Neutron Stars

Lecture 3: Dense matter and phase transitions

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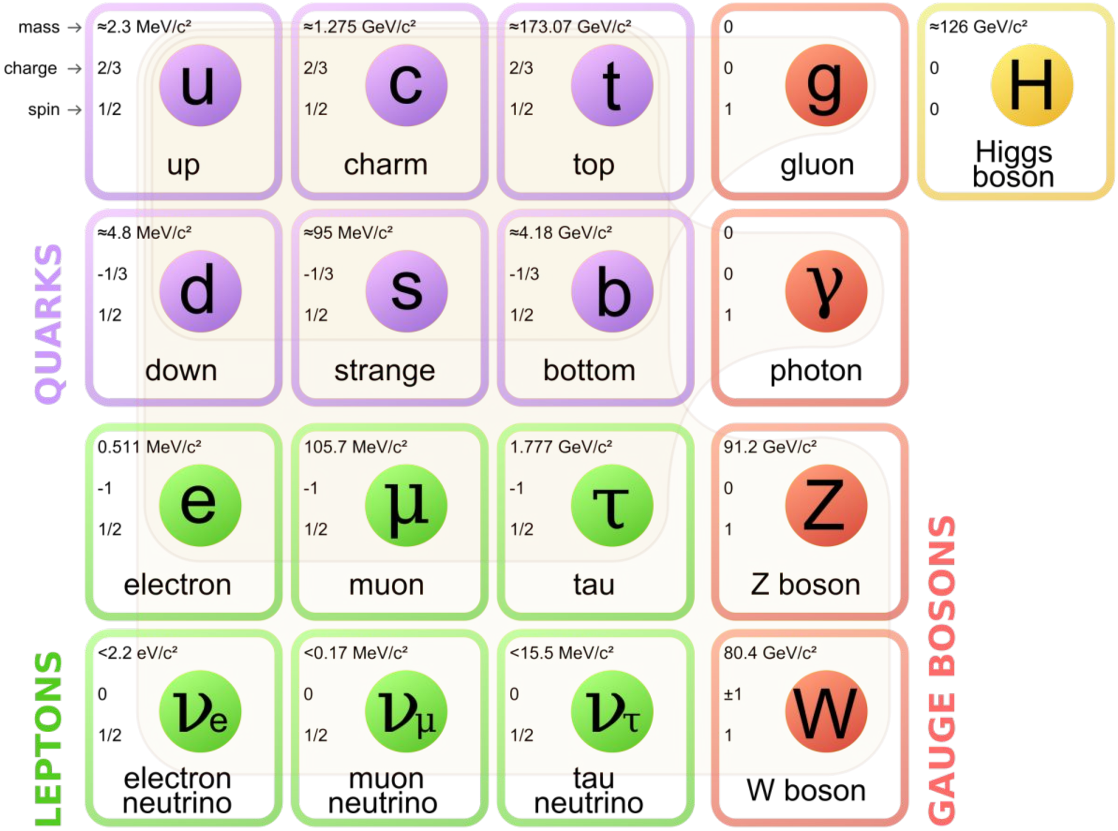
Equation of state for (extremely) dense matter

First things first

Standard Model of particles and interactions

Extremely efficient to describe matter, but can not explain Dark Matter



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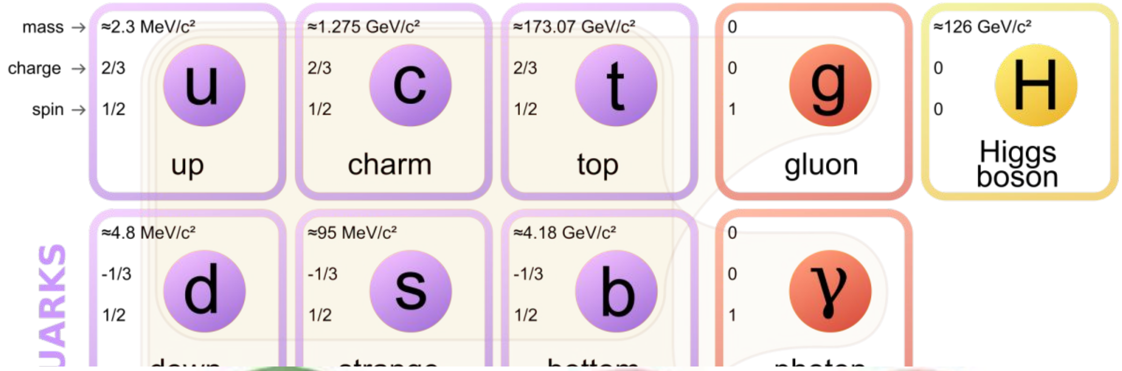
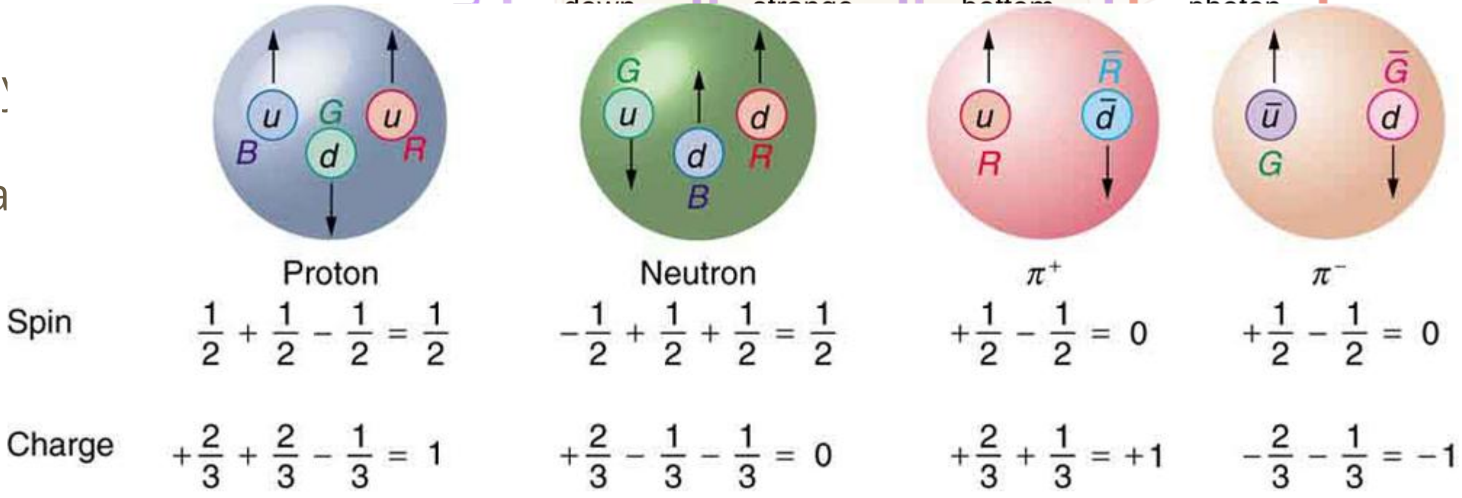


Extremel
describe
not expla



Equation of state for (extremely) dense matter

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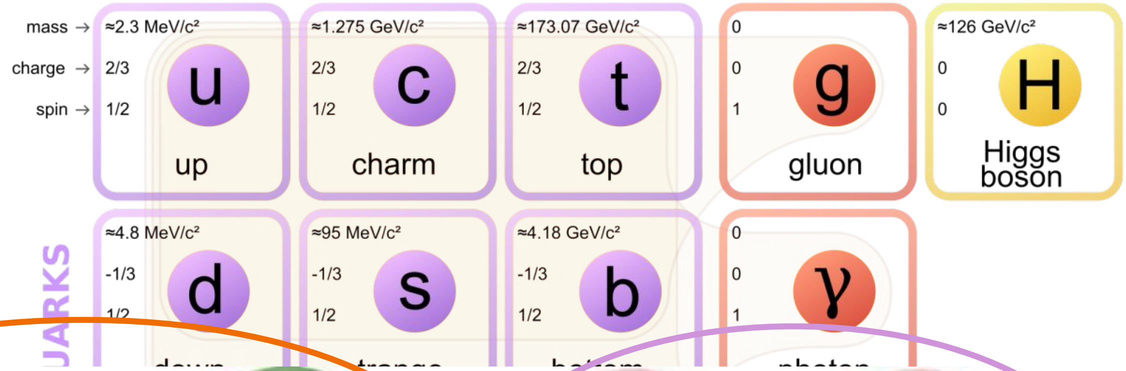

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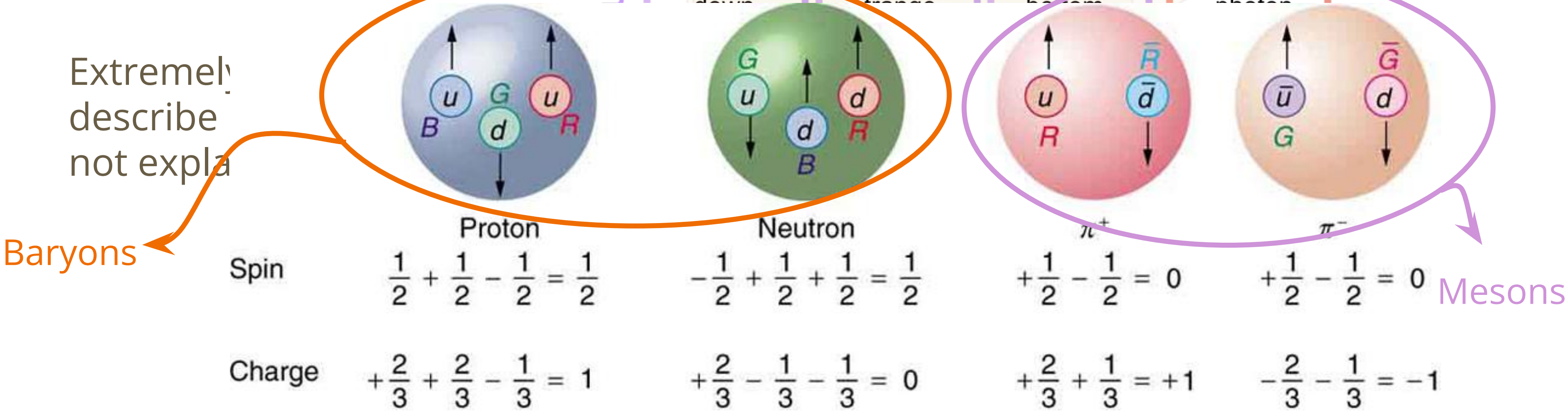

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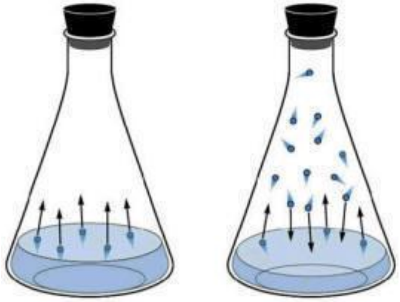


First things first

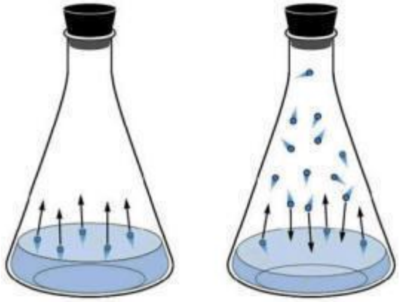
Concept of phase diagram and its relevance

Shows distinct phases at different thermodynamic conditions (of pressure and temperature)

Important definitions

- Coexistence curves
- Critical points

Equation of state for (extremely) dense matter



First things first

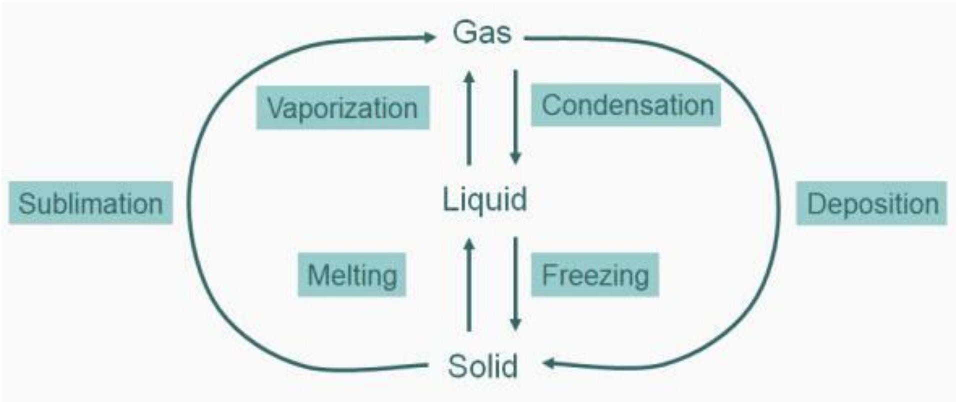
Concept of phase diagram and its relevance

Shows distinct phases at different thermodynamic conditions (of pressure and temperature)

Important definitions

- Coexistence curves
- Critical points

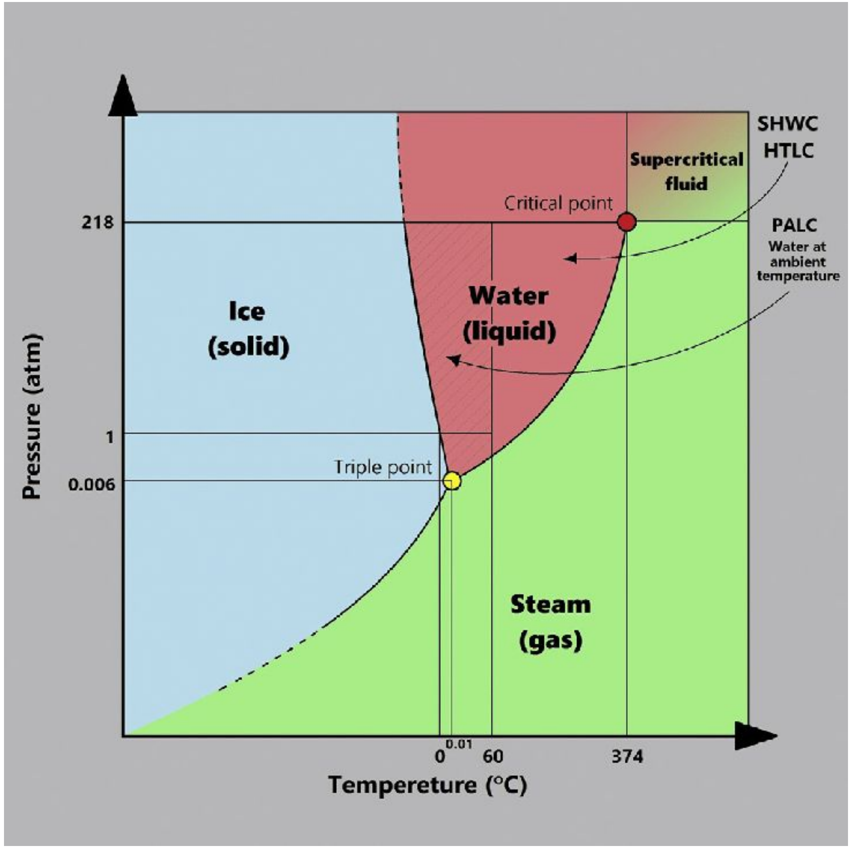
Allows to understand phase transitions



Equation of state for (extremely) dense matter

First things first

A “simple” example: water



Equation of state for (extremely) dense matter

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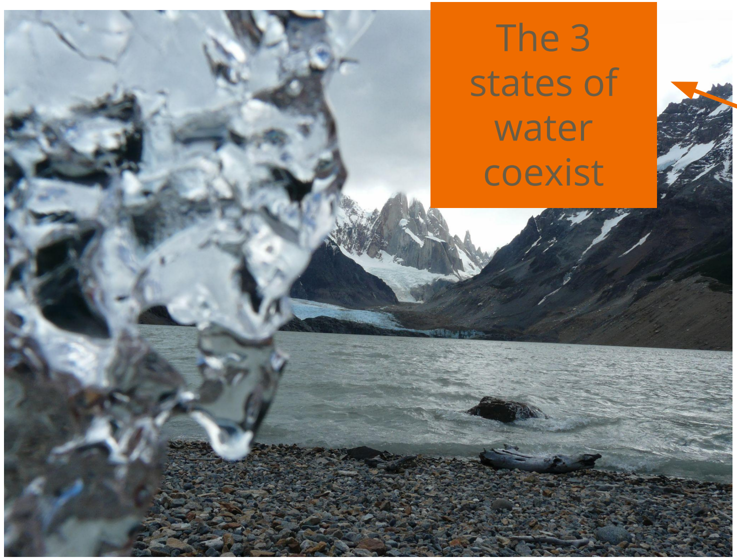
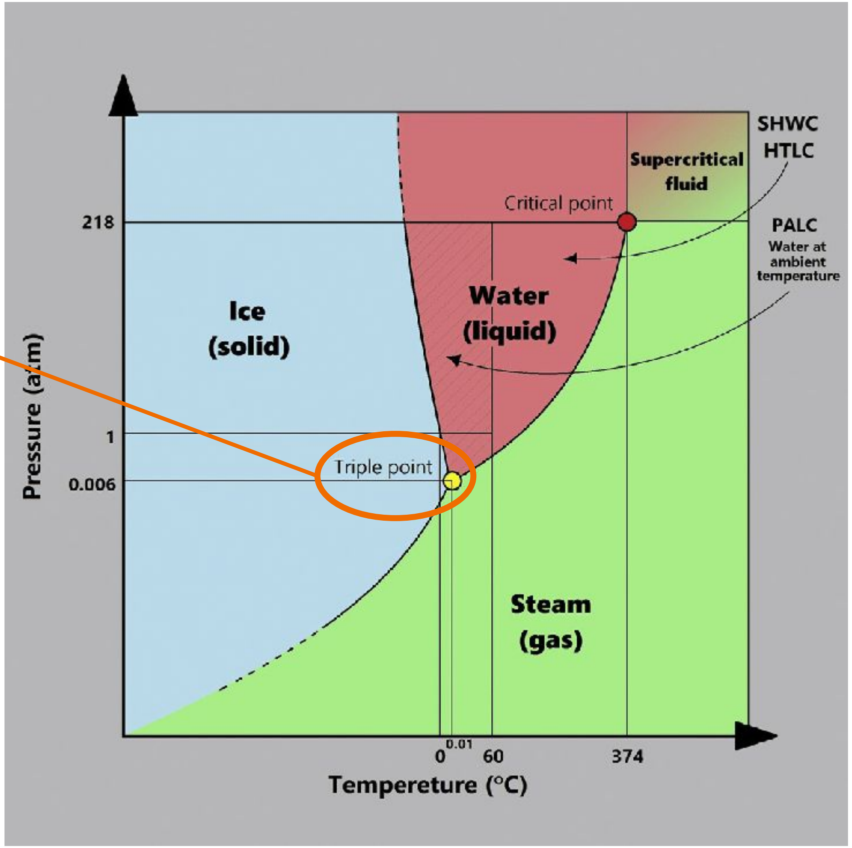


The 3 states of water coexist



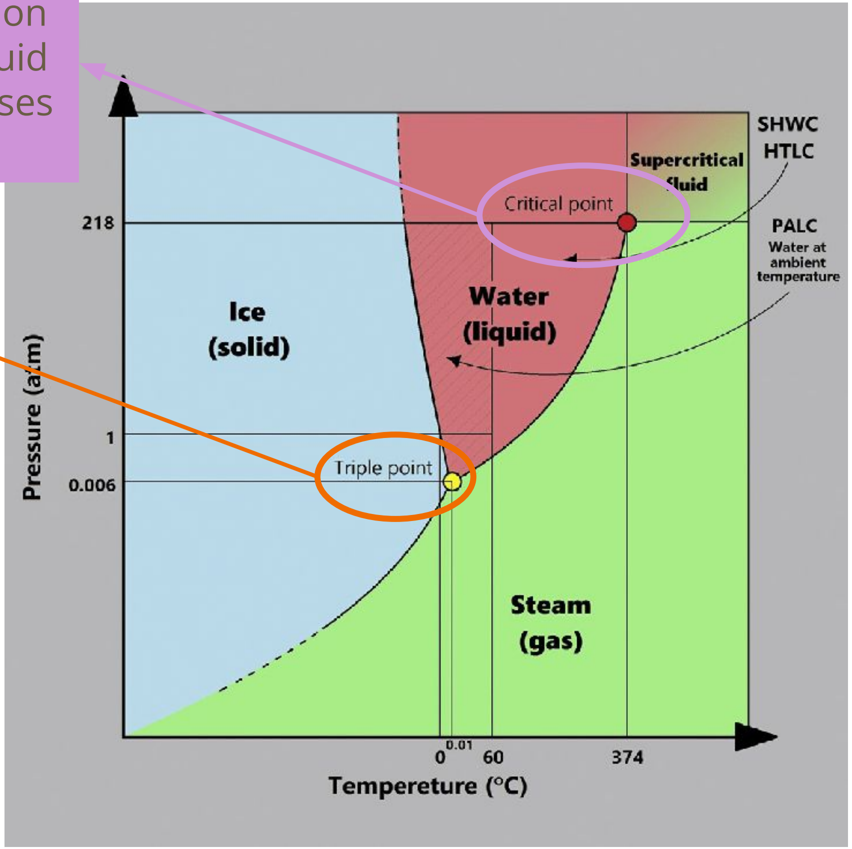
Equation of state for (extremely) dense matter

First things first

A “simple” example: water

The distinction between liquid and gas phases vanishes

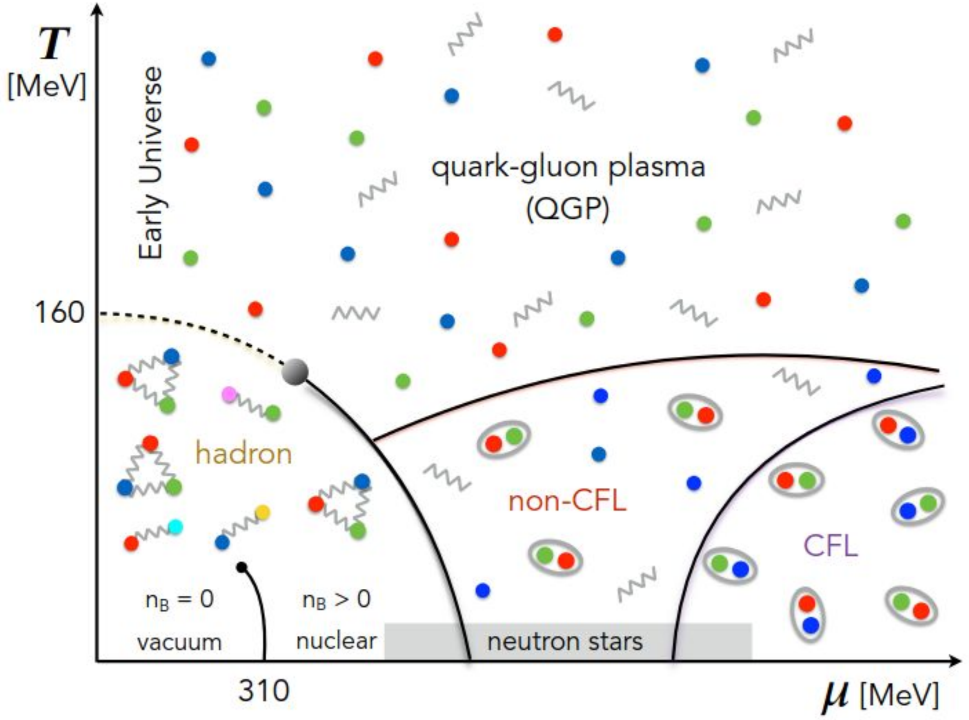
The 3 states of water coexist



Equation of state for (extremely) dense matter

QCD phase diagram

Strong interactions are governed by the quantum chromodynamic theory



Equation of state for (extremely) dense matter

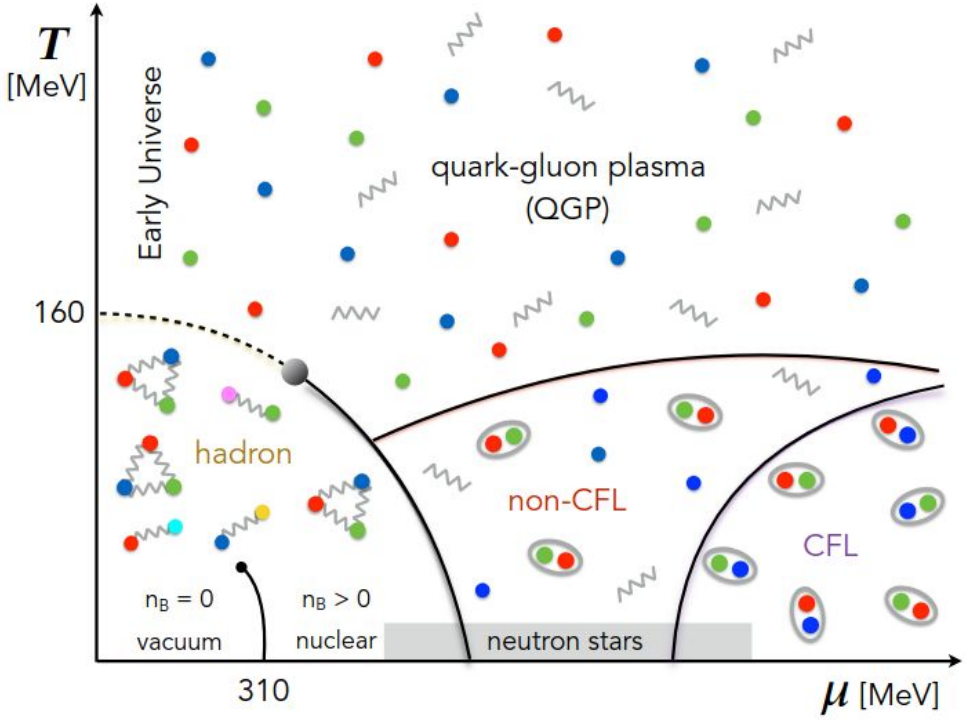
QCD phase diagram

Strong interactions are governed by the quantum chromodynamic theory

Severe issues and restrictions to perform calculations at finite densities

QCD has two fundamental features:

- asymptotic freedom
- confinement



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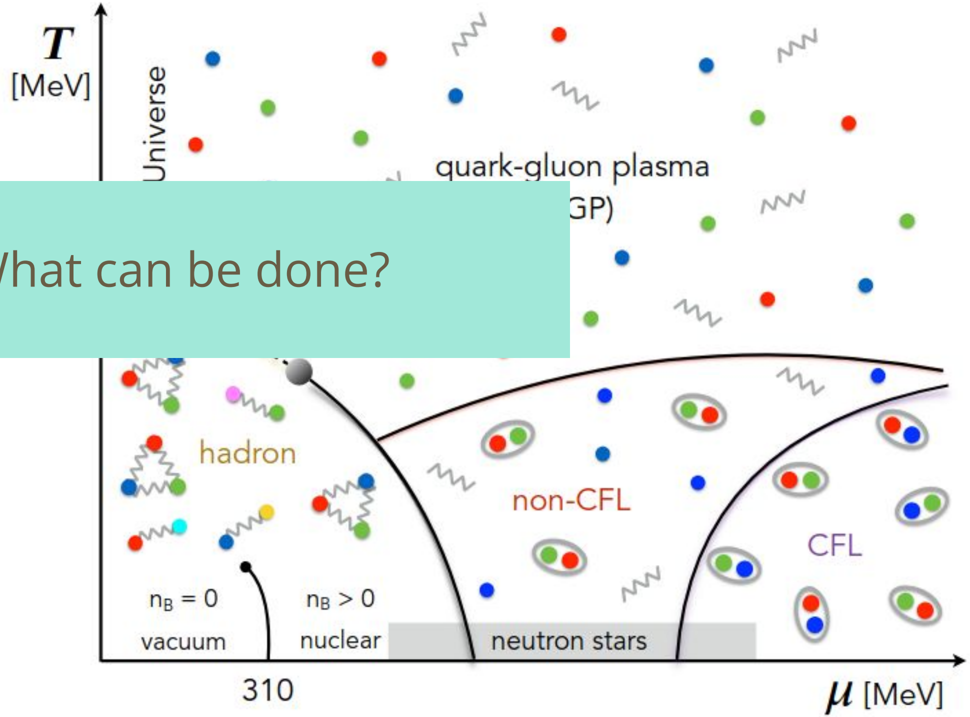


What can be done?

Equation of state for (extremely) dense matter

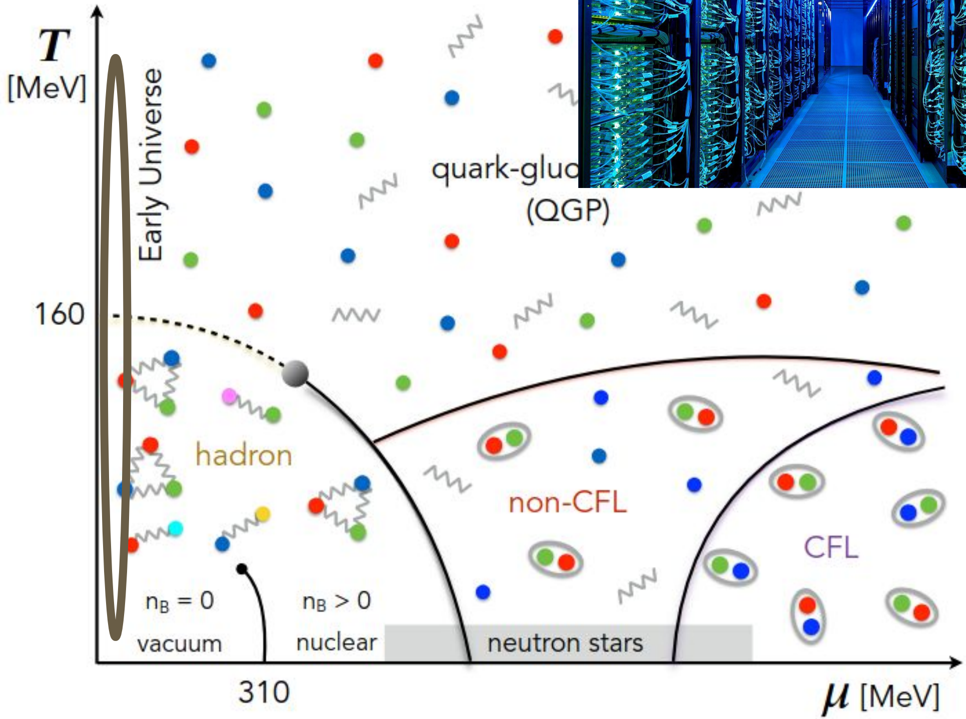
QCD phase diagram

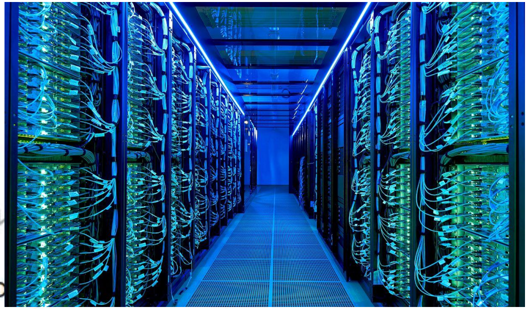
Theory: Lattice QCD where the equations can be solved after discretization of space-time.

Drawback 1: Requires the largest supercomputers available on Earth.
Drawback 2: Only for very little values of the chemical potential

But...

1. they are first principle calculations
2. the mass of light baryons have been calculated with errors ~ 2%



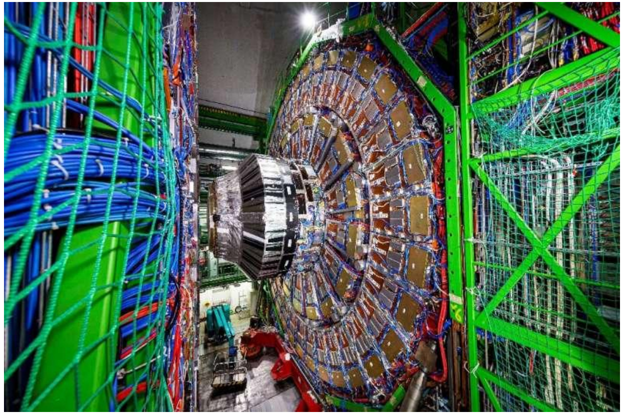


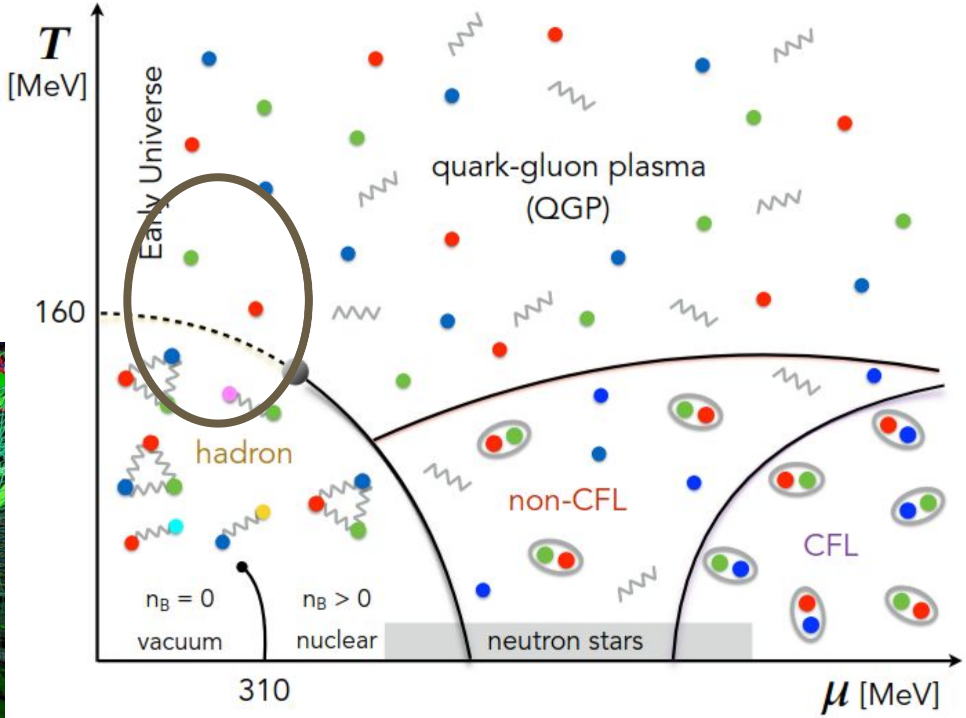
Equation of state for (extremely) dense matter

QCD phase diagram

Experiments: at particle colliders like LHC and others

Allowed to test QCD and find observational evidence of the QGP





Equation of state for (extremely) dense matter

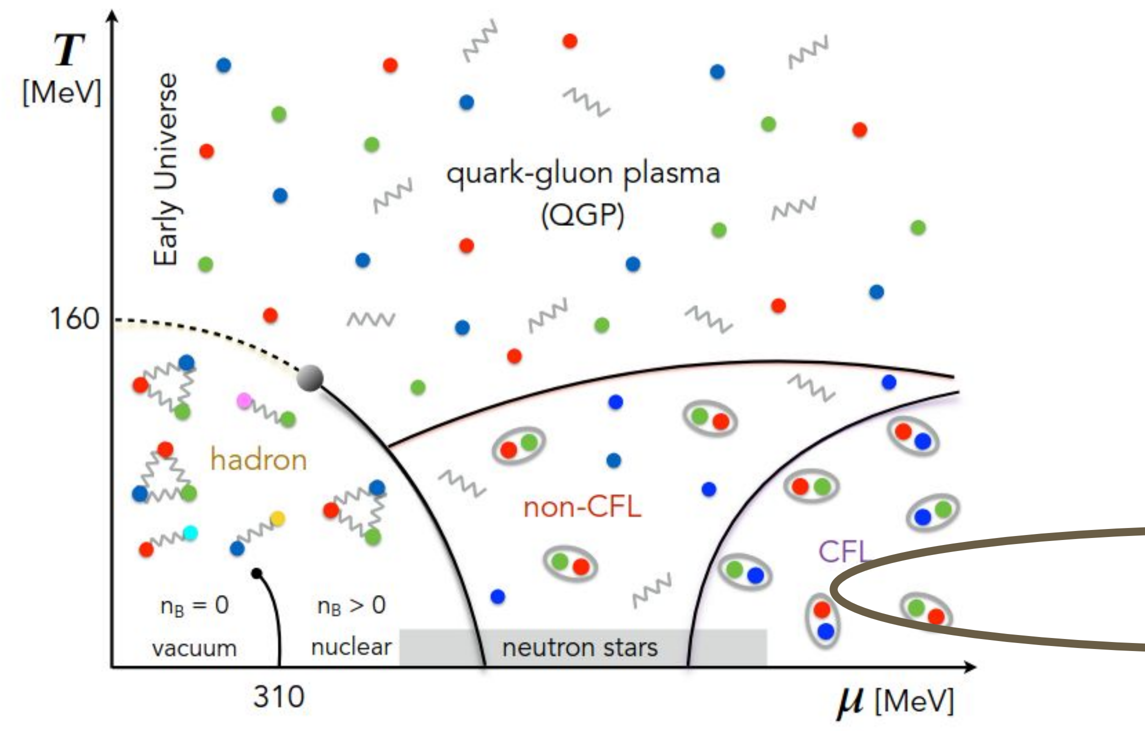
QCD phase diagram
Theory: perturbative QCD

Based on the asymptotic freedom property. In this regime quarks move almost freely if they are close to each other.

Drawback 1: Only for extremely large densities

But…

1. First principle calculations used to show that color superconductivity occurs in quark matter.



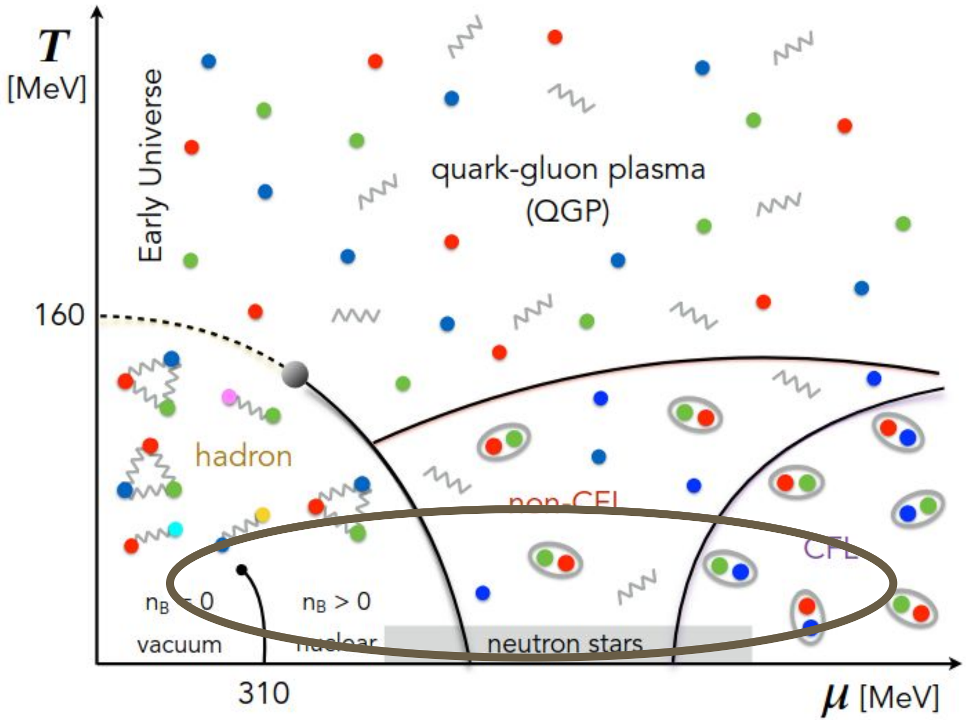
Equation of state for (extremely) dense matter

QCD phase diagram

Theory: phenomenological models

Capture some of the expected behaviour of matter from QCD results

There are lots of them!



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LOTS

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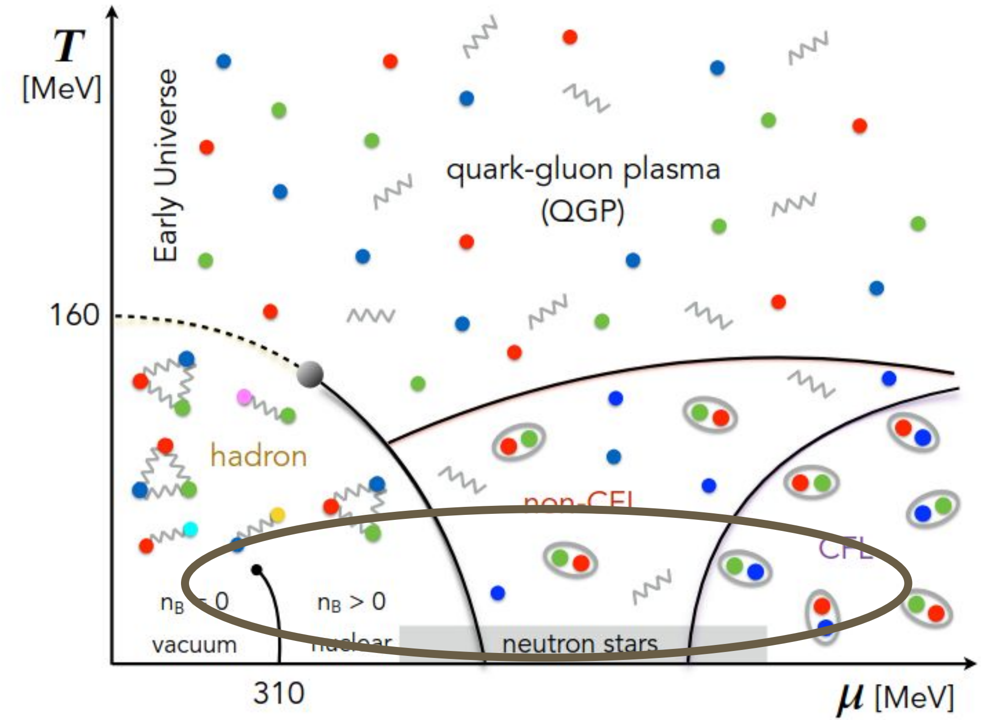
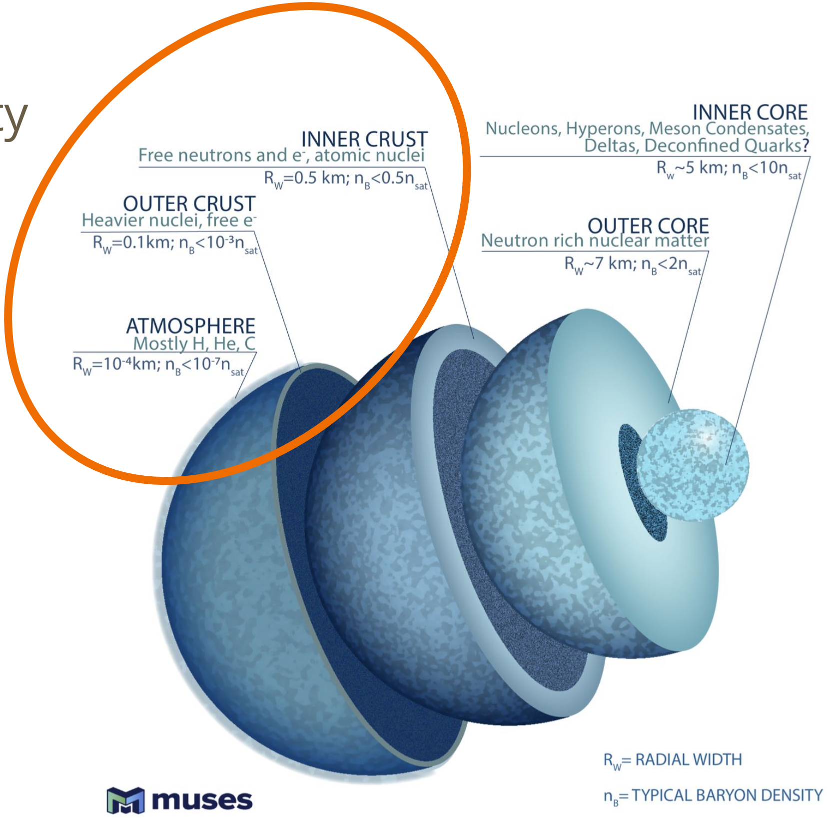


Equation of state for (extremely) dense matter

Cold hadronic EOS below saturation density

$2.3\,10^{14}$ g/cm^3

To have in mind, the densest material on Earth is Osmium. With a density of 22.59 g/cm^3



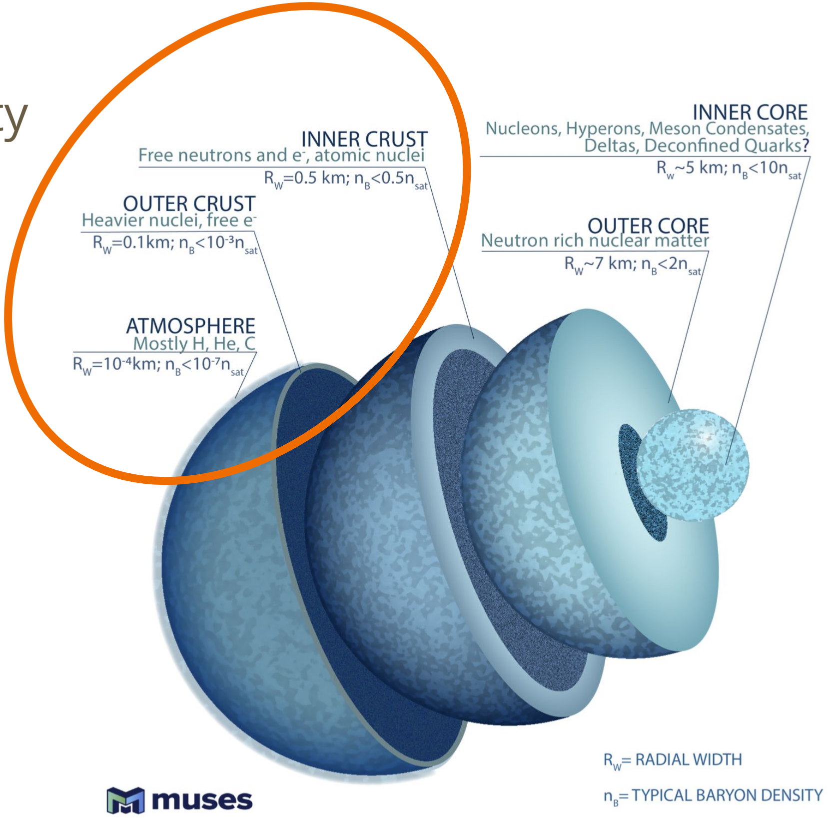
Equation of state for (extremely) dense matter

Cold hadronic EOS below saturation density

Similar description to the one needed for matter inside WDs and is more or less properly understood

Determines matter in the outermost layers (the last ~km)

Extremely relevant for studying cooling process



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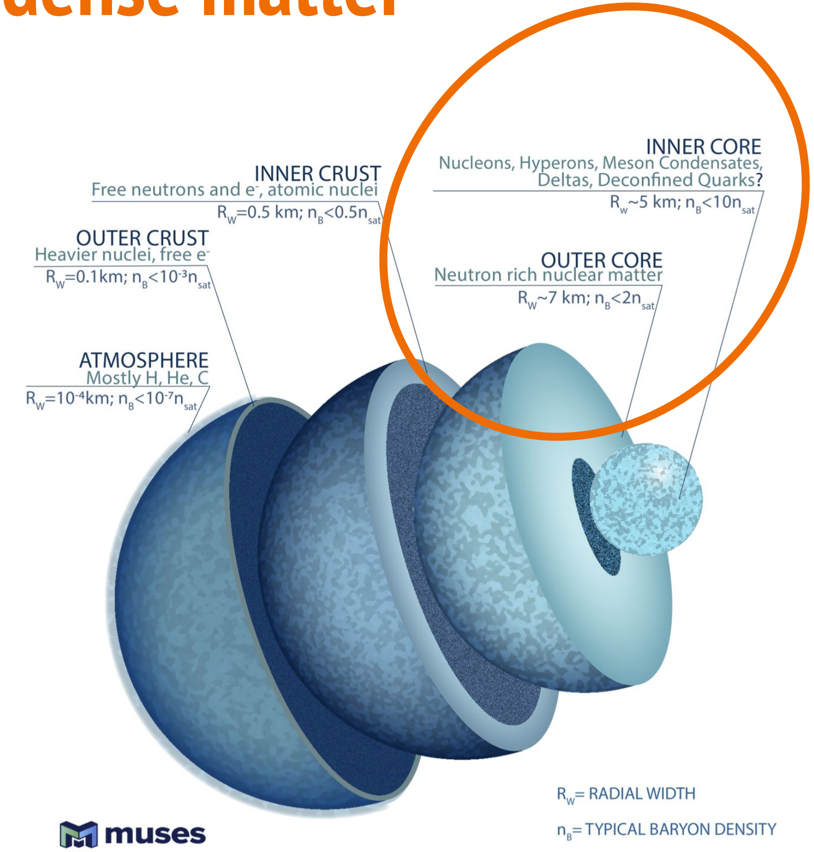

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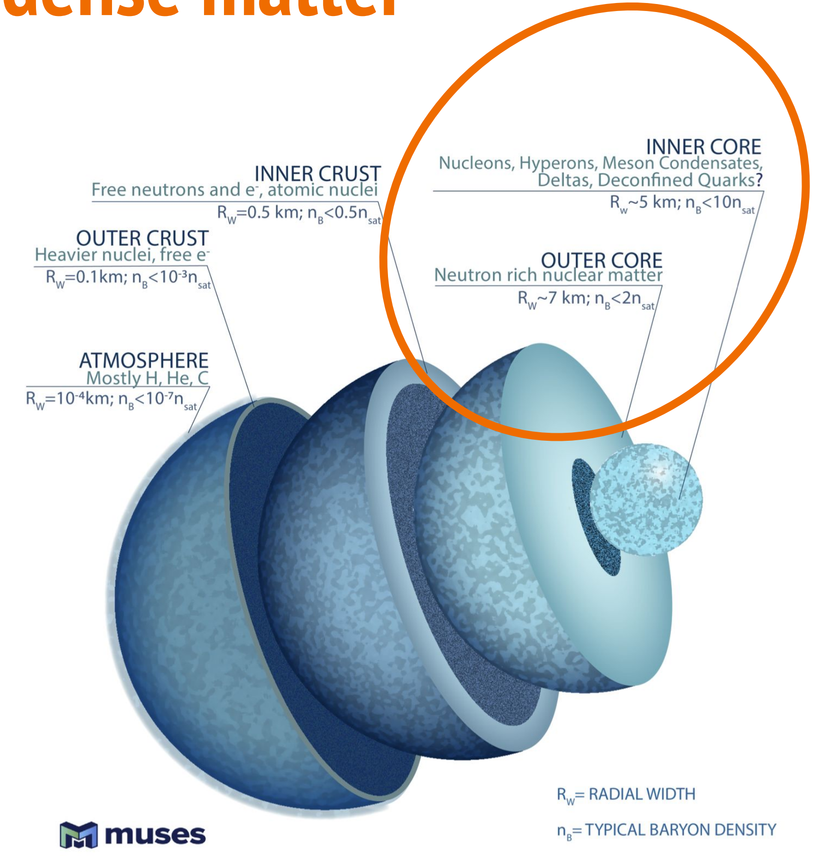
Cold hadronic EOS above saturation density

Several theoretical approaches to describe hadronic matter at such huge densities.

Above nuclear saturation density, nuclei will dissolve

We need to find a model to describe interactions under physical conditions relevant to describe NS matter



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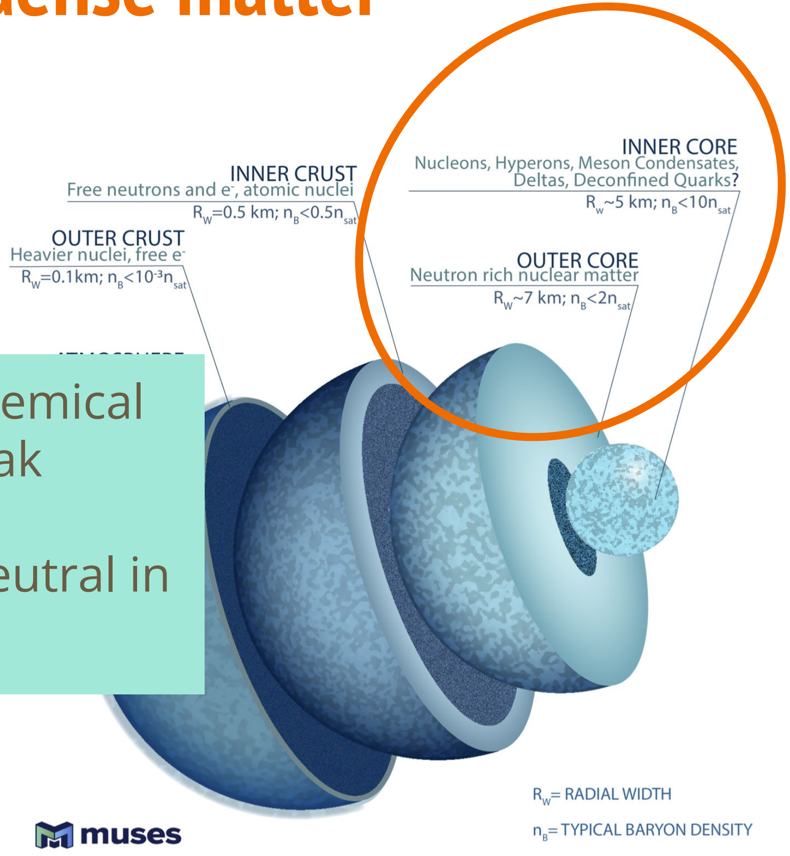
We need to find a model to describe interactions under physical conditions relevant to describe NS matter

- matter should be in chemical equilibrium (under weak interactions)
- matter is electrically neutral in a microscopic scale



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Relativistic Mean Field theory (Walecka 1974)
Lots of versions and parametizations available!

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Relativistic Mean Field theory (Walecka 1974)
The general idea:

$$\mathcal{L}_{\text{RMF}} = \mathcal{L}_{\text{bariones}} + \mathcal{L}_{\text{mesones}} + \mathcal{L}_{\text{interación}} + \mathcal{L}_{\text{auto-int}}$$

$$\begin{aligned}
\mathcal{L}_{\text{bariones}} &= \sum_b \bar{\psi}_b [i\gamma_\mu \partial^\mu - m_b]\psi_b\,, \\
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Lagrangian of free baryons and mesons

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Lagrangian of meson-baryon interactions

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Lagrangian of meson-baryon interactions

Lagrangian of meson self-interactions

Also add a similar Lagrangian for leptons
to ensure charge neutrality

Equation of state for (extremely) dense matter

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Why mean?
Change σ, ω y ρ

by their mean values at the fundamental state

$\bar{\sigma}$, $\bar{\omega}$ y $\bar{\rho}$

Lagrangian of free baryons and mesons

Lagrangian of meson-baryon interactions

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Equation of state for (extremely) dense matter

Cold hadronic EOS above saturation density

Equations for the unknowns obtained using the Euler-Lagrange equations

$$\frac{\partial \mathcal{L}}{\partial \Phi(x)} - \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \Phi(x))} = 0\,.$$

Equation of state for (extremely) dense matter

Cold hadronic EOS above saturation density

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$$m_\sigma^2 \bar{\sigma} = \sum_b g_{\sigma b} n_b^s - b_\sigma m_N g_{\sigma N} (g_{\sigma N} \bar{\sigma})^2 - c_\sigma g_{\sigma N} (g_{\sigma N} \bar{\sigma})^3 ,$$

$$m_\omega^2 \bar{\omega} = \sum_b g_{\omega b} n_b^v ,$$

$$m_\rho^2 \bar{\rho} = \sum_b g_{\rho b}(n_B) I_{3b} n_b^v ,$$

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Nonlinear system of equations that has to be solved together with charge neutrality and net baryon conservation

Equation of state for (extremely) dense matter

Cold hadronic EOS above saturation density
A particular case: density dependent RMF
developed in the group I work

Equation of state for (extremely) dense matter

Cold hadronic EOS above saturation density
A particular case: density dependent RMF developed in the group I work with

$$\mathcal{L}_{Bariones} = \sum_B \overline{\psi}_B \left\{ \gamma_\mu \left[i\partial^\mu - g_{\omega B}(n)\omega^\mu - \frac{1}{2} g_{\rho B}(n) \boldsymbol{\tau} \cdot \boldsymbol{\rho}^\mu \right] - \left[m_b - g_{\sigma B}(n)\sigma \right] \right\} \psi_B$$

$$g_{iB}(n) = g_{iB}(n_0) f_i(x)$$

coupling constants

$$f_i(x) = a_i \frac{1 + b_i (x + d_i)^2}{1 + c_i (x + d_i)^2}$$

$$f_\rho(x) = e^{[-a_\rho (x-1)]}$$

$$\mathcal{L}_{Mesones} = \frac{1}{2}\left(\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2\right) - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu + \frac{1}{2} m_\rho^2 \boldsymbol{\rho}_\mu \cdot \boldsymbol{\rho}^\mu - \frac{1}{4}\boldsymbol{\rho}_{\mu\nu} \cdot \boldsymbol{\rho}^{\mu\nu},$$

$$\mathcal{L}_{NL\sigma} = -\frac{1}{3}\tilde{b}_\sigma m_N [g_{\sigma N}(n)\sigma]^3 - \frac{1}{4}\tilde{c}_\sigma [g_{\sigma N}(n)\sigma]^4$$

$$\mathcal{L}_{Leptones} = \sum_\lambda \overline{\psi}_\lambda \left(i\gamma_\mu \partial^\mu - m_\lambda \right) \psi_\lambda,$$

It includes

$$N = \{n,\, p\}$$

$$Y = \{\Lambda,\, \Sigma^+,\, \Sigma^0,\, \Sigma^-,\, \Xi^0,\, \Xi^-\}$$

$$\Delta(1232) = \{\Delta^{++},\, \Delta^+,\, \Delta^0,\, \Delta^-\}$$

$$\lambda = \{e^-, \mu^-\}$$

$$\omega_{\mu\nu} = (\partial_\mu \omega_\nu - \partial_\nu \omega_\mu)$$

$$\boldsymbol{\rho}_{\mu\nu} = (\partial \mu \boldsymbol{\rho}_\nu - \partial \nu \boldsymbol{\rho}_\mu)$$

Equation of state for (extremely) dense matter

Cold hadronic EOS above saturation density

A particular case: density dependent RMF

develope

$$\mathcal{L}_{Bariones} = \sum_B$$

$$g_{iB}(n) = g_{iB}($$

Barión	Masa [MeV]	Spin	Carga eléctrica [e]	Isospin I_3	Quarks
n	939.6	1/2	0	-1/2	udd
p	938.3	1/2	+1	1/2	uud
Λ	1115.7	1/2	0	0	uds
Σ^-	1197.4	1/2	-1	-1	dds
Σ^0	1192.6	1/2	0	0	uds
Σ^+	1189.4	1/2	+1	1	uus
Ξ^-	1321.7	1/2	-1	-1/2	dss
Ξ^0	1314.9	1/2	0	1/2	uss
Δ^-	1232	3/2	-1	-3/2	ddd
Δ^0	1232	3/2	0	-1/2	udd
Δ^+	1232	3/2	+1	1/2	uud
Δ^{++}	1232	3/2	+2	3/2	uuu

$$\mathcal{L}_{Mesones} = \frac{1}{2}(\partial$$

$$\mathcal{L}_{NL\sigma} = -\frac{1}{3} b_\sigma m_N [g_{\sigma N}(n)\sigma]^3 - \frac{1}{4} c_\sigma [g_{\sigma N}(n)\sigma]^4$$

$$\mathcal{L}_{Leptones} = \sum_\lambda \overline{\psi}_\lambda (i\gamma_\mu \partial^\mu - m_\lambda)\psi_\lambda,$$

It includes

$$N = \{n,\ p\}$$

$$Y = \{\Lambda,\ \Sigma^+,\ \Sigma^0,\ \Sigma^-,\ \Xi^0,\ \Xi^-\}$$

$$\Delta(1232) = \{\Delta^{++},\ \Delta^+,\ \Delta^0,\ \Delta^-\}$$

$$\lambda = \{e^-, \mu^-\}$$

$$\omega_{\mu\nu} = (\partial_\mu \omega_\nu - \partial_\nu \omega_\mu)$$

$$\boldsymbol{\rho}_{\mu\nu} = (\partial\mu \boldsymbol{\rho}_\nu - \partial\nu \boldsymbol{\rho}_\mu)$$

Equation of state for (extremely) dense matter

Cold hadronic EOS above saturation density
Working with mean fields and Euler-Lagrange equations
Adding charge neutrality and baryon number conservation

$$
\begin{aligned}
\sum_B g_{\sigma_B}(n) n_B^S - m_\sigma^2 \overline{\sigma} &= 0 \\
\sum_B g_{\omega_B}(n) n_B - m_\sigma^2 \overline{\omega} &= 0 \\
\sum_B g_{\rho_B}(n) I_{3B} n_B - m_\rho^2 \overline{\rho} &= 0 \\
\sum_B \left(\frac{\partial g_{\omega B}}{\partial n} n_B \overline{\omega} + \frac{\partial g_{\rho B}}{\partial n} I_{3B} n_B \overline{\rho} - \frac{\partial g_{\sigma B}}{\partial n} n_B^S \overline{\sigma} \right) - \Sigma_r &= 0 \\
\sum_B n_B - n &= 0 \\
\sum_B n_B q_B + \sum_\lambda n_\lambda q_\lambda &= 0.
\end{aligned}
$$

After solving these nonlinear equations we get the unknowns

$\{\overline{\sigma},\, \overline{\omega},\, \overline{\rho}\}$

$\{k_n,\, k_e\}$

Σ_r

Equation of state for (extremely) dense matter

Cold hadronic EOS above saturation density

After solving the NL equations we can calculate the pressure of the system

$$
\begin{aligned}
P_{DDRMF} &= \frac{1}{3}\sum_i \langle T_{ii}\rangle \\
&= \frac{1}{3}\sum_B \frac{2J_B+1}{2\pi^2}\int_0^{k_B} \frac{k^4\,dk}{\sqrt{k^2+m_B^{*\,2}(\overline{\sigma})}} + \frac{1}{3\pi^2}\sum_\lambda \int_0^{k_\lambda} \frac{k^4\,dk}{\sqrt{k^2+m_\lambda^2}} \\
&- \frac{1}{2}\left[m_\sigma^2\overline{\sigma}^2 - m_\omega^2\overline{\omega}^2 - m_\rho^2\overline{\rho}^2\right] + n\Sigma_r .
\end{aligned}
$$

The energy density, ε, can be obtained using the Euler relationship from basic thermodynamics

$$\varepsilon = -P + \sum_i \mu_i n_i$$

Parámetros	GM1L	DD2
m_σ (GeV)	0.5500	0.5462
m_ω (GeV)	0.7830	0.7830
m_ρ (GeV)	0.7700	0.7630
$g_{\sigma N}$	9.5722	10.6870
$g_{\omega N}$	10.6180	13.3420
$g_{\rho N}$	8.9830	3.6269
$\tilde{b}_\sigma$	0.0029	0
$\tilde{c}_\sigma$	- 0.0011	0
a_σ	1	1.3576
b_σ	0	0.6344
c_σ	0	1.0054
d_σ	0	0.5758
a_ω	0	1.3697
b_ω	0	0.4965
c_ω	0	0.8177
d_ω	0	0.6384
a_ρ	0.3898	0.5189

Fixed to reproduce experimental results at nuclear saturation density

Equation of state for (extremely) dense matter

Hot hadronic EOS above saturation density

Can be generalized to finite temperature (similar -but a little harder- as for the Fermi gas)

Equation of state for (extremely) dense matter

Hot hadronic EOS above saturation density

Can be generalized to finite temperature (similar -but a little harder- as for the Fermi gas)

$$
\begin{aligned}
P_{DDRMF}(T) &= \frac{1}{3}\sum_B \frac{2J_B+1}{2\pi^2}\int_0^\infty \frac{k^4\,dk}{E_B^*(k)}\left[n_B^+ + n_B^-\right] + \frac{1}{3\pi^2}\sum_\lambda \int_0^\infty \frac{k^4\,dk}{E_\lambda(k)}\left[n_\lambda^+ + n_\lambda^-\right] \\
&- \frac{1}{2}\left[m_\sigma^2\overline{\sigma}^2 - m_\omega^2\overline{\omega}^2 - m_\rho^2\overline{\rho}^2\right] + n\Sigma_r,
\end{aligned}
$$

$$
\begin{aligned}
n_B^\pm(k,T) &= \left[1 + e^{\frac{E_B^*(k)\pm\mu_B^*}{T}}\right]^{-1} \\
n_\lambda^\pm(k,T) &= \left[1 + e^{\frac{E_\lambda(k)\pm\mu_\lambda}{T}}\right]^{-1}
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$$

$$
\begin{aligned}
S(T) &= \frac{\partial P}{\partial T} = \\
&= \sum_B \gamma_B \int_0^\infty \frac{k^4\,dk}{E_B^*}\left[\left(n_B^+ - {n_B^+}^2\right)\left(\frac{E_B^* + \mu_B^*}{T^2}\right) + \left(n_B^- - {n_B^-}^2\right)\left(\frac{E_B^* - \mu_B^*}{T^2}\right)\right] \\
&+ \sum_\lambda \gamma_\lambda \int_0^\infty \frac{k^4\,dk}{E_\lambda}\left[\left(n_\lambda^+ - {n_\lambda^+}^2\right)\left(\frac{E_\lambda + \mu_\lambda}{T^2}\right) + \left(n_\lambda^- - {n_\lambda^-}^2\right)\left(\frac{E_\lambda - \mu_\lambda}{T^2}\right)\right],
\end{aligned}
$$

$$\gamma_B = (2J_B+1)/(6\pi^2) \text{ y } \gamma_\lambda = 1/(3\pi^2)$$

Equation of state for (extremely) dense matter

Hot hadronic EOS above saturation density

Can be generalized to finite temperature (similar -but a little harder- as for the Fermi gas)

$$
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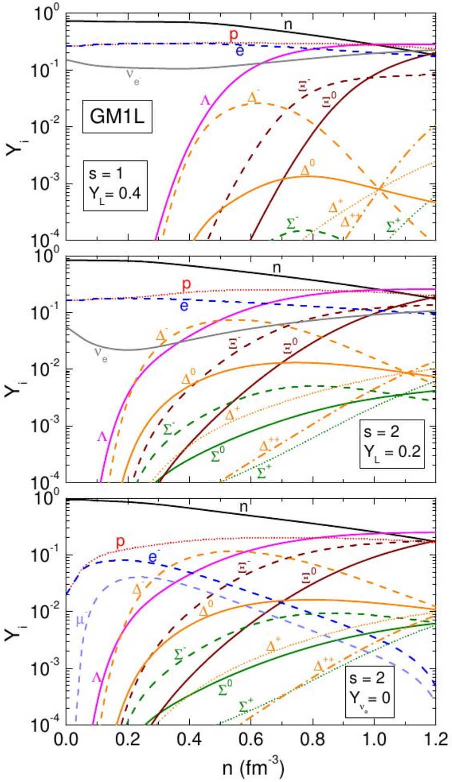
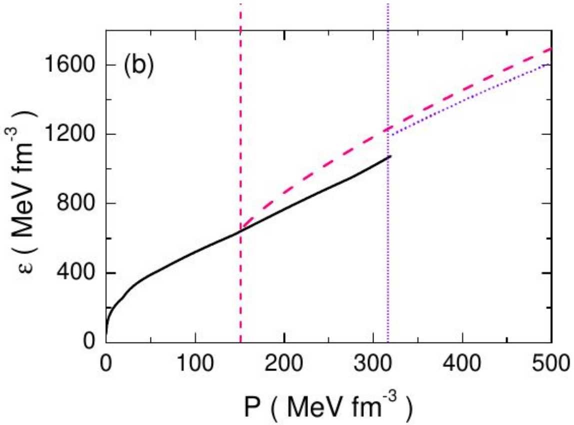
&+ \sum_\lambda \gamma_\lambda \int_0^\infty \frac{k^4\,dk}{E_\lambda}\left[\left(n_\lambda^+ - n_\lambda^{+\,2}\right)\left(\frac{E_\lambda+\mu_\lambda}{T^2}\right) + \left(n_\lambda^- - n_\lambda^{-\,2}\right)\left(\frac{E_\lambda-\mu_\lambda}{T^2}\right)\right],
\end{aligned}
$$

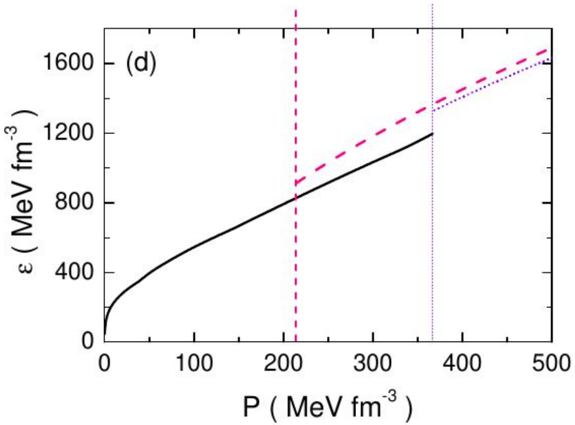
$$
\begin{aligned}
\varepsilon &= -P + TS + \sum_i \mu_i n_i \\
E_{G/B} &= \frac{\varepsilon - TS + P}{n},
\end{aligned}
$$

$$\gamma_B = (2J_B+1)/(6\pi^2) \text{ y } \gamma_\lambda = 1/(3\pi^2)$$

Equation of state for (extremely) dense matter



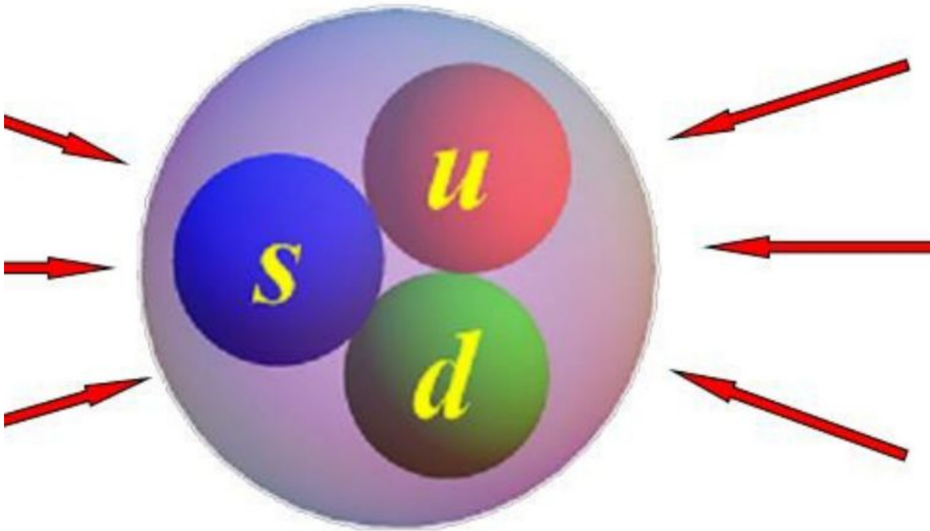




Equation of state for (extremely) dense matter

Cold quark matter EOS

The simplest model: MIT BAG model
Developed in 1974 at the MIT.



Spherical Bag

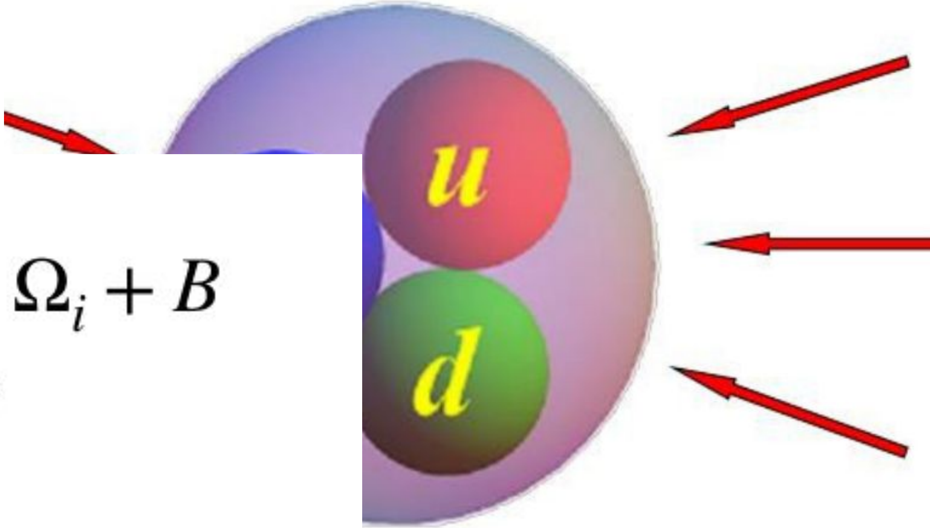
External (Bag) pressure forces quarks to move inside the cavity (ensures confinement)

Inside the Bag, quarks move freely

Equation of state for (extremely) dense matter

Cold quark matter EOS

The simplest model: MIT BAG model
Develo



$$\Omega = \sum_{i=u,d,s,e,\nu} \Omega_i + B$$

Spheric

Externa
to move
confine

Inside t

$$\Omega_i(T,\mu_i) = -\frac{g_i T}{2\pi^2}\int dk k^2 \ln\left[1 + e^{-(E_k - \mu_i)/T}\right]$$

Equation of state for (extremely) dense matter

Cold 3-flavour quark matter EOS

Generalization of MIT BAG model at zero temperature
Developed in 2005 by Alford et al.

$$\Omega = -\frac{3}{4\pi^2}a_4\mu^4 + \frac{3}{4\pi^2}a_2\mu^2 + B + \Omega_e$$

$$\mu \equiv (\mu_u + \mu_s + \mu_d)/3$$

Equation of state for (extremely) dense matter

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Accounts for strong interactions
$a_4 = 1$ (MIT)

Equation of state for (extremely) dense matter

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$a_4 = 1$ (MIT)

$$a_2 = m_s^2 - 4\Delta^2$$

The gap parameter Δ is the energy of the quark pairing and so the model accounts for color superconductivity
$\Delta = 0$ (MIT)

Equation of state for (extremely) dense matter

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Bag

electrons

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Electrons can be ignored and all the thermodynamic quantities can be obtained more easily

Equation of state for (extremely) dense matter

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$$p = -\Omega = \frac{3}{4\pi^2}a_4\mu^4 - \frac{3}{4\pi^2}a_2\mu^2 - B$$

$$n_b = -\frac{1}{3}\frac{\partial\Omega}{\partial\mu} = \frac{1}{2\pi^2}(2a_4\mu^3 - a_2\mu)$$

$$\epsilon = 3\mu n_b - p = \frac{9}{4\pi^2}a_4\mu^4 - \frac{3}{4\pi^2}a_2\mu^2 + B$$

Equation of state for (extremely) dense matter

Cold quark matter EOS

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charge neutrality and chemical equilibrium

$$\frac{2}{3}n_u - \frac{1}{3}n_d - \frac{1}{3}n_s = n_e$$

$\mu_d = \mu_u + \mu_e$ and $\mu_s = \mu_d$

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All the thermodynamic quantities can b… ined

$$p = -\Omega = \frac{3}{4\pi^2}a_4\mu^4 - \frac{3}{4\pi^2}a_2\mu^2 \ldots$$

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Exercise: check that these expressions are correct!

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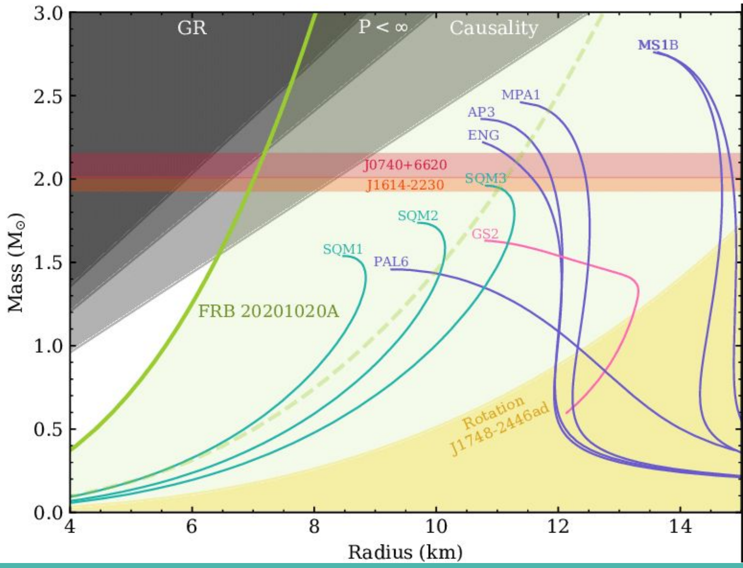
Equation of state for (extremely) dense matter

Cold quark matter EOS

Generalization of MIT BAG model
Developed in 2005 by Alford et al.

All the thermodynamic quantities can be obtained

$$p(\epsilon) = \frac{1}{3}(\epsilon - 4B) - \frac{a_2^2}{12\pi^2 a_4}\left[1 + \sqrt{1 + \frac{16\pi^2 a_4}{a_2^2}(\epsilon - B)}\right]$$



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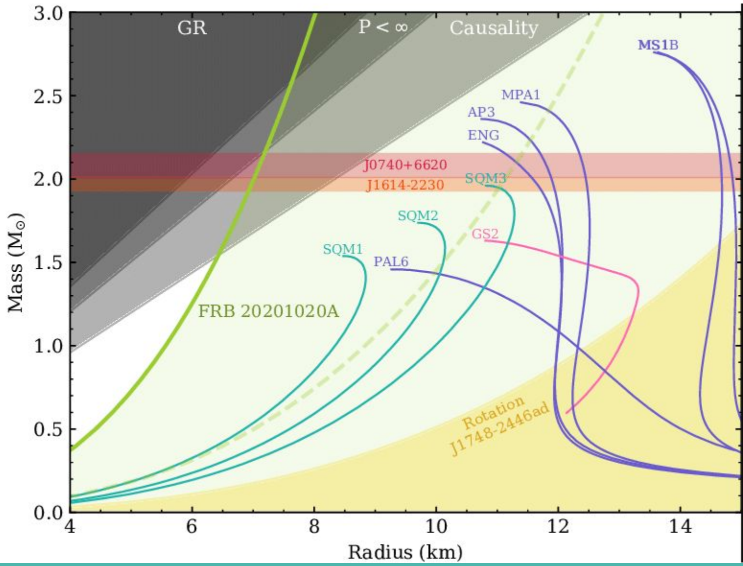
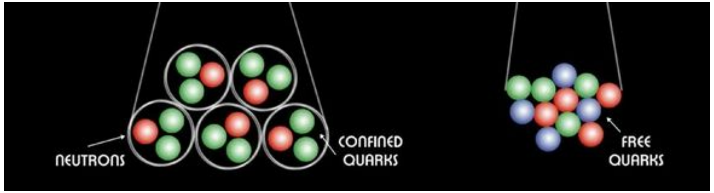
Exercise: check that the expression for the EOS is correct!

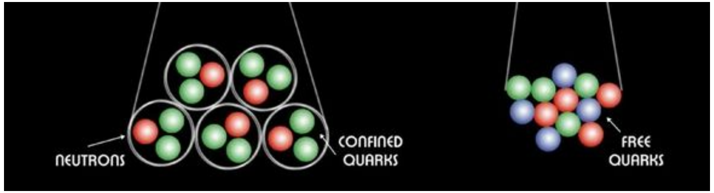


Quarks or Hadrons?



Despite some advances, we do not have a unified model capable of describing hadronic matter together with quarks.

Quarks or Hadrons?

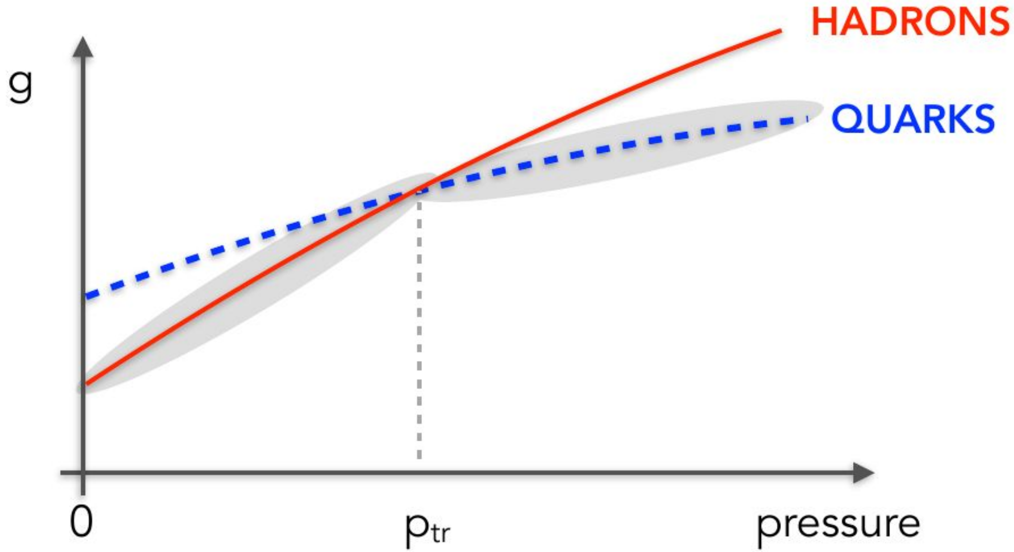


Despite some advances, we do not have a unified model capable of describing hadronic matter together with quarks.

We have to "glue" different models

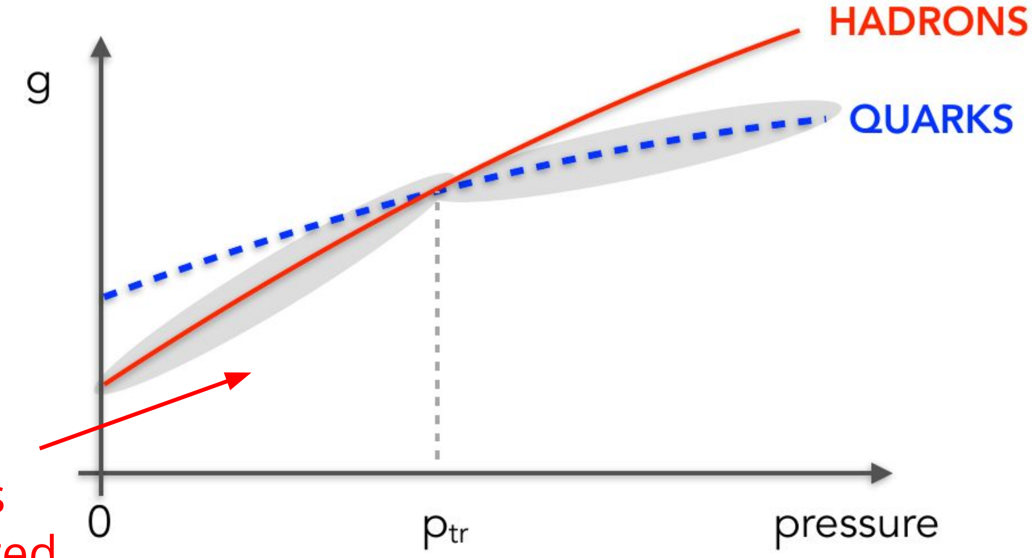
How is this "gluing" performed?

Quarks or Hadrons?



$$g = \frac{G}{n_b} = \frac{\epsilon - Ts + p}{n_b}$$

Quarks or Hadrons?



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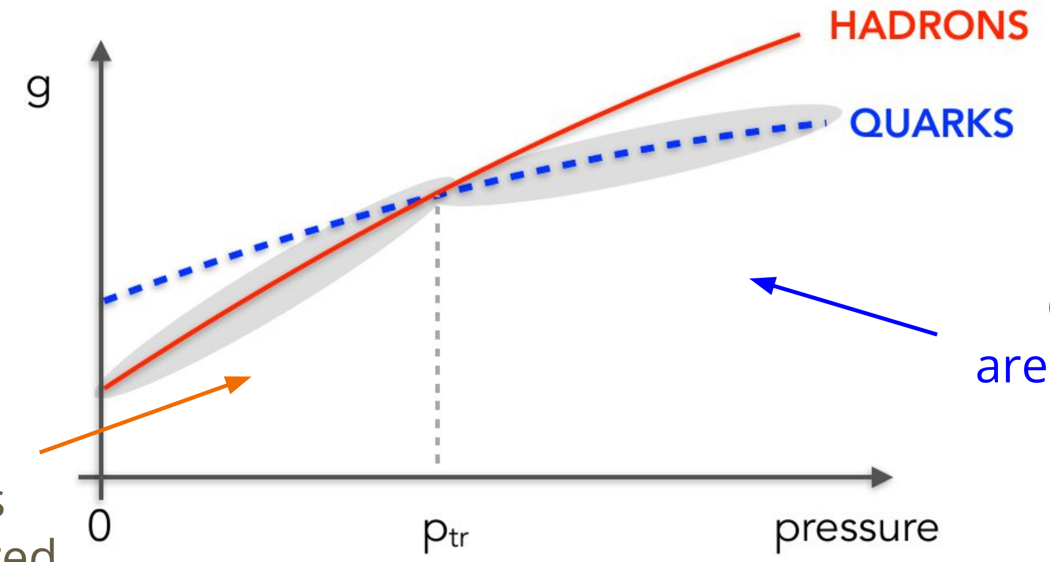
Hadrons
are preferred

Quarks or Hadrons?



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Quarks
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Hadrons
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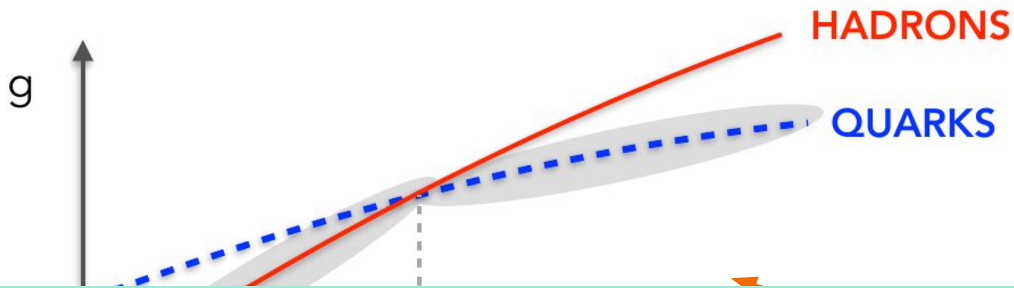
Quarks or Hadrons?

We do not have a unified model capable of describing hadronic matter together

We have

How is th



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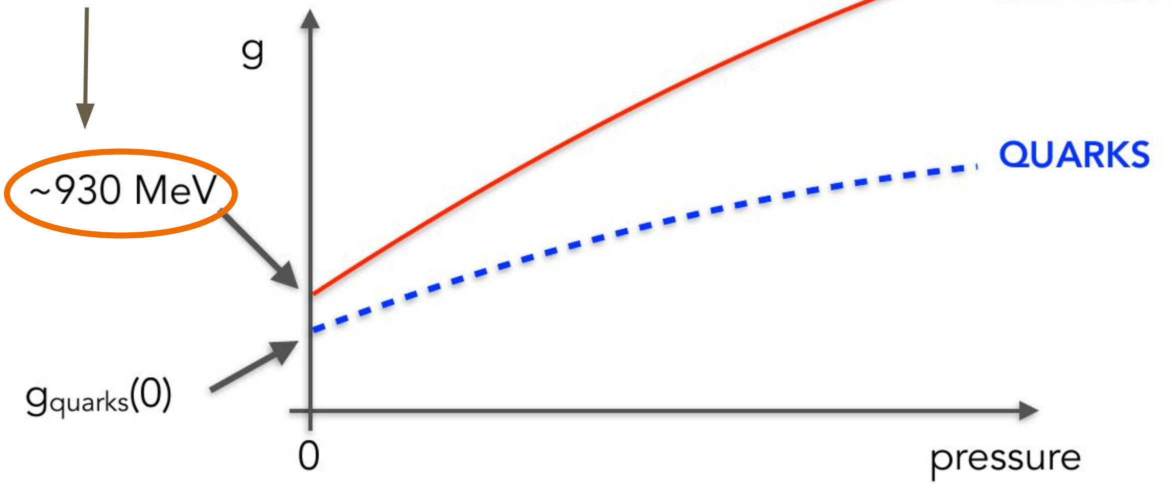
Hybrid EOS

Quarks or Hadrons?

The Bodmer (1971) - Terazawa (1979) - Witten (1984) hypothesis

^{56}Fe most stable element in nature!



Quark matter might be preferred at all pressures!

SQM hypothesis

This depends on the free parameters of the quark EOS

Quarks or Hadrons?

SQM hypothesis contradicts the existence of nuclei?

Quarks or Hadrons?

SQM hypothesis contradicts the existence of nuclei?

Not necessarily!

2-flavour quark matter should not be favored

$$\left.\frac{\epsilon}{n_b}\right|_{u,d} > 930\ \mathrm{MeV}$$

$$\Omega_{2f} = -\tilde{p} = -\frac{24a_4}{4\pi^2(1+2^{1/3})^3}\tilde{\mu}^4 + \frac{2a_2}{4\pi^2}\tilde{\mu}^2 + B$$

$$\tilde{\mu} \equiv (\mu_u + \mu_d)/2$$

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$$\tilde{\mu} \equiv (\mu_u + \mu_d)/2$$

but 3-flavour quark matter might fulfil

$$\left.\frac{\epsilon}{n_b}\right|_{u,d,s} < 930 \text{ MeV}$$

Quarks or Hadrons?

SQM hypothesis contradicts the existence of nuclei?

N

2-flavour quark matter shoulc

$$\left.\frac{\epsilon}{n_b}\right|_{u,d} > 930\ \mathrm{MeV}$$

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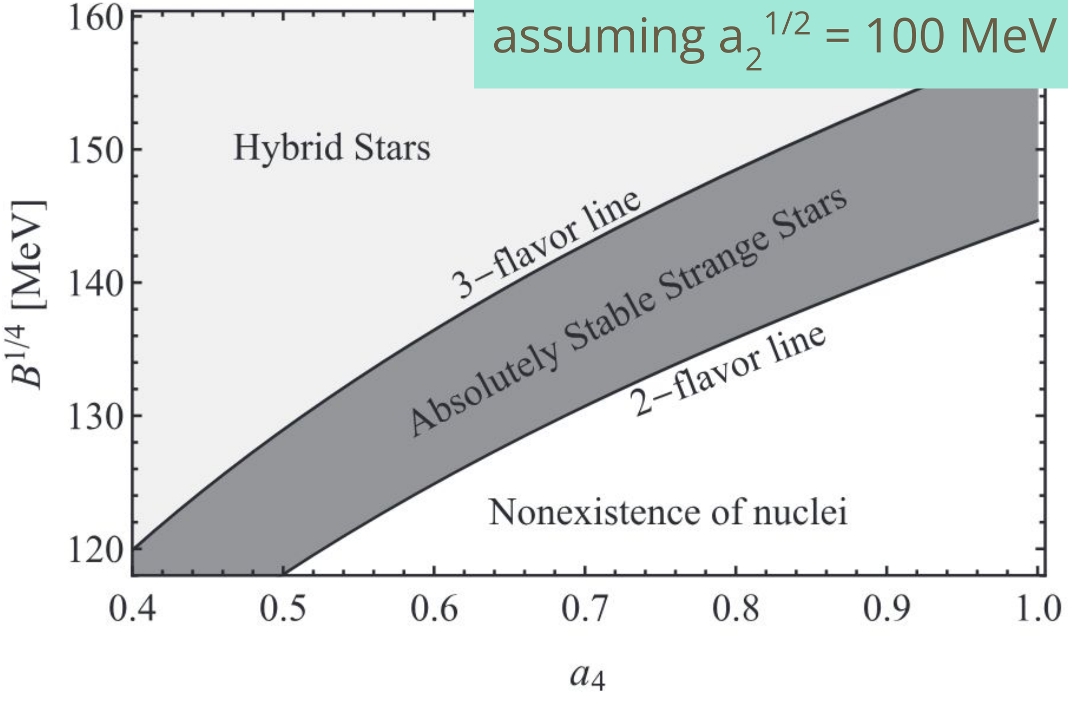

From Pereira et al. 2018 assuming $a_2^{1/2}$ = 100 MeV

Quarks or Hadrons?

SQM hypothesis cont... the existence of nuclei?

Exercise: construct the stability window presented here

2... matter should

$$\left.\frac{\epsilon}{n_b}\right|_{u,d} > 930 \text{ MeV}$$

but 3-flavour quark matter mi

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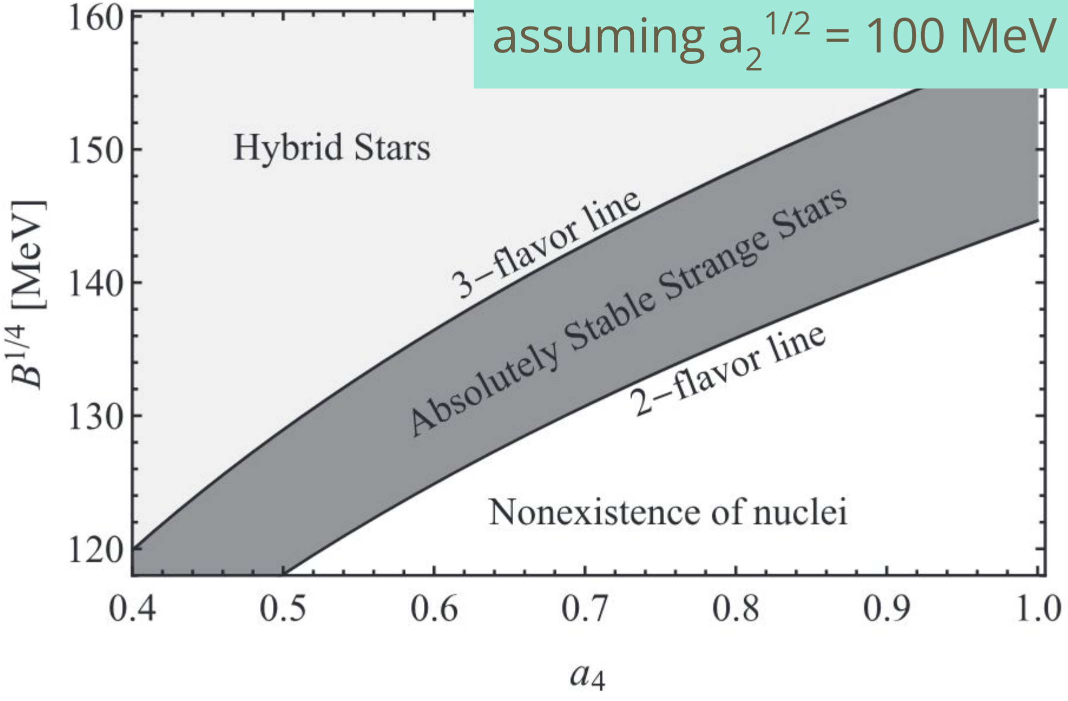
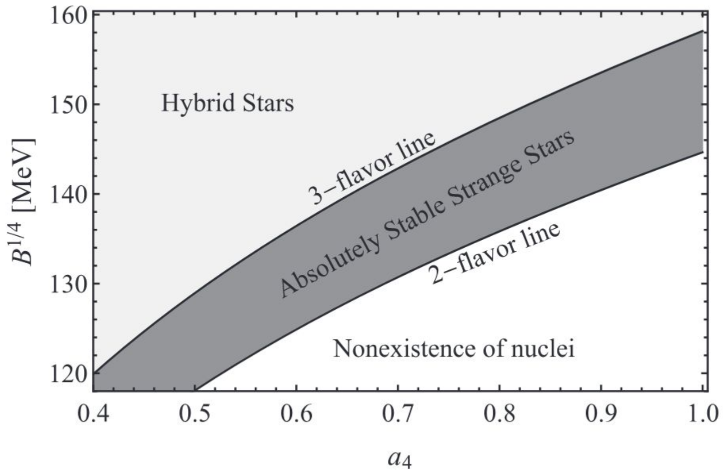


Quarks or Hadrons?

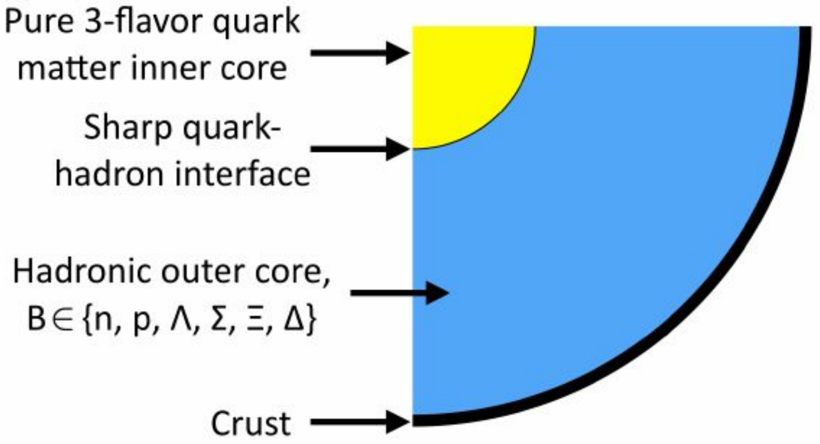
Takeaway of central differences between quark and neutron/hybrid stars



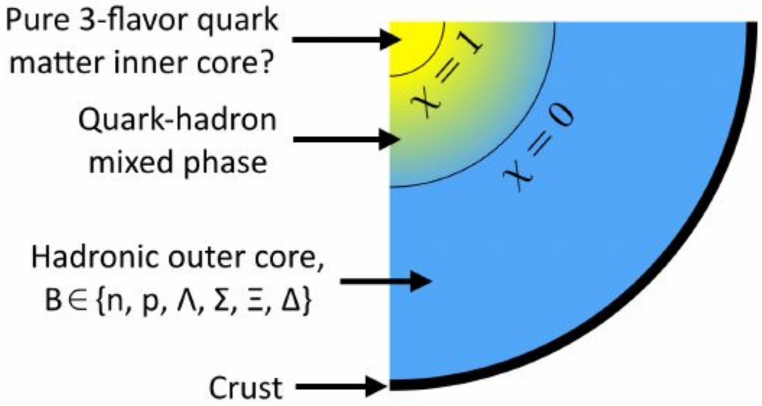
NS - HS	QS
Bound by gravity	Bound by gravity and strong interactions
If energy density is finite, pressure too	Pressure vanishes at finite energy density
Small central pressure, large radii	Small central pressure, small radii

Nature of the hadron-quark phase transition

It is not known. Two main ideas are possible



(a) Hypothetical NS cross section with a pure quark phase.



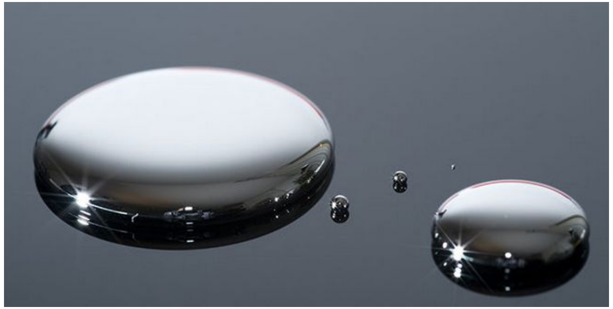
(b) Hypothetical NS cross section with a mixed phase.

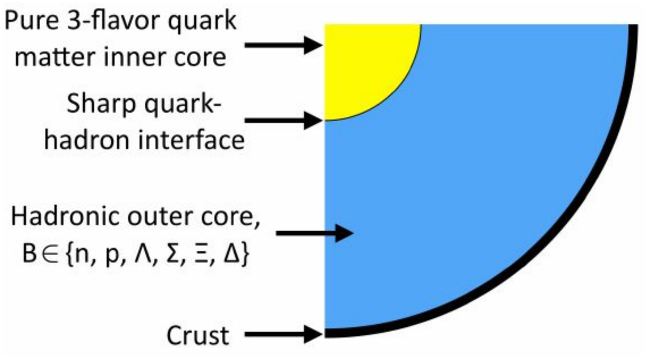
From Orsaria et al., 2018

Nature of the hadron-quark phase transition

It seems that hadron-quark phase surface tension is key to decide which scenario is favoured

- Theoretical values are highly EOS dependent. Large spread and inconsistent between them!





(a) Hypothetical NS cross section with a pure quark phase.

- Quark and hadronic matter do not mix
- Pressure and Gibbs energy are continuous at the interface
- Energy density has a discontinuity
- Electrical neutrality is locally achieved

Nature of the hadron-quark phase transition

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- Theoretical values large spreac
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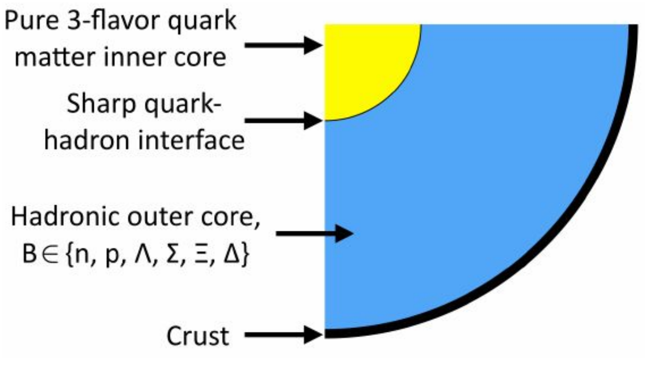

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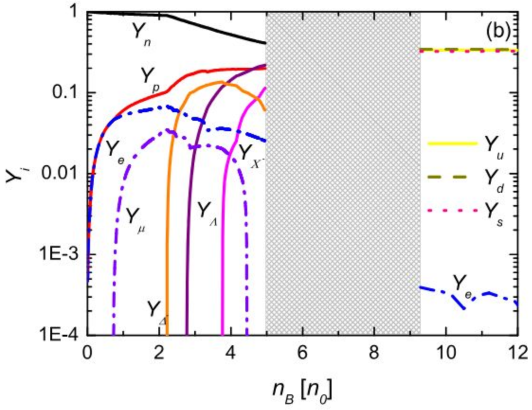

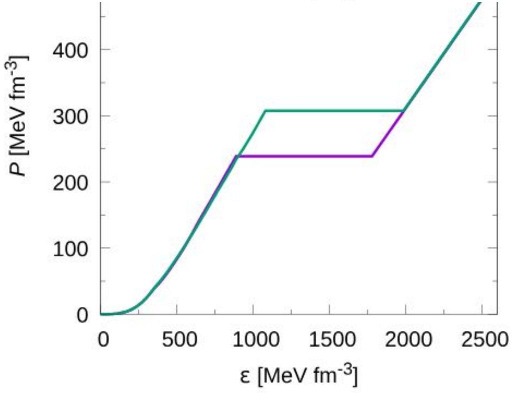

From Mariani et al. 2019

Nature of the hadron-quark phase transition

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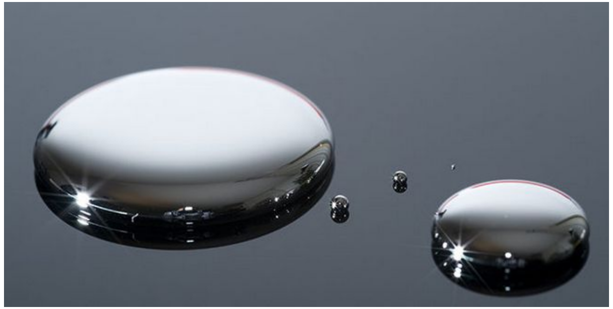

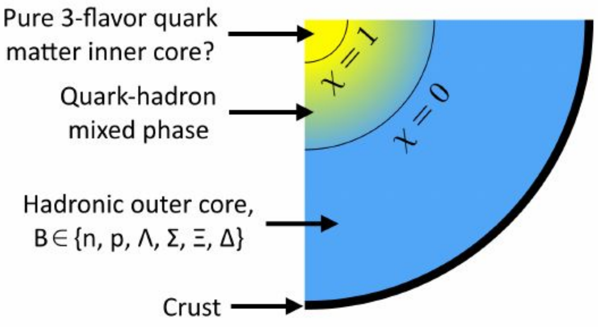

(b) Hypothetical NS cross section with a mixed phase.

- Quark and hadronic matter are mixed
- No thermodynamic quantity presents discontinuities
- Electrical neutrality is globally achieved, one phase is positive while the other negatively charged

Nature of the hadron-quark phase transition

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- Theoretical values large
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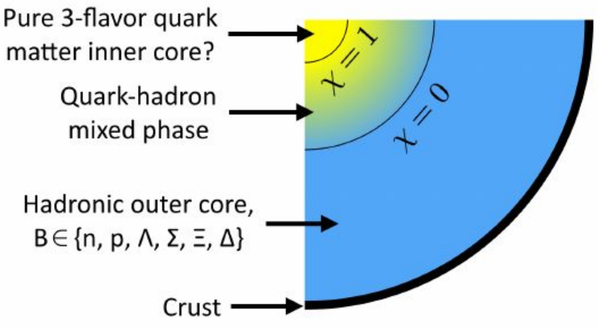

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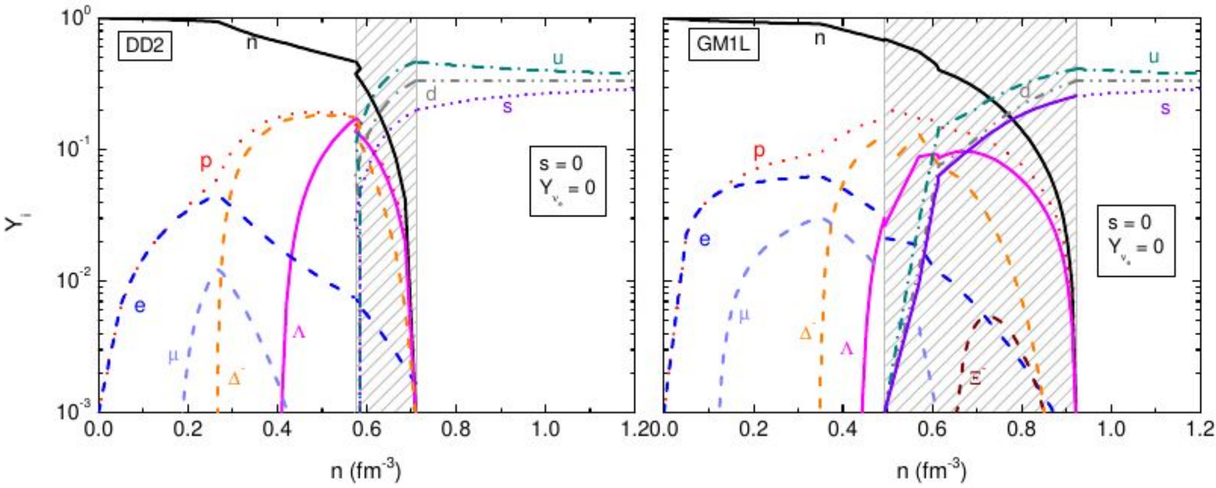

From Malfatti et al. 2019

one phase is positive while the other negatively charged

Nature of the hadron-quark phase transition

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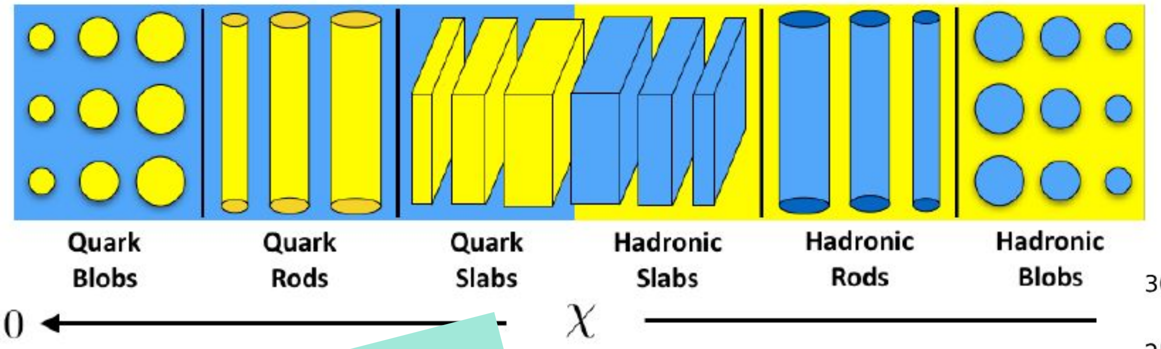
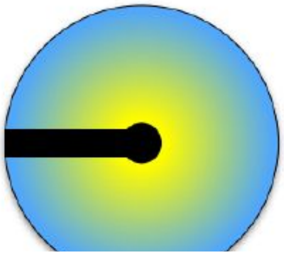
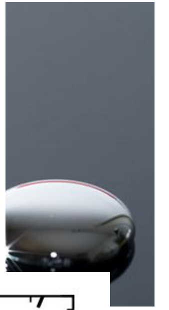
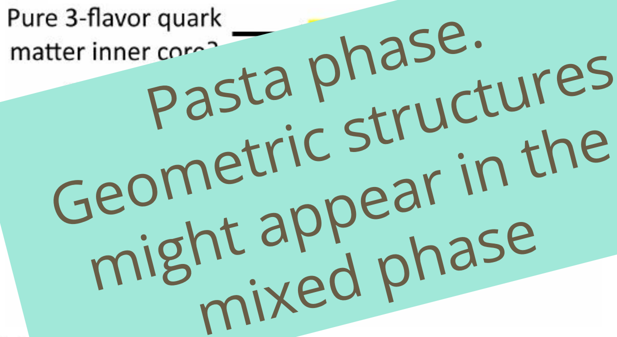




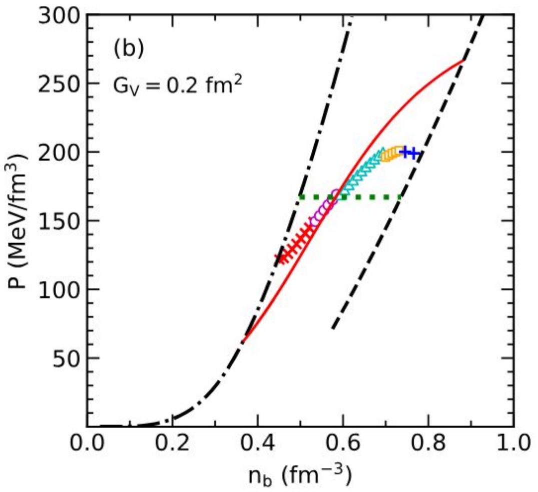




(b) … cross section with a mixed phase.

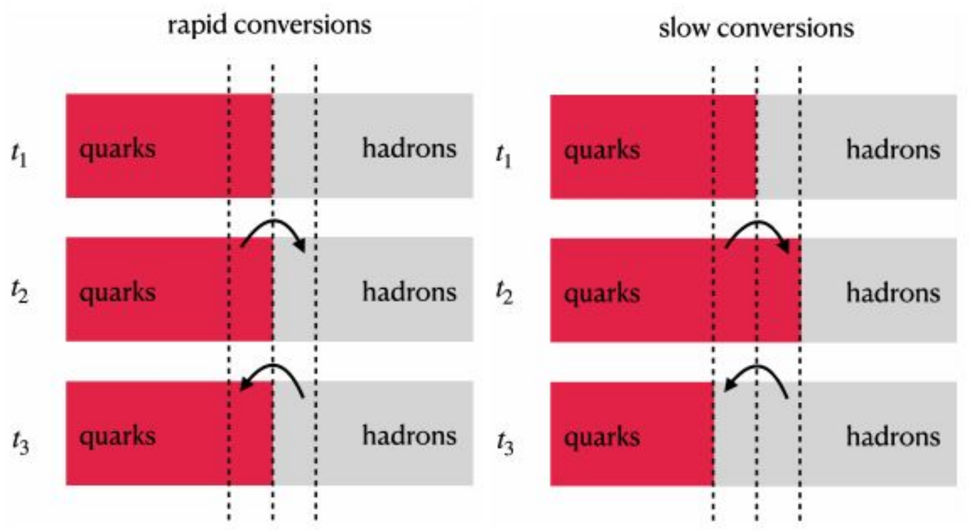
Pasta phase.
Geometric structures might appear in the mixed phase

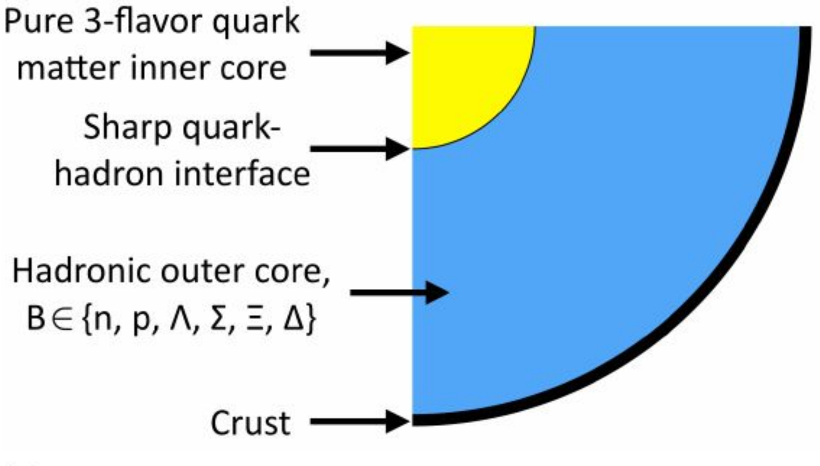
- Quark and hadron
- No thermodynam discontinuities
- Electrical neutralit one phase is posit negatively charge



Nature of the hadron-quark phase transition

If is sharp it is important to understand the timescale of hadron-quark conversion





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Nature of the hadron-quark phase transition

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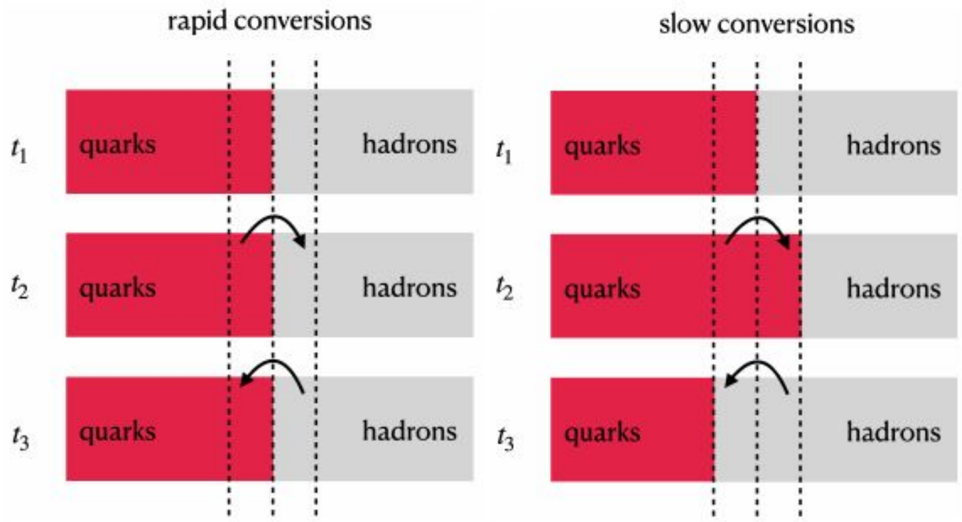
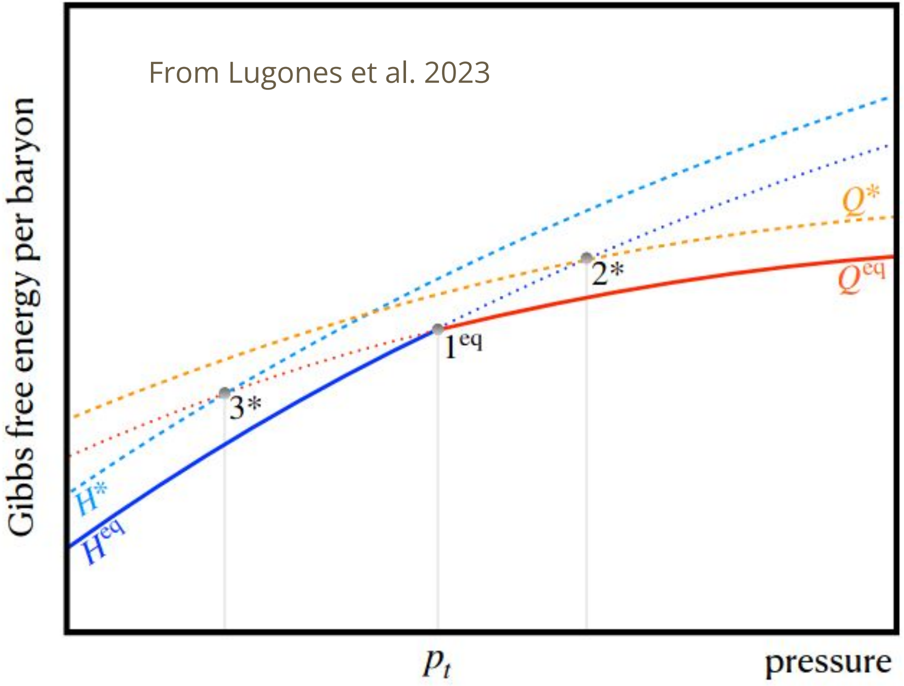


Not clear from a theoretical point of view.

Some arguments favour slow conversions are related to the fact that phase transitions are collective highly non-linear phenomena

Nature of the hadron-quark phase transition

If is sharp it is important to understand the timescale of hadron-quark conversion



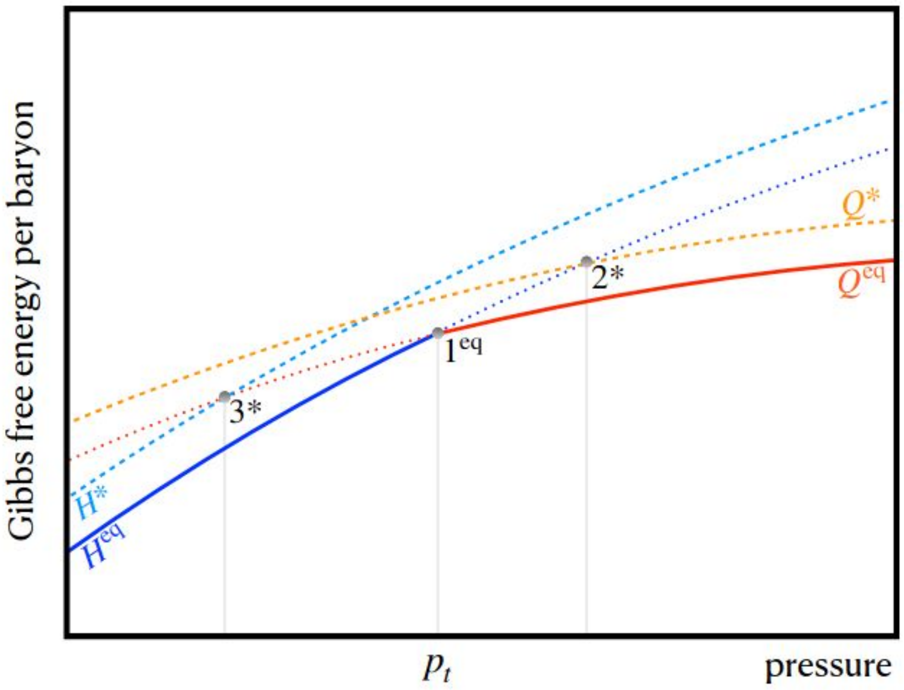
eq curves represent matter in chemical equilibrium

***** curves are out of equilibrium states of matter

Conversion at point 1 not expected as the two phases have very different flavour composition.

Nature of the hadron-quark phase transition

If is sharp it is important to understand the timescale of hadron-quark conversion



Hadronic matter in chemical equilibrium deconfines at point 2* where the out of equilibrium quark matter has the same flavour composition.

The inverse conversion occurs at point 3*.

Nature of the hadron-quark phase transition

Understanding this is extremely important, not only to shed some light into the behaviour of dense matter but also for astrophysics!

Nature of the hadron-quark phase transition

Understanding this is extremely important, not only to shed some light into the behaviour of dense matter but also for astrophysics!

If hadron-quark phase transition takes place in the inner core of hybrid stars and

the phase transition is sharp and slow then

new family of potential compact objects arise!

Radial perturbations of compact objects and stability

Stability of compact objects under linearized radial perturbations is a common criteria to study astronomical relevance of TOV solutions

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Why?

A brief introduction for tomorrow

Difference between stationary and stable configurations