The Lady Tasting Wine

Joachim Vandekerckhove

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- It is possible for you to disagree and still be sensible

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    - $p(P_R) = K_t(1 - P_R)$ for $P_R > 0.5$
- K_t and K_w are chosen such that the sum (or integral) over all possibilities is 1. This is always possible if the distribution is proper. The solution for wine here is easy enough (it is the sum of $(1 - P_R)(P_R - 0.5)$ for all values of P_R), but it isn't in general

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- K_* is the inverse of the sum of everything else over values of P_R

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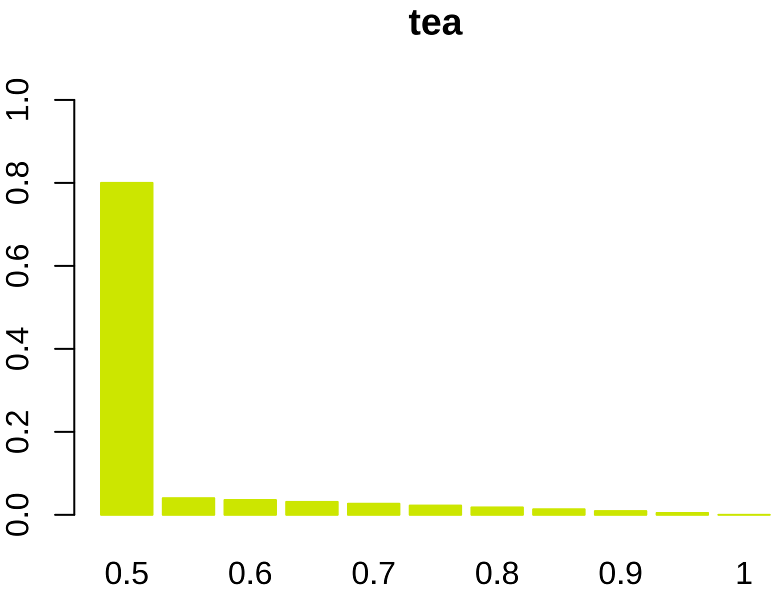

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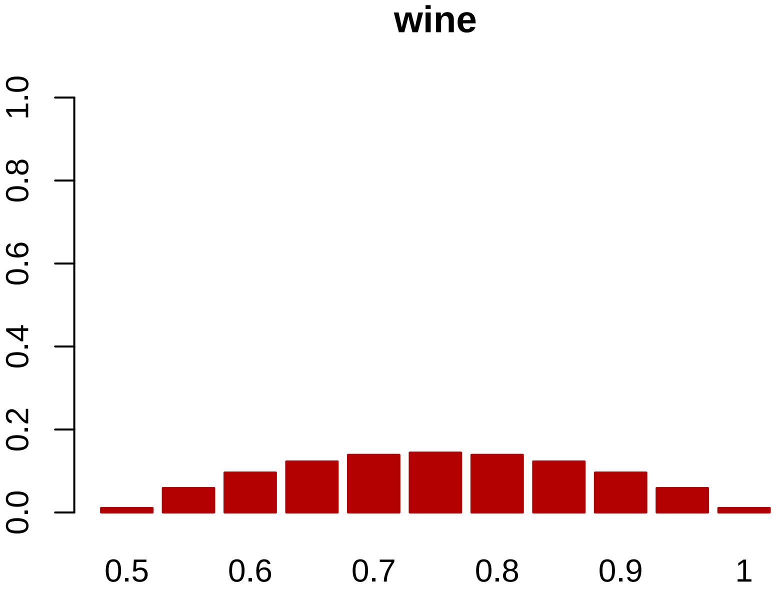

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- I usually make the proportionality explicit to avoid confusion

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- Also, make them pretty.

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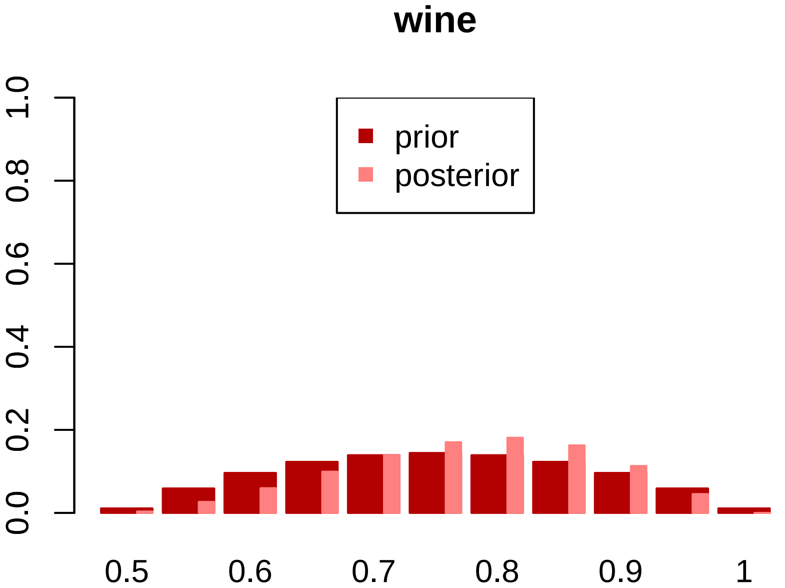

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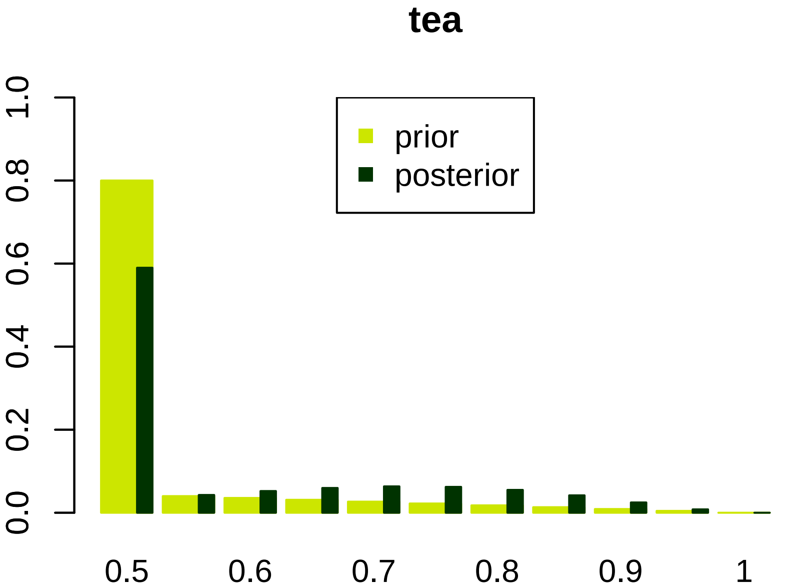

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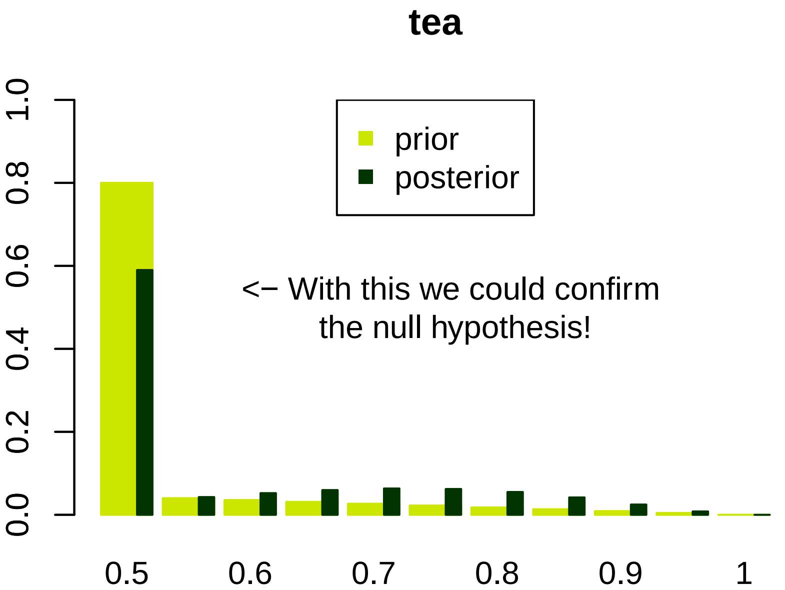

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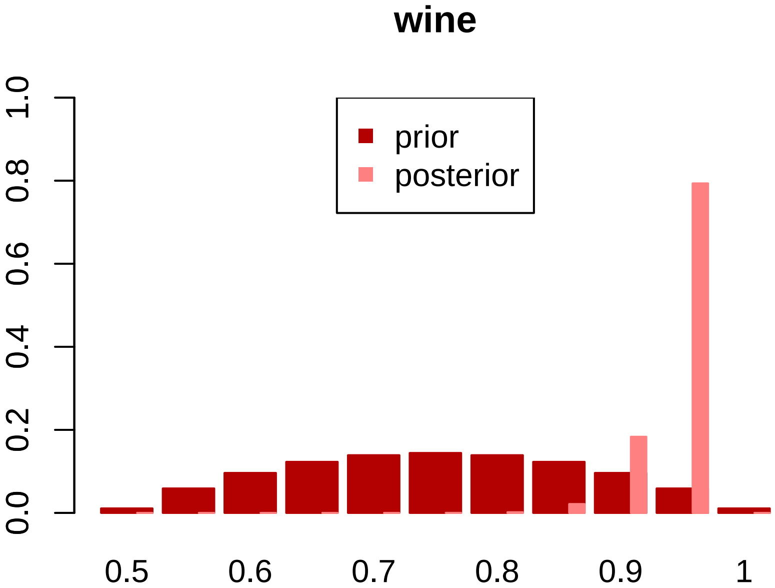

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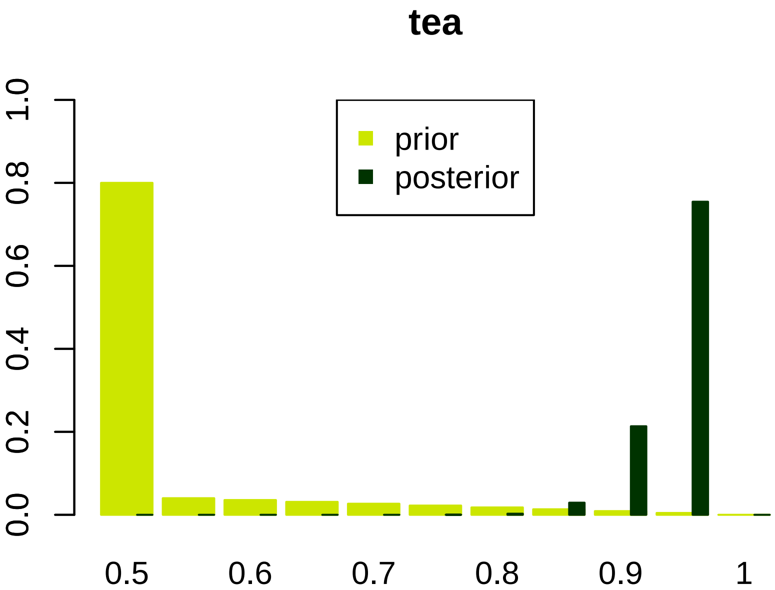

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    - ... but nothing Dr. Muriel does will convince us that $P_R = 1$, because a priori, $p(P_R = 1) = 0$.
    - Cromwell's Rule is a general recommendation to give a prior nonzero mass at any point that is not a logical impossibility.

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  - Different hypotheses weighted by prior beliefs
  - Priors are modified by the data to yield posterior beliefs
  - Then compare the various possible explanations for what has happened, and compare posterior beliefs with priors

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  - H_0 will be more easily discounted using Fisher's method than with the Bayesian approach
    - The vast number of significance tests that are used today will encourage specious beliefs in the efficacy of drugs, treatments, or experimental manipulations
    - Whenever you read some effect having been detected, remember that it probably refers to significance, which too easily suggests an effect when none exists

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    - if H_0 is true (as with tea), or
    - how big an effect is (as with wine)
  - The posterior tells us exactly what we need to know
    - In contrast to the p-value, which is a probability for something that did not happen under the assumption of a hypothesis that may not be true

Conclusions

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  - The Bayesian view recognizes that ones opinion of tasting the two liquids may be different or that the ladies may have different skills

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