

$$\begin{array}{c}
 \text{hyp} \qquad \qquad \qquad \text{hyp} \\
 \hline
 P \rightarrow Q, Q \rightarrow R, P \vdash P \qquad P \rightarrow Q, Q \rightarrow R, P \vdash P \rightarrow Q \\
 \hline
 \qquad \qquad \qquad \rightarrow -E \\
 P \rightarrow Q, Q \rightarrow R, P \vdash Q \qquad \qquad P \rightarrow Q, Q \rightarrow R, P, Q \vdash R \\
 \hline
 \qquad \qquad \qquad \text{cut} \\
 P \rightarrow Q, Q \rightarrow R, P \vdash R
 \end{array}$$