

$$\begin{array}{c}
 \frac{}{\text{hyp}} \quad \frac{}{\text{hyp}} \quad \frac{}{\text{hyp}} \quad \frac{}{\text{hyp}} \quad \frac{}{\text{hyp}} \\
 \frac{P \rightarrow Q, Q \rightarrow R, P \vdash P \quad P \rightarrow Q, Q \rightarrow R, P \vdash P \rightarrow Q}{\rightarrow -E} \quad \frac{P \rightarrow Q, Q \rightarrow R, P, Q \vdash Q \quad P \rightarrow Q, Q \rightarrow R, P, Q \vdash Q \rightarrow R}{\rightarrow -E} \quad \frac{}{\text{hyp}} \\
 \frac{P \rightarrow Q, Q \rightarrow R, P \vdash Q}{\rightarrow -E} \quad \frac{P \rightarrow Q, Q \rightarrow R, P, Q \vdash R \quad P \rightarrow Q, Q \rightarrow R, P, Q, R \vdash R}{\text{cut}} \\
 \frac{P \rightarrow Q, Q \rightarrow R, P \vdash Q \quad P \rightarrow Q, Q \rightarrow R, P, Q \vdash R}{\text{cut}} \\
 P \rightarrow Q, Q \rightarrow R, P \vdash R
 \end{array}$$