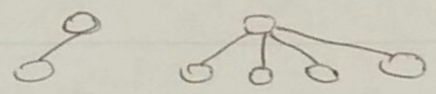
CS 3310
MARIIA KEAVTSOVA
HW 5, part 1

Part 1

Complete Quad Tree

h=1 n=1 ○ h=2 2<n<5  ....

Given h, what is n?
Since I did the summation for full tree first, please reference that. So it follows:

$$\frac{4^{h-1}-1}{3} < n \leq \frac{4^{h}-1}{3}$$

Given n, what is h?
The h will not change from the full tree, hence $h = \lceil \log_4 n \rceil$

Given m, what is h?
Same as before, h is not affected by the difference between full and complete trees. $h = \lceil \log_4 (m+1) \rceil$

Given m, what is n?
Since we have number of edges, which we know are $m = n-1$, it follows that $n = m+1$

Given h, what is L?
The leaves would be the number of leaves in h-1, but with one more. So

$$4^{h-2} < L \leq 4^{h-1}$$

Given L, what is h?
Just as we said for full tree

$$h = (\log_4 L) + 1$$

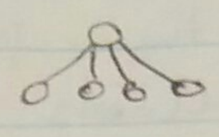
Hilroy

Part 2

Full Quad Trees

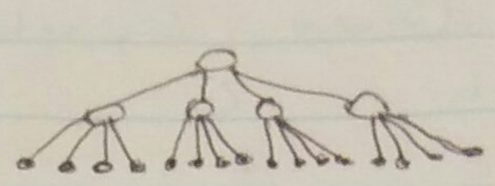
h = 1 o h = 2

n = 1 n = 5



h = 3

n = 21



Given h, what is n?

$4^0 + 4^1 + 4^2 + 4^3 + \ldots + 4^{k-1} = S_{k-1}$

$4\,S_{k-1} = 4^1 + 4^2 + 4^3 + \ldots + 4^k$

$1 - 4\,S_{k-1} = (4^0 + 4^1 + 4^2 + \ldots + 4^{k-1}) - (4^1 + 4^2 + 4^3 + \ldots + 4^k) = 1 - 4^k$

$\sum_{i=0}^{k-1} 4^i = \frac{1-4^k}{1-4} = \frac{1-4^k}{-3} = \frac{4^k-1}{3}$ where k is height h.

$$n = \frac{4^h - 1}{3}$$

Given n, what is h?

Inversly we can derive h from the n equation, where $h = \lceil \log_4 n \rceil$. To test this we can see

h	n
1	1
2	5
3	21
4	85

Given m, what is h?

There is always one less edge than the number of nodes in the tree, because of the root node. So $h = \lceil \log_4 (m+1) \rceil$.

Given m, what is n?

Just as we stated above the proportion $m = m+1$

Given h, what is L?

$L = 4^{h-1}$ because $h = 1 \quad L = 4^0 = 1$

$h = 2 \quad L = 4^1 = 4$

$h = 3 \quad L = 4^2 = 16$

Given L, what is h?

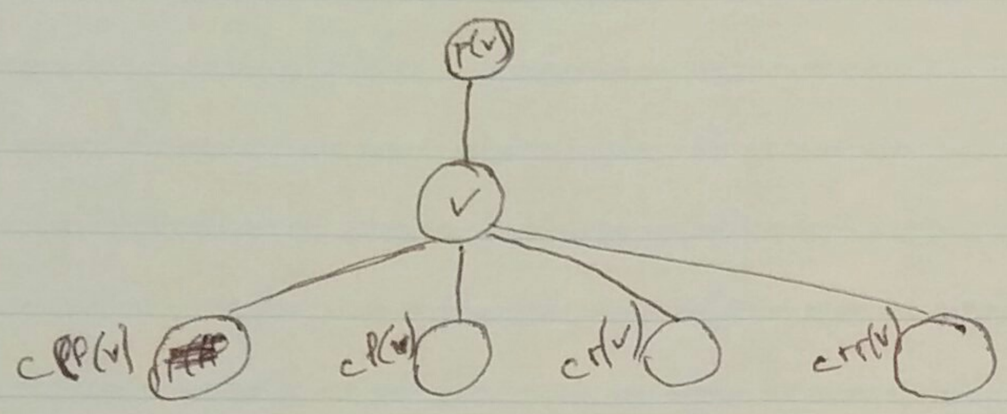
h	L
2	4
3	16
4	64

From the pattern it follows that $h = (\log_4 L) + 1$

Part 3

array of records

0	ssid 0	Name	phone Number
1	ssid 1	name	phone #
2	ssid 2	Name	phone #
3	ssid 3	Name	phone #
4	ssid 4	name	phone #
⋮	ssid ~~8~~	name	phone #
i	ssidi	name	phone #



$p(v) = \lfloor \frac{i-1}{4} \rfloor$

$cll(v) = 4i + 1$

$cl(v) = 4i + 2$

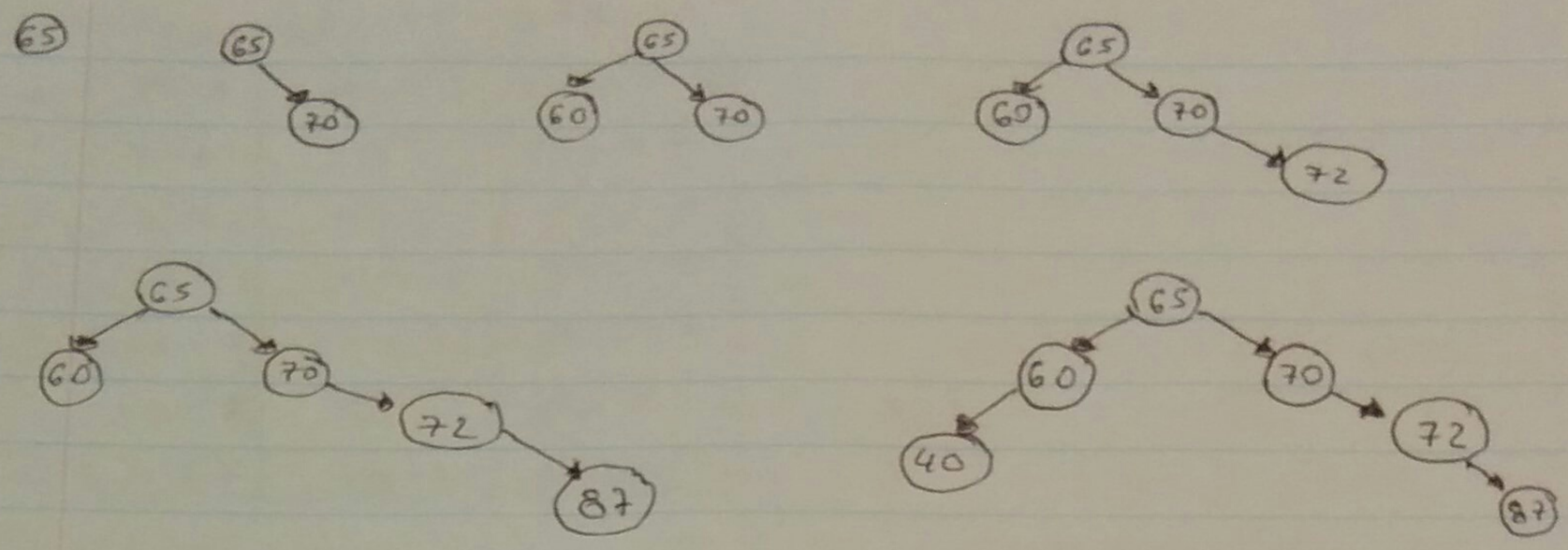
$cr(v) = 4i + 3$

$crr(v) = 4i + 4$ where v is at index i.

Declaration of the array:

```
class record {
    int ssid;
    string name;
    int phone number; }
record T = New record [n]   where n is number
    of records
```

Part 4a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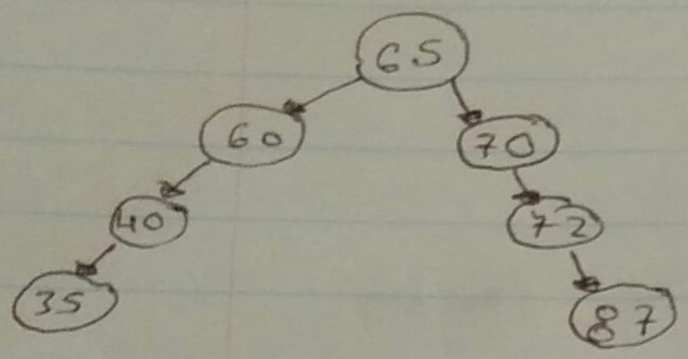
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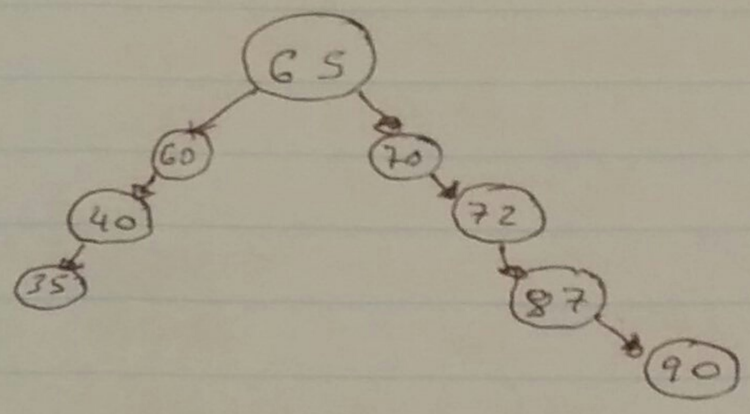
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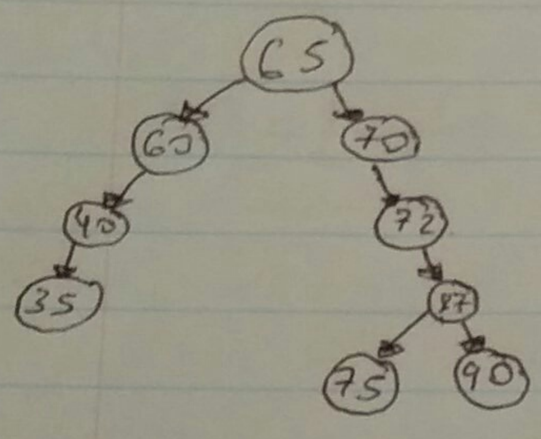
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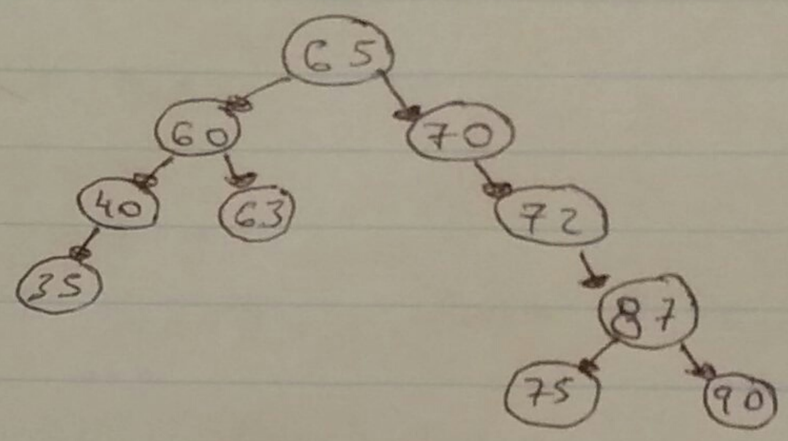
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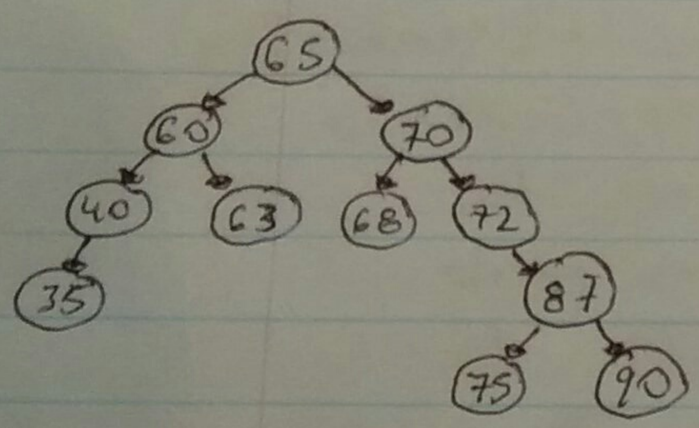
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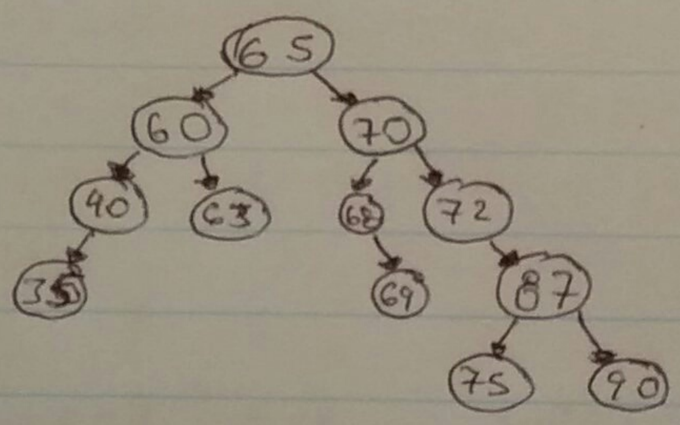
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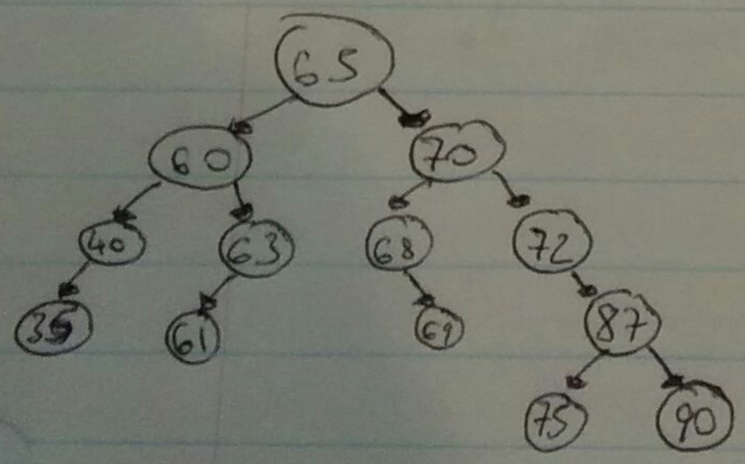
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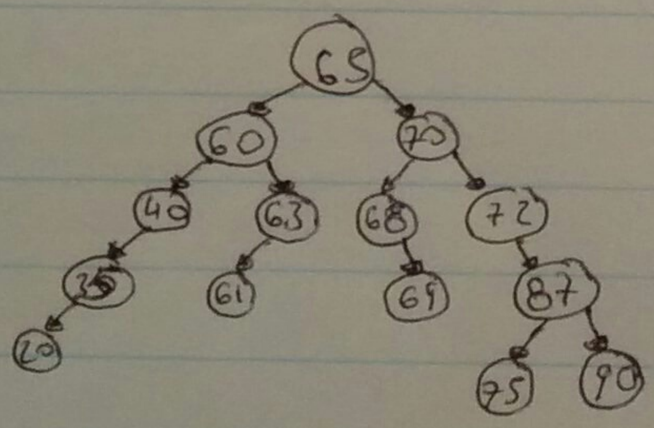
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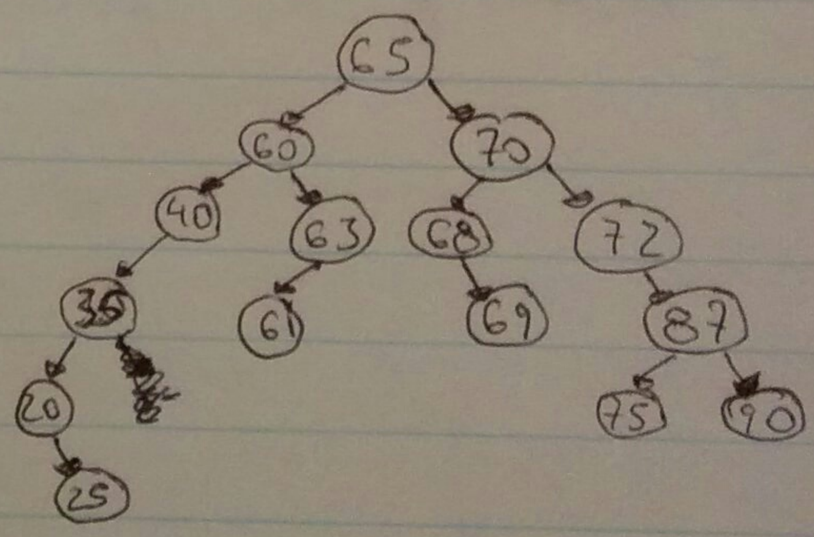
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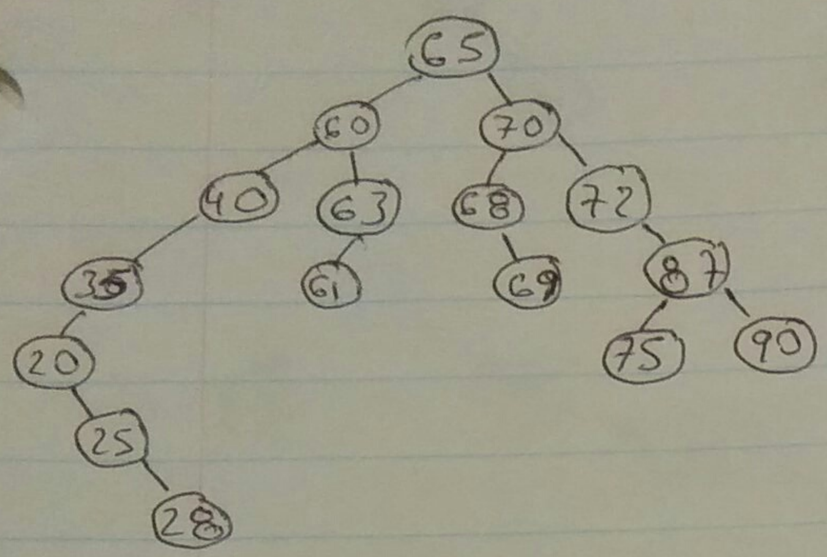
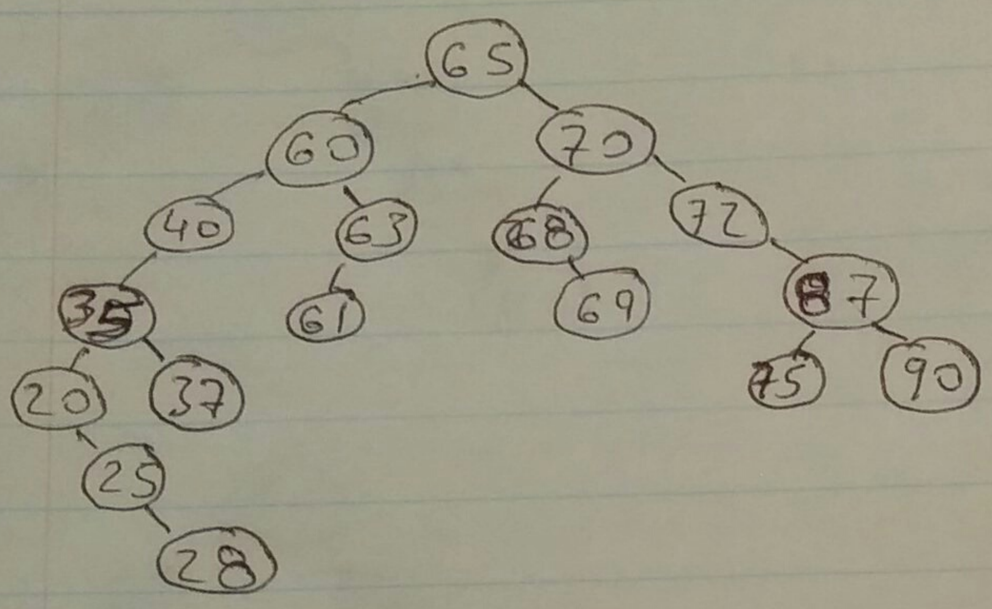




Part 4b

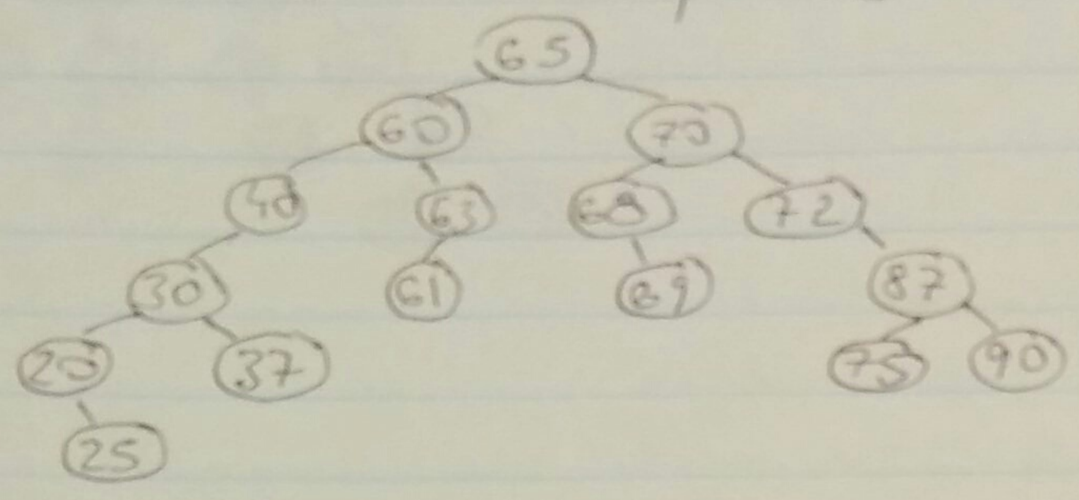
Pre Order: 65, 60, 40, 35, 20, 25, 28, 37, 63, 61, 70, 68, 69, 72, 87, 75, 90

In Order: 20, 25, 28, 35, 37, 40, 60, 63, 61, 65, 70, 68, 69, 72, 87, 75, 90

Post Order: 28, 25, 20, 37, 35, 40, 61, 63, 60, 70, 69, 68, 72, 75, 90, 87, 70, 65.

Part 4c

T after deleting 20



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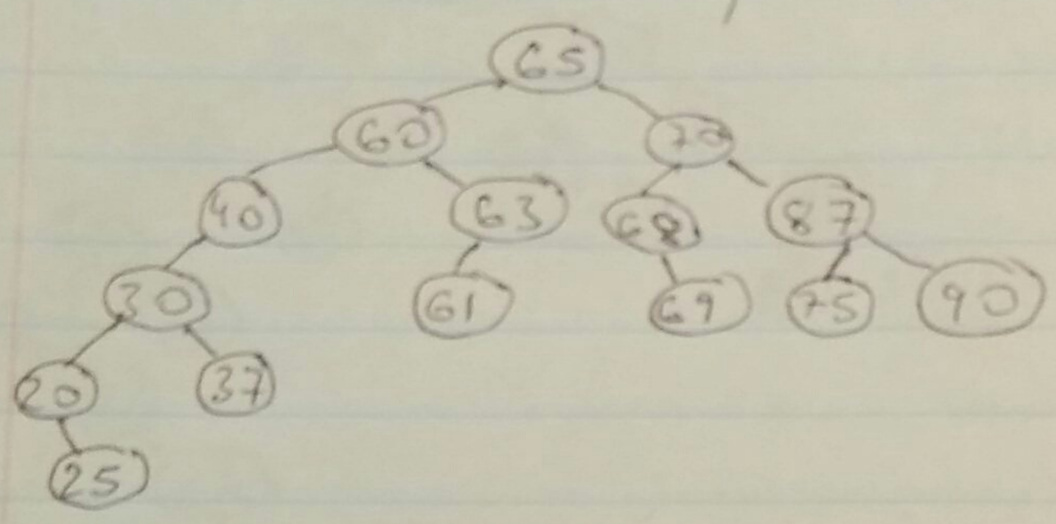

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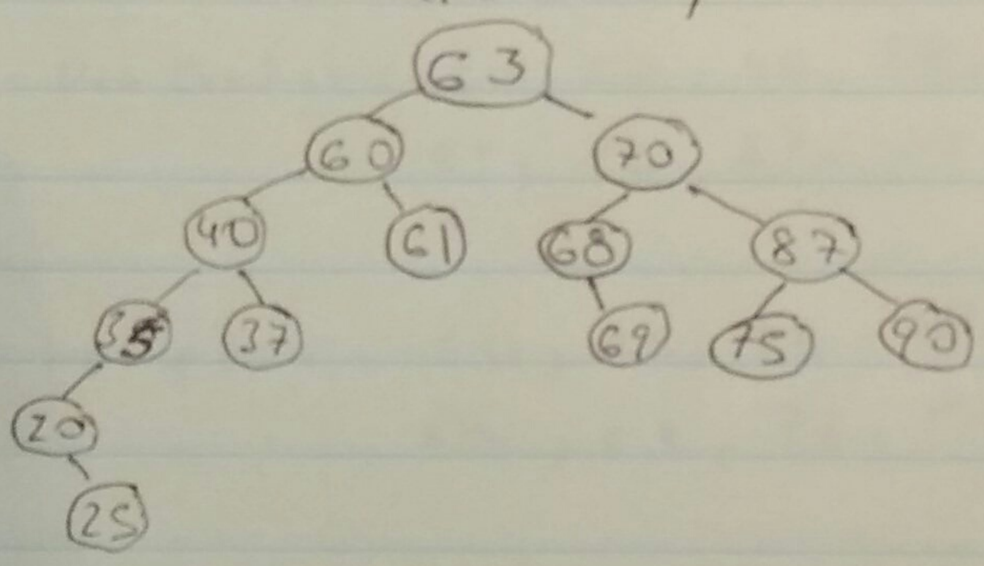

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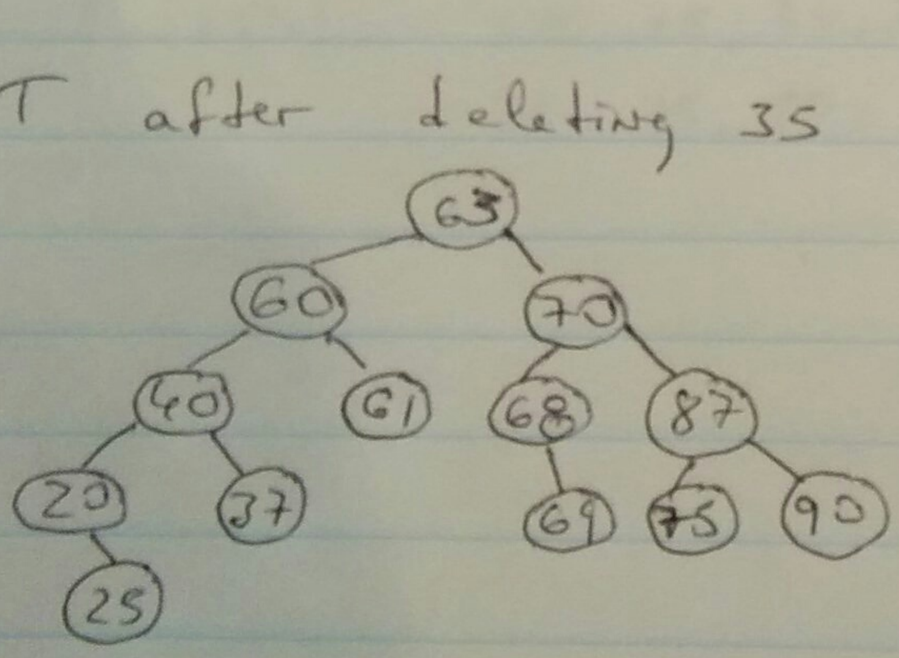

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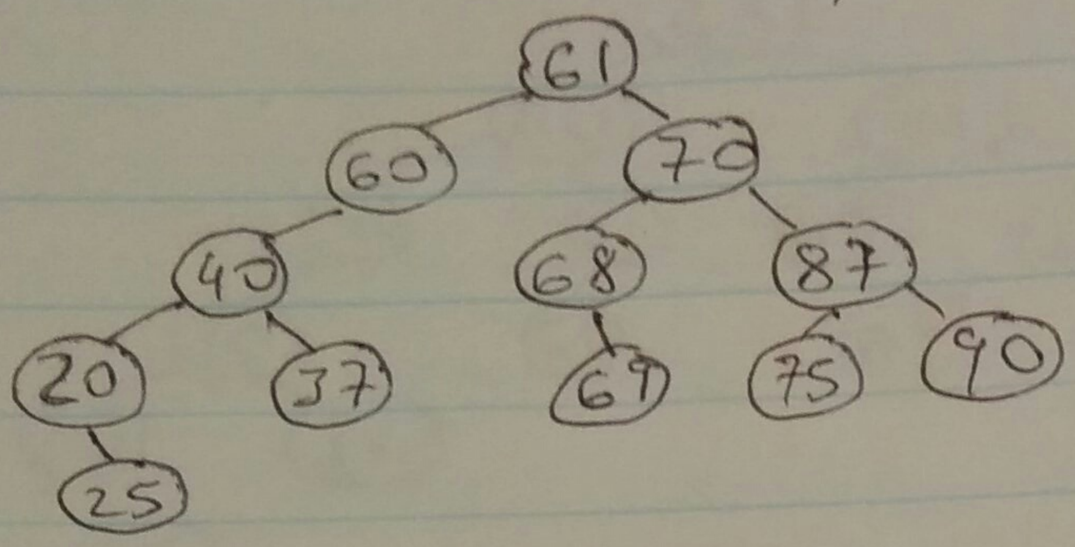

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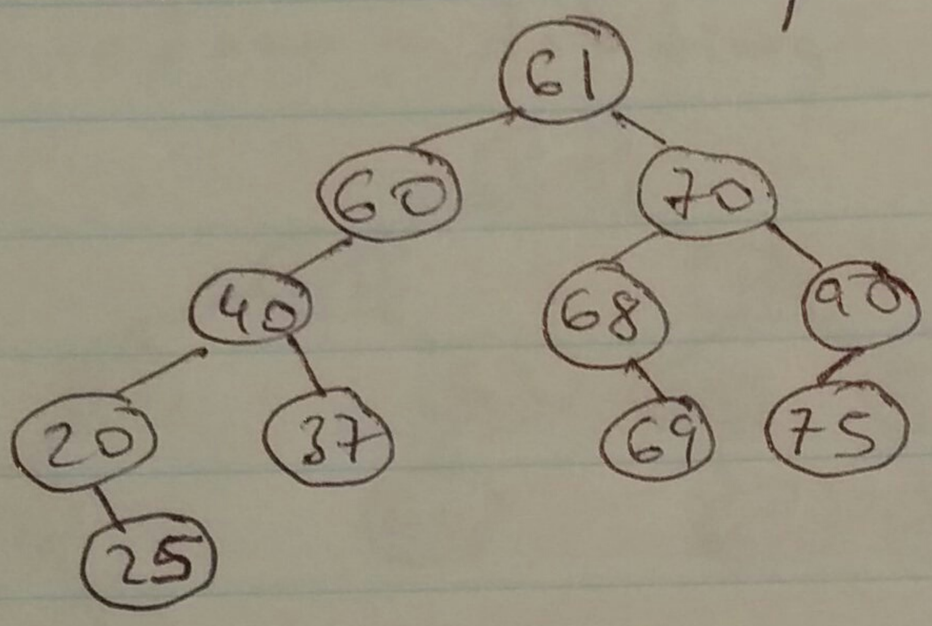

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