Comparison of BEKK GARCH and DCC GARCH Models: An Empirical Study

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Abstract. Modeling volatility and co-volatility of a few zero-coupon bonds is a fundamental element in the field of fix-income risk evaluation. Multivariate GARCH model (MGARCH), an extension of the well-known univariate GARCH, is one of the most useful tools in modeling the co-movement of multivariate time series with time-varying covariance matrix. Grounded on the review of various formulations of multivariate GARCH model, this paper estimates two MGARCH models, BEKK and DCC form, respectively, based on the data of three AAA-rated Euro zero-coupon bonds with different maturities (6 months/1 year/2 years). Post-model diagnostics indicates satisfying fitting performance of these estimated MGARCH models. Moreover, this paper provides comparison on the goodness of fit and forecasting performances of these forms by adopting the mean absolute error (MAE) criterion. Throughout this application, the conclusion can be drawn that significant fitting and forecasting performances originate from the trade-off between parsimony and flexibility of the MGARCH models.

Keywords: Volatility, Multivariate GARCH Models, BEKK/DCC Form, Quasi – Maximum Likelihood Method, Zero-Coupon Bonds.

1 Introduction

With the increase in the complexity of the instruments in the risk management field, huge demands for the various models which can simulate and reflect the characteristics of the financial time series have expanded. One of the significant features of financial data that has won much attention is the volatility; because it is a numerical measure of the risk faced by individual investors and financial institutions. It is well known that the volatility of financial data often varies over time and tends to cluster in periods, i.e., high volatility is usually followed by high volatility, and low volatility by low volatility. This phenomenon corresponds to the fluctuating volatility. The Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model and its extensions have been proved to be able to capture the volatility clustering and predict volatilities in the future.

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L. Cao, J. Zhong, and Y. Feng (Eds.): ADMA 2010, Part II, LNCS 6441, pp. 99–110, 2010.

Specifically, when analyzing the co-movements of financial returns, it is always essential to construct, estimate, evaluate, and forecast the co-volatility dynamics of asset returns in a portfolio. This task can be fulfilled by multivariate GARCH (MGARCH) models. The development of MGARCH models could be thought as a great breakthrough against the curse of dimensionality in the financial modeling. Many different formulations have been constructed parsimoniously and still remain necessary flexibility. The application fields that MGARCH models can extend to include asset pricing, portfolio theory, VaR estimation and risk management or diversification, which require the volatilities and co-volatilities of several markets [1].

In this paper, MGARCH models are estimated and evaluated for volatility and co-volatility of three zero coupon bond prices with different maturities. The data is provided by the website of the European Central Bank (ECB) which is the institution of the European Union tasked with administrating the monetary policy of the EU member states taking part in the Euro zone.

A zero coupon bond is a non-coupon-bearing bond that pays face value at the time of maturity even though it is bought at a price lower than its face value. It has no reinvestment risk and is more sensitive to interest rate change than coupon-bearing bonds. Due to these features, zero-coupon bonds can be easily used to create any type of cash flow stream and thus match asset cash flows with liability cash flows (e.g. to provide for college expenses, house-purchase down payment, or other liability funding.), and used by pension funds and insurance companies to offset, or immunize the interest rate risk of these firms' long-term liabilities.

Moreover, the return of zero coupon bond, referred to as zero rate, is a fundamental element in the field of fix-income pricing and risk evaluation. By using cash-flow-mapping method [2], any fixed cash flow can be mapped to a portfolio consisting of a few representative zero coupon bonds, which match the cash flow’s return and volatility. This viewpoint exemplifies how to generalize the specific zero coupon bond volatilities into a general case. It also motivates our study to model volatility and co-volatility of three zero-coupon bonds with three conventional maturities of 6 months, 1 year and 2 years.

The reminder of this paper is organized as follows. Section 2 reviews MGARCH models, including its different forms, diagnostics techniques and the forecasting strategy. In section 3 we present the BEKK and DCC MGARCH models of volatility and co-volatility of ECB zero coupon bond data sets. Conclusions are detailed in section 4.

2 Model Specification and Estimation Methodology

At the beginning of reviewing different formulations of MGARCH models, one should consider what specification of an MGARCH model should be imposed in contrast to the univariate case. On the one hand, it should be flexible enough to state the dynamics of the conditional variances and covariances. On the other hand, as the number of parameters in an MGARCH model increases rapidly along with the dimension of the model, the specification should be parsimonious to simplify the model estimation and also reach the purpose of easy interpretation of the model parameters. However, parsimony may reduce the number of parameters, in which situation the relevant dynamics in the covariance matrix cannot be captured. Another feature that multivariate GARCH models must satisfy is that the covariance matrix should be positive definite. Bearing these specifications in mind, one can get a review of the following formulations of multivariate GARCH models and comprehend each of their relative competence and drawbacks.

2.1 Formulations of Multivariate GARCH Models

VEC-GARCH Models

The first MGARCH model was introduced by Bollerslev, Engle and Wooldridge in 1988, which is called VEC model. In the VEC model, every conditional variance and covariance is a function of all lagged conditional variances and covariances, as well as lagged squared returns and cross-products of returns. The model can be expressed below:

$$vech(H_t) = c + \sum_{j=1}^{q} A_j vech(\varepsilon_{t-j}\varepsilon'_{t-j}) + \sum_{j=1}^{p} B_j vech(H_{t-j}) \text{ ,} \tag{1}$$

where $vech(\cdot)$ is an operator that stacks the columns of the lower triangular part of its argument square matrix, H_t is the covariance matrix of the residuals, N presents the number of variables, t is the index of the lth observation, c is an $N(N+1)/2\times1$ vector, A_j and B_j are $N(N+1)/2 \times N(N+1)/2$ parameter matrices and ε is an $N\times1$ vector.

The condition for H_t to be positive definite for all t is not restrictive. In addition, the number of parameters equals $(p+q)\times(N(N+1)/2)2+N(N+1)/2$, which is large. Furthermore, it demands a large quantity of computation.

BEKK-GARCH Models

To ensure positive definiteness, a new parameterization of the conditional variance matrix H_t was defined by Baba, Engle, Kraft and Kroner (1990) and became known as the BEKK model, which is viewed as a restricted version of the VEC model. It achieves the positive definiteness of the conditional covariance by formulating the model in a way that this property is implied by the model structure.

The form of the BEKK model is as follows

$$H_t = CC' + \sum_{j=1}^{q}\sum_{k=1}^{K} A'_{kj}\,\varepsilon_{t-j}\varepsilon'_{t-j}\,A_{kj} + \sum_{j=1}^{p}\sum_{k=1}^{K} B'_{kj}H_{t-j}B_{kj} \text{ ,} \tag{2}$$

where A_{kj}, B_{kj}, and C are $N\times N$ parameter matrices, and C is a lower triangular matrix.

The purpose of decomposing the constant term into a product of two triangular matrices is to guarantee the positive semi-definiteness of H_t. Whenever $K > 1$ an identification problem would be generated for the reason that there are not only a single parameterization that can obtain the same representation of the model.

The first-order BEKK model is

$$H_t = CC' + A'\varepsilon_{t-1}\varepsilon'_{t-1}A + B'H_{t-1}B \text{ .} \tag{3}$$

Estimation of a BEKK model still bears large computations due to several matrix transpositions. The number of parameters of the complete BEKK model is $(p+q)KN^2+N(N+1)/2$. Even in the diagonal one, the number of parameters soon reduces to $(p+q)KN+N(N+1)/2$, but it is still large. The BEKK form is not linear in parameters, which makes the convergence of the model difficult. However, the strong point lies in that the model structure automatically guarantees the positive definiteness of H_t. Under the overall consideration, it is typically assumed that $p = q = K = 1$ in BEKK form's application.

Constant Conditional Correlation Models

The Constant Conditional Correlation model was introduced by Bollerslev in 1990 to primarily model the conditional covariance matrix indirectly by estimating the conditional correlation matrix. The conditional correlation is assumed to be constant while the conditional variances are varying. Obviously, this assumption is impractical for real financial time series. Then certain modifications were made grounded on this form [3].

Dynamic Conditional Correlation Models

The Dynamic Conditional Correlation model was proposed by Engle in 2002. It is a nonlinear combination of univariate GARCH models and it is also a generalized version of the CCC model. The form of Engle's DCC model is as follows:

$$H_t = D_t R_t D_t \ , \tag{4}$$

where

$$D_t = diag(h_{11t}^{1/2}, \cdots, h_{NNt}^{1/2}) \ , \tag{5}$$

and each h_{iit} is described by a univariate GARCH model. Further,

$$R_t = diag(q_{11t}^{1/2}, \cdots, q_{NNt}^{1/2}) Q_t diag(q_{11t}^{1/2}, \cdots, q_{NNt}^{1/2}) \ , \tag{6}$$

where $Q_t = (q_{ijt})$ is the $N\times N$ symmetric positive definite matrix which has the form:

$$Q_t = (1-\alpha-\beta)\overline{Q} + \alpha u_{t-1} u_{t-1}^{'} + \beta Q_{t-1} \ . \tag{7}$$

Here, $u_{it} = \varepsilon_{it} / \sqrt{h_{iit}}$, and are non-negative scalars that + <1, $\overline{Q}$ is the $N\times N$ unconditional variance matrix of u_t.

The number of parameters to be estimated is $(N+1)\times(N+4)/2$, which is relatively smaller than the complete BEKK form with the same dimension when N is small. When N is large, the estimation of the DCC model can be performed by a two-step procedure which decreases the complexity of the estimation process. In brief, in the first place, the conditional variance is estimated via univariate GARCH model for each variable. The next step is to estimate the parameters for the conditional correlation. The DCC model can make the covariance matrix positive definite at any point in time. The shortcoming of the model is that all conditional correlations follow the same dynamic structure.

2.2 Estimation of Multivariate GARCH Models

Let $H_t(\theta)$ be a positive definite $N\times N$ conditional covariance matrix of some $N\times 1$ residual vector ε_t, parameterized by the vector θ. Denoting the available information at time t by ξ_t, we have

$$E_{t-1}[\varepsilon_t \mid \xi_{t-1}] = 0 \,, \tag{8}$$

$$E_{t-1}[\varepsilon_t \varepsilon_t' \mid \xi_{t-1}] = H_t(\theta) \,. \tag{9}$$

Generally the conditional covariance matrix $H_t(\theta)$ is well specified based on a certain MGARCH model. Suppose there is an underlying parameter vector θ_0 which one wants to estimate using a given sample of T observations. The quasi maximum likelihood approach estimates θ_0 by maximizing the Gaussian log likelihood function

$$\log L_T(\theta) = -\frac{N \cdot T}{2}\log(2\pi) - \frac{1}{2}\sum_{t=1}^{T}\log|H_t| - \frac{1}{2}\sum_{t=1}^{T}\varepsilon_t' H_t^{-1}\varepsilon_t \,. \tag{10}$$

2.3 Diagnostics of Multivariate GARCH Models

The check of the adequacy of MGARCH models is essential in identifying whether a well specified MGARCH model can obtain reliable estimates and inference.

Graphical diagnostics for MGARCH models can be fulfilled by examining plots of the sample autocorrelation (ACF) and the sample cross correlation functions (XCF). To ensure the inference from the estimated parameters in the MGARCH model is enough valid, the residuals should be exhibited as a set of white noise with features like expected zero mean vector, no autocorrelations, constant variance, and normal distribution of the residuals.

The autocorrelation and cross correlation functions for the squared process are shown to be useful in identifying and checking time series behavior in the conditional variance equation of the GARCH form.

In the literature, several tests have been developed to test the autocorrelation no matter in univariate form. Box and Pierce derived a goodness-of-fit test, called the portmaneau test.

But still, the fact is that very few tests are adaptable to multivariate models even though there are many diagnostic tests dealing with univariate models.

To summarize, once the model is assumed to catch the dynamics of the time series, the standardized residual $\hat{z}_t = \hat{H}_t^{-1}\hat{\varepsilon}_t$ should satisfy the following conditions [4]:

1) $E(\hat{z}_t \hat{z}_t') = I_N$, (11)

2) $Cov(\hat{z}_{it}^2, \hat{z}_{jt}^2) = 0$, for all pairs of the variable index $i \neq j$, (12)

3) $Cov(\hat{z}_{it}^2, \hat{z}_{j,t-k}^2) = 0$, for k > 0 . (13)

Testing 1) would find the misspecification in the conditional mean; testing 2) is to verify whether the conditional distribution is Gaussian; the purpose of testing 3) is to check the adequacy of the dynamic specification of H_t even without knowing the validity of the assumption on the distribution of z_t.

3 Construction of Multivariate GARCH Models

The original data is provided by the European Central Bank (ECB) website. It contains daily zero rates of AAA-rated euro area central government bonds, from 01/01/2007 to 30/04/2010. The data of 2007, 2008 and first half of 2009, totally 635 observations, is used to estimate MGARCH models, and the rest data, from Jul/2009, is used to evaluate model forecasting.

With given ZR_{it}, the zero rate at time t, and maturity T, the zero coupon bond price p_{it} is calculated as

$$p_{it} = S \times e^{-ZR_{it} \cdot T} \ , i = 1, 2, 3. \tag{14}$$

where S is the par value, in our case taking the value 100. The daily log return r_t is calculated as follows:

$$r_{it} = \ln(p_{it} / p_{i,t-1}) \ , i = 1, 2, 3. \tag{15}$$

Three variables (*var*1/*var*2/*var*3) correspond to three daily returns with different maturities (6*m*/1*y*/2*y*). Their descriptive statistics are given in Table 1. By viewing the value of kurtosis, one can conclude that the return series all have fat tails relative to the normal distribution, which indicates a much more possibility of extreme movements. Moreover, the result of ARCH effect [5] test proposed by Engle of each return series is given in Table 2, where "*H*" being 1 indicates rejecting of null hypothesis that there is no ARCH effect. One may see that each variable/return has significant ARCH effect.

Table 1. Descriptive Statistics of Return Series with Different Maturities (6*m*/1*y*/2*y*)

	Mean	Max	Min	Std Dev.	Skewness	Kurtosis	Jarque-Bera	Prob
*var*1	0.0023	0.1374	-0.0581	0.0166	2.5324	19.6906	8049.379	0.0000
*var*2	0.0045	0.2190	-0.1944	0.0403	0.3058	6.7716	386.2683	0.0000
*var*3	0.0082	0.3533	-0.4522	0.0979	-0.1392	5.1290	121.9801	0.0000

Table 2. GARCH Effect Testing of Return Series

	var 1		*var* 2		*var* 3	
Lag	H	pValue	H	pValue	H	pValue
1	1	0.0357	1	0.1887×10^{-5}	1	0.2385×10^{-5}
2	1	0.0121	1	0.4053×10^{-5}	1	0.1165×10^{-5}
3	1	0.0000	1	0.0157×10^{-5}	1	0.0279×10^{-5}
4	1	0.0000	1	0.0002×10^{-5}	1	0.0108×10^{-5}
5	1	0.0000	1	0.0006×10^{-5}	1	0.0086×10^{-5}

3.1 Multivariate-GARCH Modeling

As the BEKK-GARCH and DCC-GARCH models are the two most widely used multivariate GARCH models, we will restrict to model the volatility and co-volatility of the three variables by using BEKK and DCC forms.

Next we present the estimated model, and their diagnostics and forecasting are provided in the following subsections.

The estimation process is performed in our case by the econometrics software package RAT 7.0. The optimization algorithm used for the maximum likelihood estimation is BFGS proposed independently by Broyden, Fletcher, Goldfarb and Shanno in 1970. Convergence is assumed to occur if the change in the coefficients to be estimated is less than the convergence criterion option *cvcrit* specified.

The estimated BEKK-GARCH model can be obtained by substituting the following matrices into Equation 3.

$$A = \begin{pmatrix} 0.3400 & 0.1881 & 0.6015 \\ -0.0415 & -0.1082 & 0.5756 \\ 0.0245 & 0.1713 & 0.5036 \end{pmatrix}, \tag{16}$$

$$B = \begin{pmatrix} 1.2262 & 0.3803 & -0.1912 \\ -0.3316 & 0.6331 & 0.1752 \\ 0.0997 & 0.0802 & 0.9035 \end{pmatrix}, \tag{17}$$

$$C = \begin{pmatrix} 6.534\times10^{-4} & 0 & 0 \\ 3.545\times10^{-3} & -4.80\times10^{-7} & 0 \\ 7.689\times10^{-3} & -1.374\times10^{-6} & 1.21\times10^{-7} \end{pmatrix}. \tag{18}$$

The estimated DCC-GARCH model can be obtained by substituting the following numerical information into Equations 4, 5 and 7.

$$h_{11t} = 1.2142\times10^{-6} + 0.2145\varepsilon_{1,t-1}^2 + 0.8259h_{11,t-1} \cdot \tag{19}$$

$$h_{22t} = 4.6008\times10^{-7} + 0.1692\varepsilon_{2,t-1}^2 + 0.8566h_{22,t-1} \cdot \tag{20}$$

$$h_{33t} = -5.1693\times10^{-6} + 0.1502\varepsilon_{3,t-1}^2 + 0.8713h_{33,t-1} \cdot \tag{21}$$

$$Q_t = (1-0.0934-0.8971)\overline{Q} + 0.0934u_{t-1}u_{t-1}' + 0.8971Q_{t-1} \cdot \tag{22}$$

$$\overline{Q} = \begin{pmatrix} 0.9934 & 0.8301 & 0.6979 \\ 0.8301 & 1.01244 & 0.9683 \\ 0.6979 & 0.9683 & 1.0034 \end{pmatrix}. \tag{23}$$

Except for several constant terms, all the other estimated variables are statistically significant.

3.2 Model Diagnostics

Residual-based diagnostics are conducted to test the residual pattern implied by the deviation of the estimated model from underlying assumptions. As the model estimation method employed here is the Gaussian quasi MLE method. One of its assumptions is that the residuals should follow a Gaussian distribution. Hence, to test whether the estimations of the model parameters are robust, we can check whether the residuals of the estimated process are white noise.

Tables 3 and 5 show the testing results of GARCH effect on the standardized residuals of the BEKK model and the DCC model respectively. $H = 0$ represents the acceptance of the null hypothesis that no GARCH effects exist. In contrast with Table 2, we can conclude that GARCH effect has been eliminated quite a lot. The Ljung-Box test results in Tables 4 and 6 based on the autocorrelation plot test the randomness at each distinct lag. $H = 0$ means that we tend to accept the null hypothesis that the series is random.

Table 3. GARCH Effect Testing of each Standardized Residuals of the BEKK Model

	var 1		*var* 2		*var* 3	
Lag	H	pValue	H	pValue	H	pValue
1	0	0.2117	0	0.1537	0	0.8706
2	0	0.3649	0	0.3154	0	0.8924
3	0	0.4380	0	0.5108	0	0.9291
4	1	0.0309	0	0.3234	0	0.6651
5	1	0.0163	0	0.3108	0	0.6329

Table 4. LBQ Test of each Standardized Residuals of the BEKK Model

	var 1		*var* 2		*var* 3	
Lag	H	pValue	H	pValue	H	pValue
1	0	0.9488	0	0.0935	0	0.0507
2	0	0.8372	0	0.1144	0	0.1412
3	0	0.9459	0	0.2220	0	0.2655
4	0	0.9164	0	0.2025	0	0.2158
5	0	0.9477	0	0.2984	0	0.3276

Table 5. GARCH Effect Testing of each Standardized Residuals of the DCC Model

	var 1		*var* 2		*var* 3	
Lag	H	pValue	H	pValue	H	pValue
1	0	0.0500	0	0.4501	0	0.5339
2	0	0.1458	0	0.5963	0	0.6506
3	0	0.2669	0	0.1113	0	0.7538
4	0	0.3477	0	0.1851	0	0.8506
5	0	0.4898	0	0.2481	0	0.9222

Table 6. LBQ Test of each Standardized Residuals of the DCC Model

	var 1		*var* 2		*var* 3	
Lag	H	pValue	H	pValue	H	pValue
1	0	0.1951	0	0.0292	0	0.0830
2	0	0.4152	0	0.0928	0	0.2226
3	0	0.5695	0	0.1779	0	0.3635
4	0	0.2261	0	0.1891	0	0.2889
5	0	0.2339	0	0.2262	0	0.4050

3.3 Comparison of BEKK and DCC Models

The empirical measure of logarithmic daily return variability is called the realized volatility. It is calculated using the subsequent 10 observations on the log-returns in our case. In contrast realized volatility constructed from high-frequency returns with the restrictive parametric multivariate GARCH models, link between realized volatility and the diagonal elements of the conditional covariance matrix would be established [6].

In our case, the estimated volatility follows the dynamics of the realized volatility. Additionally, the volatility clustering and the relation between maturity and volatility can be clearly indicated in line graphs (e.g. Figure 1). And there is a horizontal lag between these two lines for the reason that we calculate the realized volatility by using the next ten observations.

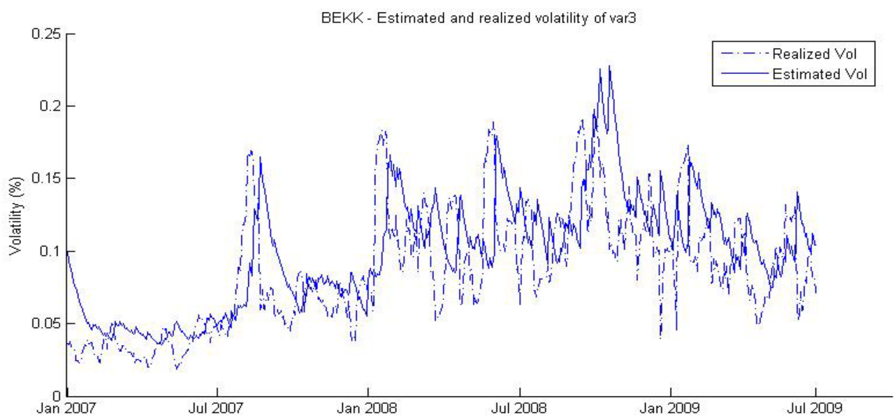

Fig. 1. Estimated Volatility of *var* 3 in the BEKK Model Compared to Realized Volatility

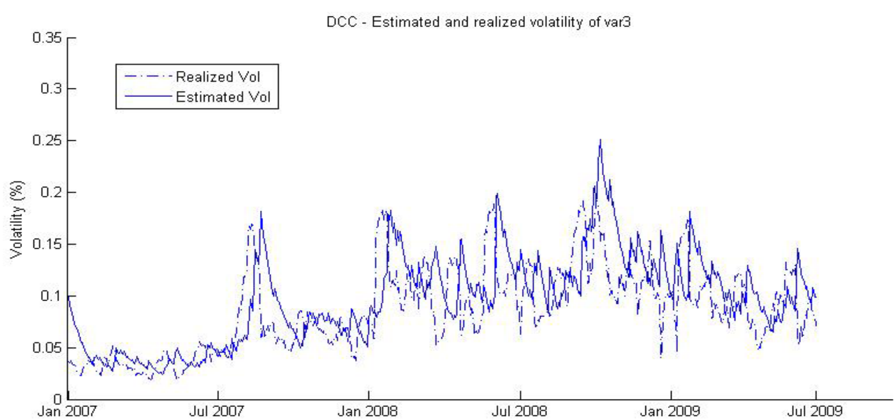

Fig. 2. Estimated Volatility of *var* 3 in the DCC Model Compared to Realized Volatility

Figure 3 presents the poor forecasting performance of the BEKK – GARCH model through the comparison of the realized correlations and the forecast correlations just on the right side of the vertical line. The forecasting performance of the DCC – GARCH model (shown in Figure 4) looks better than that of the BEKK – GARCH model. As the number of parameter estimated by BEKK – GARCH models is much more than that of DCC – GARCH models, the summation of the error accumulated by each parameter of the BEKK form tends to be greater than that of the DCC form.

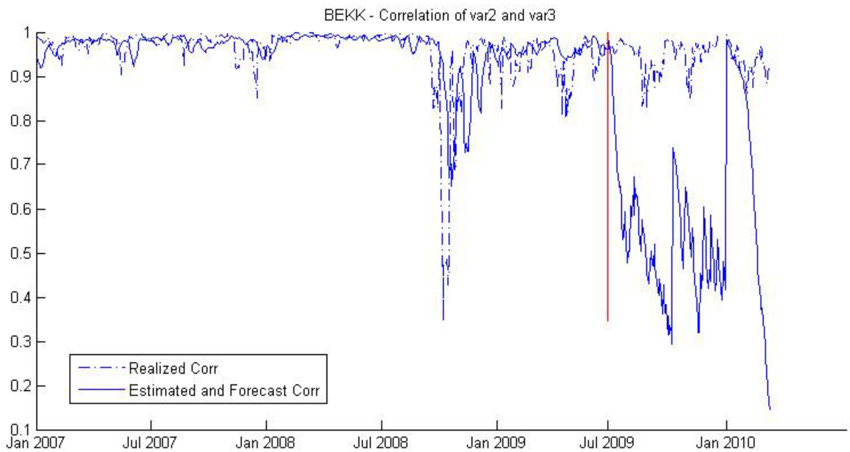

Fig. 3. Estimated and Forecast Correlation of BEKK Form Compared to Realized Correlation

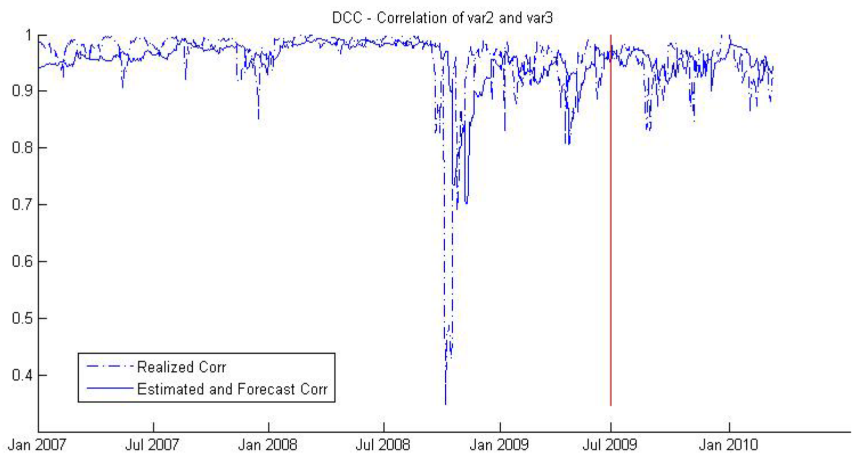

Fig. 4. Estimated and Forecast Correlation of DCC Form Compared to Realized Correlation

The mean absolute error (MAE) [7] can measure how close the estimated variables are to the realized values. It is also called the mean average error. In our case MAE is calculated by

$$MAE_{vi} = \frac{1}{n} . \sum_{k=1}^{n} \left| \sigma_{ik} - \hat{\sigma}_{ik} \right| , \tag{24}$$

for volatility where n is the total number of observations or

$$MAE_{vi} = \frac{1}{n} . \sum_{k=1}^{n} \left| \rho_{ijk} - \hat{\rho}_{ijk} \right| , \tag{25}$$

for correlation where i, j = 1, 2, 3.

Table 7. MAE in Volatility and Correlation of the BEKK Model

Average error in volatility		Average error in correlation	
*MAE v*1	0.0056	*MAE*12	0.1317
*MAE v*2	0.0132	*MAE*13	0.1990
*MAE v*3	0.0306	*MAE*23	0.0324

Table 8. MAE in Volatility and Correlation of the DCC Model

Average error in volatility		Average error in correlation	
*MAE v*1	0.0062	*MAE*12	0.1354
*MAE v*2	0.0142	*MAE*13	0.2027
*MAE v*3	0.0309	*MAE*23	0.0352

The values of the mean absolute error between these models suggest that the parameter estimation of the BEKK model is more accurate than that given by the DCC model through the magnitude of the difference between their corresponding MAEs.

4 Conclusion

The research focuses on constructing and diagnosing two formulations of multivariate GARCH models- the BEKK and DCC forms. The estimation process is fulfilled in the software package RATS 7.0 through the maximum likelihood method. As the implementation is conducted under the premise that the residual terms follow the Gaussian distribution, the diagnostics in evaluating the adequacy of modeling is operated by checking whether such assumption is credible enough.

By comparing the goodness of fit through the mean absolute error, we find that the fitting performance of the BEKK – GARCH form is much sounder than DCC – GARCH form in our example. The distinction may due to the relatively more number of parameters in BEKK – GARCH forms compared to DCC – GARCH forms. In this sense, the BEKK – GARCH model can process a well-warranted capability in explaining the information hidden in the history data. On the opposite, the DCC – GARCH model owns an advantage over BEKK – GARCH model in the area of forecasting as the DCC form is more parsimonious than the BEKK form so that the

DCC form won't possibly accumulate errors as much as the BEKK form does. To conclude, it is crucially important to balance parsimony and flexibility when modeling multivariate GARCH models.

Acknowledgments. This research was sponsored by National Natural Science Foundation of China, Ref. Nos. 40730526 and 40701142.

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