

# Cannings

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This document is not the documentation of the module `cannings`. This is only some explanations and details on the Cannings model that the module simulates.

## 1 Reproduction

We consider an haploid population of size  $N$  without recombination. A reproduction is made using the following steps:

1. Each individual produces offspring
2. If there are less offspring than the population size  $N$  then more offspring are generated.
3. Only  $N$  offspring are chosen to survive and compose the next generation.

### 1.1 Step 1: generation of the offspring with a Cannings model

Let  $p_0$  and  $\alpha$  be two reals such that  $0 \leq p_0 \leq 1$  and  $0 < \alpha$ . Let  $(X_i)_{1 \leq i \leq N}$  be identical independent  $\mathbb{N}$ -valued random variable such that:

$$\begin{aligned}\mathbb{P}(X_1 = 0) &= p_0 \\ \mathbb{P}(X_1 \geq k) &= (1 - p_0) \frac{1}{k^\alpha} \text{ for all } k \geq 1\end{aligned}$$

The number of offspring of the  $i^{\text{th}}$  individual is  $X_i$ .

### 1.2 Step 2: artificially adding offspring to reach the population size

If there are more offspring than the size of the population nothing happens in this steps. Else let assume that  $m \leq N$  oofspring have been generated with step 1. Then  $N - m$  offspring are generated using a Wright-Fisher model. It means that for these  $N - m$  offspring, the probability that the parent is the  $i^{\text{th}}$  individual is  $\frac{1}{N}$ .

### 1.3 Step 3: Drawing the surviving offspring

The  $N$  surviving offspring that will compose the next generation will be drawn without replacement from all the offspring previously generated.

Note the if the step 2 has generated offspring, then this step only consist of selecting all of them.

## 2 Adding natural selection

We consider that there are two alleles: the type  $A$  and the type  $a$ . The individuals always transfer their type to their offspring. The type  $A$  is associated with some selection coefficient.

### 2.1 Fecundity selection

Let assume that the individuals of type  $A$  have a fecundity selection advantage of coefficient  $s_f$ . Then the step 1 is replace by the following:

The number of offspring of the  $i^{\text{th}}$  individual is  $(1 + s_f)X_i$  if it has the type  $A$  and  $X_i$  otherwise where the  $X_i$  are defined as previously.

### 2.2 Viability selection

Let assume that the individuals of type  $A$  have a viability selection advantage of coefficient  $s_v$ . Then the step 3 is replace by the following:

The  $N$  surviving offspring that will compose the next generation will be drawn without replacement from all the offspring previously generated according to the non central Wallenius distribution<sup>1</sup> with a weigh  $1 + s_v$  for the offspring of type  $A$  and 1 for the offspring of type  $a$ .

## 3 Interest of the module

According the Schweinsberg, when the expectation of  $X_1$  is greater than 1 then this process converge as the population grows to infinity to a  $\beta$ -coalescent of parameter  $\alpha$ . [1]

## References

- [1] Jason Schweinsberg. Coalescent processes obtained from supercritical galton–watson processes. *Stochastic processes and their Applications*, 106(1):107–139, 2003.

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<sup>1</sup>See the [Wikipedia web page](#) for more details