Explanatory models

Joachim Vandekerckhove

Types of hierarchical models

The basic form of a hierarchy in a parametric model is:

$$
\mathcal{M}_s : \begin{cases}
\forall p \in (1, \ldots, P) : \mu_p \sim N(M, T) \\
\forall j \in (1, \ldots, J), p \in (1, \ldots, P) : x_{p,j} \sim N(\mu_p, \tau) \\
M \sim N(0, 0.1) \\
\tau \sim \Gamma(4, 0.01) \\
T \sim \Gamma(4, 0.01)
\end{cases}
$$

The hierarchical part is $\mu_p \sim N(M, T)$, but this is only one possible hierarchy.

Difference structure

Consider one alternative:

$$\mu_{p|g} \sim N(M_g, T)$$

Here the parameter M is determined by group identity g, and person p, who is a member of g, has some expected parameter value based on that.

If there are two groups, we may be interested in between-group differences $D_{21} = M_2 - M_1$.

The difference structure $D_{21} = M_2 - M_1$ is a very basic form of linear regression. The difference structure is equivalent to this:

$$\mu_p \sim N(M_1 + z_p D_{21}, T)$$

where $z_p = 0$ if p is in group 1 and $z_p = 1$ if p is in group 2.

This way, z acts as a linear predictor of the person-specific hierarchical mean.

Person-specific hierarchical mean

Hierarchical means can belong to an individual.

We then interpret it as the mean of the population of individuals who have predictors similar to this one.

But it is possible (and common) to have a hierarchical mean that is not shared with any other participant in our sample.

Consider that the predictor z could be not group membership, but some continuous predictor like age, personality, etc.

Person-specific hierarchical mean

In the case where z is a property of the person, and

$$\mu_p \sim N(\beta_0 + \beta_1 z_p, T)$$

... we have a standard linear regression structure with intercept β_0 and regression weight β_1.

Person side and item side predictors

Suppose that there are other sources of variability in the data – the difficulty of items i in a task might vary as well. Then,

$$\mu_{p,i} \sim N(\beta_0 + \beta_1 z_p + \beta_2 c_i, T)$$

Here z_p would be called a "person-side" predictor and c_i an "item-side" predictor.

You could also have an "interaction side."