Cannings

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This document is not the documentation of the module `cannings`. This is only some explanations and details on the Cannings model that the module simulates.

1 Reproduction in the class `Cannings`

We consider an haploid population of size N without recombination. A reproduction is made using the following steps:

1. Each individual produces offspring
2. If there are less offspring than the population size N then more offspring are generated.
3. Only N offspring are chosen to survive and compose the next generation.

1.1 Step 1: generation of the offspring with a Cannings model

Let $(X_i)_{1\leq i\leq N}$ be identical independent $\mathbb{N}$-valued random variable. The number of offspring of the i^{th} individual is X_i.
The distribution of X_1 can be chosen by the user when constructing a object from the class `Cannings`.

1.2 Step 2: artificially adding offspring to reach the population size

If there are more offspring than the size of the population nothing happens in this step. Else let assume that $m \leq N$ offspring have been generated with step 1. Then $N - m$ offspring are generated using a Wright-Fisher model. It means that for these $N - m$ offspring, the probability that the parent is the i^{th} individual is $\frac{1}{N}$.

1.3 Step 3: Drawing the surviving offspring

The N surviving offspring that will compose the next generation will be drawn without replacement from all the offspring previously generated.
Note that if the step 2 has generated offspring, then this step only consist of selecting all of them.

2 Adding natural selection

We consider that there are two alleles: the type 1 and the type 0. The individuals always transfer their type to their offspring. The type 1 is associated with some selection coefficient.

2.1 Fecundity selection

Let assume that the individuals of type 1 have a fecundity selection advantage of coefficient s_f. Then the step 1 is replace by the following:

The number of offspring of the i^{th} individual is $(1+s_f)X_i$ if it has the type 1 and X_i otherwise where the X_i are defined as previously.

2.2 Viability selection

Let assume that the individuals of type 1 have a viability selection advantage of coefficient s_v. Then the step 3 is replace by the following:

The N surviving offspring that will compose the next generation will be drawn without replacement from all the offspring previously generated according to the non central Wallenius distribution [1] with a weigh $1+s_v$ for the offspring of type 1 and 1 for the offspring of type 0.

3 Child class of `Cannings`

3.1 Subclass `Schweinsberg`

Let p_0 and α be two reals such that $0 \leq p_0 \leq 1$ and $0 < \alpha$.
The class `Schweinsberg` is a subclass of `Cannings` for which the law of X_1 is the following:

$$\mathbb{P}(X_1 = 0) = p_0$$
$$\mathbb{P}(X_1 \geq k) = (1-p_0)\frac{1}{k^\alpha} \text{ for all } k \geq 1$$

According the Schweinsberg, if the expectation of X_1 is greater than 1 then this process converge as the population grows to infinity to a β-coalescent of parameter α if $1 < \alpha < 2$ and to a Kingman coalescent if $\alpha = 2$. [1]

3.2 Subclass `Poisson`

Let $\lambda > 0$ be a real.
The class `Poisson` is a subclass of `Cannings` for which the law of X_1 is a Poisson distribution of parameter λ.

When λ is greater than one and close to one this model is an approximation of the Wright-Fisher model.

References

[1] Jason Schweinsberg. Coalescent processes obtained from supercritical galton–watson processes. *Stochastic processes and their Applications*, 106(1):107–139, 2003.

[1] See the Wikipedia web page for more details