

1: $\neg \forall x. \neg R(x)$ premise

2: $\neg \exists y. R(y)$ assumption

...

3: $\forall x. \neg R(x)$

4: \perp \neg elim 3,1

5: $\exists y. R(y)$ contra (classical) 2-4