Multinomial Processing Tree with JAGS

Joachim Vandekerckhove, Michael D. Lee

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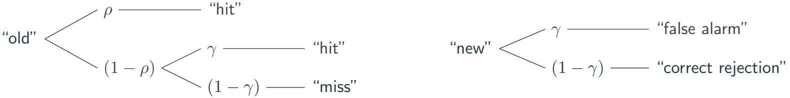

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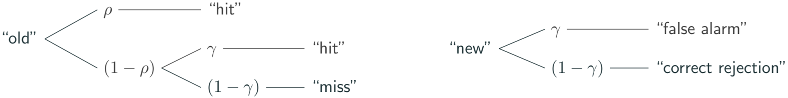

The parameters ρ and γ together determine the hit rate θ^{h} and false alarm rate θ^{f}

$$\begin{aligned} \theta^{\mathrm{h}} &= \rho + (1-\rho)\,\gamma \\ \theta^{\mathrm{f}} &= \gamma \end{aligned}$$

See Matzke et al. (2018) for more like this!

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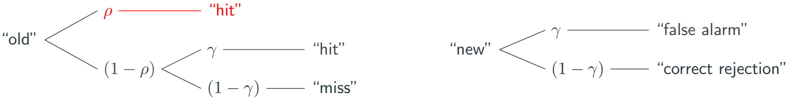

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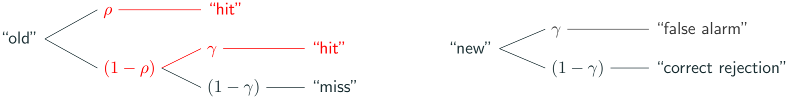

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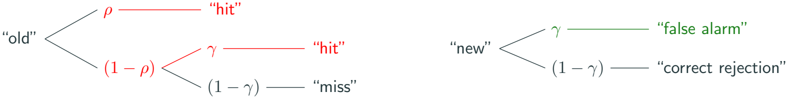

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One-High Threshold Model

The remembering parameter ρ and the old-guessing parameter γ can both take any value with equal likelihood:

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Those rates tell us how often old items lead to hits (the data are k^{h} hits out of n_o old items) and how often new items lead to false alarms (k^{f} false alarms out of n_n new items):

$$\begin{aligned} k^{\mathrm{h}} &\sim \text{binomial}\left(\theta^{\mathrm{h}}, n_o\right) \\ k^{\mathrm{f}} &\sim \text{binomial}\left(\theta^{\mathrm{f}}, n_n\right) \end{aligned}$$

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Amyloid Status	Hits	False Alarms
negative	13	0
positive	8	4
negative	12	1
negative	14	0
positive	9	4
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There were 33 participants. Each saw 15 old and 15 new stimuli.

AS	Hits	FA
+	8	4
+	9	4
+	14	0
+	14	1
+	13	2
+	8	0
+	13	3
+	12	1
+	11	3
+	4	0
+	8	0
+	13	1
+	15	0
+	12	0
+	11	0
+	9	0
+	5	1

AS	Hits	FA
+	5	0
+	6	3
+	15	0
+	11	0
+	14	1
+	12	2
+	12	1
+	11	2
+	1	0
+	14	0
+	13	0
+	7	2
+	11	1
+	12	2
+	8	0
+	11	2

Amyloid Positive Inferences

Patients remember around 60-70% of the items, and guess “old” 5-10% of the time when they do not remember

	Posterior			95% Cred. Int.	
Parameter	Mean	Median	SD	Lower	Upper
gamma	0.075	0.074	0.012	0.053	0.100
rho	0.665	0.665	0.023	0.619	0.709

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"Convergence of the MCMC procedure was good, with all $\hat{R} < 1.01$."

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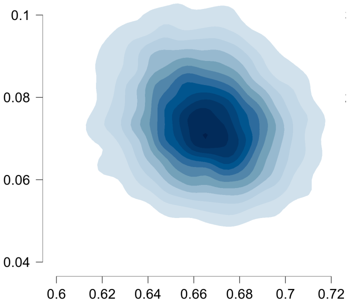

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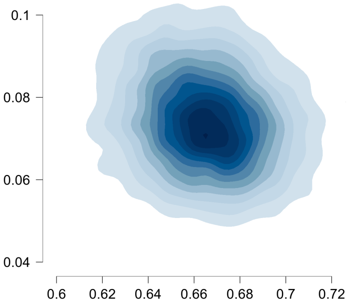

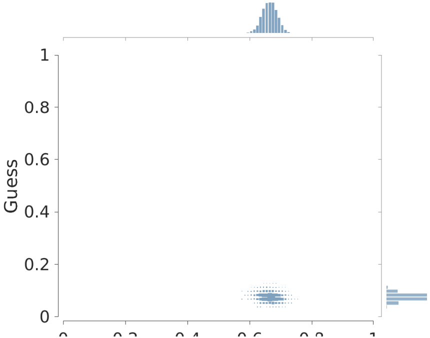

References

Matzke, D., Boehm, U., & Vandekerckhove, J. (2018). Bayesian inference in psychology, part iii: Bayesian parameter estimation in nonstandard models. *Psychonomic Bulletin & Review*, *25*, 77–101. doi: 10.3758/s13423-017-1394-5