Hidden Markov models Training session

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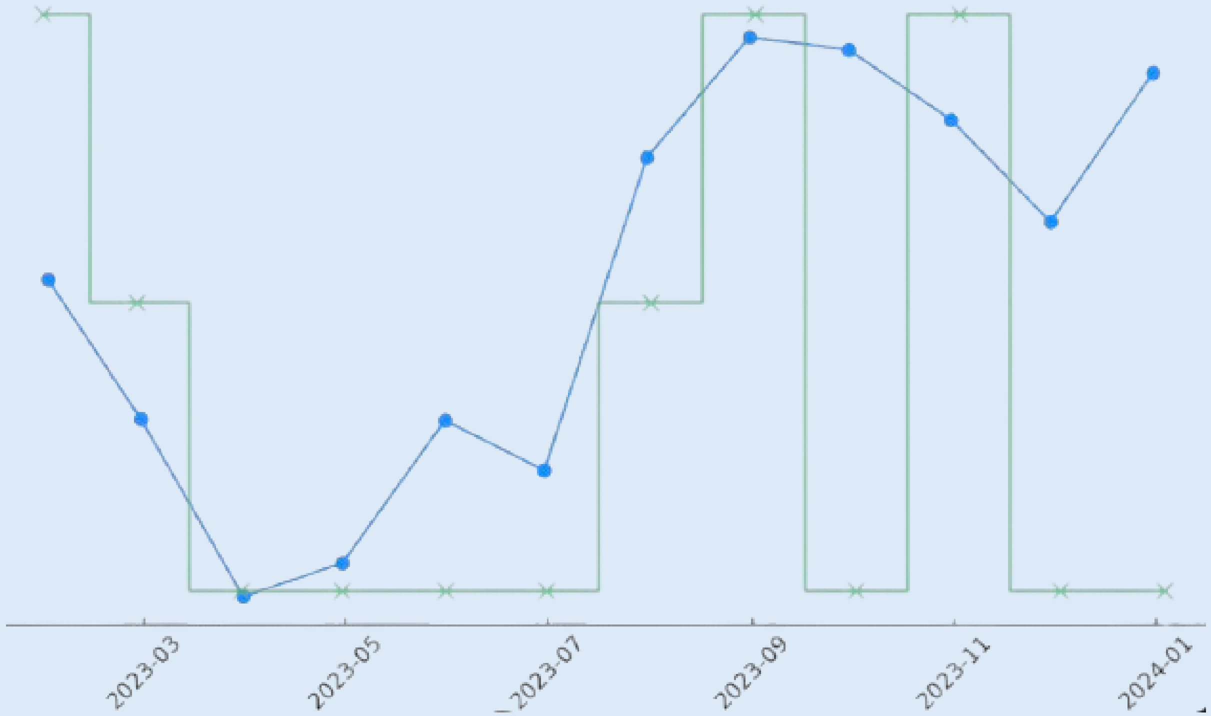
Hidden Markov models:

- Time-series models

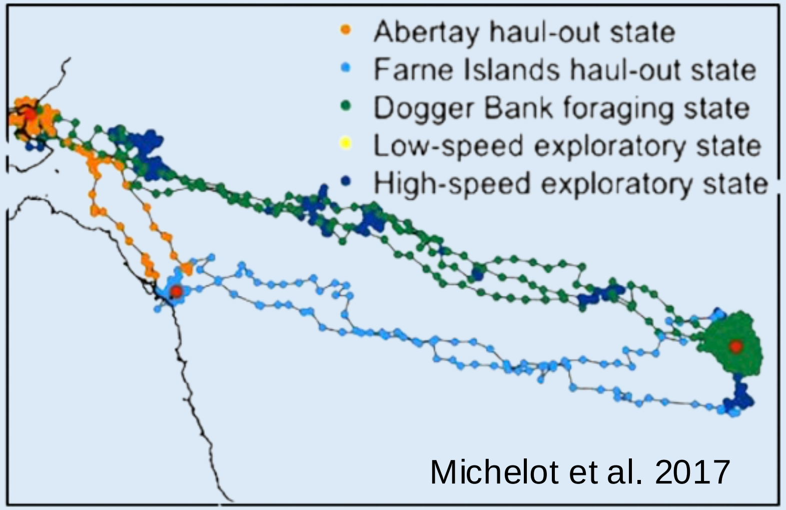


- observation arise from a hidden Markov process

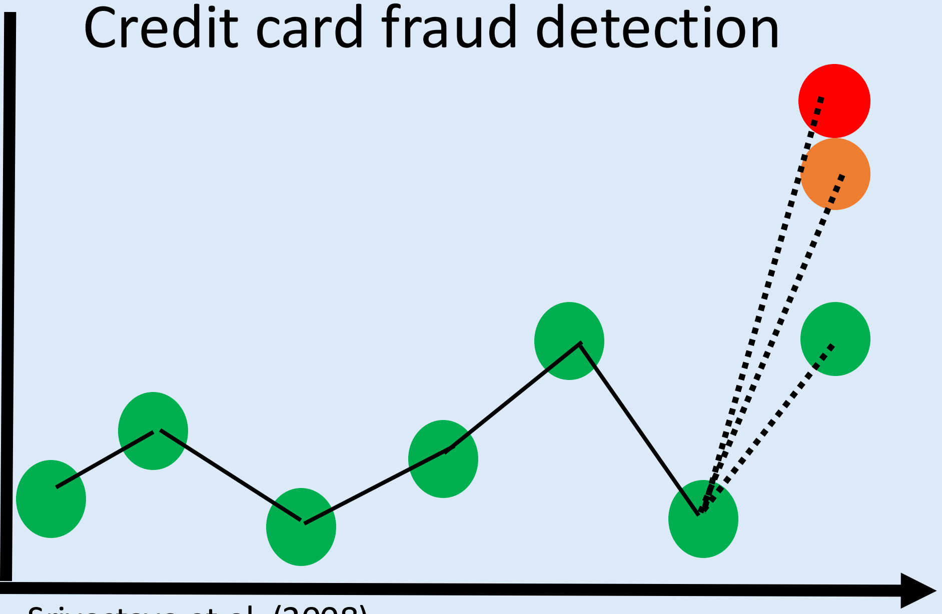
- Markov process = the future depends only on the present

Speech Recognition

Behaviour segmentation

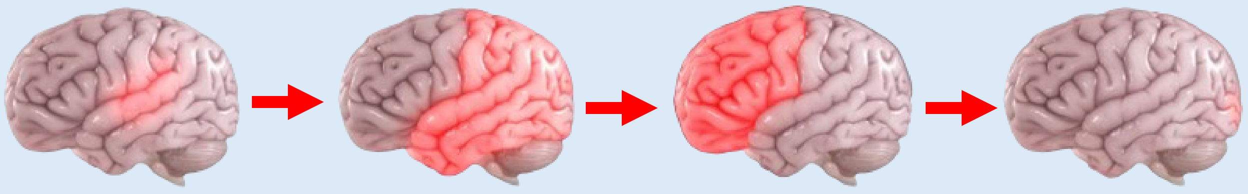


Credit card fraud detection



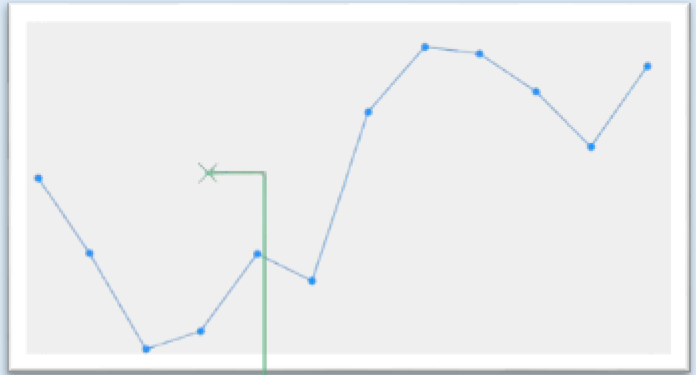
Srivastava et al. (2008)

Alzheimer's disease progression stage identification:

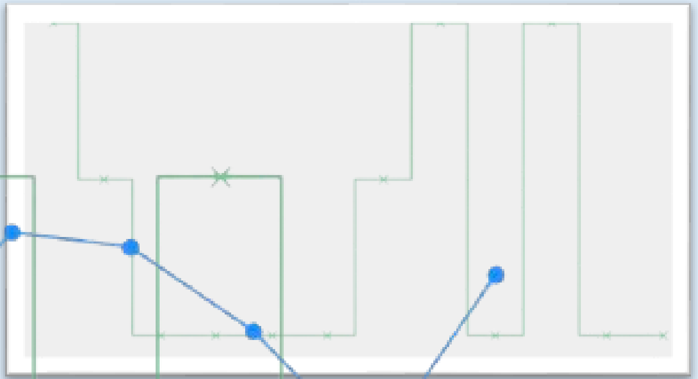


Lin and Song (2021)

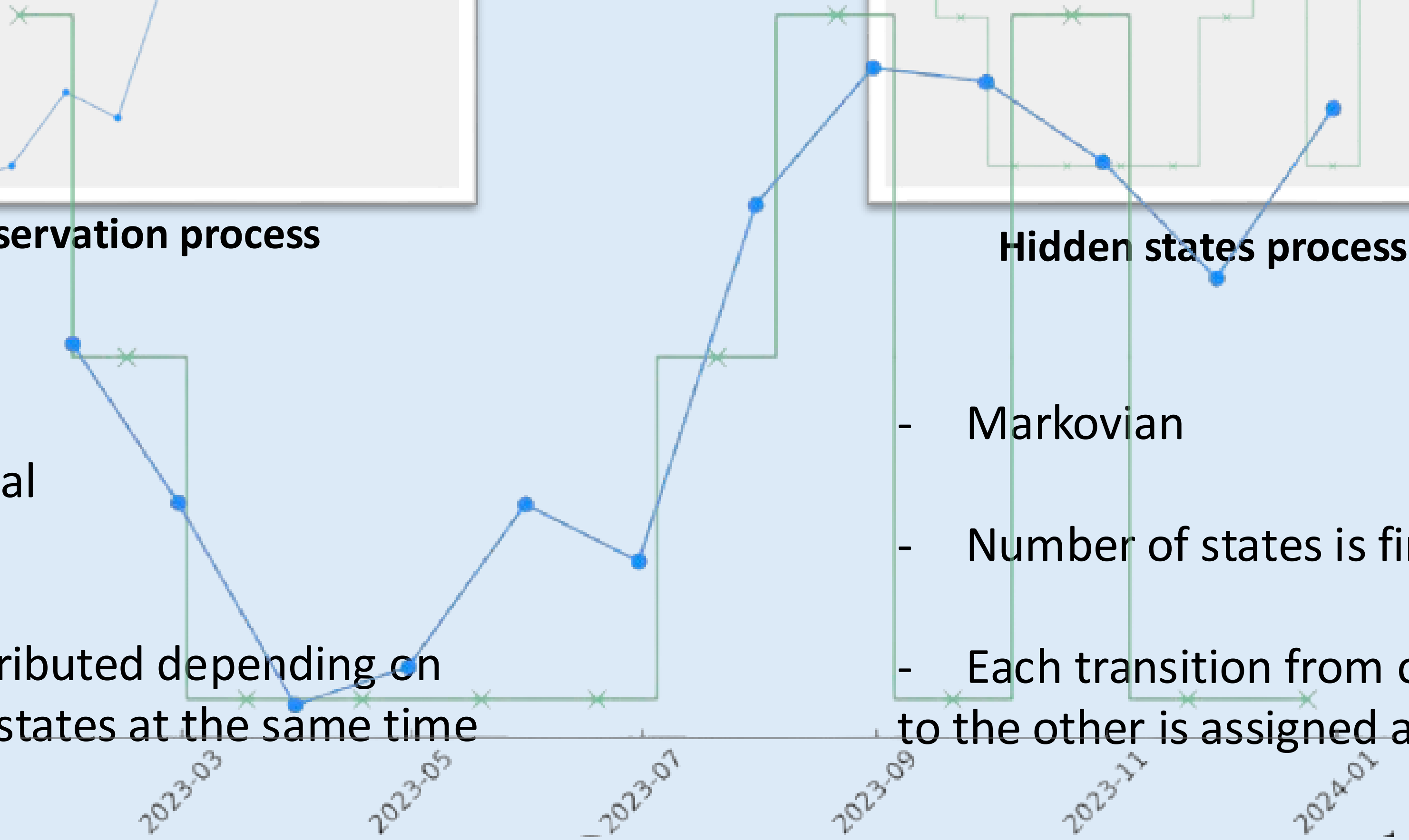
Two processes:



Observation process



Hidden states process



- Signal
- Distributed depending on the states at the same time

- Markovian
- Number of states is finite.
- Each transition from one state to the other is assigned a probability

Markovian process:

Example: weather in Vancouver

Only two types of weather:

Markovian process:

Example: weather in Vancouver on ***day t*** *is* $\boldsymbol{S_t}$

$P(S_t =$  $\mid S_{t-1} =$  $, S_{t-2} =$  $, \ldots., S_0 =$  $)$

Markovian process:

Example: weather in Vancouver on ***day t*** *is* $\boldsymbol{S_t}$

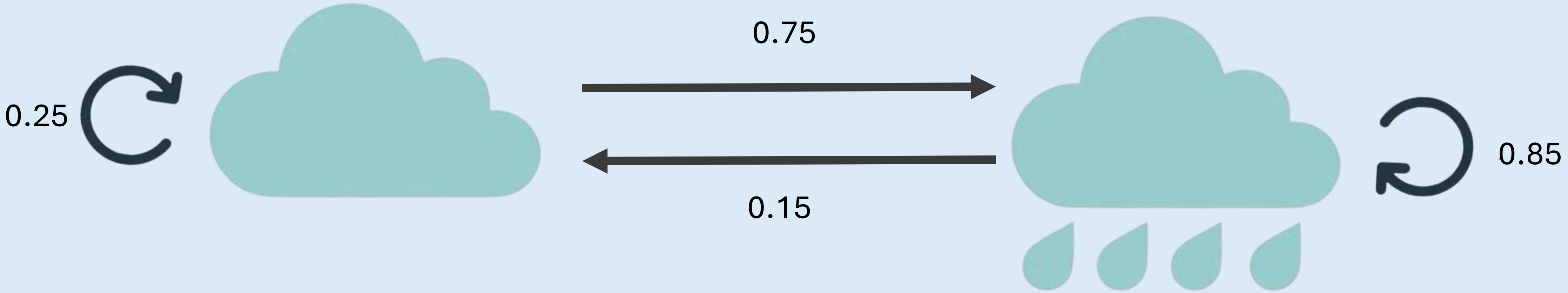
P(S_t=  | S_{t-1} =  , S_{t-2}=  , …., S_0= ) = P(S_t=  | S_{t-1} = )

Markov property: weather from tomorrow only depends on the weather from today

- The probability that it is not rainy on day *t* only depends on the weather at day *t-1*

Markovian process:

Example: weather in Vancouver



Hidden Markov model:

Example: weather in Vancouver

- Imagine you are in an appartement with a window very high up for several days

- You can't see outside, but you can see the light filtering through

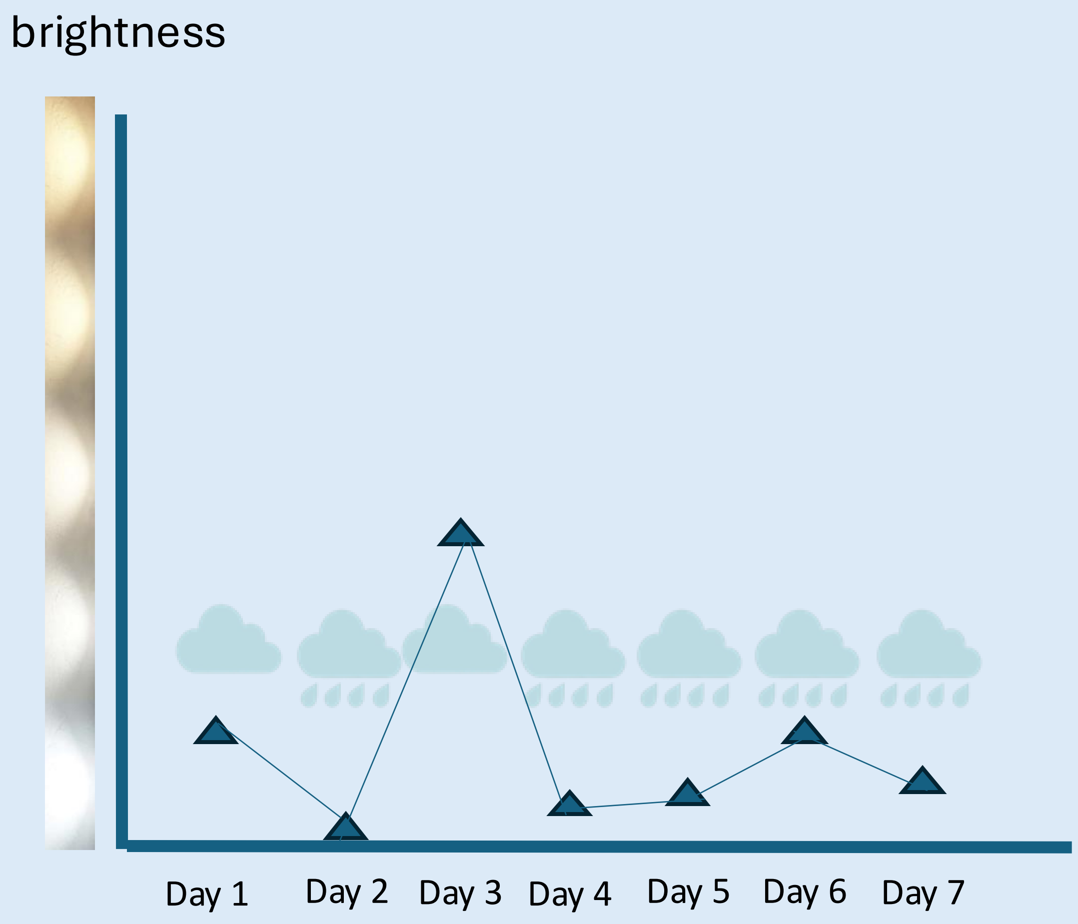
- You’re trying to figure out what the weather is like outside (rainy, cloudy).

Hidden Markov model:

Example: weather in Vancouver

Observation: brightness of the light coming through the window

Hidden state: weather outside



Hidden Markov model:

Example: weather in Vancouver

Brightness of the light coming through ≠ Weather:
Brightness is not a direct observation of rainfall:

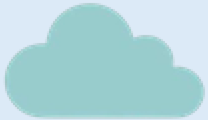
- dim lighting can be associated with rainy weather but it could also be because the sun is low or behind clouds

Hidden Markov model:

Example: weather in Vancouver

- The weather follows its own dynamic

- The distribution of the brightness depends on the weather:

  - When  , the brightness might follow a normal distribution with mean=50.

  - When  , the brightness could be much lower, around mean =30.

Hidden Markov model:

Example: weather in Vancouver

- Identify how often it is likely *rainy* versus *not rainy*

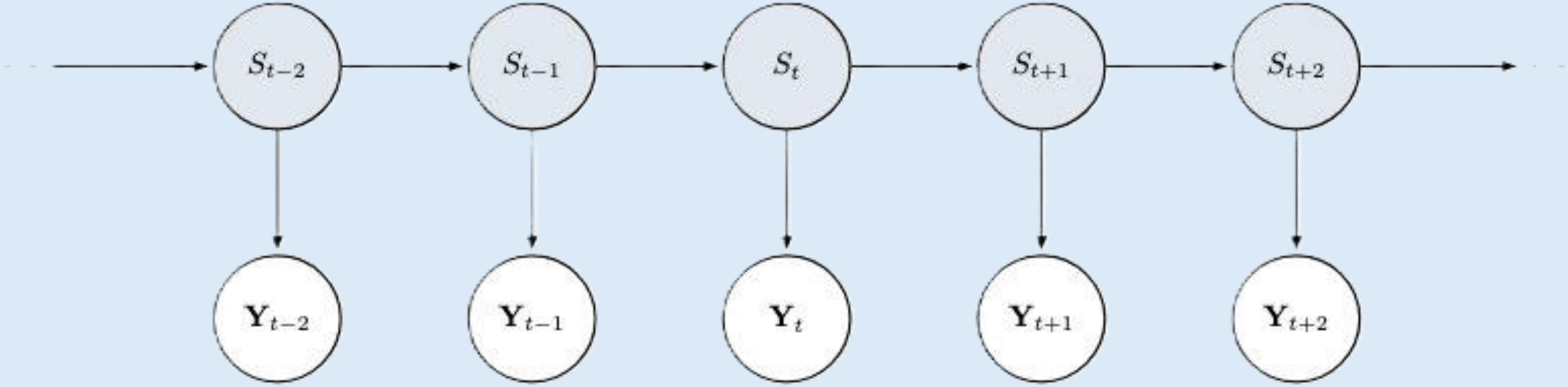
- Understand how often the weather switches between states

- Reconstruct the most likely sequence of *actual* weather states (rainy/not rainy) from the observed brightness: *state decoding*

Hidden Markov model:

Basic structure

Hidden state: weather outside

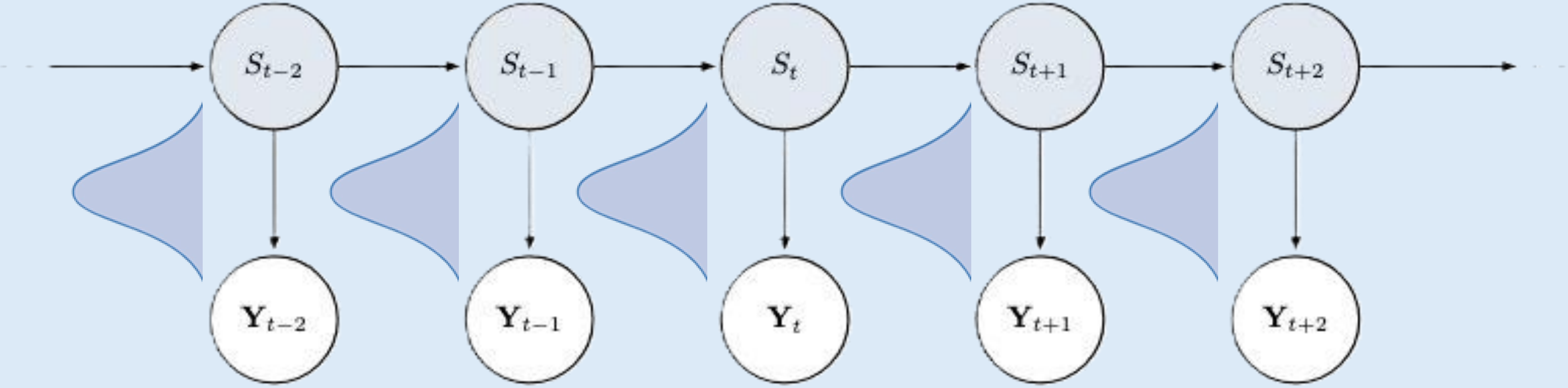


Observation: brightness of the light coming through the window

Hidden Markov model:

The aim is to learn the hidden through the observed process

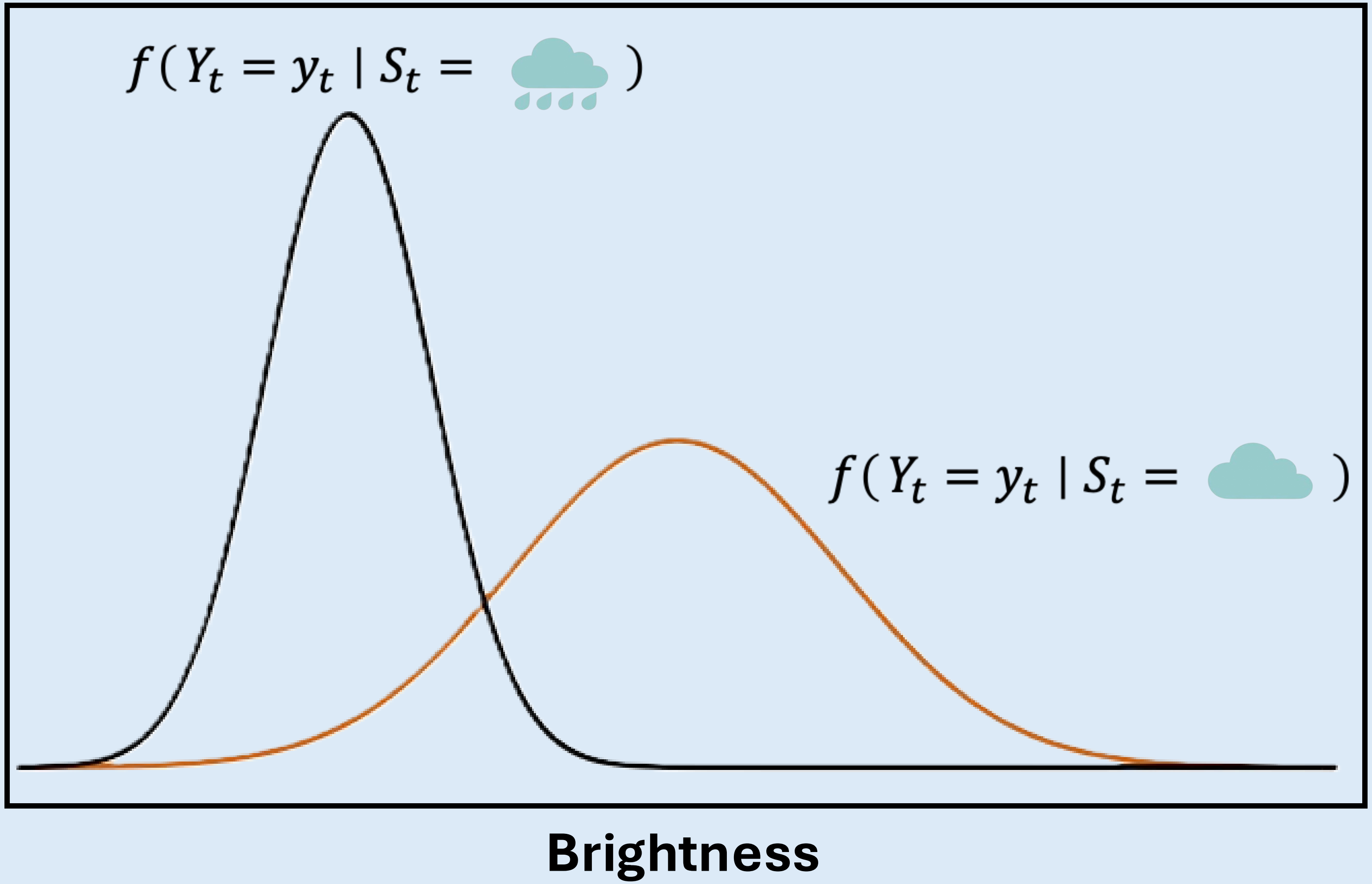
Hidden state



Observation

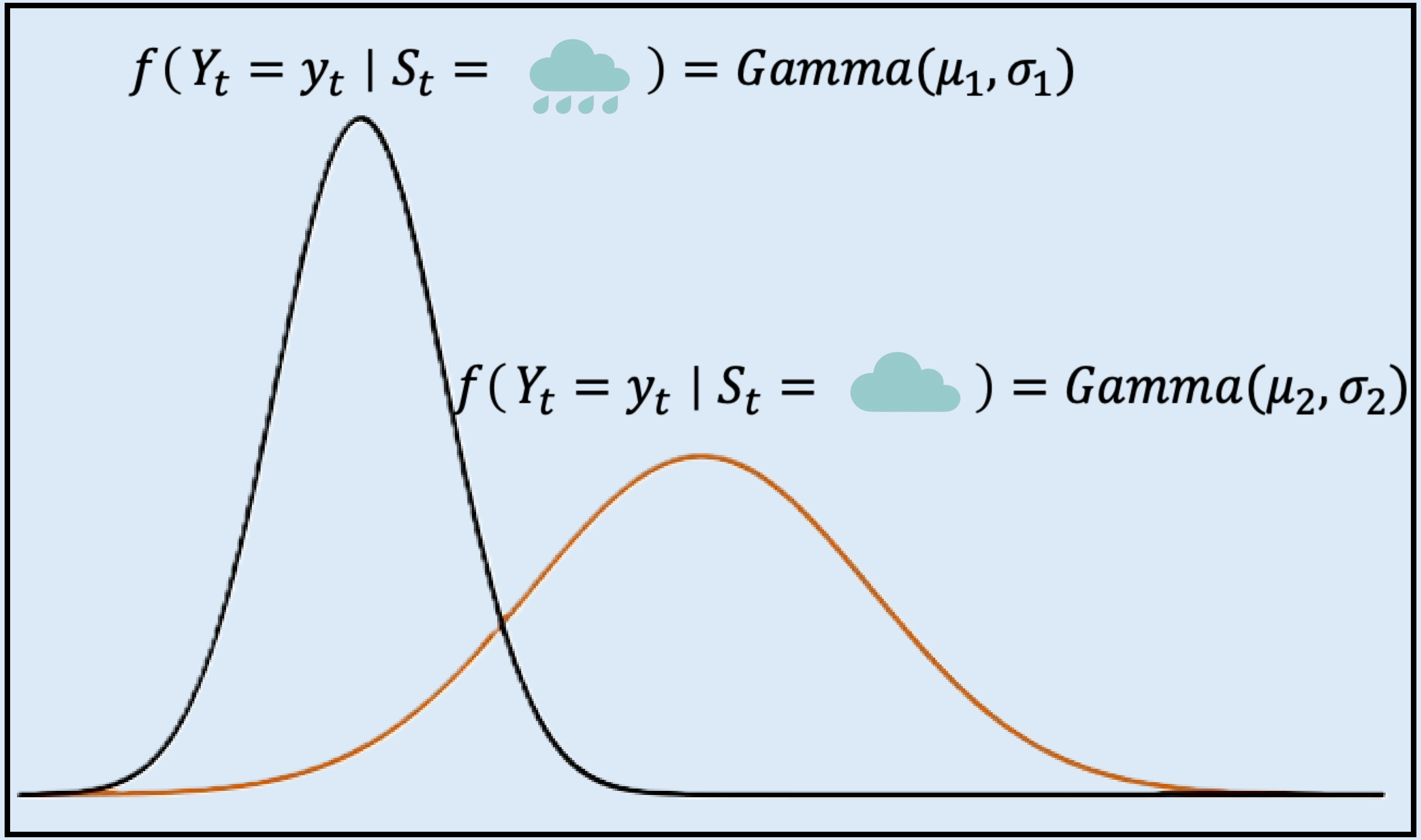
we establish a probabilistic relationship between both processes

State-dependent distributions:



State-dependent distributions:

Specified with different parameter values



Brightness

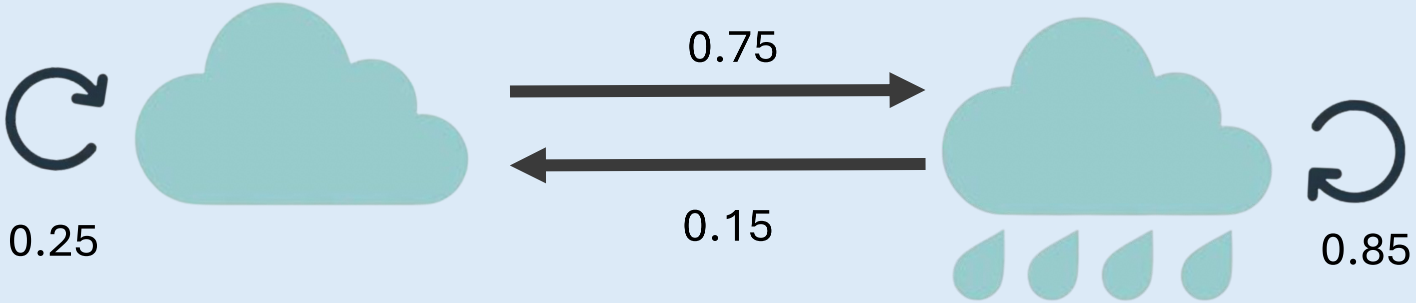
Hidden Markov model:

Can be fully characterized by:

- The number of states 

- The initial distribution of the Markov process

- The transition probabilities



- The state-dependent distributions

$f(Y_t = y_t \mid S_t =$  $)$ / $f(Y_t = y_t \mid S_t =$  $)$

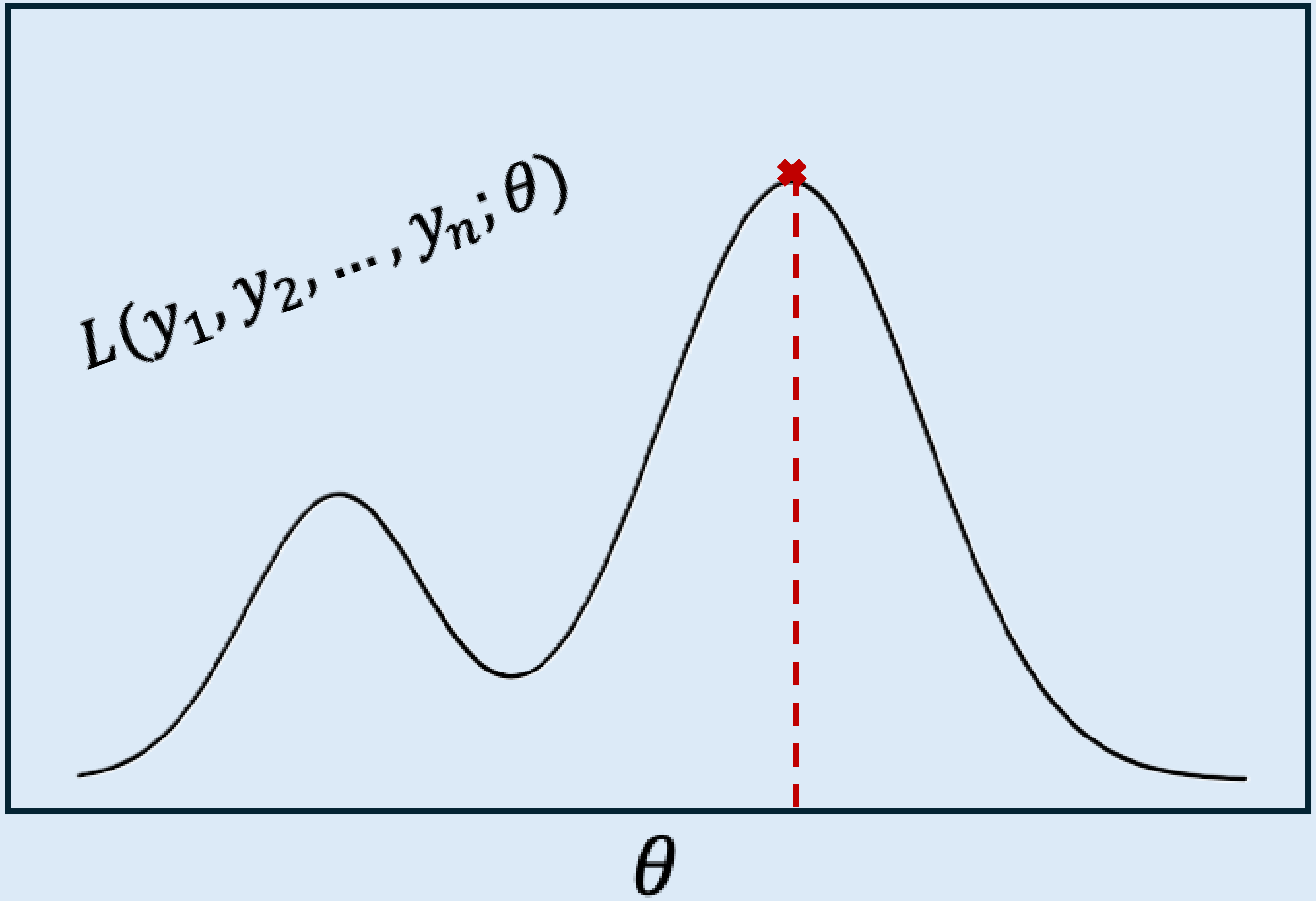
Maximum Likelihood:

Specifying the model

- We look for the parameter values (theta) that best explain our data

- The "most likely to have produced the data observed"

- The likelihood is maximized using numerical algorithms (multiple starting values)



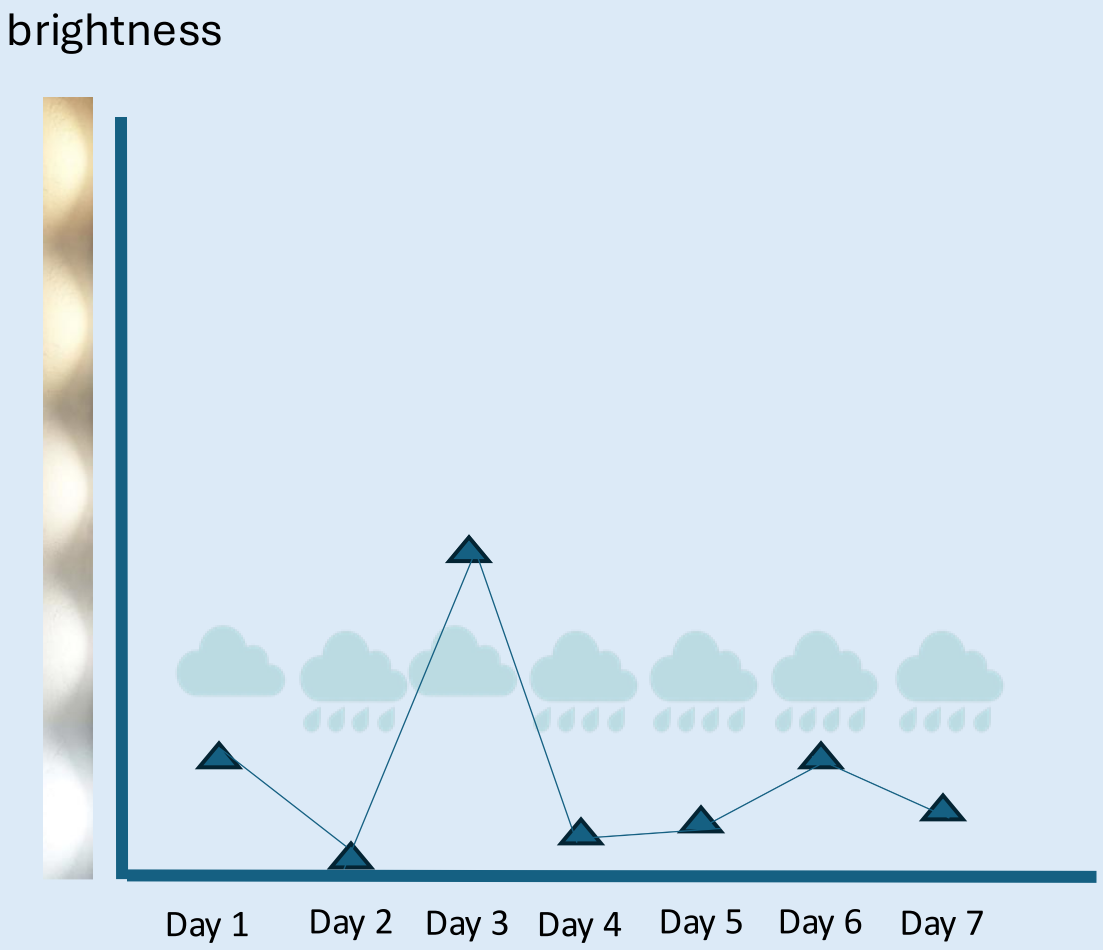
State decoding:

Making sense of the HMM

- Often the goal is to identify (classify) the state process

- Probabilistic algorithms identify the sate sequence best explaining the data observed

- Expert "interpretation" of the states



Covariates:

Incorporating additional information

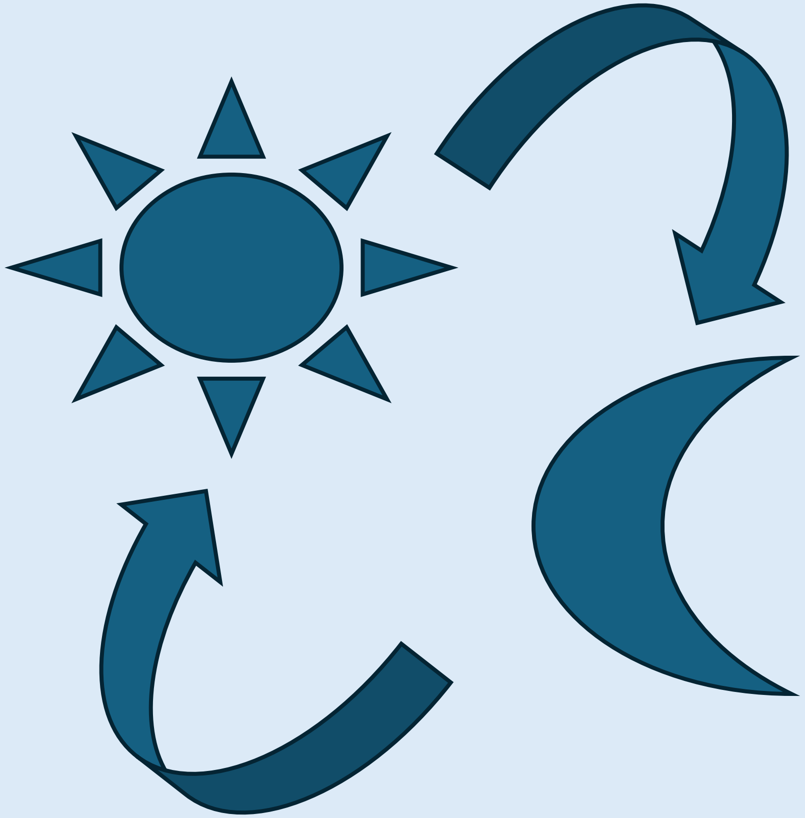
- e.g., time of the day (x)

- In state-dependent distributions:

$$\mu_1 = h(x)$$

- In transition probabilities:

$$P(S_t = \quad \mid S_{t-1} = \quad) = g(x)$$



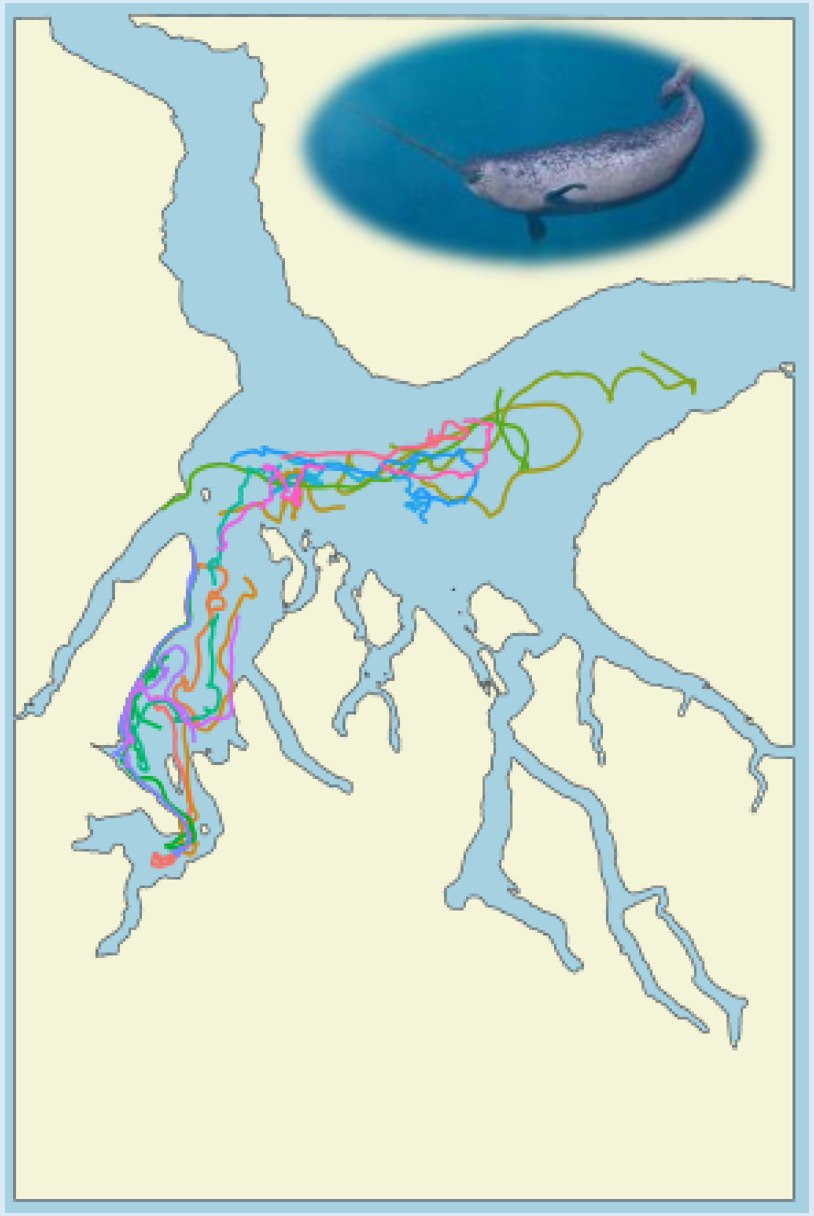
Hidden Markov models R tutorial:

Narwhal data

- Dr. Marianne Marcoux (Fisheries and Oceans, Canada)

- GPS coordinates over time (observed)

- Used to infer behaviours (hidden), e.g., travelling or foraging



Hidden Markov models R tutorial:

Narwhal data

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- GPS coordinates over time (observed)
- Used to infer behaviours (hidden), e.g., travelling or foraging

Hidden Markov models R tutorial:

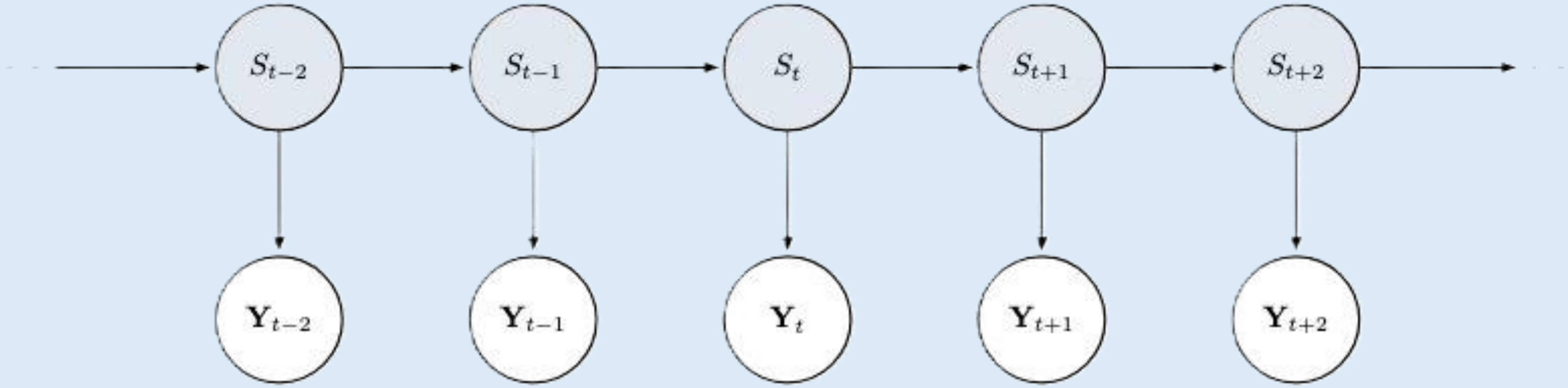
Blacktip reef shark

- Accelerometer data, activity measurements (observed)

- Used to infer behaviours (hidden), e.g., travelling

- Incorporate covariates for transitions between states

Thank you!



S_{t-2}
S_{t-1}
S_t
S_{t+1}
S_{t+2}
$\mathbf{Y}_{t-2}$
$\mathbf{Y}_{t-1}$
$\mathbf{Y}_t$
$\mathbf{Y}_{t+1}$
$\mathbf{Y}_{t+2}$