

# The Lady Tasting Wine

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Joachim Vandekerckhove

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- It is possible for you to disagree and still be sensible

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    - $p(P_R) = K_t(1 - P_R)$  for  $P_R > 0.5$
- $K_t$  and  $K_w$  are chosen such that the sum (or integral) over all possibilities is 1. This is always possible if the distribution is proper. The solution for wine here is easy enough (it is the sum of  $(1 - P_R)(P_R - 0.5)$  for all values of  $P_R$ ), but it isn't in general

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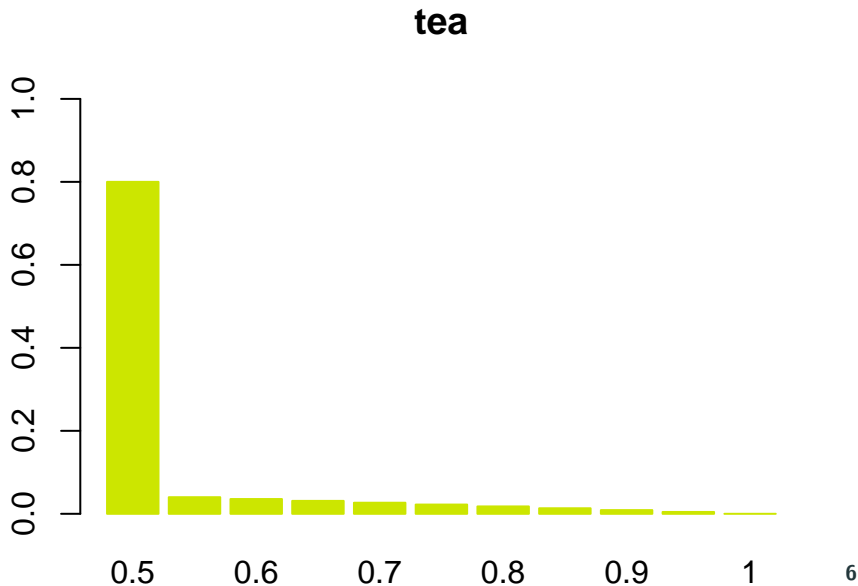
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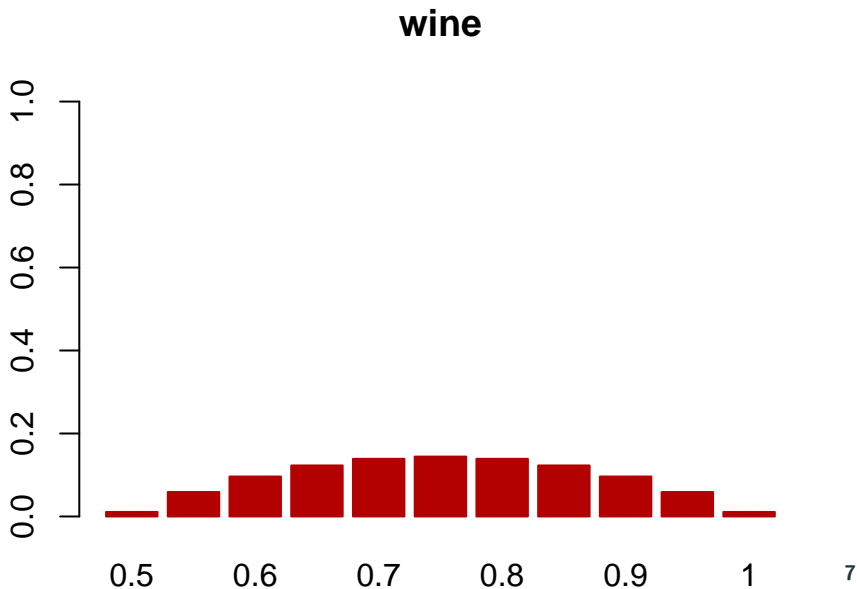
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- $K_*$  is **the inverse of** the sum of everything else over values of  $P_R$

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- I usually make the proportionality explicit to avoid confusion

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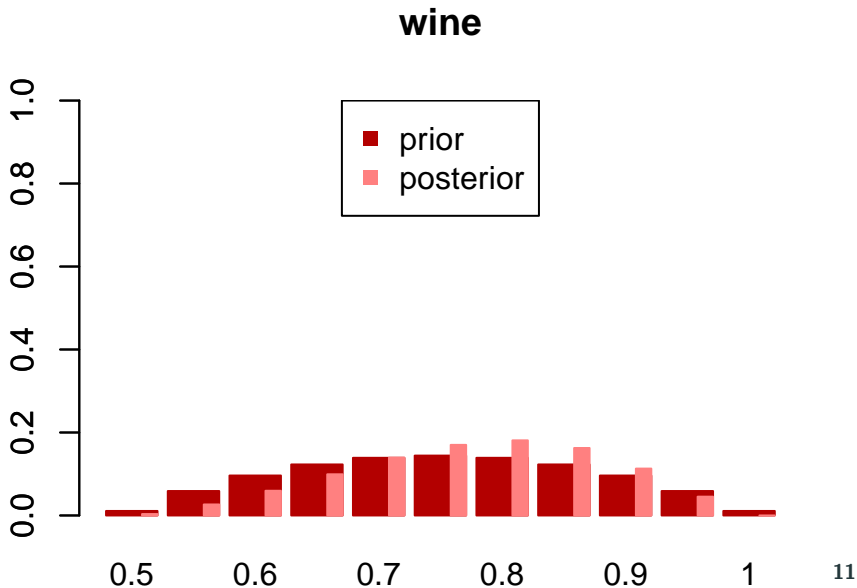
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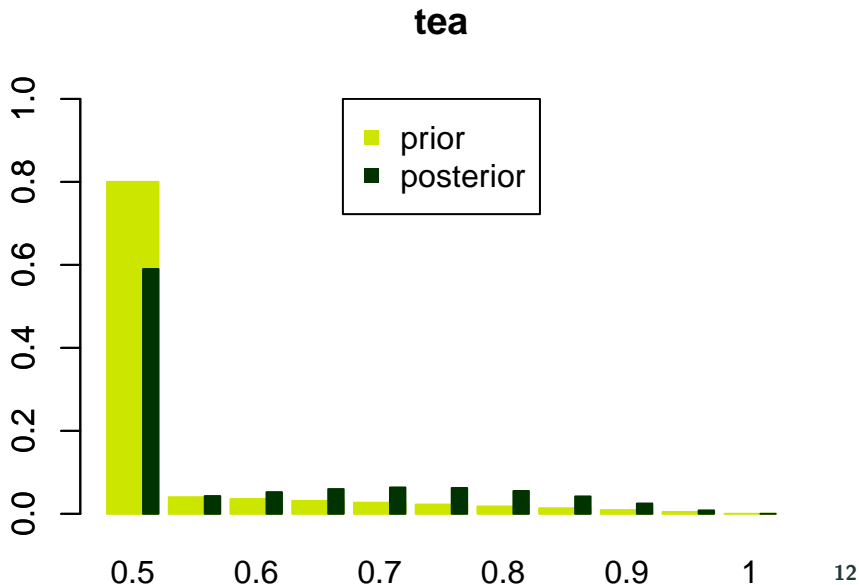
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- Also, make them pretty.



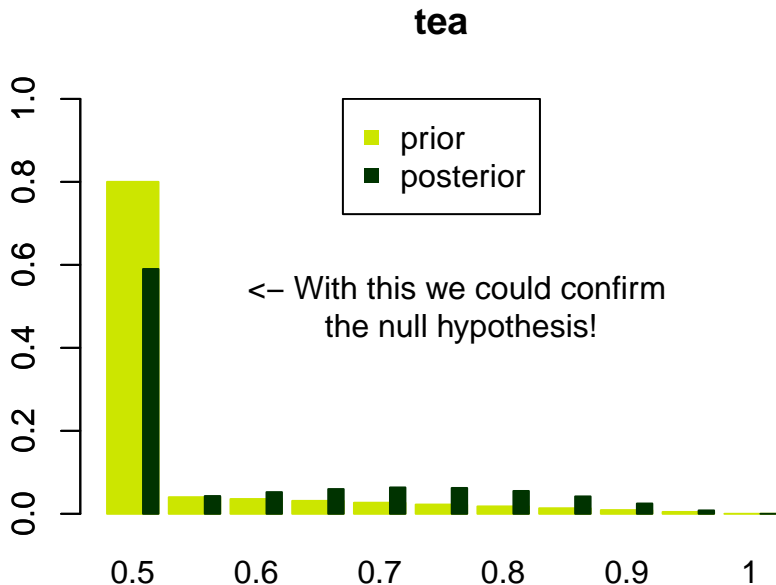
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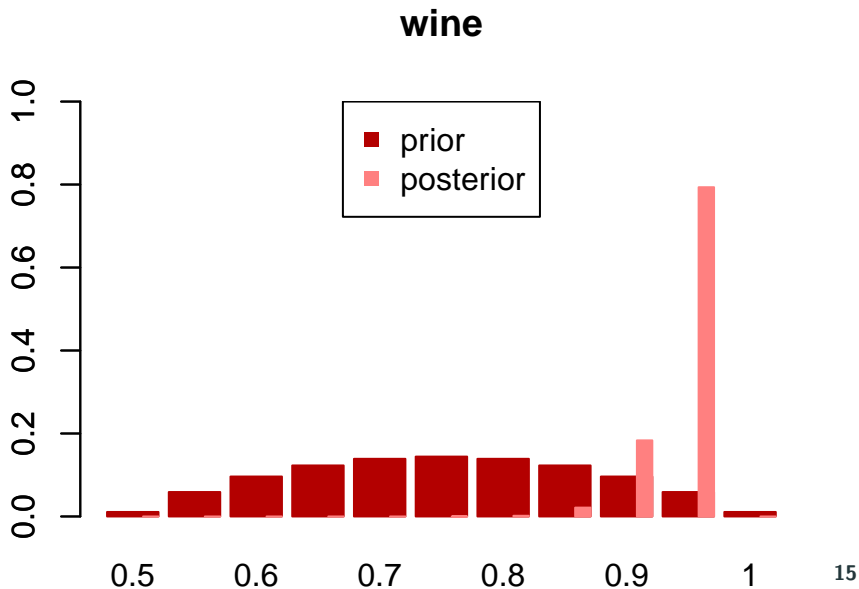
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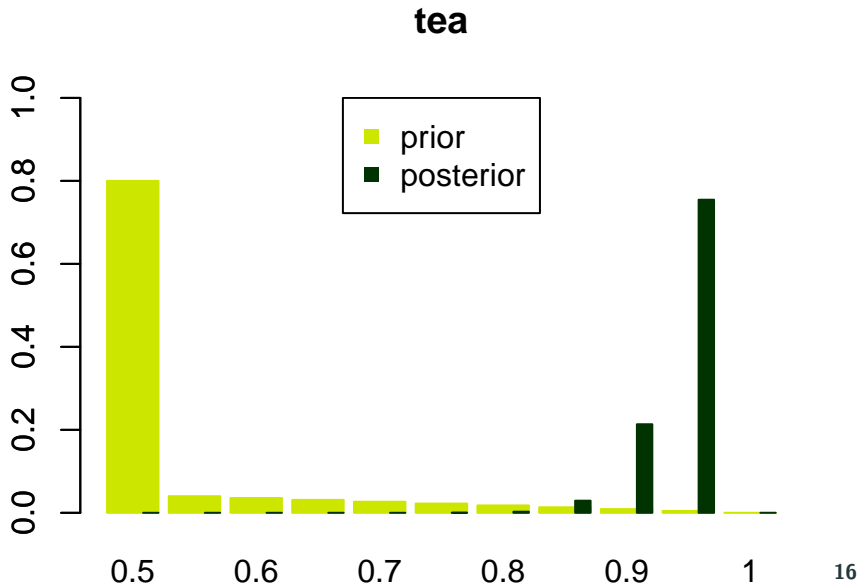
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    - After 34 corrects,  $p(P_R|data)$  for wine tasting accrues at  $P_R = 1$
    - ... but nothing Dr. Muriel does will convince us that  $P_R = 1$ , because a priori,  $p(P_R = 1) = 0$ . Cromwell's rule is the recommendation to give a prior nonzero mass at any point that is not a logical impossibility.

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  - Then compare the various possible explanations for what has happened, and compare posterior beliefs with priors

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    - The vast number of significance tests that are used today will encourage specious beliefs in the efficacy of drugs, treatments, or experimental manipulations
    - Whenever you read some effect having been detected, remember that it probably refers to significance, which too easily suggests an effect when none exists

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# Summary

- Bayesian analysis gives us everything we want
  - We (usually) either want to know
    - if  $H_0$  is true (as with tea), or
    - how big an effect is (as with wine)
  - The posterior tells us exactly what we need to know
    - In contrast to the  $p$ -value, which is a probability for something that did not happen under the assumption of a hypothesis that may not be true

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  - The Bayesian view recognizes that ones opinion of tasting the two liquids may be different or that the ladies may have different skills

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  - If evidence is produced to support some thesis, one must also consider the reasonableness of the evidence were the thesis false