Exercises in Bayesian reasoning: Proceedings of the Church of Human Potential

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- What is the probability that a subject is a mutant, when your field test says that it is mutant?
- What is the probability that a subject is a mutant, when your field test says that it is baseline?

Call the event that a specific subject is a mutant \mathcal{M} , and that it is baseline $\neg \mathcal{M}$.

Call the event that Stryker's field test diagnoses a subject as a mutant D, and that it diagnoses it baseline $\neg D$.

Senator Kelly's interest is in the probability the subject is indeed a mutant given it has been diagnosed as a mutant, or $P(\mathcal{M}|D)$, and the probability the subject is a mutant given it has been diagnosed as baseline, or $P(\mathcal{M}|\neg D)$.

The posterior probability the subject is a mutant given a mutant diagnosis is $\approx .047$.

The posterior probability that a subject is a mutant, given it is diagnosed as baseline is $\approx .000010$.

An advantage of using Bayes' Rule in this way is that it gracefully extends to more complex scenarios.

Suppose that Stryker knows that his diagnosis is statistically independent from the diagnosis of his research associate Didier Raoult, and suppose that both Stryker and Raoult return the mutant diagnosis. Now what is the posterior probability that the subject is a mutant?

It is $\approx .71$.