Signal detection theory

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A psychological model: Signal detection theory

$$\mathcal{M}_{sdt} : \begin{cases} \delta \sim N(1,1) & \beta \sim N(0,1) \\ \phi_h = \Phi(\delta/2 - \beta) & \phi_f = \Phi(-\delta/2 - \beta) \\ h \sim B(\phi_h, n_s) & f \sim B(\phi_f, n_n) \end{cases}$$

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```
model {
    d ~ dnorm(1, 1)
    b ~ dnorm(0, 1)

    phih <- phi( d / 2 - b)
    phif <- phi(-d / 2 - b)

    h ~ dbin(phih, sigtrials)
    f ~ dbin(phif, noistrials)
}
```

Signal detection theory ~ implementation

```
library(rjags)
data <- list( h = 60 , sigtrials  = 100 ,
              f = 11 , noistrials = 100 )
modelString = "
    model {
        d ~ dnorm(1, 1)
        b ~ dnorm(0, 1)

        phih <- phi( d / 2 - b)
        phif <- phi(-d / 2 - b)

        h ~ dbin(phih, sigtrials)
        f ~ dbin(phif, noistrials)
    }
"
```

Signal detection theory ∼ implementation

```
writeLines( modelString , con = "sdt.txt" )

jagsModel = jags.model( file     = "sdt.txt" ,
                        data     =    data   ,
                        n.chains =      3    ,
                        n.adapt  =    1000 )
set.seed(0)
update( jagsModel , n.iter = 1000 )  # burn-in

samples = coda.samples( jagsModel ,
                        variable.names = c("d", "b") ,
                        n.iter         =      10000 )
```
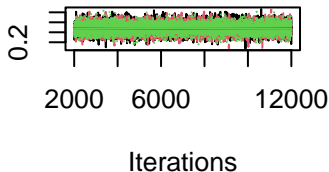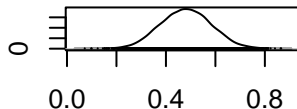
Trace of b

Density of b

0.2

0

2000   6000   12000

0.0   0.4   0.8

Iterations

N = 10000   Bandwidth = 0.0139

Trace of d

Density of d

1.0

0.0

2000   6000   12000

0.5 1.0 1.5 2.0

Iterations
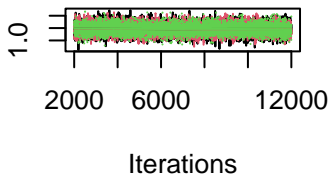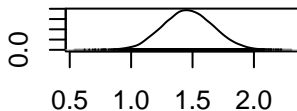
N = 10000   Bandwidth = 0.0276

5

Signal detection theory ~ summary statistics

```
summary(samples)$statistics
```

```
##        Mean        SD     Naive SE Time-series SE
## b 0.4812096 0.1032126 0.0005958984    0.0008044342
## d 1.4642849 0.2048247 0.0011825561    0.0015979583
```

```
summary(samples)$quantiles
```

```
##        2.5%       25%       50%       75%     97.5%
## b 0.2829416 0.4108845 0.4812136 0.5508675 0.6837175
## d 1.0725325 1.3251842 1.4617890 1.6014710 1.8703048
```

Signal detection theory ~ convergence

```
effectiveSize(samples)
```

```
##          b         d
## 16462.76 16428.99
```

```
gelman.diag(samples)
```

```
## Potential scale reduction factors:
##
##   Point est. Upper C.I.
## b          1          1
## d          1          1
##
## Multivariate psrf
##
## 1
```