# Multinomial Processing Tree with JAGS

Joachim Vandekerckhove, Michael D. Lee

# Recognition Memory Task

In an old/new recognition memory task, participants are asked whether stimuli are "old" or "new"

# Recognition Memory Task

In an old/new recognition memory task, participants are asked whether stimuli are “old” or “new”

| | Stimulus was old | Stimulus was new |
|---|---|---|
| Pp responds “old” | hit | false alarm |
| Pp responds “new” | miss | correct rejection |

# Recognition Memory Task

In an old/new recognition memory task, participants are asked whether stimuli are "old" or "new"

| | Stimulus was old | Stimulus was new |
|---|---|---|
| Pp responds "old" | hit | false alarm |
| Pp responds "new" | miss | correct rejection |

Probability of a hit = hit rate = $\theta^{\mathrm{h}}$

# Recognition Memory Task

In an old/new recognition memory task, participants are asked whether stimuli are “old” or “new”

| | Stimulus was old | Stimulus was new |
|---|---|---|
| Pp responds “old” | hit | false alarm |
| Pp responds “new” | miss | correct rejection |

Probability of a hit = hit rate = $\theta^{\mathrm{h}}$
Probability of a miss = miss rate = $1 - \theta^{\mathrm{h}}$

# Recognition Memory Task

In an old/new recognition memory task, participants are asked whether stimuli are "old" or "new"

| | Stimulus was old | Stimulus was new |
|---|---|---|
| Pp responds "old" | hit | false alarm |
| Pp responds "new" | miss | correct rejection |

Probability of a hit = hit rate = $\theta^{\mathrm{h}}$
Probability of a miss = miss rate = $1 - \theta^{\mathrm{h}}$

Probability of a false alarm = false alarm rate = $\theta^{\mathrm{f}}$

# Recognition Memory Task

In an old/new recognition memory task, participants are asked whether stimuli are "old" or "new"

| | Stimulus was old | Stimulus was new |
|---|---|---|
| Pp responds "old" | hit | false alarm |
| Pp responds "new" | miss | correct rejection |

Probability of a hit = hit rate = $\theta^{\mathrm{h}}$
Probability of a miss = miss rate = $1 - \theta^{\mathrm{h}}$

Probability of a false alarm = false alarm rate = $\theta^{\mathrm{f}}$
Probability of a correct rejection = correct rejection rate = $1 - \theta^{\mathrm{f}}$

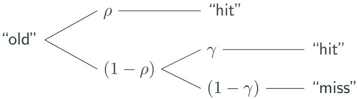
# Multinomial Processing Trees

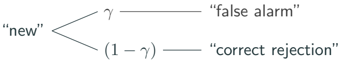
Recall that the one-high-threshold model has parameters $\rho$ (probability of remembering) and $\gamma$ (probability of guessing "old")

# Multinomial Processing Trees

Recall that the one-high-threshold model has parameters $\rho$ (probability of remembering) and $\gamma$ (probability of guessing "old")

The tree representation shows the "flow" of the process

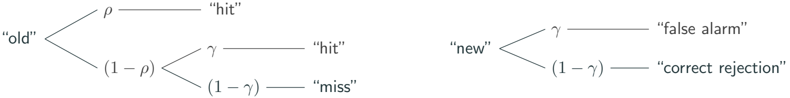


# Multinomial Processing Trees

Recall that the one-high-threshold model has parameters $\rho$ (probability of remembering) and $\gamma$ (probability of guessing "old")

The tree representation shows the "flow" of the process




The parameters $\rho$ and $\gamma$ together determine the hit rate $\theta^{\mathrm{h}}$ and false alarm rate $\theta^{\mathrm{f}}$

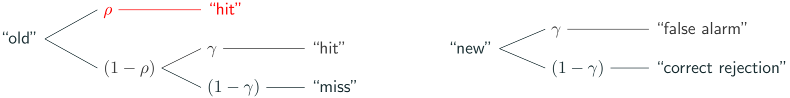
$$
\begin{aligned}
\theta^{\mathrm{h}} &= \rho + (1-\rho)\,\gamma \\
\theta^{\mathrm{f}} &= \gamma
\end{aligned}
$$

See Matzke et al. (2018) for more like this!

# Multinomial Processing Trees

Recall that the one-high-threshold model has parameters $\rho$ (probability of remembering) and $\gamma$ (probability of guessing "old")

The tree representation shows the "flow" of the process




The parameters $\rho$ and $\gamma$ together determine the hit rate $\theta^{\mathrm{h}}$ and false alarm rate $\theta^{\mathrm{f}}$

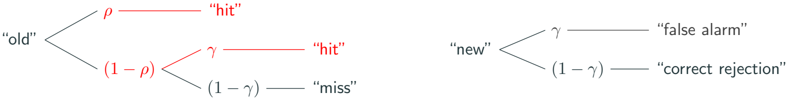
$$\begin{aligned} \theta^{\mathrm{h}} &= \rho + (1-\rho)\,\gamma \\ \theta^{\mathrm{f}} &= \gamma \end{aligned}$$

See Matzke et al. (2018) for more like this!

# Multinomial Processing Trees

Recall that the one-high-threshold model has parameters $\rho$ (probability of remembering) and $\gamma$ (probability of guessing "old")

The tree representation shows the "flow" of the process




The parameters $\rho$ and $\gamma$ together determine the hit rate $\theta^{\mathrm{h}}$ and false alarm rate $\theta^{\mathrm{f}}$

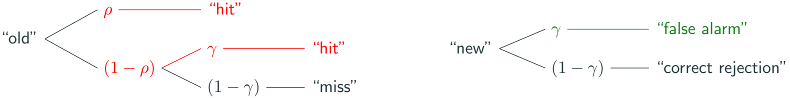
$$
\begin{aligned}
\theta^{\mathrm{h}} &= \rho + (1-\rho)\,\gamma \\
\theta^{\mathrm{f}} &= \gamma
\end{aligned}
$$

See Matzke et al. (2018) for more like this!

# Multinomial Processing Trees

Recall that the one-high-threshold model has parameters $\rho$ (probability of remembering) and $\gamma$ (probability of guessing "old")

The tree representation shows the "flow" of the process




The parameters $\rho$ and $\gamma$ together determine the hit rate $\theta^{\mathrm{h}}$ and false alarm rate $\theta^{\mathrm{f}}$

$$\begin{aligned} \theta^{\mathrm{h}} &= \rho + (1-\rho)\,\gamma \\ \theta^{\mathrm{f}} &= \gamma \end{aligned}$$

See Matzke et al. (2018) for more like this!

# One-High Threshold Model

The remembering parameter $\rho$ and the old-guessing parameter $\gamma$ can both take any value with equal likelihood:

$$\begin{aligned} \rho &\sim \text{uniform}\,(0,1) \\ \gamma &\sim \text{uniform}\,(0,1) \end{aligned}$$

# One-High Threshold Model

The remembering parameter $\rho$ and the old-guessing parameter $\gamma$ can both take any value with equal likelihood:

$$\begin{aligned} \rho &\sim \text{uniform}\,(0,1) \\ \gamma &\sim \text{uniform}\,(0,1) \end{aligned}$$

Those parameters can be transformed into hit rates and false alarm rates:

$$\begin{aligned} \theta^{\mathrm{h}} &= \rho + (1-\rho)\,\gamma \\ \theta^{\mathrm{f}} &= \gamma \end{aligned}$$

# One-High Threshold Model

The remembering parameter $\rho$ and the old-guessing parameter $\gamma$ can both take any value with equal likelihood:

$$\begin{aligned} \rho &\sim \text{uniform}\,(0,1) \\ \gamma &\sim \text{uniform}\,(0,1) \end{aligned}$$

Those parameters can be transformed into hit rates and false alarm rates:

$$\begin{aligned} \theta^{\text{h}} &= \rho + (1-\rho)\,\gamma \\ \theta^{\text{f}} &= \gamma \end{aligned}$$

Those rates tell us how often old items lead to hits (the data are $k^{\text{h}}$ hits out of $n_o$ old items) and how often new items lead to false alarms ($k^{\text{f}}$ false alarms out of $n_n$ new items):

$$\begin{aligned} k^{\text{h}} &\sim \text{binomial}\left(\theta^{\text{h}}, n_o\right) \\ k^{\text{f}} &\sim \text{binomial}\left(\theta^{\text{f}}, n_n\right) \end{aligned}$$

# One-High Threshold Model

$$
\begin{aligned}
\rho &\sim \text{uniform}\,(0,1)\\
\gamma &\sim \text{uniform}\,(0,1)\\
\theta^{\text{h}} &= \rho + (1-\rho)\,\gamma\\
\theta^{\text{f}} &= \gamma\\
k^{\text{h}} &\sim \text{binomial}\left(\theta^{\text{h}}, n_o\right)\\
k^{\text{f}} &\sim \text{binomial}\left(\theta^{\text{f}}, n_n\right)
\end{aligned}
$$

# One-High Threshold Model

$$
\begin{aligned}
\rho &\sim \text{uniform}\,(0,1) \\
\gamma &\sim \text{uniform}\,(0,1) \\
\theta^{\text{h}} &= \rho + (1-\rho)\,\gamma \\
\theta^{\text{f}} &= \gamma \\
k^{\text{h}} &\sim \text{binomial}\left(\theta^{\text{h}}, n_o\right) \\
k^{\text{f}} &\sim \text{binomial}\left(\theta^{\text{f}}, n_n\right)
\end{aligned}
$$

| Amyloid Status | Hits | False Alarms |
|---|---|---|
| negative | 13 | 0 |
| positive | 8 | 4 |
| negative | 12 | 1 |
| negative | 14 | 0 |
| positive | 9 | 4 |
| ... | ... | ... |

# One-High Threshold Model

$$\begin{aligned}
\rho &\sim \text{uniform}(0,1) \\
\gamma &\sim \text{uniform}(0,1) \\
\theta^{\text{h}} &= \rho + (1-\rho)\,\gamma \\
\theta^{\text{f}} &= \gamma \\
k^{\text{h}} &\sim \text{binomial}\left(\theta^{\text{h}}, n_o\right) \\
k^{\text{f}} &\sim \text{binomial}\left(\theta^{\text{f}}, n_n\right)
\end{aligned}$$

| Amyloid Status | Hits | False Alarms |
|---|---|---|
| negative | 13 | 0 |
| positive | 8 | 4 |
| negative | 12 | 1 |
| negative | 14 | 0 |
| positive | 9 | 4 |
| ... | ... | ... |

## One-High Threshold Model

$$
\begin{aligned}
\rho &\sim \text{uniform}(0,1) \\
\gamma &\sim \text{uniform}(0,1) \\
\theta^{\mathrm{h}} &= \rho + (1-\rho)\,\gamma \\
\theta^{\mathrm{f}} &= \gamma \\
k^{\mathrm{h}} &\sim \text{binomial}\left(\theta^{\mathrm{h}}, n_o\right) \\
k^{\mathrm{f}} &\sim \text{binomial}\left(\theta^{\mathrm{f}}, n_n\right)
\end{aligned}
$$

| Amyloid Status | Hits | False Alarms |
|---|---|---|
| positive | 8 | 4 |
| positive | 9 | 4 |
| positive | 14 | 0 |
| positive | 14 | 1 |
| positive | 13 | 2 |
| ... | ... | ... |

# One-High Threshold Model

$$
\begin{aligned}
\rho &\sim \text{uniform}\,(0,1)\\
\gamma &\sim \text{uniform}\,(0,1)\\
\theta^{\mathrm{h}} &= \rho + (1-\rho)\,\gamma\\
\theta^{\mathrm{f}} &= \gamma\\
k^{\mathrm{h}} &\sim \text{binomial}\left(\theta^{\mathrm{h}}, n_o\right)\\
k^{\mathrm{f}} &\sim \text{binomial}\left(\theta^{\mathrm{f}}, n_n\right)
\end{aligned}
$$

There were 33 participants. Each saw 15 old and 15 new stimuli.

| AS | Hits | FA |
|---|---|---|
| + | 8 | 4 |
| + | 9 | 4 |
| + | 14 | 0 |
| + | 14 | 1 |
| + | 13 | 2 |
| + | 8 | 0 |
| + | 13 | 3 |
| + | 12 | 1 |
| + | 11 | 3 |
| + | 4 | 0 |
| + | 8 | 0 |
| + | 13 | 1 |
| + | 15 | 0 |
| + | 12 | 0 |
| + | 11 | 0 |
| + | 9 | 0 |
| + | 5 | 1 |

| AS | Hits | FA |
|---|---|---|
| + | 5 | 0 |
| + | 6 | 3 |
| + | 15 | 0 |
| + | 11 | 0 |
| + | 14 | 1 |
| + | 12 | 2 |
| + | 12 | 1 |
| + | 11 | 2 |
| + | 1 | 0 |
| + | 14 | 0 |
| + | 13 | 0 |
| + | 7 | 2 |
| + | 11 | 1 |
| + | 12 | 2 |
| + | 8 | 0 |
| + | 11 | 2 |

# Amyloid Positive Inferences

Patients remember around 60-70% of the items, and guess “old” 5-10% of the time when they do not remember

| | Posterior | | | 95% Cred. Int. | |
|---|---|---|---|---|---|
| Parameter | Mean | Median | SD | Lower | Upper |
| gamma | 0.075 | 0.074 | 0.012 | 0.053 | 0.100 |
| rho | 0.665 | 0.665 | 0.023 | 0.619 | 0.709 |

# Amyloid Positive Inferences

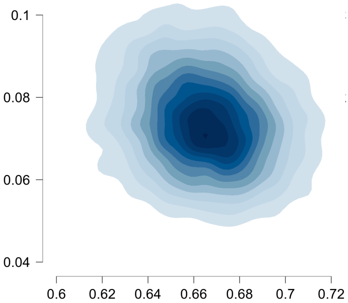
Patients remember around 60-70% of the items, and guess "old" 5-10% of the time when they do not remember

| | Posterior | | | 95% Cred. Int. | |
|---|---|---|---|---|---|
| Parameter | Mean | Median | SD | Lower | Upper |
| gamma | 0.075 | 0.074 | 0.012 | 0.053 | 0.100 |
| rho | 0.665 | 0.665 | 0.023 | 0.619 | 0.709 |

*"Convergence of the MCMC procedure was good, with all $\hat{R} < 1.01$."*

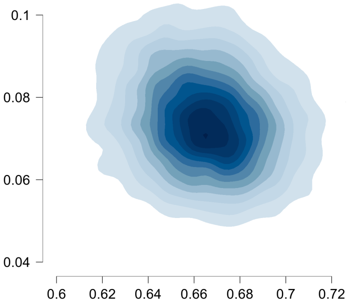
# Amyloid Positive Inferences

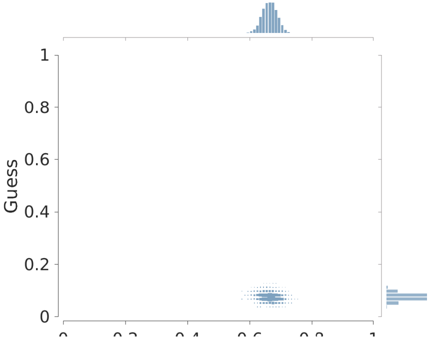
| | Posterior | | | 95% Cred. Int. | |
|---|---|---|---|---|---|
| Parameter | Mean | Median | SD | Lower | Upper |
| gamma | 0.075 | 0.074 | 0.012 | 0.053 | 0.100 |
| rho | 0.665 | 0.665 | 0.023 | 0.619 | 0.709 |



# Amyloid Positive Inferences

| | Posterior | | | 95% Cred. Int. | |
|---|---|---|---|---|---|
| Parameter | Mean | Median | SD | Lower | Upper |
| gamma | 0.075 | 0.074 | 0.012 | 0.053 | 0.100 |
| rho | 0.665 | 0.665 | 0.023 | 0.619 | 0.709 |



# References

Matzke, D., Boehm, U., & Vandekerckhove, J. (2018). Bayesian inference in psychology, part iii: Bayesian parameter estimation in nonstandard models. *Psychonomic Bulletin & Review*, *25*, 77–101. doi: 10.3758/s13423-017-1394-5