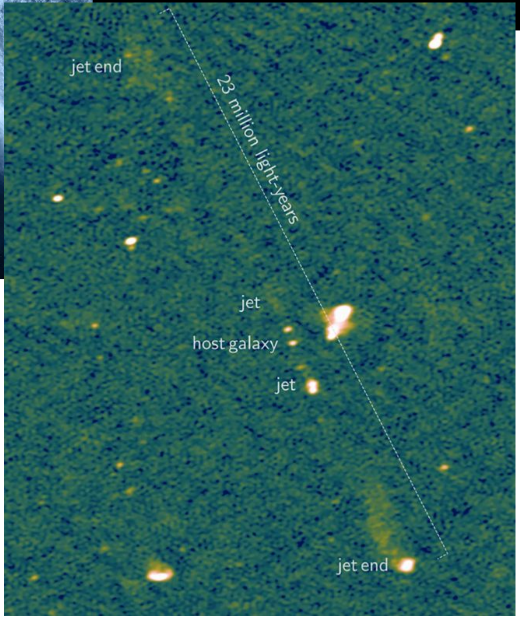
Several Solar Masses (final stage of stellar evolution of the most massive stars)

Supermassive (at the center of -almost- every galaxy in the Universe)

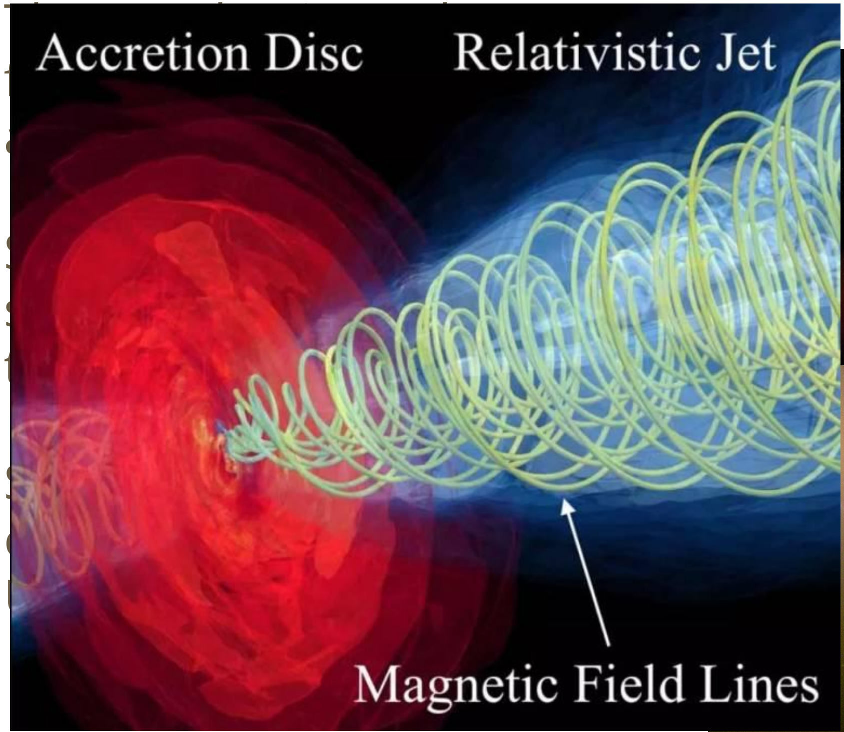




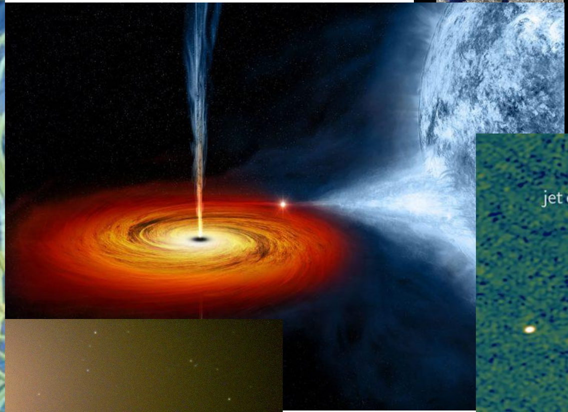


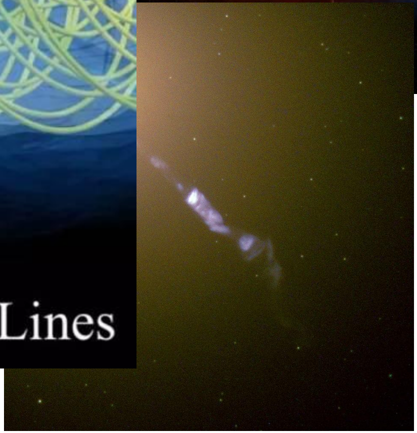


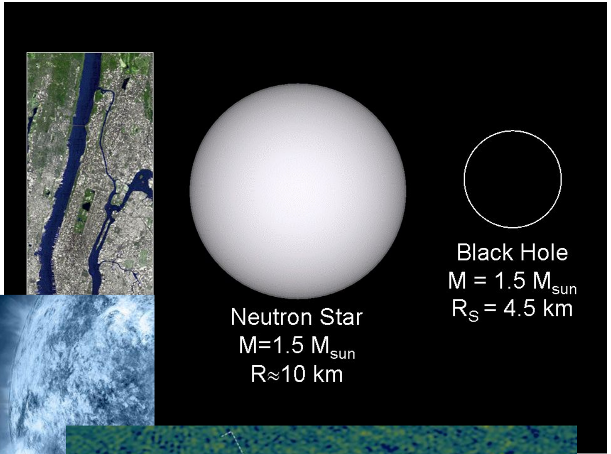
What are Black Holes?

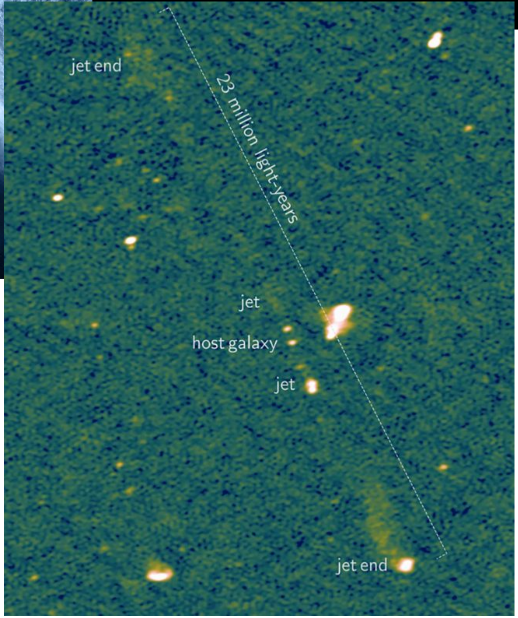


Cruz-Osorio et al. 2021







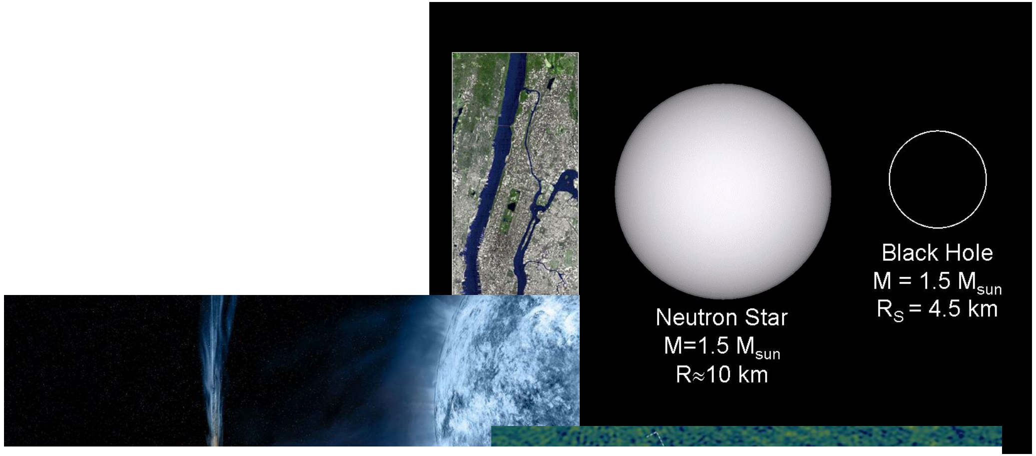


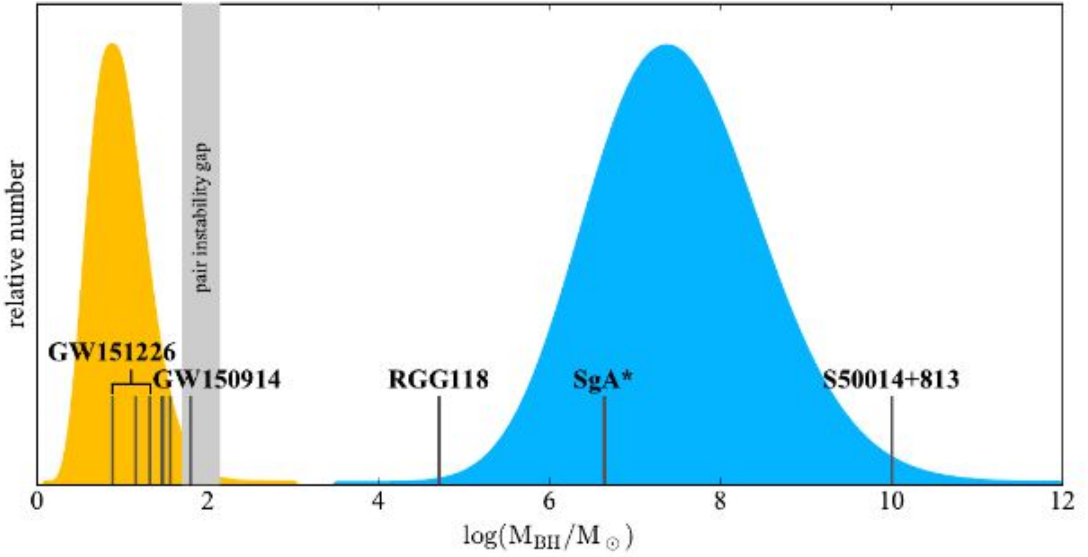
What are Black Holes?

They can be tiny or huge, we focus in those with astronomical relevance

Several Solar Masses (final stage of stellar evolution of the most massive stars)

Supermassive (at the center of -almost- every galaxy in the Universe)





What are Black Holes?

A geometrical approach to (try to) understand BHs.

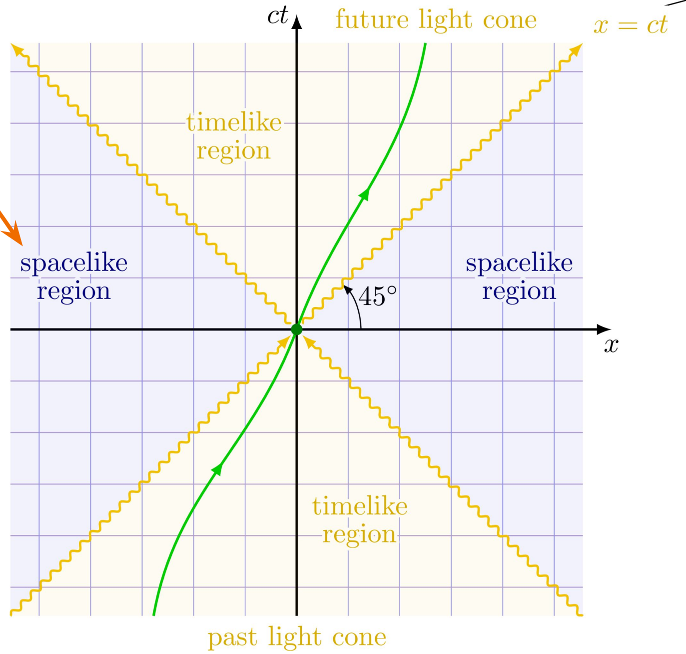
Light in vacuum has speed, c, in every inertial frame. Maximum attainable speed.

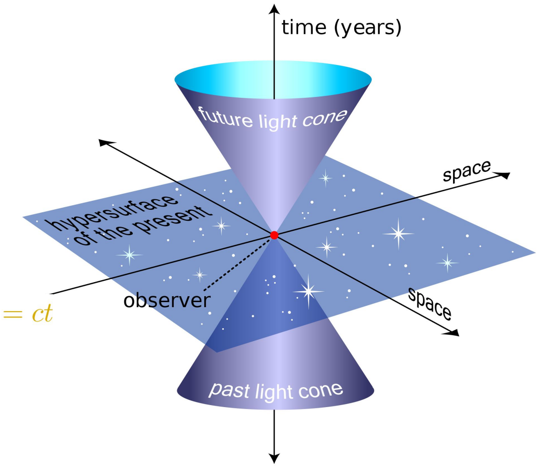
What are Black Holes?

A geometrical approach to (try to) understand BHs

Vacuum spacetime
Light has speed c
45° angle trajectory

Spacetime diagram of flat spacetime





What are Black Holes?

A geometrical approach to (try to) understand BHs

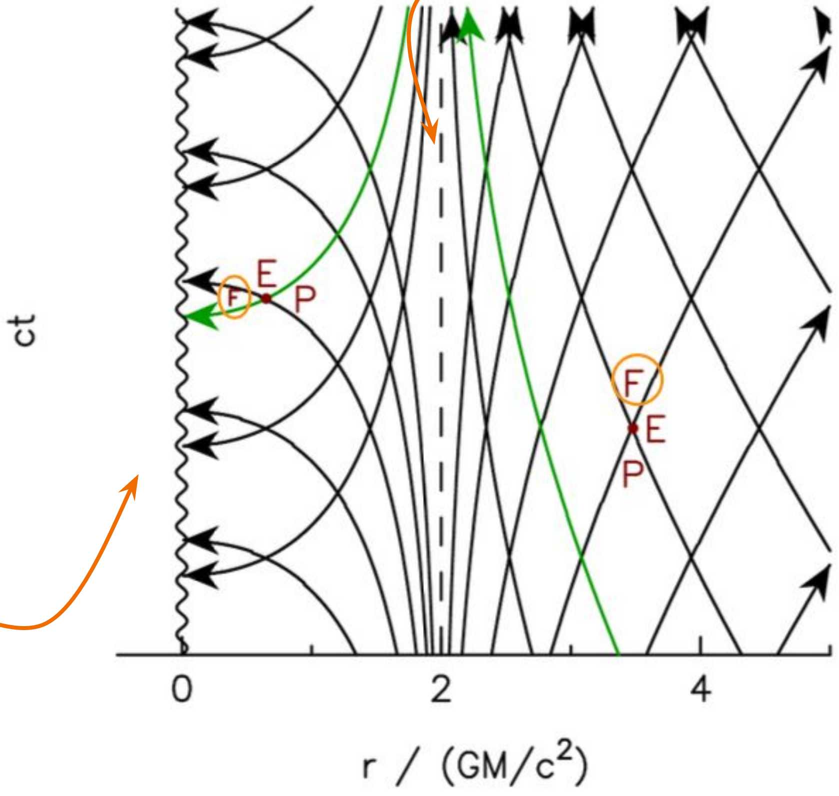
What happens when we add gravity into the mix?

What are Black Holes?

A geometrical approach to (try to) understand BHs

What happens when we add gravity into the mix?

Spacetime diagram for a non-rotating BH



Something that crosses this surface inevitably ends up in r = 0

What are Black Holes?

Black solutions to the Einstein Field Equations

What are Black Holes?

Black solutions to the vacuum Einstein Field Equations

Schwarzschild BH (non-rotating, no electric charge)

In 1916, just a month after Einstein’s GR in a hospital during WWI

$$ds^2 = -c^2 d\tau^2 = -\left(1 - \frac{r_s}{r}\right) c^2 dt^2 + \frac{dr^2}{1 - \frac{r_s}{r}} + r^2 \left(d\theta^2 + \sin^2\theta \, d\varphi^2\right)$$

$$r_s = \frac{2GM}{c^2}$$

Schwarzschild radius. Coordinate singularity (the event horizon)

$r = 0$

Physical singularity. Breakdown of the theory

What are Black Holes?

The Schwarzschild solution
a glimpse of deduction

$$\boxed{R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu}.}$$

What are Black Holes?

The Schwarzschild solution
a glimpse of deduction

In vacuum they reduce to

$$R_{\mu\nu} = 0.$$

What are Black Holes?

The Schwarzschild solution
a glimpse of deduction

Some definitions

$$\Gamma^{\lambda}_{\mu\nu} = \tfrac{1}{2} g^{\lambda\sigma}(\partial_\mu g_{\nu\sigma} + \partial_\nu g_{\sigma\mu} - \partial_\sigma g_{\mu\nu}).$$

Christoffel symbols

$$R^{\rho}{}_{\sigma\mu\nu} = \partial_\mu \Gamma^{\rho}_{\nu\sigma} - \partial_\nu \Gamma^{\rho}_{\mu\sigma} + \Gamma^{\rho}_{\mu\lambda}\Gamma^{\lambda}_{\nu\sigma} - \Gamma^{\rho}_{\nu\lambda}\Gamma^{\lambda}_{\mu\sigma}.$$

Riemann tensor

$$R_{\mu\nu} = R^{\lambda}{}_{\mu\lambda\nu}.$$

Ricci tensor

$$R = R^{\mu}{}_{\mu} = g^{\mu\nu} R_{\mu\nu}.$$

Ricci scalar

What are Black Holes?

The Schwarzschild solution
a glimpse of deduction

More general spherically symmetric static vacuum line element

$$ds^2 = -e^{2\alpha(r)}\,dt^2 + e^{2\beta(r)}\,dr^2 + r^2\,d\Omega^2.$$

What are Black Holes?

The Schwarzschild solution
a glimpse of deduction

$$ds^2 = -e^{2\alpha(r)}\,dt^2 + e^{2\beta(r)}\,dr^2 + r^2\,d\Omega^2.$$

non zero Christoffel symbols

$$\Gamma^t_{tr} = \partial_r\alpha \qquad \Gamma^r_{tt} = e^{2(\alpha-\beta)}\partial_r\alpha \qquad \Gamma^r_{rr} = \partial_r\beta$$

$$\Gamma^\theta_{r\theta} = \frac{1}{r} \qquad \Gamma^r_{\theta\theta} = -re^{-2\beta} \qquad \Gamma^\phi_{r\phi} = \frac{1}{r}$$

$$\Gamma^r_{\phi\phi} = -re^{-2\beta}\sin^2\theta \qquad \Gamma^\theta_{\phi\phi} = -\sin\theta\cos\theta \qquad \Gamma^\phi_{\theta\phi} = \frac{\cos\theta}{\sin\theta}.$$

What are Black Holes?

The Schwarzschild solution
a glimpse of deduction

$$ds^2 = -e^{2\alpha(r)}\,dt^2 + e^{2\beta(r)}\,dr^2 + r^2\,d\Omega^2.$$

non zero
Christoffel
symbols

$$\Gamma^t_{tr} = \partial_r \alpha \qquad \Gamma^r_{tt} = e^{2(\alpha-\beta)}\partial_r \alpha \qquad \Gamma^r_{rr} = \partial_r \beta$$

$$\Gamma^\theta \ \ 1 \qquad \Gamma^r \ \ -2\beta \qquad \Gamma^\phi_{r\phi} = \frac{1}{r}$$

$$\theta\cos\theta \qquad \Gamma^\phi_{\theta\phi} = \frac{\cos\theta}{\sin\theta}.$$

$$\Gamma^t_{tr} = \frac{1}{2} g^{t\sigma}\left(\partial_t g_{r\sigma} + \partial_r g_{\sigma t} - \partial_\sigma g_{tr}\right)$$

$$= \frac{1}{2} g^{tt}\,\partial_r g_{tt} = \partial_r \alpha$$

$$g_{tt} = -e^{2\alpha} \qquad g^{tt} = -e^{-2\alpha}$$

What are Black Holes?

The Schwarzschild solution
a glimpse of deduction

$$ds^2 = -e^{2\alpha(r)}\,dt^2 + e^{2\beta(r)}\,dr^2 + r^2\,d\Omega^2.$$

non zero
Riemann tensor
components

$$R^t{}_{rtr} = \partial_r\alpha\,\partial_r\beta - \partial_r^2\alpha - (\partial_r\alpha)^2$$

$$R^t{}_{\theta t\theta} = -re^{-2\beta}\,\partial_r\alpha$$

$$R^t{}_{\phi t\phi} = -re^{-2\beta}\sin^2\theta\,\partial_r\alpha$$

$$R^r{}_{\theta r\theta} = re^{-2\beta}\,\partial_r\beta$$

$$R^r{}_{\phi r\phi} = re^{-2\beta}\sin^2\theta\,\partial_r\beta$$

$$R^\theta{}_{\phi\theta\phi} = (1 - e^{-2\beta})\sin^2\theta.$$

What are Black Holes?

The Schwarzschild solution
a glimpse of deduction

$$ds^2 = -e^{2\alpha(r)}\,dt^2 + e^{2\beta(r)}\,dr^2 + r^2\,d\Omega^2.$$

non zero
Ricci tensor
components

$$R_{tt} = e^{2(\alpha-\beta)}\left[\partial_r^2\alpha + (\partial_r\alpha)^2 - \partial_r\alpha\partial_r\beta + \tfrac{2}{r}\partial_r\alpha\right]$$

$$R_{rr} = -\partial_r^2\alpha - (\partial_r\alpha)^2 + \partial_r\alpha\partial_r\beta + \tfrac{2}{r}\partial_r\beta$$

$$R_{\theta\theta} = e^{-2\beta}[r(\partial_r\beta - \partial_r\alpha) - 1] + 1$$

$$R_{\phi\phi} = \sin^2\theta\, R_{\theta\theta},$$

Ricci scalar

$$R = -2e^{-2\beta}\left[\partial_r^2\alpha + (\partial_r\alpha)^2 - \partial_r\alpha\partial_r\beta + \frac{2}{r}(\partial_r\alpha - \partial_r\beta) + \frac{1}{r^2}(1 - e^{2\beta})\right].$$

What are Black Holes?

The Schwarzschild solution
a glimpse of deduction

$$ds^2 = -e^{2\alpha(r)}\,dt^2 + e^{2\beta(r)}\,dr^2 + r^2\,d\Omega^2.$$

we are looking for vacuum solutions so R_{tt} and R_{rr} vanish independently

$$0 = e^{2(\beta-\alpha)}R_{tt} + R_{rr} = \frac{2}{r}(\partial_r\alpha + \partial_r\beta),$$

This implies that

$$\alpha = -\beta.$$

What are Black Holes?

The Schwarzschild solution
a glimpse of deduction

$$ds^2 = -e^{2\alpha(r)}\,dt^2 + e^{2\beta(r)}\,dr^2 + r^2\,d\Omega^2.$$

we are looking for vacuum solutions
so R_{tt} and R_{rr} vanish independently

$$0 = e^{2(\beta-\alpha)}R_{tt} + R_{rr} = \frac{2}{r}(\partial_r\alpha + \partial_r\beta),$$

This implies that

$$\alpha = -\beta.$$

Then from $R_{\theta\theta} = 0$

we get $$e^{2\alpha}(2r\partial_r\alpha + 1) = 1.$$

What are Black Holes?

The Schwarzschild solution
a glimpse of deduction

$$ds^2 = -e^{2\alpha(r)}\,dt^2 + e^{2\beta(r)}\,dr^2 + r^2\,d\Omega^2.$$

we are looking for vacuum solutions so R_{tt} and R_{rr} vanish independently

$$0 = e^{2(\beta-\alpha)}R_{tt} + R_{rr} = \frac{2}{r}(\partial_r\alpha + \partial_r\beta),$$

This implies that

$$\alpha = -\beta.$$

Then from $R_{\theta\theta} = 0$

we get $e^{2\alpha}(2r\,\partial_r\alpha + 1) = 1.$

$$e^{2\alpha} = 1 - \frac{R_S}{r},$$

What are Black Holes?

The Schwarzschild solution
a glimpse of deduction

$$ds^2 = -\left(1 - \frac{R_S}{r}\right) dt^2 + \left(1 - \frac{R_S}{r}\right)^{-1} dr^2 + r^2 d\Omega^2.$$

where R_S is to be determined

What are Black Holes?

The Schwarzschild solution
a glimpse of deduction

Up to now we have that

$$ds^2 = -\left(1 - \frac{R_S}{r}\right) dt^2 + \left(1 - \frac{R_S}{r}\right)^{-1} dr^2 + r^2 d\Omega^2.$$

where R_S is to be determined

Using the weak field limit of General Relativity it can be shown that

$$g_{tt} = -\left(1 - \frac{2GM}{c^2 r}\right).$$

What are Black Holes?

The Schwarzschild solution
a glimpse of deduction

Up to now we have that

$$ds^2 = -\left(1 - \frac{R_S}{r}\right) dt^2 + \left(1 - \frac{R_S}{r}\right)^{-1} dr^2 + r^2 d\Omega^2.$$

where R_S is to be determined

Using the weak field limit of General Relativity it can be shown that

$$g_{tt} = -\left(1 - \frac{2GM}{c^2 r}\right).$$

End of the proof

What are Black Holes?

Black solutions to the vacuum
Einstein Field Equations

Reissner-Nordström BH (non-rotating,
with electric charge)

$$ds^2 = -\left(1 - \frac{2GM}{r} + \frac{4\pi G Q^2}{r^2}\right) dt^2 + \left(1 - \frac{2GM}{r} + \frac{4\pi G Q^2}{r^2}\right)^{-1} dr^2 + r^2 d\Omega^2 \,,$$

Independent
findings near
1918

r = 0 Physical singularity as in the Schwarzschild
solution. Breakdown of the theory

What are Black Holes?

Black solutions to the vacuum
Einstein Field Equations

Reissner-No… öm BH (non-rotating,

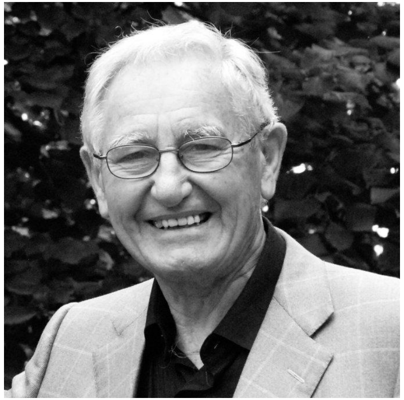
Exercise: find the position of the event horizons

$$\ldots \frac{\ }{r^2}\Big) dt^2 + \left(1 - \frac{2GM}{r} + \frac{4\pi G Q^2}{r^2}\right)^{-1} dr^2 + r^2 d\Omega^2 \ ,$$

r = 0 Physical singularity as in the Schwarzschild solution. Breakdown of the theory

What are Black Holes?

Black solutions to the vacuum
Einstein Field Equations

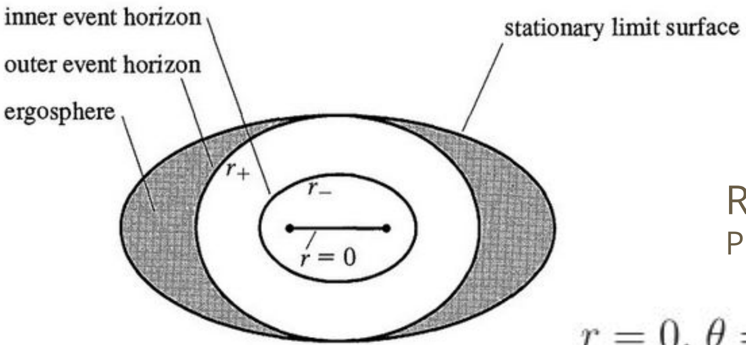


Kerr BH (rotating, no electric charge)

$$ds^2 = -\frac{\Delta\left(dt - a\sin^2\theta\, d\phi\right)^2}{\Sigma} + \frac{\sin^2\theta\left[a dt - (r^2+a^2)\, d\phi\right]^2}{\Sigma} + \Sigma\left(\frac{dr^2}{\Delta} + d\theta^2\right),$$

$$\Sigma = r^2 + a^2\cos^2\theta, \qquad \Delta = r^2 - 2mr + a^2.$$

In 1963
Almost 50 years later
that the non-rotating
solution!!

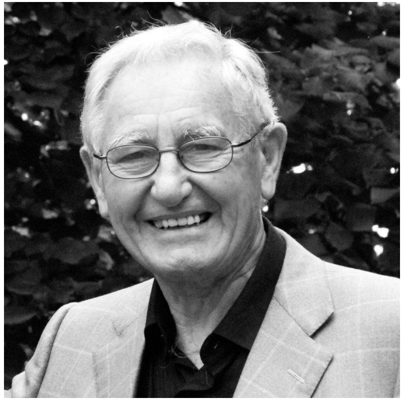


Ring singularity.
Physical

$$r = 0, \theta = \pi/2$$

Horizons.
Coordinate singularities

What are Black Holes?



Black solutions to the vacuum
Einstein Field Equations

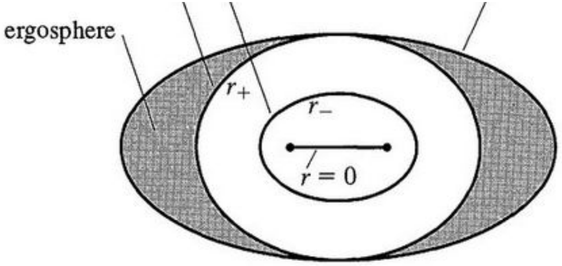
Kerr BH (rotating, no electric charge)

Nature of the singularity

$$ds^2 = -d\tau^2 + dx^2 + dy^2 + dz^2 + \frac{2Mr^3}{r^4 + a^2 z^2}\left(\frac{r(xdx + ydy) - a(xdy - ydx)}{r^2 + a^2} + \frac{zdz}{r} + d\tau\right)^2 .$$

where

$$r^4 - (x^2 + y^2 + z^2 - a^2)r^2 - a^2 z^2 = 0.$$



$$x^2 + y^2 + \left(\frac{r^2 + a^2}{r^2}\right) z^2 - (r^2 + a^2) = 0,$$

describes surfaces of constant τ and r

$r = 0, \theta = \pi/2$ corresponds to the ring $z = 0,\ x^2 + y^2 = a^2$

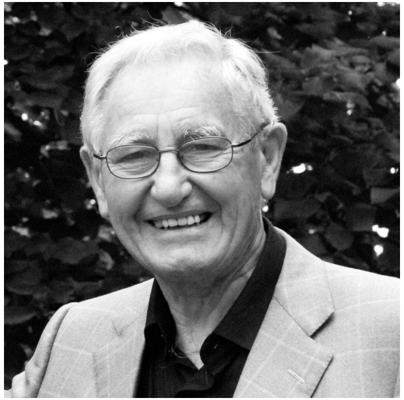
What are Black Holes?

Black solutions to the vacuum
Einstein Field Equations

Kerr BH (r

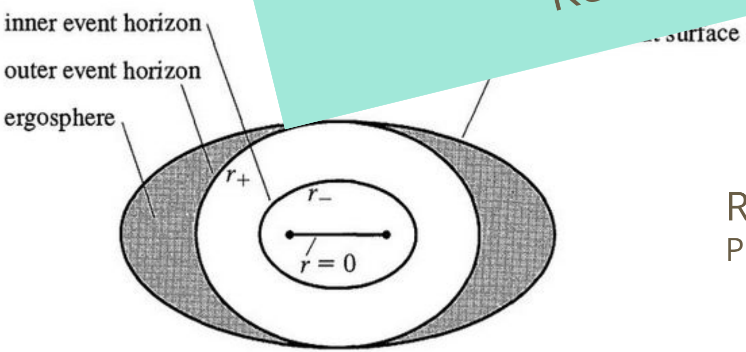
In 1965, the rotating charged solution for a black hole was found.

Kerr-Newman metric



In 1963

$$a^2 \cos^2\theta, \qquad \Delta = r^2 - 2mr + a^2.$$



Ring singularity.
Physical

Horizons.
Coordinate singularities

What are Black Holes?

What happens when we get close to a BH?

We have seen that the event horizon is just a coordinate singularity

But can it be safely crossed?

What are Black Holes?

What happens when
we get close to a BH?

Tidal acceleration

Central object of mass M
Length of the deformed object d
located at a distance r
suffers a tidal acceleration (non relativistic calculations)

$$a = 2GMd/r^3$$

What are Black Holes?

What happens when
we get close to a BH?

Example 1: The Earth

$M = 5.9\ 10^{27}$ grams

$R = 6.4\ 10^{8}$ cm

$d = 2$ m

$a = 0.0006\ cm/s^2$

$g = 979\ cm/s^2$

Tidal acceleration

Central object of mass M
Length of the deformed object d
located at a distance r
suffers a tidal acceleration (non relativistic calculations)

$$a = 2GMd/r^3$$

What are Black Holes?

Tidal acceleration

What happens when we get close to a BH?

Example 2: Stellar mass BH

$M = 1.9\ 10^{33}$ grams

r = 100 km

d = 2 m

$a = 50700000\ cm/s^2$

$g = 979\ cm/s^2$

Human gets spaghettified!

Central object of mass M

Length of the deformed object d

located at a distance r

suffers a tidal acceleration (non relativistic calculations)

$a = 2GMd/r^3$

Almost 52000 g!

What are Black Holes?

What happens when
we get close to a BH?

Exercise: Supermassive BH
M = 100 million times the Solar Mass
R_{EH} = ?? km
r = 100 km of the EH
d = 2 m

a = ?? cm/s^2
g = 979 cm/s^2

Human gets spaghettified?

Tidal acceleration

Central object of mass M
Length of the deformed object d
located at a distance r
suffers a tidal acceleration (non relativistic calculations)

$$a = 2GMd/r^3$$

What are Black Holes?

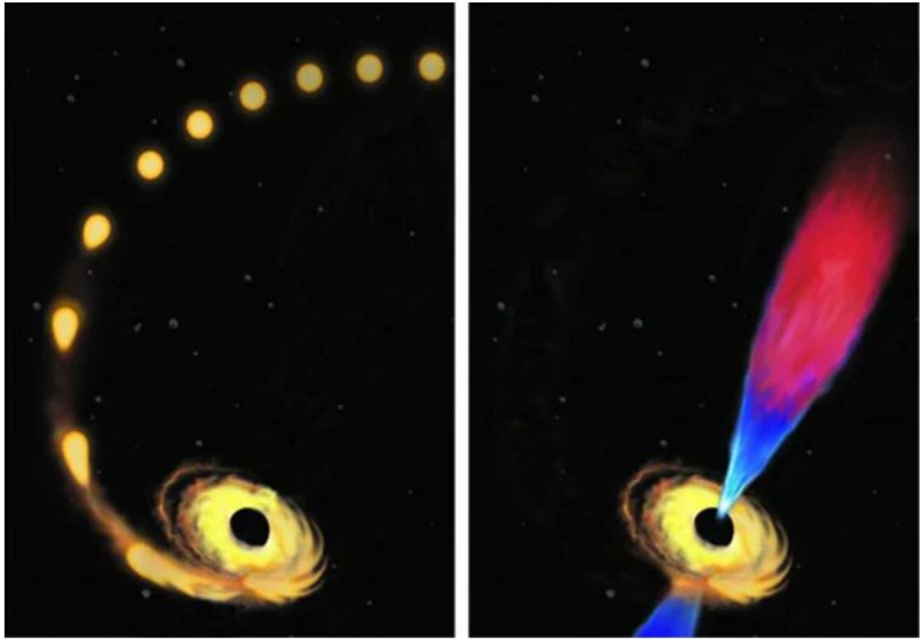
BLACK HOLE PHYSICS

A radio jet from the optical and x-ray bright stellar tidal disruption flare ASASSN-14li

S. van Velzen,[1*] G. E. Anderson,[2,3] N. C. Stone,[4] M. Fraser,[5] T. Wevers,[6] B. D. Metzger,[4] P. G. Jonker,[6,7] A. J. van der Horst,[8] T. D. Staley,[2] A. J. Mendez,[1] J. C. A. Miller-Jones,[3] S. T. Hodgkin,[5] H. C. Campbell,[5] R. P. Fender[2]

The tidal disruption of a star by a supermassive black hole leads to a short-lived thermal flare. Despite extensive searches, radio follow-up observations of known thermal stellar tidal disruption flares (TDFs) have not yet produced a conclusive detection. We present a detection of variable radio emission from a thermal TDF, which we interpret as originating from a newly launched jet. The multiwavelength properties of the source present a natural analogy with accretion-state changes of stellar mass black holes, which suggests that all TDFs could be accompanied by a jet. In the rest frame of the TDF, our radio observations are an order of magnitude more sensitive than nearly all previous upper limits, explaining how these jets, if common, could thus far have escaped detection.

Although radio jets are a ubiquitous and well-studied feature of accreting compact objects, it remains unclear why only a subset of active galactic nuclei (AGNs) are radio-loud. A stellar tidal disruption flare (TDF) presents a novel method with which to study jet production in accreting supermassive black holes. These flares occur after perturbations to a star's orbit have brought it to within a few tens of Schwarzschild radii of the central supermassive black hole and the star gets torn apart by the black hole's tidal force. A large amount of gas is suddenly



Artist's conception of a star being drawn toward a black hole and destroyed (left), and the black hole later emitting a "jet" of plasma composed of the debris left from the star's destruction. Modified from an original image by Amadeo Bachar.

What are Black Holes?

Naked singularities

What are Black Holes?

Naked singularities (there are many of them)
Case of Kerr spacetime

$$r_\pm = M \pm \sqrt{M^2 - a^2}.$$

Horizons disappear in the case a > M

Ring singularity causally connected with observers!

What are Black Holes?

Naked singularities
Case of Kerr spacetime

$$r_\pm = M \pm \sqrt{M^2 - a^2}.$$

Horizons disappear in the case a > M

Ring singularity causally connected with observers!

Can not accept these objects in the Universe!
Cosmic Censorship hypothesis

What are Black Holes?

Naked singularities
Case of Kerr spacetime

r

No proof yet

In a series of works with my PhD advisor and collaborators have shown that, contrary to BH solutions, the most relevant naked singularities are unstable against linear perturbations and for that reason they lack astronomical relevance

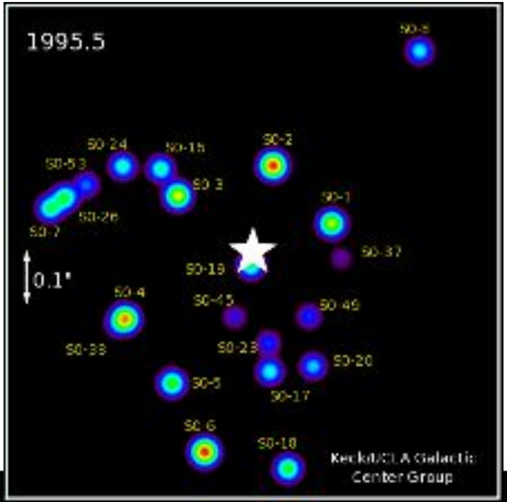
Horizons disappear in the case a > M

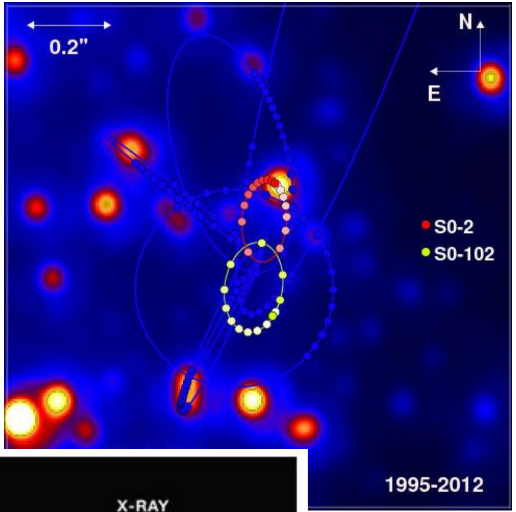
Ring singularity causally connected with observers!

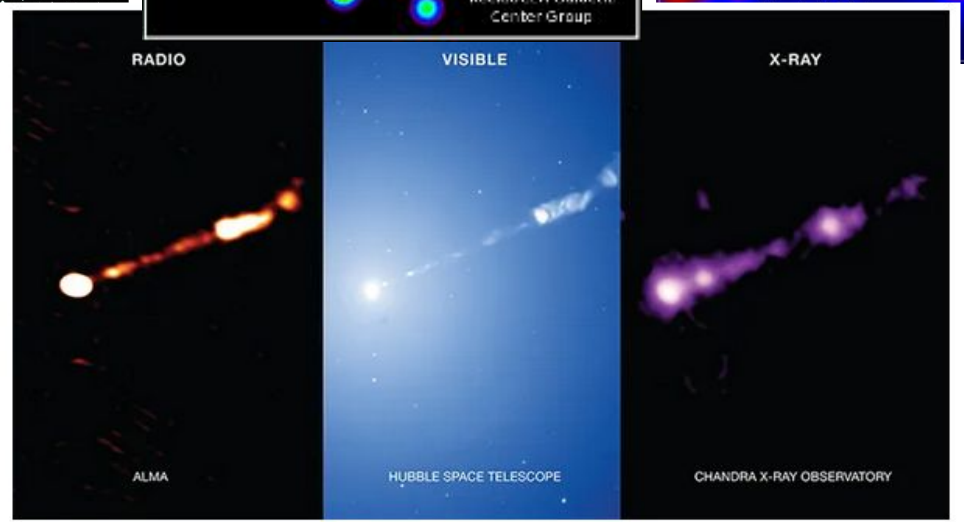
Can not accept these objects in the Universe!
Cosmic Censorship hypothesis

What are Black Holes?

Observational evidence
of what can not be
seen

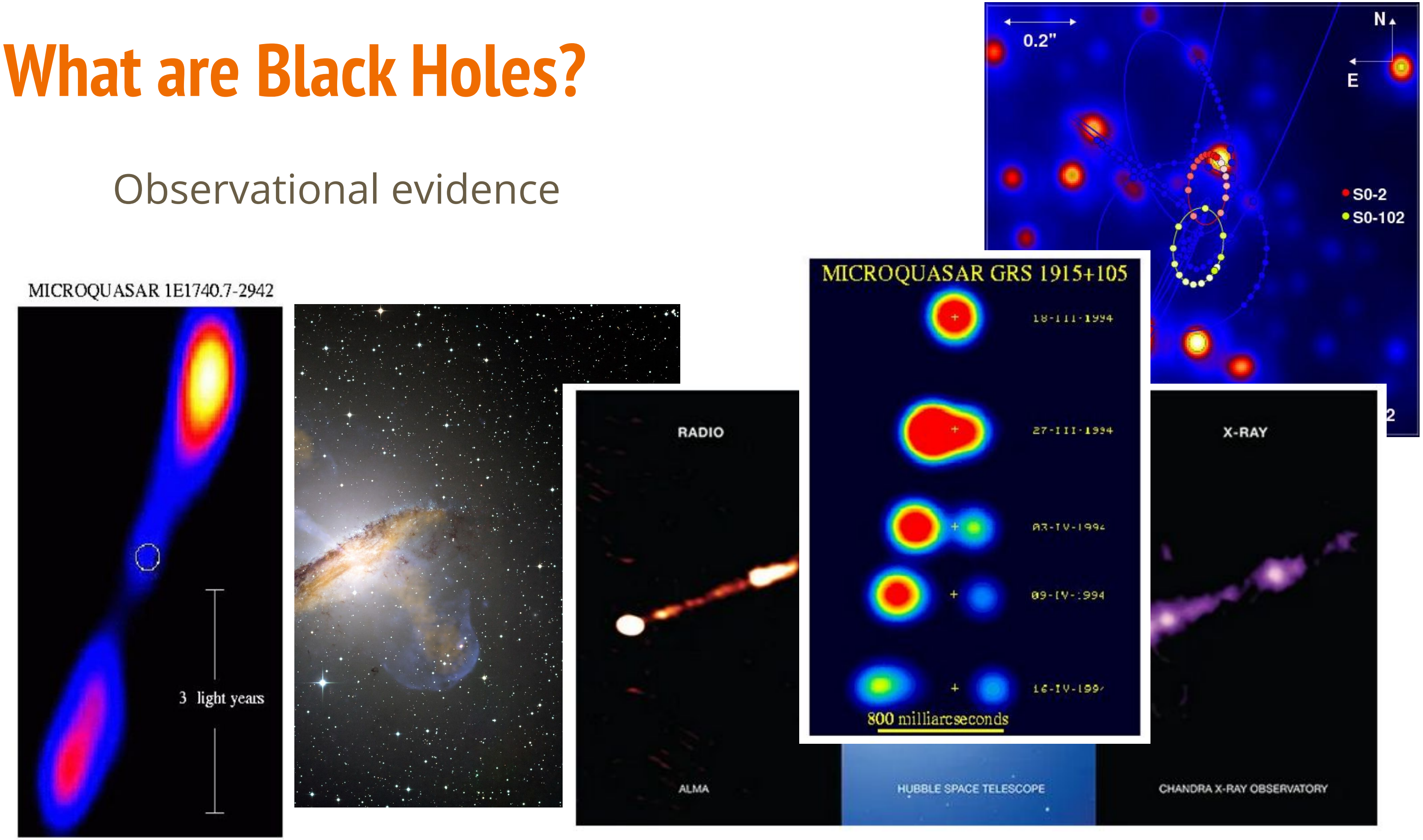






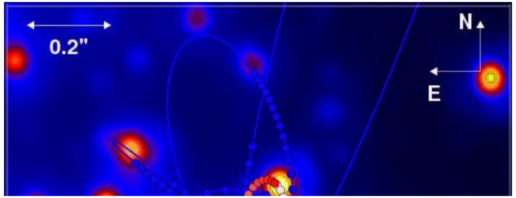
What are Black Holes?

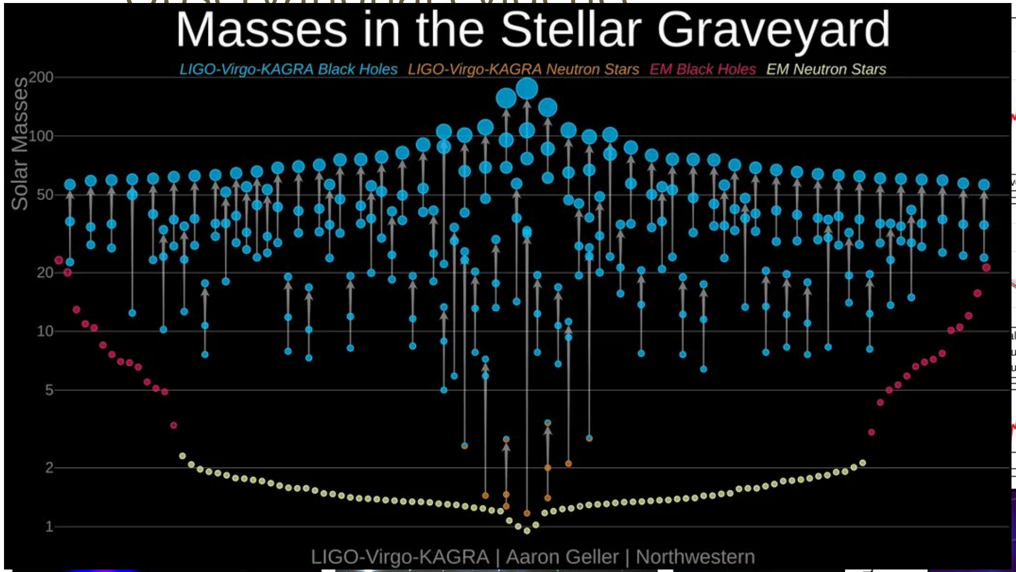
Observational evidence

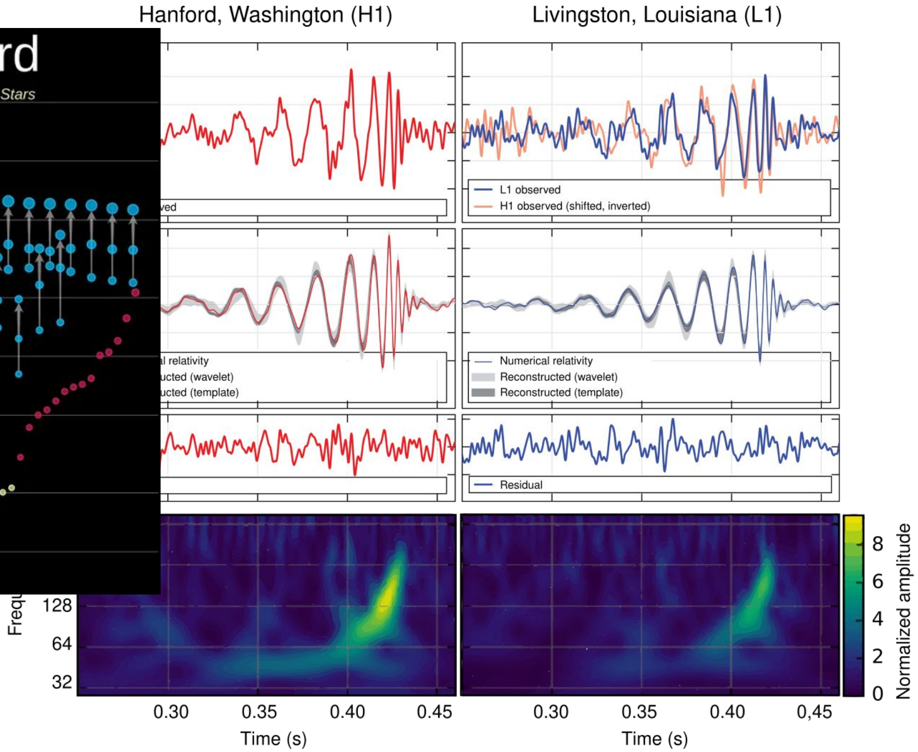


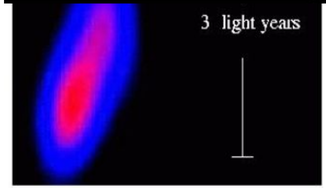
What are Black Holes?

Observational evidence

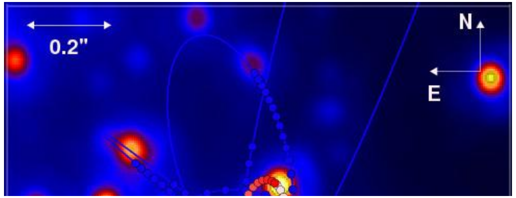


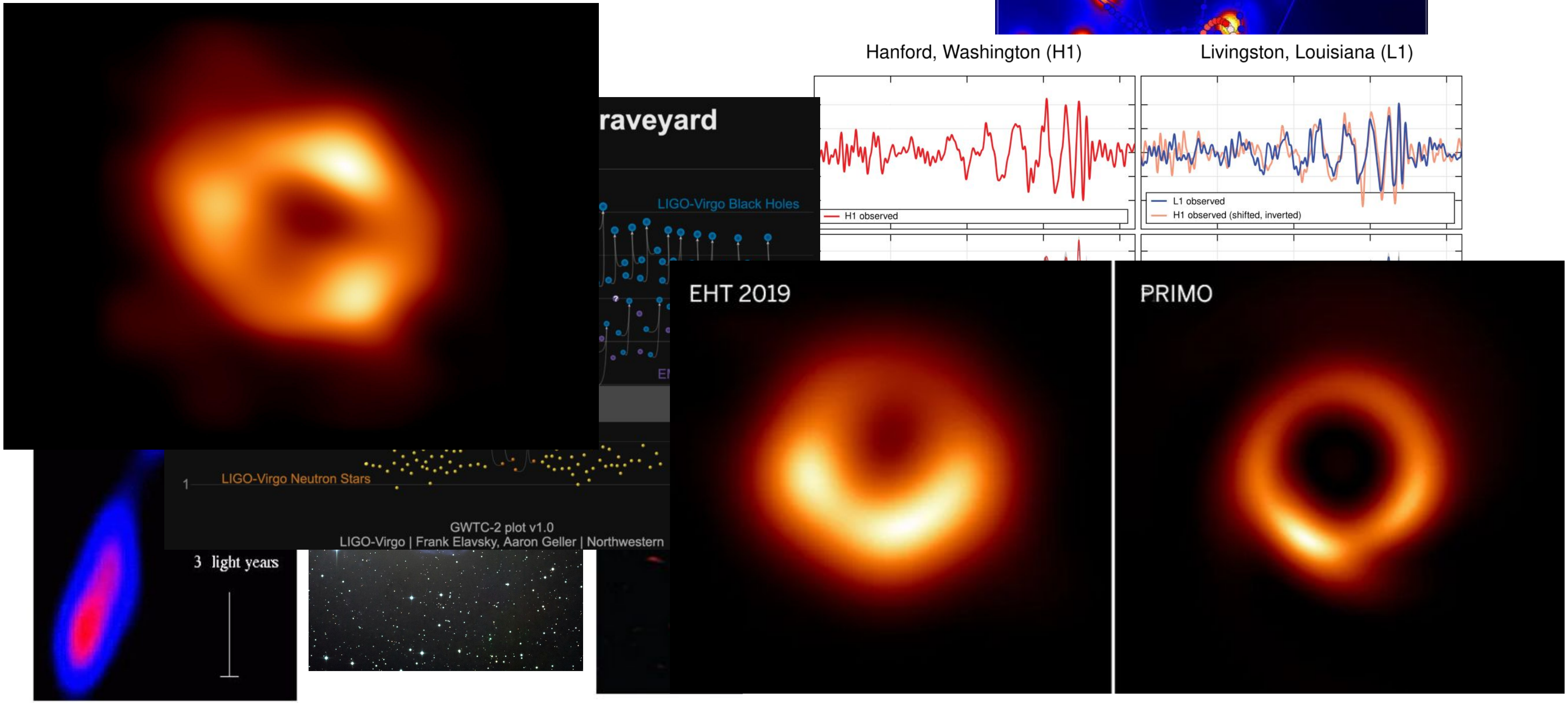




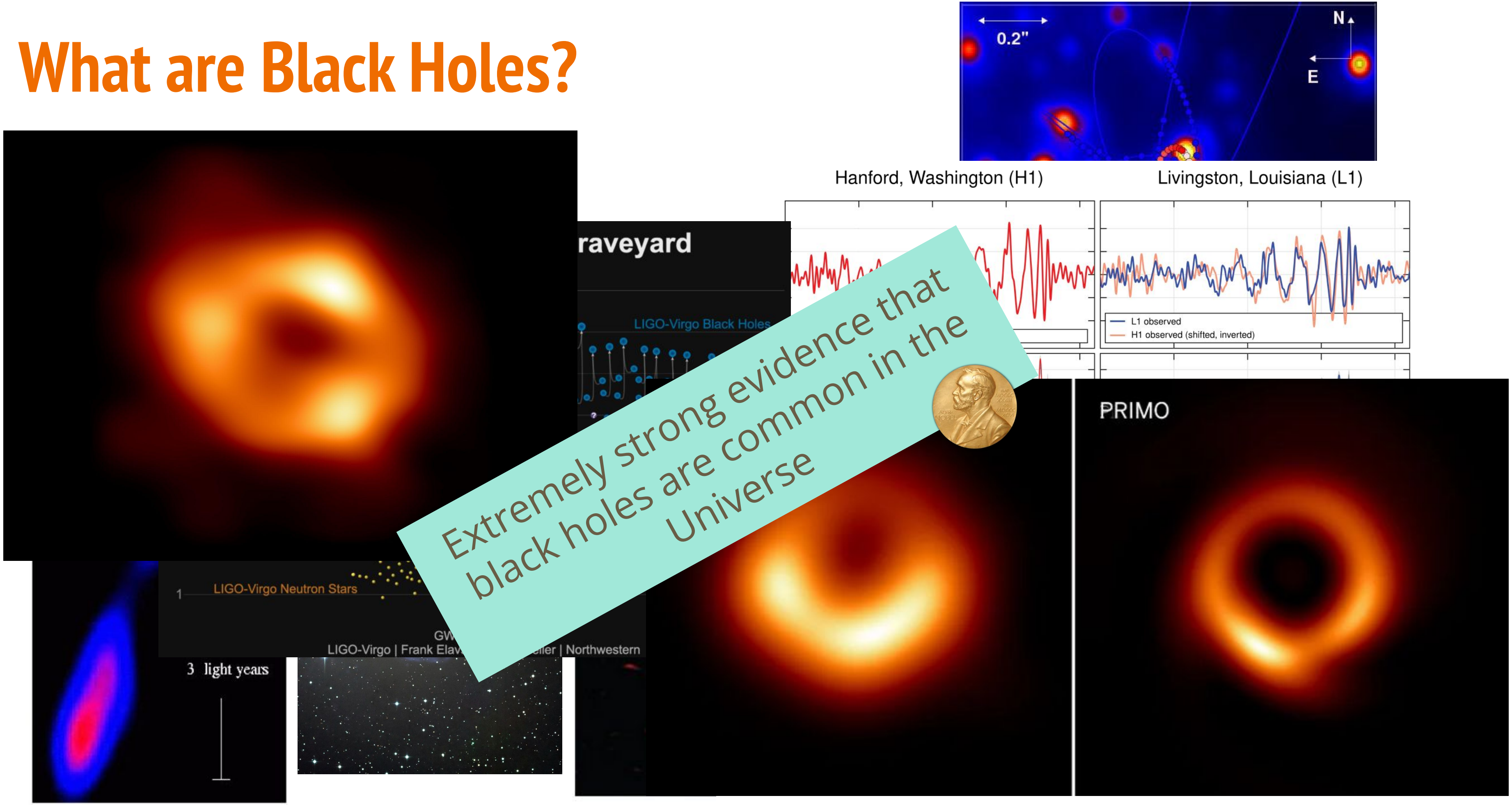


What are Black Holes?





What are Black Holes?

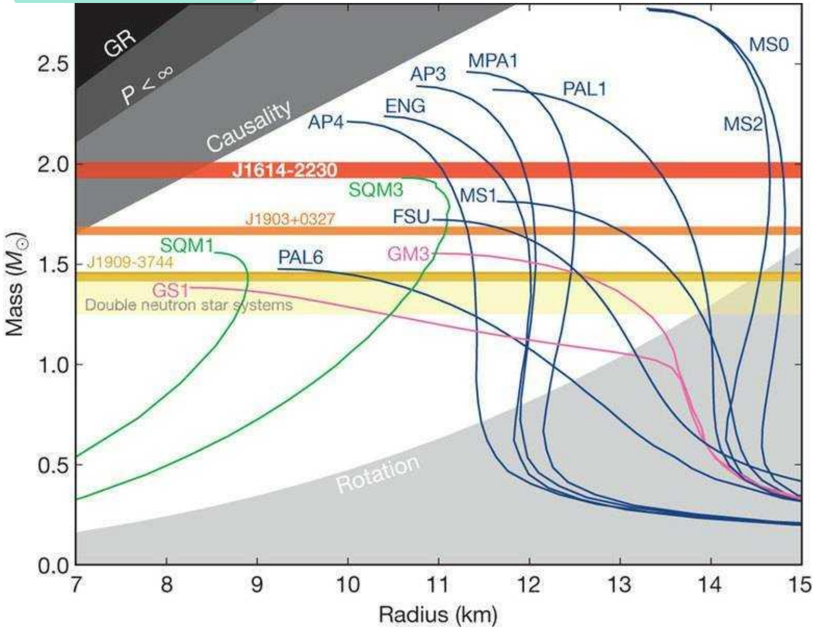


Extremely strong evidence that black holes are common in the Universe

How are Black Holes formed?

We will focus in stellar mass black holes...

We have seen that as NSs get more massive, they shrink



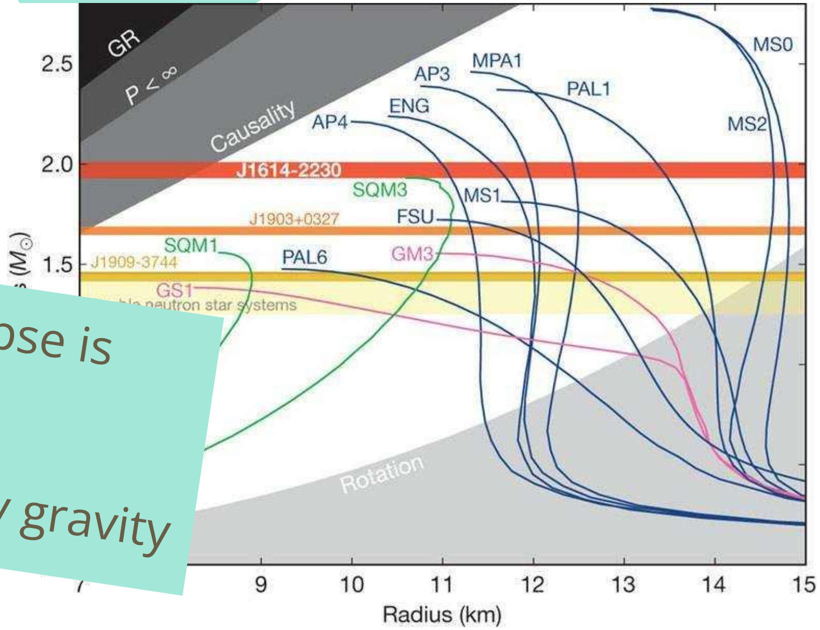
How are Black Holes formed?

We will focus in stellar mass black holes...

We have seen that as NSs get more massive, they shrink

Until complete collapse is inevitable

Matter is overpowered by gravity



How are Black Holes formed?

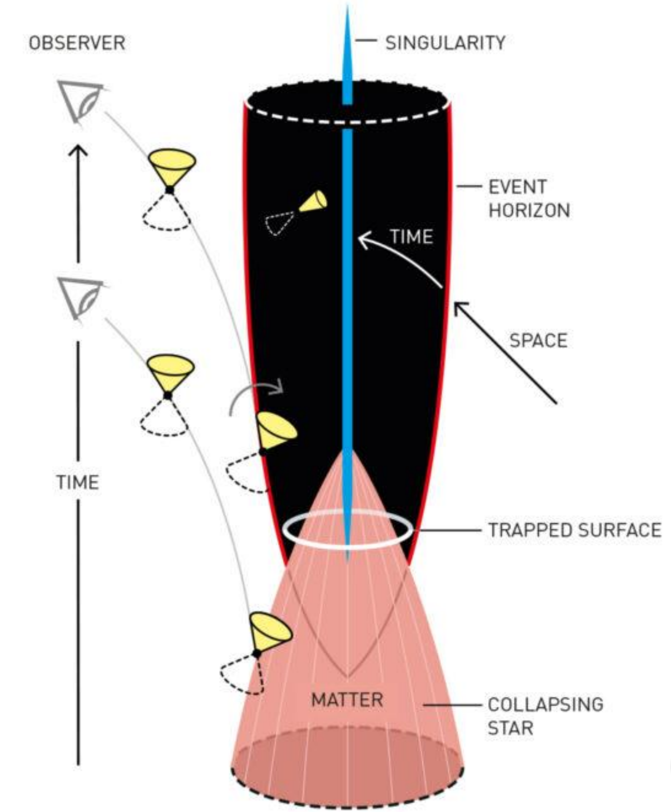
stellar mass

We wi

We have seen that as NSs get more massive, they shrink

Until complete collapse is inevitable

Matter is overpowered by gravity



Accretion disks around BHs

Geodesics around BHs

Accretion disks around BHs

Geodesics around BHs

But first….
What is a geodesic?

Accretion disks around BHs

Geodesics ~~around BHs~~

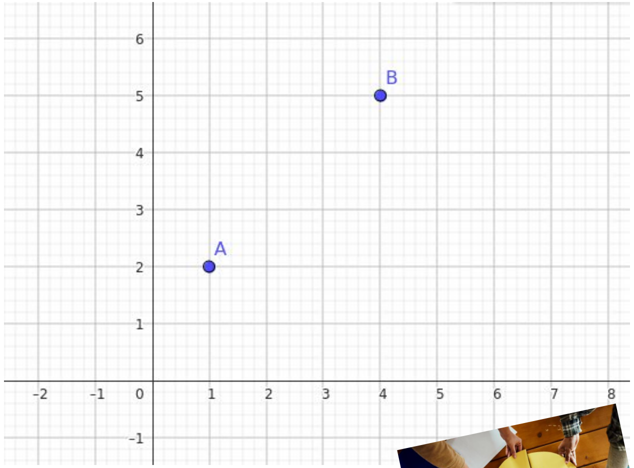
A geodesic is a curve that represents the shortest path between two points in a surface

Accretion disks around BHs

Geodesics ~~around BHs~~

A geodesic is a curve that represents the shortest path between two points of a surface

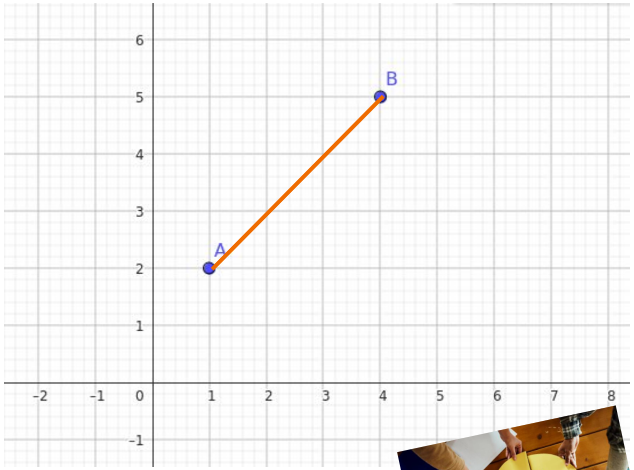
Example: plane



Accretion disks around BHs

Example: plane

Geodesics ~~around BHs~~



A geodesic is a curve that represents the shortest path between two points in a surface

Line segment

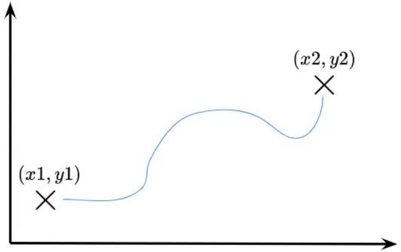
Accretion disks around BHs

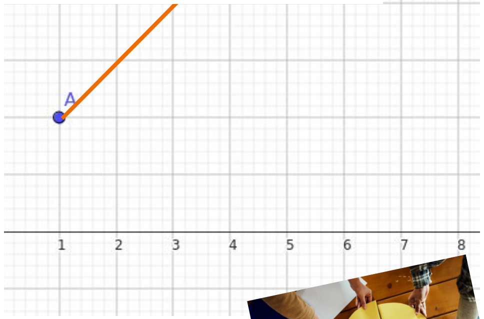
Variational approach

Geodesics

$$J[y] = \int_{x_1}^{x_2} dS$$
$$= \int_{x_1}^{x_2} \sqrt{dx^2 + dy^2}$$
$$= \int_{x_1}^{x_2} \sqrt{1 + \frac{dy}{dx}^2} dx$$
$$= \int_{x_1}^{x_2} \sqrt{1 + y'(x)^2} dx$$
$$= \int_{x_1}^{x_2} L(x, y, y') dx$$

y is the function that minimizes functional J between points x_1 and x_2





A geodesic shortest pa surface

ıent

Lagrangian

Accretion disks around BHs

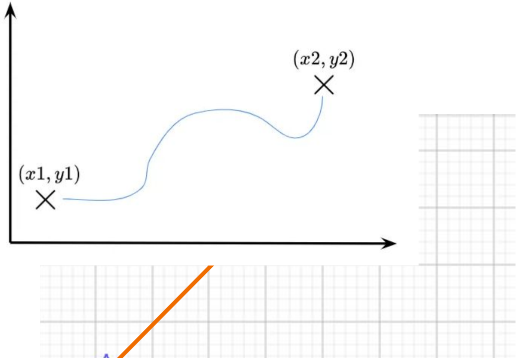
Variational approach

Geodesics

y is the function that minimizes functional J between points x_1 and x_2

$$
\begin{aligned}
J[y] &= \int_{x_1}^{x_2} dS \\
&= \int_{x_1}^{x_2} \sqrt{dx^2 + dy^2} \\
&= \int_{x_1}^{x_2} \sqrt{1 + \frac{dy}{dx}^2}\, dx \\
&= \int_{x_1}^{x_2} \sqrt{1 + y'(x)^2}\, dx \\
&= \int_{x_1}^{x_2} L(x, y, y')\, dx
\end{aligned}
$$

A geodesic
shortest pa
surface



$$\frac{\partial L}{\partial y} - \frac{d}{dx}\left(\frac{\partial L}{\partial y'}\right) = 0$$

$=$

$y(x$

$y(x$ $_1) + y_1$

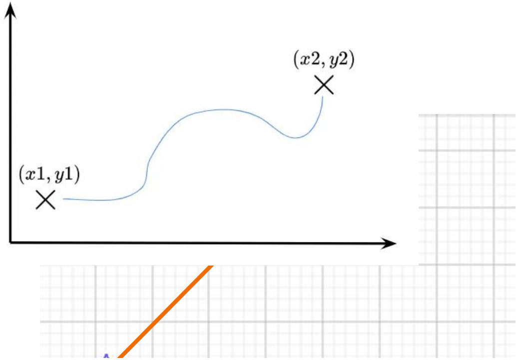
Accretion disks around BHs

Variational approach

Geodesics

$$
\begin{aligned}
J[y] &= \int_{x_1}^{x_2} dS \\
&= \int_{x_1}^{x_2} \sqrt{dx^2 + dy^2} \\
&= \int_{x_1}^{x_2} \sqrt{1 + \frac{dy}{dx}^2}\, dx \\
&= \int_{x_1}^{x_2} \sqrt{1 + y'(x)^2}\, dx \\
&= \int_{x_1}^{x_2} L(x, y, y')\, dx
\end{aligned}
$$

y is the function that minimizes functional J between points x_1 and x_2



$$\frac{\partial L}{\partial y} - \frac{d}{dx}\left(\frac{\partial L}{\partial y'}\right) = 0$$

$$\implies y''(x) = 0$$

$$y(x) = ax + b$$

$$y(x) = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) + y_1$$

A geodesic shortest pa surface

Accretion disks around BHs

Variational approach

Geodesics

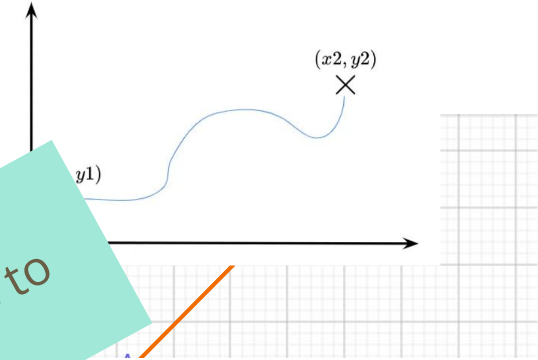
y is the function that minimizes functional J between points x_1 and x_2

$$J[y] = \int_{x_1}^{x_2} dS$$

$$= \int_{x_1}^{x_2} \sqrt{dx^2 + dy^2}$$

$$= \int_{x_1}^{x_2} \ldots dx$$

$$= \int_{x_1} L(x, y, y')dx$$



Exercise
Perform these calculations to check the result

$$\frac{\partial L}{\partial y} - \frac{d}{dx}\left(\frac{\partial L}{\partial y'}\right) = 0$$

$$\implies y''(x) = 0$$

$$y(x) = ax + b$$

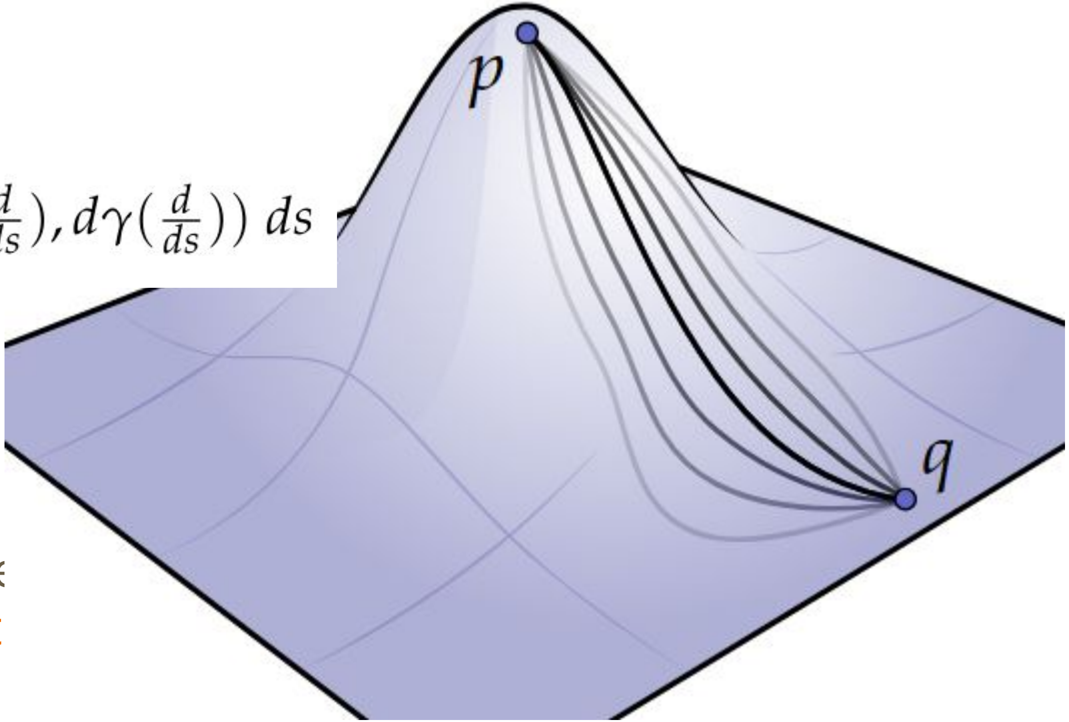
$$y(x) = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) + y_1$$

A geodesic shortest pa
surface

Accretion disks around BHs

Geodesics ~~around BHs~~

$$L(\gamma) := \int_a^b |d\gamma|^2 = \int_a^b g(d\gamma(\tfrac{d}{ds}), d\gamma(\tfrac{d}{ds}))\, ds$$



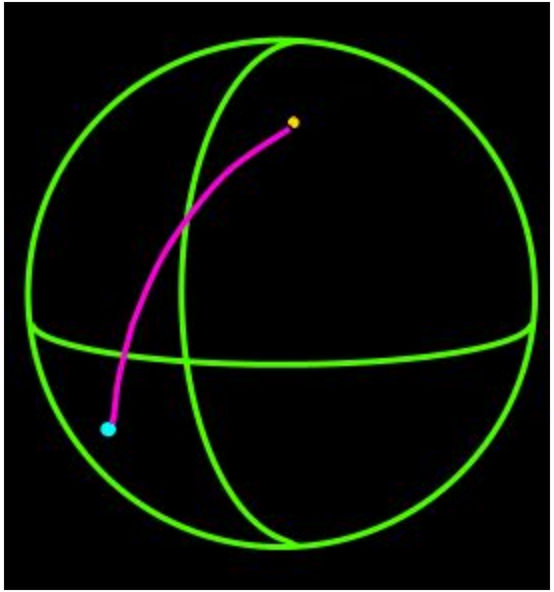
A geodesic is a curve that represe
shortest path between two point
surface

Accretion disks around BHs

Geodesics ~~around BHs~~

A geodesic is a curve that represents the shortest path between two points in a surface

Example: sphere

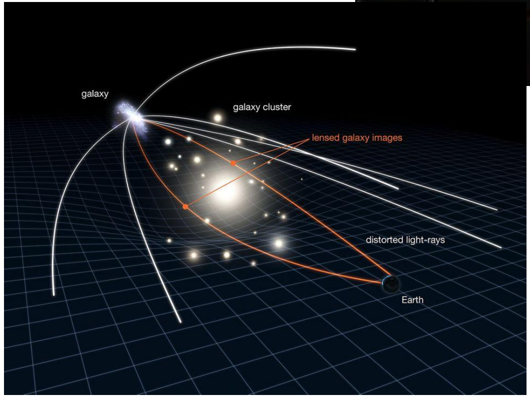


Great arc on circle

Accretion disks around BHs

Example: light path

Geodesics ~~around BHs~~



A geodesic is a curve that represents the shortest path between two points in a surface

Gravitational lensing

Accretion disks around BHs

Geodesics around BHs

different for massive particles that for photons

we would not go into the details…. long calculations

Accretion disks around BHs

Geodesics in Kerr metric

we would not go into the details…. long calculations

$$E \equiv -k^\mu u_\mu = -u_t = -p_t \quad \text{constant along geodesics}$$

$$L \equiv m^\mu u_\mu = u_\phi = p_\phi \quad \text{constant along geodesics}.$$

$$g_{\mu\nu} u^\mu u^\nu = \kappa$$

$$\begin{aligned} \kappa &= -1 && \text{for timelike geodesics} \\ \kappa &= 1 && \text{for spacelike geodesics} \\ \kappa &= 0 && \text{for null geodesics}. \end{aligned}$$

Accretion disks around BHs

Geodesics in Kerr metric

we would not go into the details.... long calculations

$$E \equiv -k^\mu u_\mu = -u_t = -p_t \quad \text{constant along geodesics}$$

$$L \equiv m^\mu u_\mu = u_\phi = p_\phi \quad \text{constant along geodesics}\,.$$

$$g_{\mu\nu} u^\mu u^\nu = \kappa$$

$$\kappa = -1 \qquad \text{for timelike geodesics}$$

$$\kappa = 0 \qquad \text{for null geodesics}\,.$$

movement in the equatorial plane $\theta \equiv \frac{\pi}{2}\,.$

$$\begin{aligned}\mathcal{L} &= \frac{1}{2} g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu = \frac{1}{2}\left\{-\left(1-\frac{2Mr}{\Sigma}\right)\dot{t}^2 - \frac{4Mr}{\Sigma} a \sin^2\theta\, \dot{t}\dot{\phi} + \frac{\Sigma}{\Delta}\,\dot{r}^2\right. \\ &+ \left.\Sigma\,\dot{\theta}^2 + \left[r^2 + a^2 + \frac{2Mra^2}{\Sigma}\sin^2\theta\right]\sin^2\theta\,\dot{\phi}^2\right\}\end{aligned}$$

Accretion disks around BHs

Geodesics in Kerr metric

we would not go into the details.... long calculations

$$\boxed{\dot{r}^2 = \frac{(r^2+a^2)^2 - a^2\Delta}{r^4}(E - V_+)(E - V_-) + \frac{\kappa\Delta}{r^2}},$$

$$\boxed{V_\pm = \frac{2Mar \pm r^2\sqrt{\Delta}}{(r^2+a^2)^2 - a^2\Delta}L}.$$

The conserved quantities read

$$\begin{aligned} E &= -g_{t\mu}u^\mu = \left(1 - \frac{2M}{r}\right)\dot{t} + \frac{2Ma}{r}\dot{\phi} \\ L &= g_{\phi\mu}u^\mu = -\frac{2Ma}{r}\dot{t} + \left(r^2 + a^2 + \frac{2Ma^2}{r}\right)\dot{\phi}\,. \end{aligned}$$

Equations of motion

$$\Delta = r^2 - 2Mr + a^2$$

$$\boxed{\begin{aligned} \dot{t} &= \frac{1}{\Delta}\left[\left(r^2 + a^2 + \frac{2Ma^2}{r}\right)E - \frac{2Ma}{r}L\right] \\ \dot{\phi} &= \frac{1}{\Delta}\left[\left(1 - \frac{2M}{r}\right)L + \frac{2Ma}{r}E\right] \end{aligned}}$$

Accretion disks around BHs

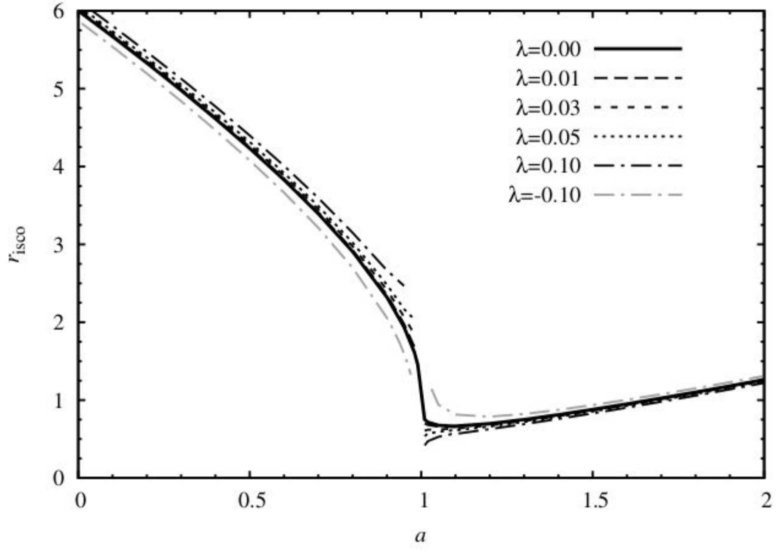
Geodesics in Kerr metric

Innermost circular orbit
Important feature for X-ray astronomy

we would not go into the details…. long calculations

$$\dot{r}^2 = \frac{(r^2+a^2)^2 - a^2\Delta}{r^4}(E - V_+)(E -$$

$$V_\pm = \frac{2Mar \pm r^2\sqrt{\Delta}}{(r^2+a^2)^2 - a^2\Delta}L$$



Accretion disks around BHs

Geodesics in Kerr
metric

For photons

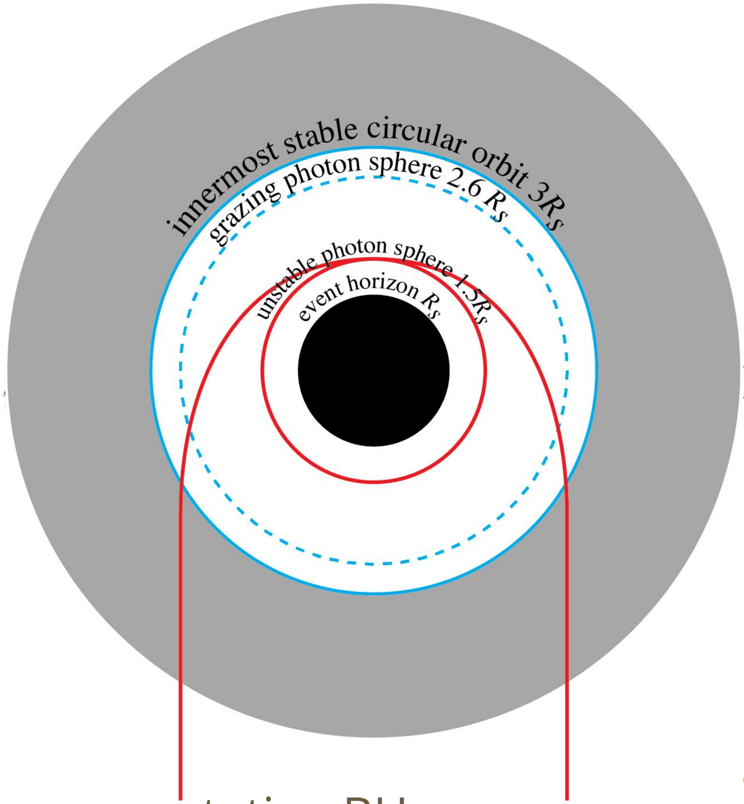
$$\dot{r}^2 = \frac{C}{r^2}(E - V_+)(E - V_-) = \frac{(r^2 + a^2)^2 - a^2\Delta}{r^4}(E - V_+)(E - V_-)$$

$$\Delta = r^2 - 2Mr + a^2$$

$$r(r - 3M)^2 - 4Ma^2 = 0\,.$$

Photon sphere

Accretion disks around BHs



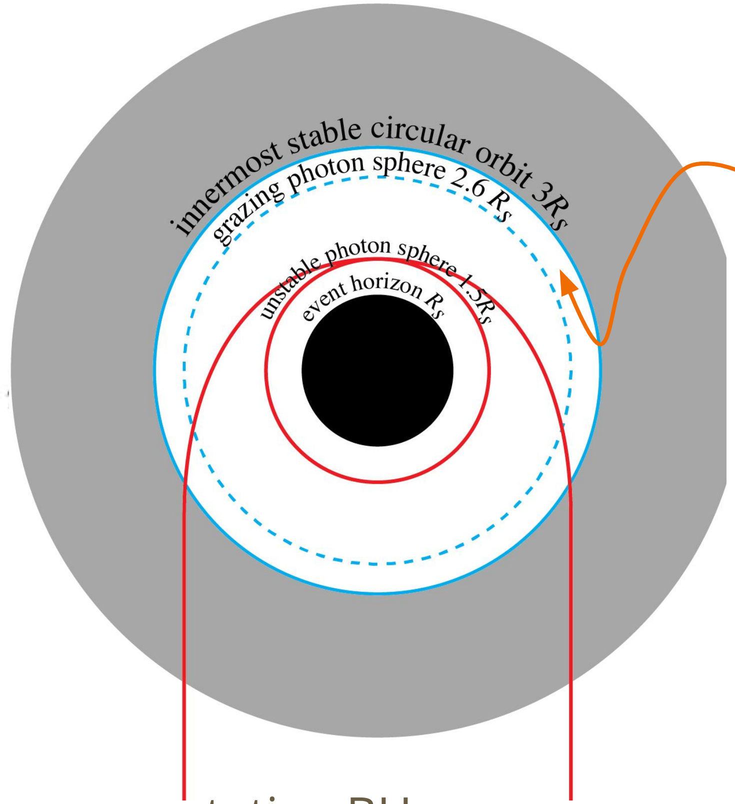
$$\frac{-\, a^2\Delta}{\;}(E - V_+)(E - V_-)$$

$$\Delta = r^2 - 2Mr + a^2$$

Photon sphere

For non-rotating BHs

Accretion disks around BHs

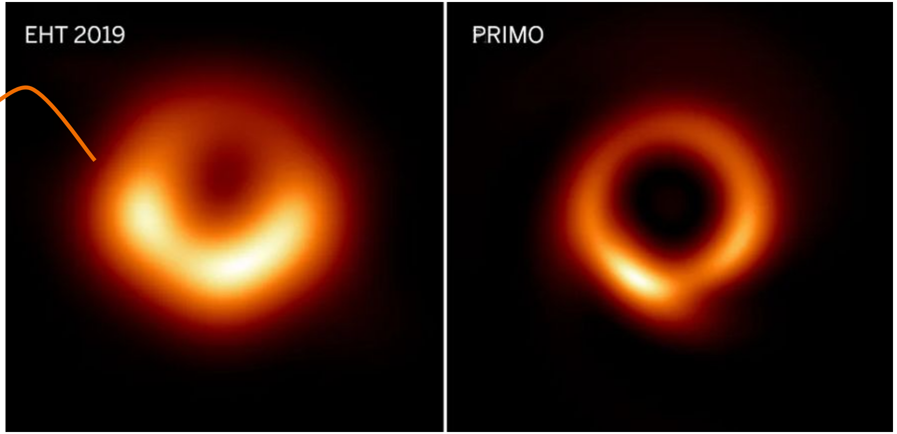


For non-rotating BHs

It can be generalized to Kerr BH

$$r^{\circ}_{\pm} = r_s \left[1 + \cos\left(\frac{2}{3} \arccos \frac{\pm|a|}{M}\right)\right]$$

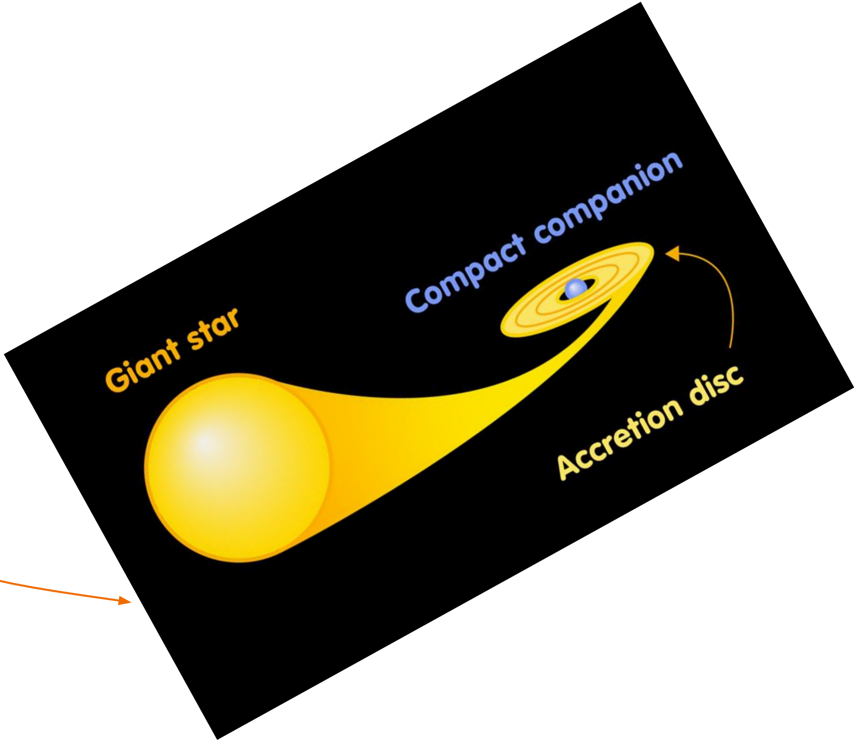
and observationally tested!!!!



Accretion disks around BHs

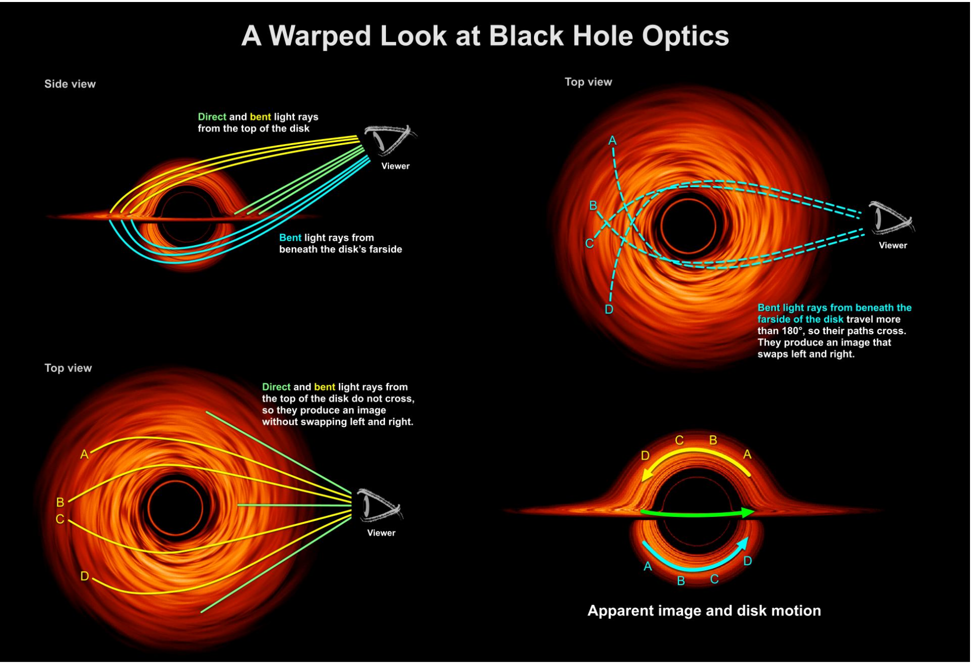
How are accretion disks seen

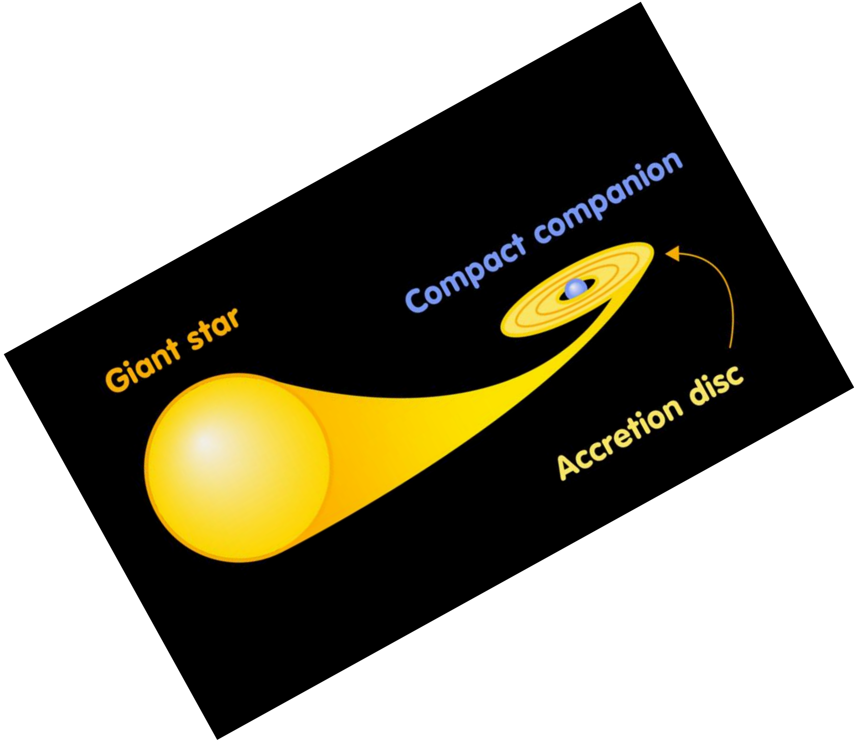
A giant star is eaten by the gravitational pull of the black hole and an accretion disk around it is formed



Accretion disks around BHs

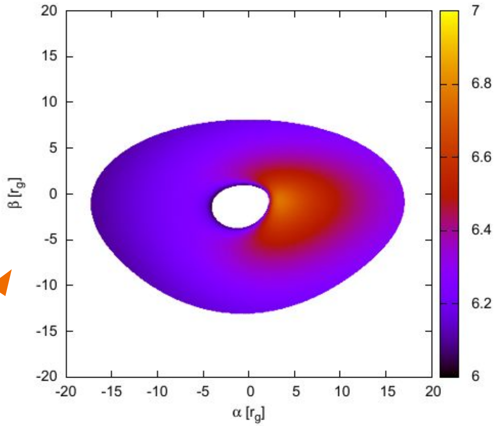
How are accretion disks seen

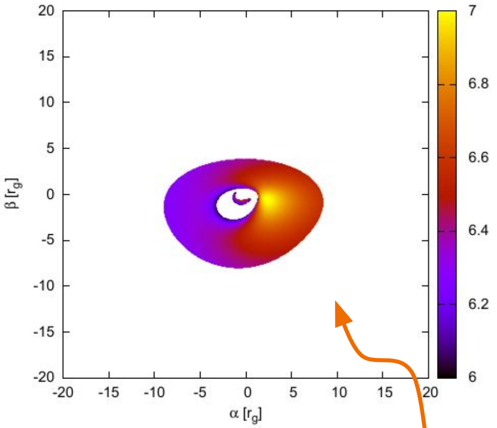




Accretion disks around BHs

How are accretion disks
seen



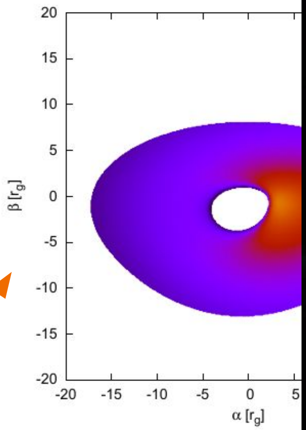


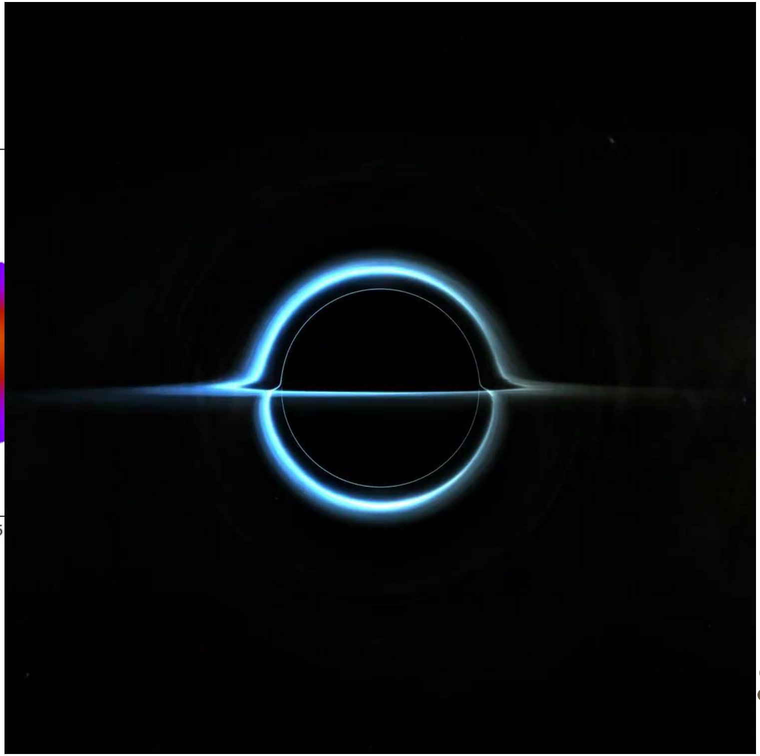
almost extreme Kerr BH

almost extreme Kerr naked singularity

Accretion disks around BHs

How are accretion disks
seen





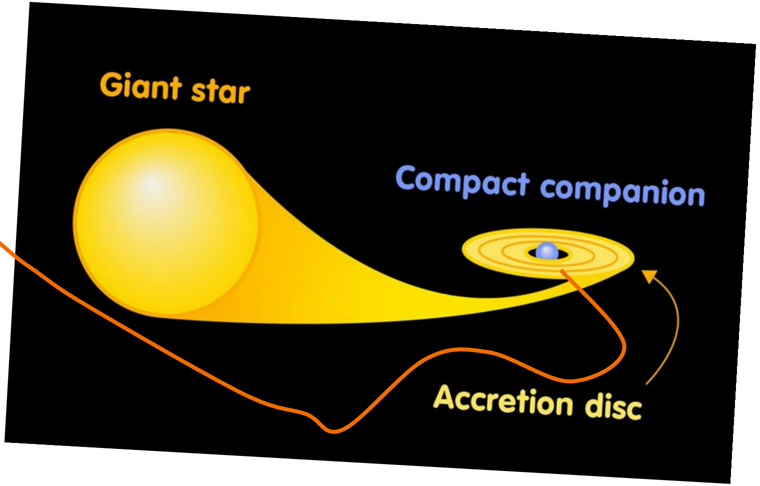
almost extreme Kerr BH

gularity

Accretion disks around BHs

Thermal radiation

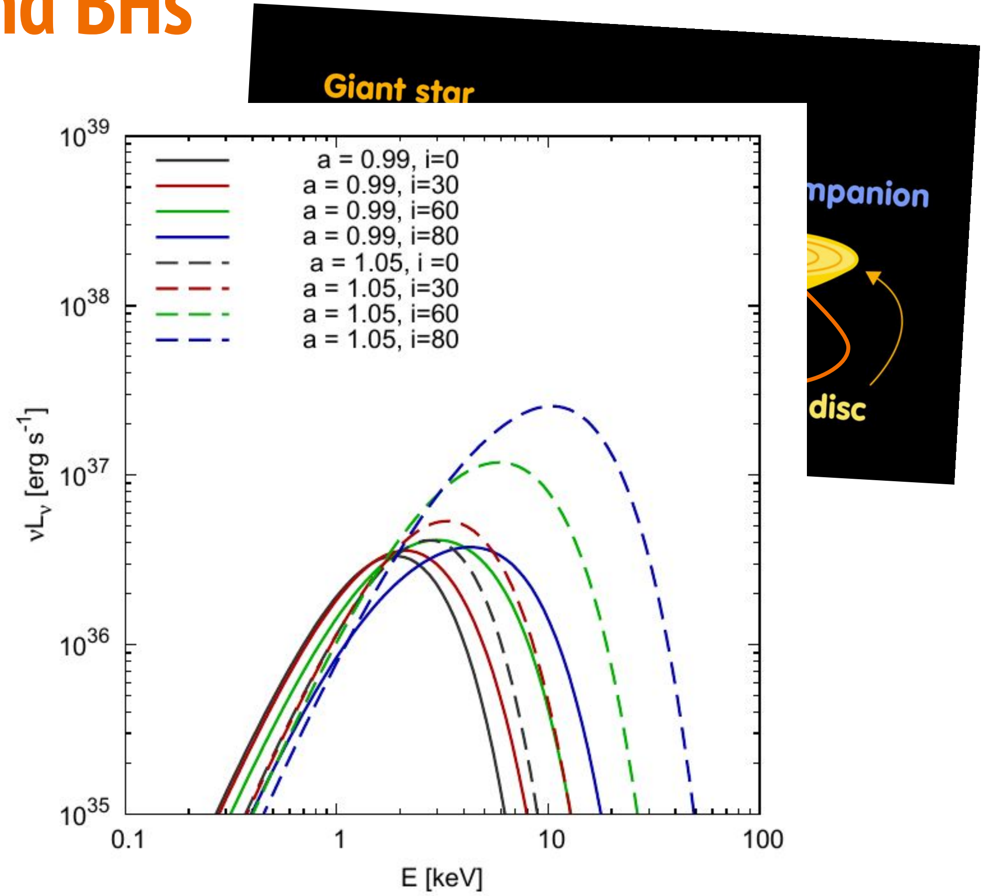
$$B(\nu; T_{\mathrm{ef}}) = \frac{2h\nu^3}{c^2} \frac{1}{e^{\frac{h\nu}{k_B T_{\mathrm{ef}}}} - 1}.$$

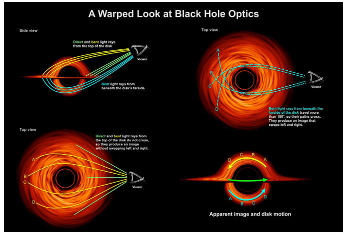


Accretion disks around BHs

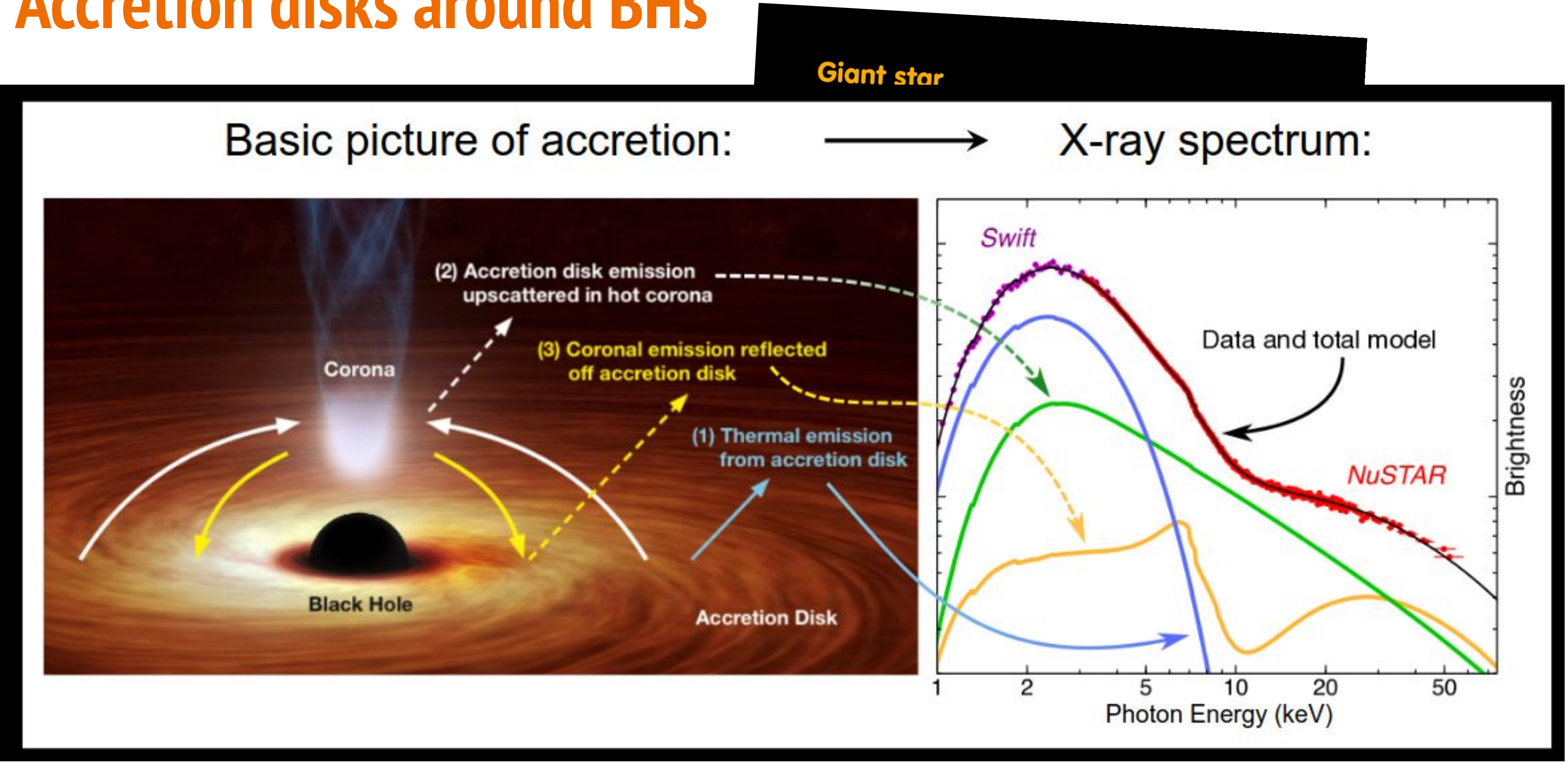
Thermal radiation

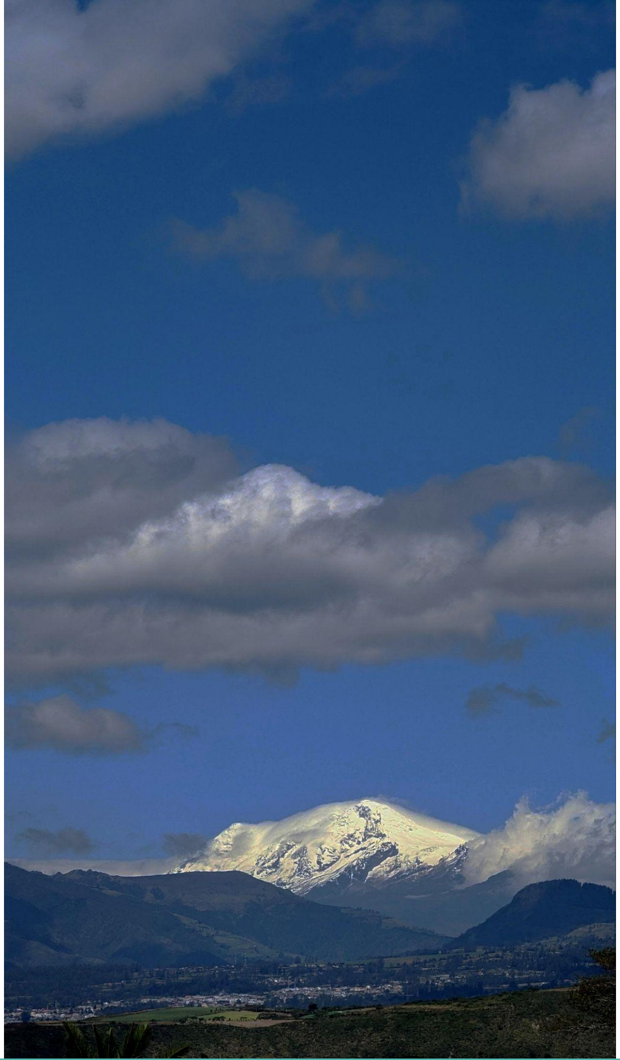
$$B(\nu; T_{\rm ef}) = \frac{2h\nu^3}{c^2}\frac{1}{e^{\frac{h\nu}{k_B T_{\rm ef}}} - 1}.$$





Accretion disks around BHs





Thanks for all!!
enjoy the rest of ISYA 46!!

If you want to contact me feel free to write an e-mail

ifrs82@gmail.com